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A native of Canada, Cecil J. Nesbitt did his undergraduate and graduate work at the University of Toronto, where he received his Ph.D. in 1937 as a student of Richard Brauer. After a postdoctoral year at the Institute for Advanced Study, he took a position at the University of Michigan and remained there until his retirement in 1980. His early research was in algebra, but at both Toronto and Michigan his primary bent was to actuarial mathematics. With Carl H. Fischer he led a flourishing actuarial program in the Mathematics Department at Michigan, while publishing actively and serving the Society of Actuaries in various capacities. His main work has been in the areas of pension funding and social insurance.

Personal Reflections on Actuarial Science in North America from 1900

CECIL J. NESBITT

1. Introduction

At the outset, it should be made clear that this article does not pretend to be a definitive history of actuarial science developments in North America since the beginning of the century. Deadlines, and my own available time and energy, do not permit such an undertaking, worthy as it may be. Instead I shall draw on memories of almost 60 years as an actuarial student, teacher, practitioner, and researcher, to indicate actuarial highlights of that period, and also sources for further review if readers become so inclined. Such readers should turn first to *Actuarial Mathematics* (Proc. Symp. Appl. Math. 35, 1986) and peruse it alongside this article to gain detailed, introductory overviews of the diverse actuarial models that will be mentioned here. The non-exhaustive list of references at the end of the article is selected to aid such review by pointing the way to other more complete lists in regard to various topics mentioned herein. The body of ideas, known and unknown,

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is infinite, and even in one special area, such as the intersection of actuarial science and mathematics, can be covered only by broad strokes.

Actuarial science has a major role in the guidance of financial security systems, developed to protect individuals and groups against a multiplicity of risks such as impairment of health, premature death, destruction of property, and extended old age. The systems may range from self-insured groups to national programs of social security. Some of these systems operate on an international basis, and more such development may lie in the future. These systems during my life have made much progress despite economic, financial, and political disturbances and disasters. The systems have been facing fast-growing environmental hazards, and military potentialities of incredible magnitude. Actuarial science has a role to play, as do all fields, in finding viable equilibria in a fast-changing world.

In the following section, there will be brief discussion of the main fields of knowledge on which actuarial science draws. Those to be mentioned are mathematics, statistics, probability, accounting, computer science, demography, economics, finance theory, law and medical science. Some of these fields were relatively undeveloped at the beginning of this century. The section on sources will be followed by one on distinctively (although not exclusively) actuarial theories. These are: estimation of mortality and other rates, life tables (now broadened to survival models), graduation theory, risk theory, credibility theory, actuarial finance theory, life insurance mathematics including growth models and stochastic models of life contingencies. The application of these theories to various fields of practice will come next, with a final summary overview.

2. Sources of Actuarial Science

In a broad sense, all portions of actuarial science relate to some form of mathematical theory or application. The theory may be relatively elementary, but the application may be extremely detailed and numerical. Of prime importance are the actuarial assumptions from which the mathematical model is developed. For short-term insurances, there may be a large volume of current data for statistical and probability analysis. Such current data may also be available for long-term insurances, as for example, whole life insurance, pension systems, and social security, but must be extended by projection factors to guide the future growth of the financial security system.

From the data analysis, one may estimate probabilities needed for the model of the system. The mathematical model may draw heavily on probability theory for its structure, or for the longer term it may be deterministic in character, following out the consequences of assumed rates of growth and eligibility for benefits. Statistical and probability theories, which have grown rapidly in this century, are playing an expanding role in actuarial science.

Another main source of actuarial science is the mathematics of finance. Until recent years, this has been an elementary theory, defining various discrete and continuous rates of interest, and utilizing a constant rate compound interest model to calculate present and accumulated values of series of payments. Still more recently, the turbulence of financial markets, has led to simulation studies being conducted by committees of the Society of Actuaries under the leadership of C. L. Trowbridge and D. D. Cody (Cody, 1987). About the same time there appeared Phelim Boyle's "Immunization under stochastic models of the term structure" (Boyle, 1978). Also, finance theory, with application to the pricing of options, has been advancing strongly. (For a comprehensive view of this last work, see Pedersen, Shiu and Thorlacius, forthcoming, and D'Arcy and Doherty, 1988.)

Computer science has greatly empowered actuaries in regard to: estimation of rates or probabilities, the calculation of premiums or contributions, the projection of future benefit outgo and of premium or contribution income, and the corresponding accumulation of reserves. A notable example is provided by the annual actuarial projections for old-age, survivors and disability insurance in the United States (see Andrews and Beekman, 1987).

Other bodies of knowledge or practice which impinge on actuarial practice are indicated by the examples below:

> **Accounting**. To get a feel for some of the discussion preceding (Financial Accounting Standards No. 87, 1985), see E. L. Hicks and C. L. Trowbridge, *Employer accounting for pensions* (Hicks and Trowbridge, 1985).
>
> **Demography**. A major actuarial concern here is in regard to the development of national life tables. An early reference was H. H. Wolfenden's *Population statistics and their compilation* (Wolfenden, 1925). This was followed by M. Spiegelman's *Introduction to demography* (Spiegelman, 1955). One of the current references in the actuarial education syllabus is *Demography through problems* (Keyfitz and Beekman, 1984). See also A. Wade's *Social security area population projections* (Wade, 1988) and J. Wilkins' *OASDI long-range beneficiary projection, 1987* (Wilkins, 1988).
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> **Economics**. Recently, the Office of the Actuary, Social Security Administration, has published Actuarial Study No. 101, *Economics projections for OASDHI cost and income estimates*: 1987 (Goss, 1988).
>
> **Law and Regulation**. Life insurance companies are supervised by the State Insurance Departments in the United States, and by

federal and provincial departments in Canada. In the United States, the Employee Retirement Income Security Act of 1974 and subsequent legislation have been a major influence on pension funds.

Medicine. This last comes to the fore in the underwriting of life insurance, in health insurance, in the projection of future mortality improvements, and in regard to current epidemics such as AIDS.

3. Some Actuarial Theories

This section overviews some of the theories used by actuaries in their professional practices.

3.1. Estimation of Mortality and Other Rates

From chapter III of J. S. Elston's *Sources and characteristics of the principal mortality tables* (Elston, 1932), we have the quotation:

"United States Life Tables, 1910

United States Life Tables 1890, 1901, 1910 and 1901–1910

These tables are the first of any scientific value prepared by the U. S. Government from census returns. When the census of 1910 was taken, the Bureau of the Census called into consultation a committee of The Actuarial Society of America, and this committee gave general advice with reference to the taking of the census, the tabulation of the data, and the preparation of life tables. Although not all the committee's recommendations were followed, these tables, which were prepared under the supervision of Professor James W. Glover, mark a notable epoch in the history of mortality."

In all, 69 life tables were prepared from the censuses, and the death statistics of the ten original registration states and the District of Columbia (Glover, 1916 and 1921). In view of the status of computing facilities in the 1910–1920 decade, the preparation and publication of these tables was a monumental task.

Another mathematician who has been a principal and innovative consultant for U. S. Life Tables for 1939–1941, and for subsequent intervals around the decennial censuses, is T. N. E. Greville (Greville, 1946). The problems he met and solved for the 1939–1941 Tables led him to many later developments in theories of interpolation, graduation, splines, generalized inverses of matrices, and life tables, as indicated in bibliographies of (Bowers et al. 1986), (London, 1985), and (Shiu, 1984 and 1986).

Turning now to insured lives mortality, we have as an early reference, *Construction of mortality tables from the records of insured lives* (Murphy and Papps, 1922). This was followed by a number of papers in the actuarial literature which plateaued in the texts (Gershenson, 1961) and (Batten, 1978). Concurrently, beginning in 1934, committees of the Society of Actuaries, or its predecessors, have published a series of annual *Reports of mortality, morbidity and other experience* based on data mainly contributed by a number of life insurance companies (Society of Actuaries, 1984).

In regard to annuitant mortality, a landmark paper by W. A. Jenkins and E. A. Lew developed the idea of scales of projection factors to allow for future mortality improvements (Jenkins and Lew, 1948).

Much statistical work has gone into the estimation of mortality rates from clinical data, and the subject has broadened to that of survival models. Simultaneously, the computer evolution has greatly facilitated the calculation of exposed to risk from the records of the individuals observed in the estimation process. These new approaches, are presented in Dick London's text, *Survival models and their estimation* (London, 1988). See also J. D. Broffitt's paper "Maximum likelihood alternatives to actuarial estimators of mortality rates" (Broffitt, 1984). It should be added that census methods used in the estimation of population mortality also have application to estimation of mortality of insured lives or of pension fund participants. There are, of course, distinctive differences in the data for the various studies.

3.2. Graduation Theory

This topic is concerned with the systematic revision of estimates of series of rates, in particular, those to be used as bases for survival models. A fine survey is given by E. S. Shiu in the 1986 *Proceedings*, volume 35. His abstract is as follows: "Graduation is the process of obtaining from an irregular set of observed values, a corresponding smooth set of values consistent in a general way with the observed values. This is a survey of various methods of graduation used by actuaries."

Some early work goes back to E. L. DeForest in the 1870s which was later brought to life by H. H. Wolfenden in (Wolfenden, 1925). R. Henderson, who was prominent in the early history of both the actuarial and the mathematical professions, prepared the monograph *Mathematical theory of graduation* (Henderson, 1938). Let us pause for a moment to pay tribute to this distinguished man.

His life spanned from 1871 to 1942. He graduated from the honors mathematics program of the University of Toronto in 1891. He became a Fellow of the Actuarial Society of America and of the Casualty Actuarial Society, and was elected president of the former organization. Robert Henderson rose to become actuary of the Equitable Life Assurance Society. He served as trustee of the Mathematical Association of America and of the American Statistical Association. In addition, "For a number of years, Mr. Henderson served as a member of the Board of Trustees of the American Mathematical Society. He felt the keenest interest in the place which the Society was taking in scientific progress and lent earnest assistance to the raising of funds in order that its work might continue unimpaired despite the economic difficulties of recent years. He also served from 1935 until shortly before his death as a Director of the Teachers Insurance and Annuity Association." (Quotation from the obituary for Robert Henderson in *Transactions of the Actuarial Society of America*, **43** (1942)).

We should also note that Robert Henderson delivered the second Gibbs Lecture on "Life insurance as a social science and as a mathematical problem" (Henderson, 1925). The entire principal of his estate was received by the American Mathematical Society in 1961 for its Endowment Fund.

Another notable author was C. A. Spoerl, a summa cum laude graduate of Harvard University. See, for instance, his paper "Whittaker-Henderson graduation formula A" (Spoerl, 1937).

For a number of years following 1950, a new monograph, *Elements of graduation*, by M. D. Miller served as education reference (Miller, 1946). Meanwhile, a succession of papers were coming from the pen of T. N. E. Greville, which may be well seen in the book, *Selected papers of T. N. E. Greville, 1984*. These have influenced the work of G. S. Kimeldorf and D. A. Jones in "Bayesian graduation" (Kimeldorf and Jones, 1967), and E. S. Shiu in "Minimum-R_z moving-weighted-average formulas" (Shiu, 1984). Another approach is exemplified by D. R. Schuette's "A linear programming approach to graduation", (Schuette, 1978).

3.3. Risk Theory

We consider first the simpler case of short-term insurances. Here one may be concerned with the distribution of total claims in a given period for a given portfolio of insurance policies. The approach in individual risk theory is to set up a random variable

$$Y_j = I_j B_j \quad j = 1, 2, \ldots, n \tag{3.3.1}$$

for each of the n insurance policies in the portfolio under consideration. Here I_j is 1 if policy j leads to a claim and is 0 otherwise; B_j is the amount of such a claim, given that it occurs. On the assumption that I_j, B_j, $j = 1, 2, \ldots, n$ are mutually independent, one proceeds to approximate the distribution of aggregate claims for the period, that is, the distribution of

$$S_{IR} = \sum_{j=1}^{n} Y_j. \tag{3.3.2}$$

For this we need knowledge of the probability that $I_j = 1$ and of the distribution of B_j, for each j.

In the collective risk model, the basic concept is that of a random process that generates claims for a portfolio of policies. This process is in terms of a portfolio as the whole rather than in terms of the individual policies. Let N be the random number of claims for a portfolio of policies in the given period. If X_1 is the random amount of the first claim, X_2, the random amount of the second claim, and so on, then

$$S_{CR} = X_1 + X_2 + \cdots + X_N \tag{3.3.3}$$

is the random amount of aggregate claims. The random variable N is referred to as frequency of claims and the random variables X_j measures the size of claims. In order to proceed, one makes the assumptions that:

1. $X_1, X_2 \ldots$ are identically distributed.
2. The random variables $N, X_1, X_2, \ldots$ are mutually independent.

An overview of risk theory, with emphasis on the collective theory, is given by H. Panjer's, "Models in risk theory" (Panjer, 1986). See also H. Gerber's *An introduction to mathematical risk theory* (Gerber, 1979). Both of these references provide bibliographies which indicate the historical development of risk theory. A major figure is H. L. Seal, as the bibliographies attest (see Seal, 1969). The reader interested in connecting the two approaches to risk theory for short-term insurances is referred to Section 13.5 of (Bowers et al, 1986). For information about estimating the probability distribution of the X_j's one can refer to S. A. Klugman's "Loss Distributions" (Klugman, 1986), or to the book by R. V. Hogg and S. A. Klugman with the same title (Hogg and Klugman, 1984).

An early discussion of risk theory for individual insureds under long-term life insurance and annuity contracts was given by W. O. Menge, a later-year colleague of J. W. Glover, in the paper "A statistical treatment of actuarial functions" (Menge, 1937). An extensive development of individual risk theory for such contracts is a major theme of (Bowers et al, 1986).

3.4. Credibility Theory

Since the early papers of F. A. Perryman (Perryman, 1937) and A. L. Bailey (Bailey, 1950), an extensive literature has grown up. Successive overviews of this literature have been presented by P. M. Kahn in 1967, 1968, 1975 and 1986. The reader is referred to this last paper, and its bibliography (Kahn, 1986).

In brief, credibility theory applies mainly to short-term insurances such as group life insurance, or those in various casualty lines, or the year-to-year risks under individual life insurances. The theory studies the revision of premium rates in the light of current claim experience. To quote from (Kahn, 1986):

"In the classical approach the actuary must first determine the size of the experience which warrants full credibility, i.e. a credibility factor $Z(t)$ of 1, where t measures the size of the exposure or insurance experience which generated the level of chaims x. The next step is to determine partial weights, or partial credibility factors for some smaller groups. Then the adjusted estimate of claims may be expressed as

$$Z(t)x + [1 - Z(t)]m(t) \tag{3.4.1}$$

where $m(t)$ is the prior estimate of expected claims."

A. L. Mayerson's paper "A Bayesian view of credibility" (Mayerson, 1964) was a stimulus for much further research. In 1975, J. C. Hickman drew a distinction between classical theory where the parameters of the claims process are considered as fixed constants, and the newer theories where the parameters are themselves random variables (Hickman, 1975). As with much actuarial theory, the newer concepts of credibility must undergo validation and refinement in actual insurance experience.

3.5. Mathematics of Compound Interest

In Section 2, Sources of Actuarial Science, reference has been made already to the mathematics of finance and the direction in which it is headed. Here, and in the next section, we refer to some of the classical actuarial mathematics texts. For further information on these texts, and how they became incorporated into the education and examination processes of the profession in North America, the reader is referred to the chapter on actuarial education in E. J. Moorhead's forthcoming 1809–1979 history of the actuarial profession, entitled *Our yesterdays* (Moorhead, forthcoming). This chapter, from a different viewpoint, gives insight about the professors and universities that have contributed to actuaral education and science.

From the University of Toronto, we have had M. A. Mackenzie's *Interest and bond values* (Mackenzie, 1917), and N. E. Sheppard and D. C. Baillie's *Compound interest* (Sheppard and Baillie, 1960). From the University of Michigan, there has appeared M. V. Butcher and C. J. Nesbitt's *Mathematics of compound interest* and, as one of the leading more elementary texts, P. R. Rider and C. H. Fischer's *Mathematics of investment* (Rider and Fischer, 1951). Since 1970, the Society of Actuaries has benefitted from S. G. Kellison's *The Theory of Interest*. The newest text in the English language is J. J. McCutcheon and W. F. Scott's *An introduction to the mathematics of finance* (McCutcheon and Scott, 1986). These texts treat basic finance concepts which go far back into the mists of history of civilization.

3.6. Mathematics of Life Contingencies

As a major part of the core of actuarial mathematics is the subject of this subsection, it is in order to discuss the principal textbooks that have appeared from time to time in the English language. If one refers to (Moorhead, forthcoming), one reads about such early works as R. Price's *Observations on reversionary payments* (1771), William Morgan's *The doctrine of annuities and assurances on lives and survivorships* (1779), Francis Bailey's *Doctrine* (1812–1813), Joshua Milne's *Treatise* (1815), and David Jones' *Value of annuity and reversionary payments* (1843). My own acquaintance goes back to G. King's *Institute of actuaries textbook, Part II* (King, 1902), and I endured through examinations on E. F. Spurgeon's *Life contingencies* (Spurgeon, 1922).

It is probably little known by now that C. H. Fischer and myself were invited in the late 1940s to undertake for the Society of Actuaries a new textbook. At that time concepts about the probability distributions of random variables were not well organized, at least in my mind, but nevertheless, it seemed to me then to be the way to proceed. The Society was not ready for what appeared to be a novel approach, and turned the project over to C. W. Jordan who by 1952 produced a book which served the profession well for over thirty years (Jordan, 1952). His book began with the notion of survival function but soon settled down to deterministic formulas. This was followed by P. F. Hooker and L. H. Longley-Cook's two-volume text, *Life and other contingencies* (Hooker and Longley-Cook, 1953, 1957). This text had brief discussion of variance around the expected values, as did also the successor book, A. Neill's *Life contingencies* (Neill, 1977). In 1978, the author team of N. L. Bowers, H. U. Gerber, J. C. Hickman, D. A. Jones and C. J. Nesbitt began work on a new textbook, entitled *Actuarial mathematics*, which emerged in final form by 1986 (Bowers et al, 1986). An enlightening overview is given by J. C. Hickman's paper "Updating life contingencies" (Hickman, 1988). This textbook goes way beyond what Fischer and I attempted forty years earlier. It intertwines individual risk theory and individual life insurance mathematics, and introduces collective risk theory, with various practical applications in group insurance and reinsurance. It ends with a chapter on "Theory of pension funding," using a mathematical deterministic model, generalizing the work of C. L. Trowbridge in "Fundamentals of pension funding" (Trowbridge, 1952). The extensive bibliography lists the many authors whose works have helped to shape the text.

For some time, a new direction in actuarial mathematics has been appearing in Europe. This is exemplified by J. Hoem's "The versatility of the Markov chain as a tool in the mathematics of life insurance" (Hoem, 1988), and by H. Wolthuis' doctoral thesis, *Savings and risk processes in life contingencies* (Wolthuis, 1988). To a considerable extent, this direction runs counter to American practice which models separately each state that an insured may enter, for example, the state of disability, rather than use an integrated model covering all states, and transfers among them. It remains for the future to determine the usefulness of the integrated models. One indication is that the work of M. J. Cowell and W. H. Hoskins (Cowell and Hoskins, 1987), and of H. J. Panjer (Panjer, 1988) on projections regarding the AIDS epidemic, and recent work of J. Beekman in modeling decline of activity of the aged, are related thereto.

Meanwhile, actuaries like myself who are interested in the long-term guidance of pension funds and social security, are prone to use what I term mathematical deterministic (or growth) models, and to utilize a range of actuarial assumptions which are monitored regularly. This viewpoint is reflected in B. N. Berin's *The Fundamentals of pension mathematics* (Berin, 1978), and in the long-range projections for U.S. Social Security (Andrews and Beekman, 1987). This approach is also exploited in A. W. Anderson's *Pension mathematics for actuaries* (Anderson, 1985).

Another example of theory developments which have not gained much usage yet in practice is given by W. S. Bicknell's thesis "Premiums and reserves in multiple decrement theory (Bicknell and Nesbitt, 1956). This discusses three systems for premiums and reserves for the case of multiple forms of termination and benefits related thereto, as in pension plans. The second and third systems involved somewhat complex composition of the actuarial bases for the several benefits. This, we have noted, is not the American way in practice. The third system, which goes back to Alfred Loewy, has considerable possibilities, but has practical and throretical subtleties which have been explored by (Schuette and Nesbitt, forth coming in *ARCH*).

It seems fitting to end this subsection with a tribute to H. L. Rietz who was from 1918 to 1962 influential in the development of mathematical statistics and actuarial science at the University of Iowa. He served as vice president of the American Institute of Actuaries, 1919–1920, as president of the Mathematical Association of America in 1924, as vice president of the American Statistical Association in 1925, and as vice president of the American Association for the Advancement of Science in 1929. He was the first president of the Institute of Mathematical Statistics, organized in 1935, and the 1943 volume of the *Annals of Mathematical Statistics* was dedicated to him on the occasion of his retirement. Among his doctoral students was C. H. Fischer, my long-time colleague at the University of Michigan.

4. APPLICATIONS

From 1900 through 1987, life insurance in force in the United States has grown from a little over \$7.5 billion to almost \$7.5 trillion. Some \$3 trillion of this latter amount is classified as group insurance, a form which did not exist in 1900. This period saw the development of retirement income policies, variable life insurance and several forms of flexible life insurance. Discussions of these may be found in chapter 16 of (Bowers et al, 1986). Some initial papers were authored by E. G. Fassel (1930), J. C. Fraser, W. N. Miller and C. M. Sternhell (1969), W. L. Chapin (1976), and S. A. Chalke and M. F. Davlin (1983). (See bibliography in Bowers et al, 1986 for references.)

During the same period, the growth of pension funds is indicated by the increase from about \$20 billion of assets in 1900 to about \$2 trillion in 1986. A notable development during this period was the concept of variable annuities and the formation in 1952 of the College Retirement Equities Fund (CREF). The actuarial basis for that fund was pioneered by R. M. Duncan's "A retirement system granting unit annuities and investing in equities" (Duncan, 1952). I recall one lunchtime where Carl Fischer and I pressed Robert Duncan on the theory of dollar averaging for accumulating purchases of units by a series of regular contributions to CREF. When asked what would happen if the stock market collapsed, he thought for a moment and then with a smile said "They might not be worth very much, but you would have a lot of accumulation units."

For further information about the Teachers Insurance and Annuity Association (TIAA) and CREF, see my paper, "On the performance of pension plans" (Nesbitt, 1986). In particular, note the graded benefit annuity option which has in recent years become available from TIAA.

This section concludes with a few comments on the Old Age, Survivors and Disability Insurance (OASDI) system, popularly called Social Security but this latter also embraces the insurances under Medicare. OASDI is an extremely large system with annual benefit outgo now at the level of \$235 billion, and projected level of \$8 trillion by year 2045 under moderate growth assumptions (Annual Report, 1988). With good reason, the actuaries of the System prefer to project benefit outgo as a percent of projected taxable payroll for the System. On this basis, projected OASDI outgo in 2045 is 16.25 percent of taxable payroll. The actuarial guidance of this huge system is a major challenge for the actuarial profession.

An ackowledged leader in such guidance has been R. J. Myers who has written very extensively on Social Security (see, for instance, Myers, 1985). He set the pattern for the short-range and long-range projections, the processes for which are continuously evolving. An overview of these processes is given in (Andrews and Beekman, 1987) and (Annual Report, 1988). A recently formed National Academy of Social Insurance, with Alicia Munnell of the Federal Reserve Bank of Boston as president, will form a common ground for persons from different fields who are interested in Social Security. OASDI developments over the past fifty years have been of major importance as a foundation of benefits to be supplemented by nonfederal life insurance and pension-funding, and should remain so.

5. Concluding Comments

This paper has been written mainly as personal reactions to actuarial developments of the last fifty years. As I got further into it, I had more and more occasion to refer to (Bowers et al, 1986) and (*Proceedings*, 1986). For the reader interested in going beyond this paper, I recommend a perusal of the latter reference. I draw special attention to J. C. Hickman's introduction, and to his paper "Updating life contingencies" which enlarges on the concepts underlying the textbook (Bowers et al, 1986).

I have tried to make at least one reference to many, but by no means all, contributors to actuarial science in North America. Most of the omitted names may be found in the bibliographies of (Bowers et al, 1986) and (*Proc. Symp. Appl. Math.*, 1986). Some omissions relate to young men whose work is in process of recognition, and some to special areas of expertise. I have depended on the useful bibliographies in the various references cited to provide a more complete picture of developments, including much work not cited here.

Two references I wish to add here are to W. O. Menge and J. W. Glover's *An introduction to the mathematics of life insurance* (Menge and Glover, 1935), and its later revision, *Mathematics of life insurance* (Menge and Fischer, 1965). In a very real sense, these helped to clarify life insurance actuarial practice.

While risk theory and credibility theory are major elements in the actuarial mathematics of nonlife insurance, beyond these two theories and reference to (D'Arcy and Doherty, 1988) no attempt has been made to cover that field further, as it has not been part of my experience. A similar remark applies for the large field of health insurance on an individual, group or national basis.

It should be recorded that there are some actuaries who have realized that we have undergone in the last forty years the risk of incredible destruction by nuclear war. Among these was Edmund C. Berkeley who included this topic in his address "Society, computers, thinking and actuaries" to the 16th Annual Actuarial Research Conference, University of Manitoba (see Berkeley, 1982). In papers presented to the 22nd and 23rd annual actuarial research conferences, I have indicated a simple model for recognising nuclear holocaust hazard, and its pervasive effect on all longterm actuarial calculations such as those regarding average length of life, or for mortgage amortization over a term of years (Nesbitt, 1987 and 1988). This is the actuarial science that must be communicated to protect life, and to counter the weight of science that could destroy life. These tasks really fall upon teachers in all fields, but actuarial science should do its part.

Finally, this review, which was mainly retrospective but had some updating and prospective aspects, has encouraged me about this century's progress in actuarial science, and has increased my awareness of developments to come.

ADDENDUM

Here I present some information and reflections on the education and examination of actuaries. This could be a very large assignment, and I shall resolve it mainly by pointing to sources of information. An immediate problem is that there is not just one, but four, actuarial organizations directly involved in actuarial education and examination, and three others that cosponsor some or all of the examinations. The two oldest organizations, the Society of Actuaries and the Casualty Actuarial Society, have leading roles but have been supplemented by the Joint Board for the Enrollment of Actuaries (a unit of the U.S. Department of the Treasury), and the American Society of Pension Actuaries. Cosponsoring organizations are the American Institute of Actuaries, the Canadian Institute of Actuaries (CIA has a different connotation in Canada than in the U.S.), and the Conference of Actuaries in Public Practice. This may seem confusing but there is considerable coordination among the seven bodies through the Council of Presidents, and also through overlapping memberships. My own experience has been mainly with the Society of Actuaries, and I shall use the Society as my information source. The catalogs of the Society list the addresses of all seven organizations.

A second problem is that culminating in the years since 1985, there has been a restructuring of the Society's education process into a Flexible Education System (FES), and a follow-up by proposed Future Education Methods (FEM). The multi-membered Education and Examination Committee distributed two white papers, on FES in 1986, and on FEM in 1987, setting forth the proposed changes and their rationale. As Vice President for Research and Studies in 1985–1987, I witnessed the presentation of these documents and both the general support and the counter-reactions that they gathered. The new emphasis is on education that can adapt itself to our fast-changing world and that achieves a better balance with the discipline of the actuarial examinations.

FES is now in place and some steps have been taken in regard to FEM. These are reflected in the booklets, 1989 Associateship Catalog, and Spring 1989 Fellowship Catalog, where associateship is the first level and fellowship is the second level of qualification for membership in the Society. Both booklets state the following:

Principles Underlying the Education and Examination System

The Society of Actuaries administers a series of self-study courses and examinations leading to Associateship and Fellowship. The principles underlying the Society's education and examination system are the following:

(1) To provide the actuary with an understanding of fundamental mathematical concepts and how they are applied, with recognition of the dynamic nature of these fundamental concepts in that they must remain up-to-date with developments in mathematics and statistics;

(2) To provide the actuary with an accurate picture of the socio-demographic, political, legal and economic environments within which financial arrangements operate, along with an understanding of the changing nature and potential future directions of these environments;

(3) To expose a broad range of techniques that the actuary can recognize and identify as to their application and as to their inherent limitations, with appropriate new techniques introduced into this range as they are developed;

(4) To expose a broad range of relevant actuarial practice, including current and potential application of mathematical concepts and techniques to the various and specialized areas of actuarial practice; and

(5) To develop the actuary's sense of inquisitiveness so as to encourage exploration into areas where traditional methods and practice do not appear to work effectively."

Under FES, a number of self-study courses are available, each providing a certain number of credits. Completion of the Series 100 requirements now satisfies the education requirements for the Associate of the Society of Actuaries (ASA) designation. A candidate must obtain 200 units of credit prior to 1995 for courses listed in Table A to satisfy the Series 100 requirements.

Table A. Course Description

Course	**Description**	**Credits**	**Type**
100	Calculus and Linear Algebra	30	Required
110	Probability and Statistics	30	Required
120	Applied Statistical Methods	15	Required
130	Operations Research	15	Elective
135	Numerical Methods	10	Elective
140	Mathematics of Compound Interest	10	Required*
141	EA-1, Segment A	10	Required*
150	Actuarial Mathematics	40	Required
151	Risk Theory	15	Required
160	Survival Models	15	Required
161	Mathematics of Demography	10	Elective
162	Construction of Actuarial Tables	10	Elective
165	Mathematics of Graduation	10	Elective

Each course is designated as required or elective. A candidate must obtain 155 credits in "required" courses and 45 credits in "elective" courses to satisfy the Series 100 requirements for this catalog.

Credit for courses 140, 150, and 151 must be obtained by examinations offered by the Society of Actuaries. Credit for course 141 must be obtained by passing EA-1, Segment A of the Enrolled Actuary (EA) Examinations. Credit for all other courses must be obtained by examinations offered by the Society of Actuaries or by an alternative method which has been approved by the Board of Governors. For fall 1988 and spring 1989, credit for course 100 may be obtained by an alternative method (an appropriate score on the Graduate Record Examination Mathematics Test)."

While each 10 credits usually implies one hour of multiple-choice examination, there are exceptions. Course 140 has a one-and-one-half-hour examination, and course 150 has a four-and-one-half-hour examination split into two sessions, and including some written-answer questions.

The written-answer examinations I took years ago had algebra based on the classical Hall and Knight textbook, had analytic geometry and calculus together, and scarcely touched linear algebra. Probability was mainly combinatorics based on Whitworth's Choice and Chance, and statistics was at a precalculus descriptive level. Now course 110 includes topics among those proposed for a one-year college course in probability and statistics by the Committee on the Undergraduate Program in Mathematics. Course 120 covers analysis of variance, regression analysis and time-series analysis which were largely omitted from the syllabus of my examination-writing years. Course 130 on linear programming, project scheduling, dynamic programming, relates to topics that came to the fore during World War II.

Course 135, Numerical Methods, replaces the former examination on finite differences. The finite (as opposed to the infinitesimal) calculus was one of my teaching joys. It was always a pleasure to define divided differences, proceed to the Lagrange interpolation formula with remainder, relate divided differences under prescribed conditions to derivatives at intermediate points, and pull out Newton's divided difference interpolation formula with remainder, and as special cases obtain Taylor's series and the various classical polynomial interpolation formulas, all with remainders. One then was set to make applications to summation, approximate integration, and difference equations. Now, the impact of computers has greatly expanded the subject to modern numerical analysis with its algorithmic approach. This is reflected in course 135 which covers iteration, interpolation, numerical integration and linear systems.

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*Candidates must receive credit for either course 140 or 141 but will not receive credit for both.

Compound interest theory, the subject of course 140, while benefitting from some refinement of basic concepts, and from the enormous improvement in computing facilities, has been well established for many decades. Course 141, administered by the Joint Board for the Enrollment of Actuaries, has a two-and-one-half-hour examination covering the mathematics of compound interest and of life contingencies.

Course 150 is an extensive coverage of the mathematics of life contingencies. It is based on the new textbook (Bowers et al, 1986) which employs future lifetime as the underlying random variable. For the development of this central subject of life actuarial science over the past two centuries, and its updated setting in *Actuarial mathematics*, see Section 3.6 of my foregoing Reflections and also (Hickman, 1986).

While "Economics of insurance", and "Individual risk models for a short term," (chapters 1 and 2 of *Actuarial mathematics*) are recommended as background readings for course 150, these topics together with "Collective risk models for an extended period", and "Applications of risk theory," comprise the examination subjects for course 151. Random variables were only vaguely elaborated in the syllabus when I was a student, and much of this theory has developed since.

The subjects of courses 160, 161, 162 and 165 have been touched upon in Sections 3.1 and 3.2 of my foregoing paper. Only course 160 is required, the others are elective, but 45 credits, as of now, must be chosen from 55 available. In many cases, actuaries have very large amounts of data available (relative to insureds and deaths) in the form of policies of insurance, amounts of insurance, annuity or pension incomes, census counts, and vital statistics of births and deaths. Methods for analyzing such data may then differ in some degree from those for smaller, more detailed studies of clinical data, or of impaired lives. In any case, the actuarial profession seeks reasonable understanding of the various estimation procedures that are feasible and available.

After attaining ASA designation, many actuarial students aspire to complete the education requirements for the Fellow of the Society of Actuaries (FSA) designation. To do so, they must undertake the Series 200–500 courses. In the Fellowship Catalog, we read these Series "are divided into four groups; the common Core and three specialty tracks: the group Benefits (GB) Track, the Individual Life and Annuity (ILA) Track and the Pension (P) Track. All candidates must earn 100 credits from the core courses, 90 credits from the required courses in one of the tracks with a single national emphasis, and 60 credits from other Fellowship courses. Within a track, some courses are designed to be national in emphasis (either Canada or U.S.)".

To give a little more insight to the nature of these requirements, I quote from the Fellowship Catalog.

COURSE 200.
INTRODUCTION TO FINANCIAL SECURITY PROGRAMS

(40 Credits) Required

The examination for this course is a four-hour multiple-choice and written-answer examination. The course covers: design, regulation and taxation of the major voluntary financial security programs involving life insurance, health insurance, property and casualty insurance, and employee benefit and pension programs; characteristics of the major social insurance programs in Canada and the U.S.; description of the providers of financial security programs; and an introduction to taxation of insurance companies in both Canada and the U.S."

Some hardy souls, after attaining the FSA designation proceed to the ACAS and FCAS designations of the Casualty Actuarial Society. Canadian FSAs take whatever additional steps may be needed for the Fellow of the Canadian Institute of Acturies (FCIA) designation.

Future Education Methods (FEM) are in progress. In October 1987, the Board of Governors of the Society approved implementation of five programs, namely:

(i) a Fellowship Admissions Course, a two-and-one-half-day course focusing on professional ethics and integration of syllabus material.
(ii) a research paper option for 30 elective Fellowship credits (details in the Fellowship Catalogs).
(iii) credit for examinations of other actuarial organizations and complete designations of non-actuarial organizations.
(iv) elective credit for an Intensive Seminar at the Associate level.
(v) an experiment in allowing credit for college courses, approved by the Society of Actuaries Education and Examination Committee, covering the topics of applied statistics. operations research and numerical methods.

These programs are at various stages of implementation.

In my fellowship student days, I was required to pass three six-hour examinations, each of which had a number of subjects. Fortunately, one examination was in my special fields of interest of pensions and of social insurance. The current requirement of 250 credits may require more examination hours, although FEM programs may effect such increase.

Of 112 members of the Society of Actuaries who hold appointments in U.S. and Canadian colleges and universities, 40 are in departments of mathematics or mathematical sciences; 27, in departments of statistics and actuarial science; 22, in schools of business administration; and 10, in actuarial science and insurance programs. The remaining 13 are in miscellaneous or unstated units. An additional 8 members of the Society are in foreign colleges and universities. As of November 1, 1988, total membership of the Society was 11,157, consisting of 6,039 Fellows and 5,118 Associates. Some 721 members reside outside Canada and the U.S.A. Additional membership statistics can be found in the 1989 Yearbook of the Society.

Much detailed information about the Society of Actuaries courses and examinations can be obtained by writing to:

Society of Actuaries
475 N. Martingale Road
Schaumburg, IL 60173
(708)706-3500

and requesting a copy of the 1989 Associateship Catalog and of the spring 1989 Fellowship Catalog.

Education in the topics of courses 100–135 can be acquired at many colleges and universities in the United States and Canada. A list of schools which offer degree programs covering much of courses 150–165 can be obtained from the Society of Actuaries. In addition, the catalogs give information about study manuals and study groups that a student may wish to utilize.

Throughout my teaching career, and in following years, there has been a strong demand for actuarial students. These may find employment in insurance companies, consulting firms, state and federal government agencies (including insurance departments, the Internal Revenue Service and the Social Security Administration). Such organizations cover the tremendous range of financial security systems such as property-liability insurance, health insurance, life insurance, annuities and social insurance.

Actuarial education and examinations provide a rigorous but equitable process, manned by many dedicated volunteers who are complemented by an able, growing staff. The process provides a pathway to a challenging life devoted to making our financial security systems truly effective. My part therein has been a major satisfaction of my life.

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