THE IMPLICATIONS OF CHANGES IN SURVIVAL PROBABILITY AND ATTITUDES TOWARDS RISK IN A MODEL OF ADVERSE SELECTION

by

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Abstract: Consider a model in which risks of various types are lumped together by annuity contract issuing companies due to asymmetry of information between the companies and their customers. If firms behave nonstrategically, an equilibrium exists in which the firms charge uniform price to all customers. In such an equilibrium, I study how changes in (a) the fractions of various groups of individuals, (b) survival probabilities of various types and (c) attitudes towards risks of the customers affect the equilibrium price of the annuity contracts.

1. Introduction

Insurance companies (and annuity issuing firms) classify individuals in different risk groups based on some cnaracteristics. They offer policies to customers who are "similar" in some respects (for example, term life insurance policies offered in the United States by TIAA-CREF for University Professors). However, there are some characteristics that are hard to detect (for example, a person developing Alzheimer's disease at a later date) and some other characteristics cannot be used for classification by law (for example, it is known that distribution of longevity for blacks is different from that of whites, but such a knowledge cannot be used by insurance companies to charge different premia for blacks). The result is "lumping" of various risk types. If, for some exogenous reason, the proportions of various risk types change, the equilibrium price of insurance will change. Using the concepts developed in this paper, it is possible to evaluate the effect of some exogenous changes on the equilibrium prices of annuities. If the risk perceptions of the population change, there will be changes in the demand for annuities, and consequently, the equilibrium rate of return on annuities will change as well. I shall shed some light on this issue in this paper. In summary, this paper studies the effects of movements of various parameters on the equilibrium price of annuities in a model with adverse selection (where risk classification is imperfect).

The paper is organized as follows: The second section describes the model. Section three shows how various parameters like the proportion of risk types and attitudes towards risk affect the equilibrium allocation of resources. The conclusion is contained in the final section.

2. The Model

The model I study is a variant of Abel (1986) discrete time, one good economy, whose specifications are given below.

Birth/death: At each date t, $t \geq 1$, the population consists of (old) members of generation t-1 (who die with certainty at the end of that period), and (young) members of generation t. Each generation t is partitioned into two distinct groups H (high survival type) and L (low survival type) who appear in relative sizes of α and $1-\alpha$ respectively. Members of generation t in each group live for at most for two periods, the first of which (date t) they survive with certainty. Death can occur at the beginning of the second period (date t+1) with probability $1-p_i$, $0 < p_i < 1$, i = H, L. where $p_H > p_L$. With a continuum of agents of each type, there is no aggregate uncertainty.

Consumers: Following the literature, each young person of type i, at time t, is assumed to maximize

$$U[c^i_t, c^i_{t+1}] = u(c^i_t) + p_i u(c^i_{t+1}), \quad i = H,L \tag{2.1}$$

with $u'(x) > 0$ and $u''(x) < 0$ for all positive x and $u'(x) \to 0$ as $x \to \infty$, $u'(x) \to \infty$ as $x \to 0$. Each old of type i person at t maximizes c^i_{t-1}. Each young member of every generation t is endowed with w units of the t good at birth which is storable: if $x \geq 0$ units are stored at t, the results is xR units at t+1 where $R > 0$. Thus, R can be thought of as the (constant) rate of return on investment. Let R_a be the rate of return on annuities. It is then possible to write the budget constraints:

$$c^i_t = w - a^i_t, \tag{2.2}$$

$$c^i_{t+1} = a^i_t R_a, \text{ for } i=H,L. \tag{2.3}$$

The relationship between R and R_a is spelled out below.

Firms: Annuity issuing firms are risk neutral, perfectly competitive, and maximizer of profits.

Information: Each person of generation t perceives correctly the probability of her survival but the annuity supplying firms cannot distinguish between the two types. Moreover, no firm can find out the quantity of annuity contracts a given individual has purchased from other firms.

The assumptions regarding birth, death and preferences of the consumers follow Eckstein et al.. Information assumption is taken from Abel (1986). The equilibrium described below was implicit in Pauly (1974). Abel has used the model to explore the effects of social security on the capital accumulation. However, his model has an explicit bequest motive for the individuals.

Equilibrium: An equilibrium in the steady state is characterized by that rate of return R^*_a for the annuity (R^*_a is the ratio of the benefit and the premium) such that (a) the firms maximize their expected profits and (b) the individuals maximize their utility functions (2.1) subject to their budget constraints (2.2) and (2.3). Abel has shown under the information assumption above, an equilibrium exists such that the equilibrium rate of return of annuities, R^*_a, lies between R/p_H and R/p_L. Denoting the demand function of type i individual by $D(R_a, p_i)$, i=H, L., the profit (π) of a representative firm can be written as follows:

$$\pi(R_a; p_H, p_L, \alpha) = \alpha(R - p_H R_a)D(R_a, p_H) + (1-\alpha)(R - p_L R_a)D(R_a, p_L) \qquad (2.4)$$

The equilibrium rate of return on annuity can be solved from

$$\pi(R^*_a; p_H, p_L, \alpha) = 0. \qquad (2.5)$$

In other words, R^*_a is implicitly determined by (2.5). Abel has shown that $\pi(R/p_H; p_H, p_L, \alpha) > 0$ and $\pi(R^*_\infty; p_H, p_L, \alpha) < 0$. To summarize, the equilibrium rate of return described by (2.5) depends on a number of parameters in the economy: (a) the fraction of type H in the population (α); (b) the survival probabilities p_H and p_L; (c) the degree of risk aversion (which is implicit in the utility function u)and therefore, also implicit in the demand functions. In this paper, I shall explore the effects of these factors on R^*_∞.

In the absence of any additional restrictions on the utility function, equation (2.5) may have many solutions for R^*_∞. In the rest of the paper, I shall assume that there is, in fact, a unique equilibrium. These comparative statics results will be pointless if all utility functions generating solutions for the equation (2.5) do not produce a unique values of R_∞. However, for $u(x) = \ln x$, I can show that (2.5) can be reduced to

$$\alpha k_H(R-p_H R_\infty) + (1-\alpha)k_L(R - p_L R_\infty) = 0, \qquad (2.6)$$

where $k_i = p_i/(1+p_i)$, $i = H,L$. Clearly, this is a linear equation in R_∞. Consequently, it has a unique solution for R_∞. Therefore, the class of utility functions (satisfying the conditions of section two) producing a unique solution to (2.5) is nonempty (and contains the loglinear utility function). For this class, the profit function can be depicted as in figure 1:

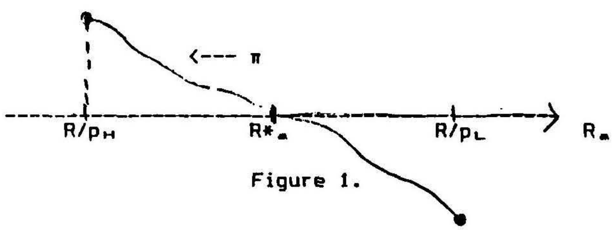

Figure 1.

3. The Effects of The Parameters on the Equilibrium Rate of Return for Annuity

The equilibrium described in section two depend on a number of parameters in the economy: (a) the fraction of type H individuals in the population (and consequently, on the fraction of type L individuals); (b) the survival probabilities p_H and p_L; (c) the degree of risk aversion of the individuals of both types (the "shapes" of the utility functions); (d) endowment pattern (which could be altered by government tax/transfer schemes). In the first subsection, I show that the effect of a change in the population proportion on the equilibrium rate of return for annuity.

3.1 Change in the population proportion

Consider a change in the proportion of the two types (α and $1-\alpha$) without altering the survival probabilities (p_H and p_L) and any other parameter of the economy. Note that the individual decision processes will not alter: for each type (H or L), the demand for annuity would change only as a consequence of the change in the equilibrium rate of return. The effect can be traced follows: consider an increase in the type H in the population. From the type H (L), there is a net loss (gain) to the annuity industry. A rise in α will therefore increase the loss and reduce the gain to the industry at the same time. Therefore, the industry has to reduce the rate of return on annuity to bring back the zero profit condition. This argument is formalized in the following

Proposition 1: Consider two economies where the fractions of type H are α_0 and α_1 ($\alpha_1 > \alpha_0$) respectively. If the economies are otherwise identical and each has a unique equilibrium, then the economy with α_1

fraction of type H has a lower equilibrium rate of return than the economy with α_0 fraction of type H.

<u>Proof</u>: Let the equilibrium rate of return in α_0 be R^*_a. Therefore, $\pi(R^*_a; p_H, p_L, \alpha_0) = 0$. *To complete the proof, I shall show that* $\pi(R^*_a; p_H, p_L, \alpha_1) < 0$. This will imply that the equilibrium rate of return for α_1 economy will be lower (see figure 1).

I know from Abel (1986, p. 1085) that R^*_a is in $(R/p_H, R/p_L)$. Therefore $R - p_H R^*_a > 0$ and $R - p_L R^*_a < 0$. Thus, the equilibrium condition (2.5) for α_0 economy can be rewritten as

$$\pi(R^*_a; p_H, p_L, \alpha_0) = \alpha_0.N + (1 - \alpha_0).P = 0$$

where $N = (R-R^*_a p_H)D(R^*_a, p_H)$ and $P = (R-R^*_a p_L)D(R^*_a, p_L)$. Note that N (P) is a negative (positive) number. By hypothesis, $\alpha_1 > \alpha_0$. Thus,

$$\alpha_1.N < \alpha_0.N < 0$$

and

$$0 < (1-\alpha_1)P < (1-\alpha_0)P.$$

Therefore, it follows that

$$\pi(R^*_a; p_H, p_L, \alpha_1) = \alpha_1 N + (1 - \alpha_1)P$$

$$< \alpha_0 N + (1 - \alpha_0)P = 0. \qquad \text{Q.E.D.}$$

Consider two regimes: regime zero (one) with α_0 (α_1) fraction of type H($\alpha_0 < \alpha_1$). By proposition 1, I concluded that the equilbrium rate of return is lower in regime one. In fact, I can easily show that individuals of <u>both</u> types are better off under regime zero. The reason is simple: regime zero has a higher rate of return for annuity contracts. Thus, in (2.1)-(2.3), nothing changes except for an increase in R^*_a.

If different survival types are "lumped together" in a pension plan and if it breaks even now, the proposition 1 tells us that it will not do so in the future as the proportion of longer living individuals increase. In the United States, this type of problem is rampant among private pension funds. As a consequence, the government pension insurance agency, Pension Benefit Guarantee Corporation (PBGC), had severe strains on it's resources. Partly for this reason, PBGC is scheduled to increase the premium for it's members by 400%.

3.2 Changes in the Probabilities of Survival

Any change in p_H and/or p_L could potentially alter the equilibrium rate of return. However, such changes do not necessarily change the equilibrium rate of return unmbiguously. Let us first consider the simplest case: change in p_H only. If p_H inreases (α, P_L and all other parameters remaining unchanged), it will increase the demand for annuity for type H (because $dD(Ra, p)/dp > 0$). It will also produce a larger "loss per dollar" (I call it "loss" in the sense of deviations from an actuarially fair annuity market) for type H. As a consequence, $R - p_H R_a$ will increase in magnitude. Therefore, the rate of return has to fall to restore the equilibrium. This heuristic argument is formalized below.

Proposition 2: Consider two economies: one with type H with p_H and the other with type H with $p_H + e$ $(e>0)$ probabilities of survival (otherwise identical). If they have unique equilibria, then the economy with $p_H + e$ will have a lower rate of return for annuity than the economy with p_H.

Proof: Let the equilibrium rate of return with p_H be R^*_a. Then, by (2.5), $\pi(R^*_a; p_H, p_L, \alpha) = 0$. I shall show that $\pi(R^*_a; p_H+e, p_L, \alpha) < 0$. This will prove the result (see figure 1).

$\pi(R^*_a; p_H+e, p_L, \alpha)$

$= \alpha D(R^*_m, p_H + e)\ (R-(p_H + e)R^*_m) + (1-\alpha)D(R^*_m, p_L)\ (R-R^*_m p_L)$.

I know that $dD/dp > 0$. Therefore, $D(R^*_m, p_H+e) = D(R^*_m, p_H) + d$ for some $d > 0$.

Now, $\pi(R^*_m; p_H+e, p_L, \alpha)$

$= \alpha D(R^*_m, p_H)(R - p_H R_m^*) + (1-\alpha)D(R^*_m, p_L)(R - p_L R_m^*) + A$

$= \pi(R^*_m; p_H, p_L, \alpha) + A = A$ by (2.5), where,

$A = -eR^*_m.(D(R^*_m, p_H) + d) + d(R-p_H R^*_m)$.

But A is negative since $R^*_m < R/p_H$. Thus, $\pi(R^*_m; p_H+e, p_L, \alpha) < 0$. Q.E.D.

Unfortunately, an analogous argument cannot be made to prove that a rise in p_L will have a similar effect. the reason is that $D(R_m, p_L)$ will increase if p_L risesand $R-p_L R^*_m$ will decrease. Therefore, in order to assess the net effect, I need to find out the extent of increase in D and decrease in $R-R^*_m p$ (because, an expression, analogous to A will have an ambiguous sign). Thus, it produces an ambiguous effect on the equilibrium rate of return.

Precisely because of the reason outlined above, a simultaneous rise in p_H and p_L will not result in an unambiguous change in the rate of return. Consider the case of loglinear utility function with w=1, R=1 in equation (2.6). The following table summarizes the effects of changing p_H and p_L and α on R_m.

α =	.3	.5
p_H=.6, p_L=.1	2.3846	1.9900
p_H=.7, p_L=.2	2.8572	1.7987

The example demonstrates that with a simultaneous increase in p_H and p_L, increases R^*_m if α =.3 but decreases R^*_m if α = .5.

The results can be summarized as follows: In a pension plan, where two groups "lumped together" have different survival probabilities, an increase in the probability of survival for the high survival type is unambigously welfare improving. However, such a conclusion cannot be arrived at, if both groups have improved survival probability distributions.

3.3 Change in the attitudes towards risk

Finally, I shall investigate the effect of attitudes towards risk on the equilibrium rate of return. I have shown in the appendix that if a "more risk averse" individual is defined as the one with more concave single period utility function ($u(c_i)$, $i = 1, 2$) then a more risk averse individual buys more (less) annuity (than a less risk averse individual, other things being the same), if and only if the annuity market faced by the individual is less (more) than actuarially fair. This logic will be exploited here. The equilibrium described by the equation (2.5) implies that it is individually actuarially less (more) than fair for type H (L) because $R/p_H < R^*_a < R/p_L$. Therefore, the demand for annuity of type H (L) will decrease (increase) as a result of higher risk aversion. Thus, there will be an increase in the equilibrium rate of return. This idea is formalized below:

Let R^*_a be the equilibrium rate of return when the utility function of type i individual is given by

$$u(c_t) + p_i\, u(c_{t+1}) \qquad \text{with } i = H, L.$$

By adding the utility function u to the argument of the profit function, I write the profit function of the "type u economy" as $\pi(R_a, u)$[1]. Thus, the zero profit condition for the type u economy can be rewritten as:

$\pi(R^*_m, u) = 0.$

Following Yaari (1984), define "a more concave utility function" $v = f(u)$ where $f' > 0$ and $f'' < 0$. Then, I can prove the following

<u>Proposition 3</u>: $\pi(R^*_m, v) > 0$.

<u>Proof</u>: By the theorem in the appendix, I have shown that

$$D(R^*_m, p_H, u) > D(R^*_m, p_H, v) \qquad (3.3.1)$$

$$D(R^*_m, p_L, v) > D(R^*_m, p_L, u) \qquad (3.3.2)$$

Let $f(R^*_m, p_i) = R - R^*_m p_i$, for i=H, L.

By (3.3.1) and (3.3.2) and by the fact $f(R^*_m, p_H) < 0 < f(R^*_m, p_L))$,

$\pi(R^*_m, v) = \alpha D(R^*_m, p_H, v)\ f(R^*_m, p_L) + (1-\alpha)D(R^*_m, p_L, v)f(R^*_m, p_L)$

$> \alpha D(R^*_m, p_H, u)\ f(R^*_m, p_H) + (1-\alpha)D(R^*_m, p_L, u)\ f(R^*_m, p_L) = \pi(R^*_m, u).$

By (2.5), it follows that $\pi(R^*_m, v) > 0$. Q.E.D.

With the assumption of a unique equilibrium, the above proposition says the higher the risk aversion, the higher the equilibrium rate of return (see figure 1). To see the quantitative importance of proposition 3, consider the following example:

Let the one period utility be given by $u(x) = \exp(-\sigma x)$. (This is the so called constant absolute risk aversion (CARA) type utility function, where the coefficient of risk aversion is σ). There are two individuals with one period utility function $u_i(x) = \exp(-\sigma_i x)$, i = 1, 2. Then, σ_2 is more risk averse than σ_1 if and only if $\sigma_2 > \sigma_1$. This definition is covered under our definition of "more risk averse" because σ measures the concavity of u. For this special type of utility function, I calculate the equilibrium rate of return for two values of σ.

First, by simplifying the equilibrium condition (2.5) for the CARA family of utility functions:

$$\alpha(w + \ln p_H/\sigma)\ (R - p_H R_a) + (1-\alpha)\ (w + \ln p_L R_a/\sigma)(R - p_L R_a) = 0. \quad (3.3.3)$$

For given values of α, w, p_H, p_L and R, this equation is nonlinear in R_a. Letting $\alpha = 1/2$; $w = 1$; $p_H = 1$; $p_L = 1/e$; $R = 1$, I have solved for R_a by setting $\sigma = 2$ and $\sigma = 10$. The solutions for the rates of return are: $R^*_a(\sigma = 2) = 1.2921$ and $R^*_a(\sigma = 10) = 1.4285$. It shows that a fivefold increase in the coefficient of risk aversion leads to a 10% increase in the equilibrium rate of return. Thus, a change in the risk aversion can have significant impact on the equilibrium rate of return on annuities.

4. Conclusion

This paper discusses three different comparative statics results. The effect of a change in the proportion of different types of individuals has an unambiguous effect on the equilibrium rate of return of the annuity contracts. The effects of changes in the probability distribution themselves may not result in unambiguous changes. However, a change in the attitude towards risk will have an unambiguous effect on the equilibrium rate of return of the annuity contracts.

These results rest on two crucial assumptions: (a) the particular equilibrium concept used in the model; and (b) the equilbrium generated is unique. What will happen to the results described if I considered "more strategic" equilibrium concepts like Wilson (1977)? The answer is still open. As for the second assumption, I have shown that the uniqueness holds for at least loglinear utility function. With additional algebra, it is possible to show the uniqueness for $u(x)=\beta \ln x$. Does it generalize any further? Once again, it is an open question.

Appendix

Consider two utility functions

$$U(c_1, c_2) = u(c_1) + pu(c_2) \quad \text{(A1)}$$

$$V(c_1, c_2) = v(c_1) + pv(c_2) \quad \text{(A2)}$$

V is said to be more risk averse than U if and only if $v(x) = f(u(x))$ for $x \geq 0$, where f is some function with $f'(x) > 0$ and $f''(x) < 0$ for all $x \geq 0$. (This definition follows Yaari (1984)).

To simplify notations, in this section, I shall write $a^*_j(p)$ for $D(R^*_m, p, j)$ (where, $j = u, v$ and $p = p_L, p_H$). As a further simplification, I shall also drop the argument p. The budget constraints (2.2) and (2.3) shall be rewritten as

$$c_1 = w - a \quad \text{(A3)}$$

and

$$c_2 = aR/pk \quad \text{(A4)}$$

It may seem awkward to write the budget constraints in terms of R rather than in terms of R^*_m. But, it greatly simplifies the derivation of the theorem.

<u>Theorem</u>: a^*_u is greater, equal or less than a^*_v as k is less, equal or greater than unity.

<u>Proof</u>: The optimal first order conditions for optimization of (A1) and (A2) subject to (A3) and (A4) can be written as:

$$\frac{u'(w-a^*_u)}{u'(a^*_u/pk)} = \frac{1}{k} = \frac{v'(w-a^*_v)}{v'(a^*_v/pk)} \quad \text{(A5)}$$

First, let $k = 1$. Then by $u'' < 0$ and $v'' < 0$,

$$w - a^*_u = a^*_u/p,$$

i.e.,

$$a^*_u = w/(1 + 1/p)$$

and

$$w - a^*_v = a^*_v/p,$$

i.e., $a^*_v = w/(1+1/p)$.

Therefore, $a^*_v = a^*_u$.

Let $k > 1$. Then by (A4),

$$v'(a^*_v/pk) > v'(w-a^*_v),$$

i.e., $a^*_v/pk < w-a^*_v$

or, $u'(a^*_v/pk) > u'(w-a^*_v)$ (because $u'', v'' < 0$).

Therefore, $f'(u'(a^*_v/pk)) < f'(u'(w-a^*_v))$,

i.e.,

$$1 < f'(u'(a^*_v/pk))/f'(u'(w-a^*_v)). \qquad \text{(A6)}$$

From (A5), by using the fact $v(x) = f(u(x))$,

$$\frac{u'(w-a^*_u)}{u'(a^*_u/pk)} = \frac{f'(u'(w-a^*_v))\ u'(w-a^*_v)}{f'(u'(a^*_v/pk))\ u'(a^*_v/pk)} \qquad \text{(A7)}$$

By using (A6) in (A7), I get

$$\frac{u'(w-a^*_u)}{u'(a^*_u/pk)} < \frac{u'(w-a^*_v)}{u'(a^*_v/pk)} \qquad \text{(A8)}$$

Suppose now $a^*_u \geq a^*_v$. Then

$$u'(a^*_u)pk) \leq u'(a^*_v/pk) \qquad \text{(A9)}$$

and

$$u'(w-a^*_u) \geq u'(w-a^*_v). \qquad \text{(A10)}$$

Combining (A9) and (A10) I get

$$\frac{u'(a^*_v/pk)}{u'(a^*_u/pk)} \geq 1 \geq \frac{u'(w-a^*_v)}{u'(w-a^*_u)} \qquad \text{(A11)}$$

Rewriting (A11),

$$\frac{u'(w-a^*_u)}{u'(a^*_u/pk)} \geq \frac{u'(w-a^*_v)}{u'(a^*_v/pk)} \qquad \text{(A12)}$$

But (A12) contradicts (A8). Therefore $a^*_u < a^*_v$. Following a similar argument, I can show that if $k < 1$ then $a^*_u > a^*_v$. Q.E.D.

I have noted earlier that $R/p_L > R^*_a > R/p_H$. This means $R^*_a = R/p_L k_L$ for some $k_L > 1$ and $R^*_a = R/p_H k_H$ for some $k_H < 1$. Thus, for type H (L), I can apply the theorem because $k = k_H < 1$ ($k = k_L > 1$). Therefore, it follows that

$$D(R^*_*, p_H, u) > D(R^*_*, p_H, v)$$

$$D(R^*_*, p_L, v) > D(R^*_*, p_L, u).$$

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