Interest Rate Hedging on Traditional Life and Health

A Research Project with the Society of Actuaries

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Abstract

Traditional life and health products have long been regarded as non-interest sensitive. Pricing has often been done with only a single interest scenario projection. Asset liability matching is sometimes confusing because of the very long or sometimes even negative duration measures of net liability cash flows. In this paper, we summarize the practices of some major industry players with respect to how they manage the interest-rate risk of their traditional business. We have found that there is a general lack of industry attention to managing the interest-rate risk on traditional products. We show by stochastic pricing on these products that interest-rate risk may warrant more attention than it receives. We measure interest sensitivity in terms of DV01 and Dollar Partial Duration, both of which are calculated by applying a one-basis-point shock to the corresponding Par Yields. We then simulate hedging strategies using cash and bonds or derivatives to hedge the interest-rate changes. We analyze a nonpar whole-life product as well as a long-term care product, observing very similar results for both products. We conclude that there is substantial interest-rate risk in these products, and hedging may provide significantly more protection against such risk than does a simple duration-match strategy.

1. Introduction

This research paper is prepared in response to the request for proposals from the Society of Actuaries' Committee on Finance Research to explore the design and use of interest-rate hedging on traditional health and life products. In particular, we attempt to answer questions such as:

- What are the appropriate measures for interest sensitivity?
- How should renewal premiums be reflected in the calculations?
- What interest-hedging strategy can insurers follow?
- What instruments can or should insurers adopt to manage interest-rate risk?
- What is the impact of the hedging on financial results?

We have modeled a typical nonpar whole-life product as a representative traditional life product in our analysis. We have also extended the same analysis to a typical long-term-care product. All our work has been done using Milliman's financial forecasting system, MG-ALFA®.

We start the paper with a summary of a survey we conducted of a few selected major industry players. The survey gathered information on their current hedging, analysis, reporting, and risk-management techniques for traditional products with respect to interest-rate risk.

We then look at the pricing results of a nonpar whole-life product using a stochastic interest-rate model. The results indicate that the product might be more interest sensitive than most would expect.

The next section discusses the appropriate interest-sensitivity measures. We analyze duration by both considering and not considering future renewal premiums. After identifying problems related to duration, we propose alternative measures, DV01 and Dollar Partial Duration.

We devote a separate section to discussion of both theories and simulation of the two measures. The results of simulations show that with a bond-only strategy, some short selling of bonds is required to make the strategy work. We will explain why this happens and then look for more practical solutions.

Instead of short selling bonds, insurers can short sell bond futures, which also provide the interest sensitivity insurers may need to offset their own interest sensitivity. With the much better liquidity provided by the futures market, insurers can assume bond positions without having to worry about matching the interest sensitivity. Bonds maximize yields and futures mitigate any balancing interest risk.

We then performed the same analysis on a long-term care product and made similar observations and conclusions.

The hedging strategy we introduce in this paper is aimed at reducing the volatility in statutory earnings for the insurer when the interest-rate environment changes. The hedge instruments provide potential offset to changes in the net value of liabilities. We define net value of liabilities as the present value of cash outgo minus the present value of premiums. The present value calculation is based on the current U.S. Treasury yield curve at each calculation point. Policy lapses are also taken into consideration. This approach aims to reflect the market value of the liabilities.

However, the hedging strategy may appear less effective and attractive to insurers on traditional life and health products under the current statutory reserve methodology, which is rather insensitive to changes in interest and ignores expenses and policy lapses. Nonetheless, we believe that a discussion and study of the interest-rate sensitivity of traditional life and health products from a market-value perspective helps to illustrate their inherent interest-rate risk, a risk that is perhaps not fully recognized by the industry, partly due to the lack of statutory reinforcement. Understanding the risk and how to manage it will be important to insurers with the potential adoption of principles-based reserves or fair value accounting in the future.

All effective ALM is associated with a particular accounting framework. Even in the absence of a market-value-based accounting system, there may be value in hedging the ability of the assets to mature the liabilities on a market-value basis.

2. Industry Practice

We approached five major industry players with questions related to size, pricing practice, and hedging strategies of their traditional products. The survey form is provided in Appendix 1.

It is perhaps not surprising that only one company we spoke with has adopted a hedging strategy for its traditional product portfolio. However, that strategy is more a result of the uniqueness of this particular company's product portfolio. The primary liabilities of this company are linked to an index, and the hedging strategy is in place to make sure the increase in reserve does not exceed the increase in the index.

Companies may forgo hedging mostly due to the insignificant size of the block of the business relative to the total liabilities. One company indicates that although it purposely does not hedge the risk, both duration and convexity targets for assets are in place and adjusted when necessary. They have experienced very infrequent adjustments to date on whole-life business. Our sense is that hedging is much more common for other lines of business, such as variable annuities and GICs.

Most product pricing for these companies was done with a single scenario for the interest environment. One company indicated that it has always conducted sensitivity testing on the results to changes in interest rates and is moving towards stochastic pricing.

It is obvious that interest-rate risk of traditional life business is not a major concern to these industry players, unless the product design makes the liability pattern more interest sensitive. Our sense is that interest-rate risk is more actively managed for interest-sensitive products. The diminishing relative size of the traditional life portfolio at most companies certainly could be one of the major reasons. It may also be that industry players have underestimated the impact of interest-rate volatility on traditional business. Having priced these products under a deterministic interest assumption, companies might not have fully understood the impact on reported profits when interest rates change. As an example, in the late 1970s many insurers experienced large losses related to fixed-interest policy-loan activity. This research is motivated by a desire to avoid similar interest-rate-risk losses in the future.

In this analysis, stochastic pricing of a hypothetical nonpar whole-life product will show the extent to which the profit is sensitive to interest rates with and without various types of hedging.

3. Stochastic Pricing

In this section, we will look at the pricing results from a stochastic simulation for a typical nonpar whole life product without hedging.

3.1 Liability Model Description

This is a plain vanilla nonpar regular premium whole-life product. The premium size and pricing assumptions are hypothetical. They are given in Appendix 2 for reference. The liability profile is assumed to consist of newly issued policies as of December 2005 males only, at ages 25, 35, 45, 55, and 65. Their sales distribution is as follows:

Age	Sales Distribution
25	6.20%
35	41.50%
45	41.50%
55	10.40%
65	0.40%

3.2 Scenario description

We utilized the interest scenarios generated by the American Academy of Actuaries for C3Phase2 capital purposes, dated December 2005. Though for a different purpose, the scenarios were generated based on historical economic data and apply to all businesses. We extracted 1,000 scenarios using the academy's scenario picking tool. Then for the

stochastic testing, we ran the first 100 scenarios for illustrative purposes. In a real hedging exercise, more scenarios might be appropriate.

3.3 Stochastic results

The table below shows the NPV of statutory pretax book profits at the discount rate of 12%, as well as the profit margin, which is defined as the NPV of pretax statutory book profits divided by NPV of premium income. The projection covered a 30-year period. This time period was chosen, rather than the life of the contract, because the interest rate scenarios from the Academy cover only 30 years of projected rates. The results pertain to a cash-only model, which means that we assume all assets are in cash with an investment return of the 90day rates on the yield curve.

Note particularly that these results do not reflect any dynamic policyholder behavior. Were we to model, for example, interest-sensitive lapse behavior, results would be even more volatile.

Statistical Measures of Profits over 100 scenarios		
	PV of Pre-Tax Book Profits	Profit Margin
Mean	$670,385	10%
Standard Deviation	$435,070	6%
Minimum	($265,307)	-4%
Maximum	$1,918,077	27%
Quartile 1	$344,741	5%
Median	$656,761	9%
Quartile 3	$940,135	13%
Maximum - Minimum	$2,183,383	31%
Quartile 3 - Quartile 1	$595,394	8%

The volatility of the profit is surprisingly large. The difference between the best-case result ($1,918,077) and the worst-case result (-$265,307) spans a very wide range. Given that the liability cashflows are the same across all scenarios, the volatility is driven purely by the different investment returns from the different yield curve in each scenario.

It appears that the stochastic pricing indicates much more interest sensitivity than one might expect for a nonpar whole-life product. It also suggests that further analysis into how hedging can be implemented on traditional products is appropriate.

4. DV01 and Dollar Partial Duration Hedge

4.1 Duration

Duration is a very common measure of interest-rate sensitivity. There are different definitions of duration, such as effective duration, modified duration, and Macaulay duration. In this paper, we define duration as a percentage change in price due to change in interest rate. In mathematical form, denoting D as duration:

$D = - (1/P) \bullet (\Delta P/\Delta i)$, where P is the price of the asset and i is the interest rate.

To address any potential asymmetry of interest-rate sensitivity to upward/downward shock, we calculate D as

$-(P_u - P_d) / (2 * P * \Delta i)$, where P_u represents price of the asset after upward shock to interest rates and P_d represents price of the asset after downward shock to interest rates.

In our liability modeling, the cash flows are not affected by the change in interest rates because we have not modeled policyholder behavior that is linked to the interest-rate environment. Therefore, the difference between the values of liabilities when interest rates change comes only from the discount impact. We believe there is ample market evidence that even traditional life can experience notable interest-sensitive policyholder behavior. Further research into the impact of such behavior on hedging effectiveness would be appropriate.

In addition, we have also calculated Macaulay duration (MaD) for reference.

$$\text{MaD} = \frac{\sum_{t=1}^{n} \frac{t \cdot CF_t}{(1+i)^t}}{\sum_{t=1}^{n} \frac{CF_t}{(1+i)^t}}$$

Where,
n = number of years till end of projection period
t = time from start of projection
CF_t = cash flow at time t

Table 4.1.1 captures the P, D, and MaD calculated for the liability model described in 3.1 for the first 10 years, with all values calculated at the beginning of the year. The P represents the present value of net-liability cash flows, i.e., cash outgo-premium income. Interest rate, y, is the spot yield curve as of December 2005. The change in y is 1BP.

Table 4.1.1 Duration of Net Liabilities			
Year	P(Value of Net Liabilities)	D (Duration of Net Liabilities)	MaD (Macaulay Duration)
1	($898,398)	(59)	(60)
2	($166,099)	(350)	(354)
3	$534,635	113	114
4	$1,192,371	52	53
5	$1,790,783	35	36
6	$2,332,092	27	28
7	$2,817,860	23	23
8	$3,250,189	19	20
9	$3,633,432	17	17
10	$3,970,287	15	16

Duration and Macaulay duration are similar, and we will thus focus our discussion on duration. The duration values for this product are rather strange. The durations start with large negative values and then progress in large steps to positive durations. A quick look at this might lead one to conclude that assets with negative durations are required for the first two years, followed by assets with long durations. One way to utilize negative duration is to short sell bonds, but insurance companies are not allowed to short sell. Besides, it is nearly impossible to find assets with durations as long as 350 or 113. It appears that we have hit a dead end, and hedging duration is not possible.

The strange pattern of durations might seem to result from the deduction of future renewal premiums. The duration of liabilities should be calculated based on pure cash outgo, and future renewal premiums should be grouped with the assets to derive asset durations. Table 4.1.2 captures the P, D, and MaD calculated on the premium income only, and Table 4.1.3, the pure cash outgoes, both for the first 10 years.

Table 4.1.2 Duration of Premium			
Year	P(Value of Premiums)	D (Duration of Premiums)	MaD (Macaulay Duration)
1	$9,585,751	8	9
2	$9,227,572	8	8
3	$8,620,963	7	8
4	$8,042,082	7	8
5	$7,490,214	7	7
6	$6,964,628	7	7
7	$6,464,631	7	7
8	$5,989,528	7	7
9	$5,538,608	7	7
10	$5,111,116	7	7

Table 4.1.3 Duration of Cash Outgo			
Year	P(Value of Cash Outgo)	D (Duration of Cash Outgo)	MaD (Macaulay Duration)
1	$8,687,353	15	16
2	$9,061,474	14	14
3	$9,155,598	14	14
4	$9,234,453	13	13
5	$9,280,997	13	13
6	$9,296,721	12	12
7	$9,282,492	12	12
8	$9,239,717	11	12
9	$9,172,040	11	11
10	$9,081,403	11	11

The duration patterns of both premiums and cash outgo are now more like the durations one would observe on a fixed-income asset. The result suggests that very long duration assets are required to match the duration of cash outgo. By moving premiums to the asset side, we see that we need very long durations for the other assets in order to allow us to match the liabilities, since the total asset duration is a weighted average of the duration of all assets, and the premiums have relatively short duration.

Nonetheless, we have received conflicting messages from the different grouping of renewal premiums. One might wonder what the right decision is.

A closer look at results in Table 4.1.1 shows that the negative durations in the first two years are really due to the negative sign of the value of liabilities. Using the same notation, $(\Delta P/\Delta y)$ is actually positive for all the first 10 years, and it is the sign of 1/P that determines the sign of the duration. In other words, the value of the liability cash flow always increases when interest rates fall, regardless of the sign of the value itself.

To end the sign confusion, we would simply modify the definition of D to be $\Delta P/\Delta y$. Beyond simply solving the sign problem, this is a direct measure of how much the value of the liabilities changes when interest rates change, and it is thus a direct target for the change in the asset value. This is referred to as DV01 when Δy is 1 basis point.

4.2 DV01

4.2.1 Concepts

DV01 is an acronym for *dollar value of an '01 (i.e., 0.01%)*. It gives the change in the value of a fixed income security for a one-basis-point decline in interest rates. In its mathematical form, $DV01 = -\Delta P / (10000 \bullet \Delta y)$; 10000 is in the denominator because Δy is usually 0.0001. More specifically in our calculation, $DV01 = -(P_u - P_d) / 2$, where P_u represents price of the asset after upward shock to the spot-yield curve, and P_d represents price of the asset after downward shock to the spot-yield curve. The negative sign makes sure that DV01 is positive when the price increases after interest declines and negative when the price decreases after interest declines. This convention is adopted for the same

reason as for duration: All fixed coupon bonds and most other fixed income securities do rise in price when interest rates decline. Another common feature it shares with duration is that the change in interest rates assumes a parallel shift in the entire yield curve. Table 4.2.1 shows DV01 and Duration of a five-year noncallable bond with $100 principal amount and semiannual coupon rate of 5% at par at the beginning of the year.

Table 4.2.1 DV01 & Duration

Year	DV01	Duration
1	0.044	4.4
2	0.036	3.6
3	0.028	2.8
4	0.019	1.9
5	0.010	1.0

It should be noted that the duration happens to be 100 * DV01 in Table 4.1.1 because the bond is at par.

Having defined DV01, the hedging of the interest-rate sensitivity on the whole-life product is straightforward. The entire procedure can be described in six steps:

Step 1: Calculate DV01 of assets, or ADDur, currently on the books.

Step 2: Calculate DV01 of the net liabilities, or LDDur, currently on the books.

Step 3: Calculate balancing DV01, or LDDur - ADDur, to be hedged.

Step 4: Calculate DV01 of one unit of the hedging instrument.

Step 5: Calculate the amount of assets to add in this cycle, by dividing balancing DV01 in Step 3 by DV01 in Step 4.

Step 6: Update the asset portfolio with the newly purchased/sold/matured assets.

Some clarifications are called for at this stage.

First, with DV01, we do not have to struggle with the renewal premium income. The DV01 of a portfolio equals the sum of the DV01 of each individual asset in the portfolio. Therefore, it makes no difference whether we deduct renewal premiums from liabilities or group premiums into the asset portfolio.

Second, theoretically, DV01 hedging allows us to use any fixed-income asset as a hedging instrument. However, when insurance companies purchase bonds, they will have to make sure that the book value of the bonds matches the statutory reserve plus target surplus. This additional constraint implies that at least two assets are required to complete DV01 hedging for insurance companies.

Third, at each projection cycle, we will capture the future cash-flow streams from the current cycle to policy maturity. These cash flows will then be discounted at current treasury yield as well as current yield shocked by +/- 1BP. The average of the difference between the discounted value of cash flows at shocked yields results in DV01. We have not modeled any policyholder behavior linked to the interest rate environment, so the cash flows are the same under different interest yields.

In the next section, we will see DV01 hedging at work under artificial deterministic scenarios.

4.2.2 DV01 Hedging under Deterministic Scenarios

Assuming a base scenario of 5% level yield curve for all time, we simulate the DV01 hedging strategies using two assets: cash and a 30-year noncallable bond with a semiannual coupon rate of 5%. With zero DV01, cash acts as a balancing item to meet the book-value constraint. Two bonds of different maturities would work just as well, but cash is fine from a theoretical discussion perspective.

We thought about using callable bonds instead of noncallable bonds. Though insurance companies use callable bonds most of the time, the embedded option gives them extra uncertainty not desirable in a hedge instrument. Callable bonds offer higher yields, which may explain their popularity. Since this is a hedge analysis, we chose not to use callable bonds.

Table 4.2.2.1 in Appendix 3 captures the hedge positions required for the first 10 projection years. The table shows that to hedge against DV01 of the liabilities, we would initially need a negative position in cash. Intuitively, it implies that liability DV01 is so large that we would need more 30-year bonds than cash available on the books. We would have to borrow to finance the purchase of the 30-year bonds necessary for a DV01 hedge. It can be inferred that should we hedge with a shorter-term bond with smaller DV01, more initial borrowing would be necessary. Similarly, if we were to invest with a combination of a long bond and a short bond, rather than a long bond and cash, we would need even larger negative positions on the short bond than we do with cash to obtain the appropriate leverage effects.

To assess the effectiveness of hedging, we look at the market value of the surplus under six sample scenarios. The market value of the surplus is defined as

Market Value of Assets–Present Value of Liabilities

We considered the present value of liabilities as a proxy for the market value of liabilities. DV01 hedge essentially hedges the change in market value of liabilities, and the market-value measure best reflects the effectiveness of hedging.

We also compared the results of DV01 hedging with results of two other strategies. The first assumes all assets are in cash. The second has a mixed pool of callable bonds and mortgages following a rigid percentage allocation strategy, as shown below. This reflects

a strategy that buys when available, possibly more for maximizing yields than for duration matching. The only change that different strategies would have on the result is the investment return.

	Percentage
1-year Bond	14%
2-year Bond	14%
5-year Bond	14%
10-year Bond	21%
30-year Bond	7%
30-year Mortgage	30%

We thus ran each strategy through six scenarios:

Scenario 1 (Base): base scenario with 5% level yield curve throughout future projections

Scenario 2 (SU): shock-up scenario with 5.01% level yield curve throughout future projections

Scenario 3 (SD): shock-down scenario with 4.99% level yield curve throughout future projections

Scenario 4 (HSU): high shock-up scenario with 6% level yield curve throughout future projections

Scenario 5 (HSD): high shock-down scenario with 4% level yield curve throughout future projections

Scenario 6 (Tilt): tilt scenario with 1% increase to a two-year rate, 0.7% increase to five-year rate, 0.5% increase to 10-year rate, 0.5% decrease to 30-year rate, and other points on the yield curve increased by linear interpolation.

Table 4.2.2.2 summarizes each strategy during the first six projection years.

Table 4.2.2.2
Summary of Portfolio Market Value

		MV of Surplus							Change from Base					
Unhedged		0	1	2	3	4	5	6	1	2	3	4	5	6
Cash Assets	MV of portfolio Base	$1,172,401	$392,907	-$282,377	-$918,637	-$1,499,341	-$2,027,470	-$2,504,758	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
	MV of portfolio SU	$1,172,401	$397,540	-$277,470	-$913,510	-$1,494,047	-$2,022,058	-$2,499,276	1.2%	-1.7%	-0.6%	-0.4%	-0.3%	-0.2%
	MV of portfolio SD	$1,172,401	$388,259	-$287,299	-$923,778	-$1,504,648	-$2,032,894	-$2,510,253	-1.2%	1.7%	0.6%	0.4%	0.3%	0.2%
	MV of portfolio HSU	$1,172,401	$791,550	$144,410	-$468,789	-$1,031,416	-$1,546,197	-$2,014,621	101.5%	-151.1%	-49.0%	-31.2%	-23.7%	-19.6%
	MV of portfolio HSD	$1,172,401	-$152,175	-$853,323	-$1,509,730	-$2,105,037	-$2,642,559	-$3,124,350	-138.7%	202.2%	64.3%	40.4%	30.3%	24.7%
	MV of portfolio Tilt	$1,172,401	$19,384	-$618,705	-$1,219,136	-$1,765,647	-$2,261,440	-$2,708,256	-95.1%	119.1%	32.7%	17.8%	11.5%	8.1%

		MV of Surplus							Change from Base					
Unhedged		0	1	2	3	4	5	6	1	2	3	4	5	6
Mixed Pool of Bonds & Mortgages	MV of portfolio Base	$1,172,401	$392,907	-$282,377	-$918,637	-$1,499,341	-$2,027,470	-$2,504,758	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
	MV of portfolio SU	$1,172,401	$397,405	-$304,279	-$934,356	-$1,506,244	-$2,059,617	-$2,492,365	1.1%	7.8%	1.7%	0.5%	1.6%	-0.5%
	MV of portfolio SD	$1,172,401	$388,394	-$253,355	-$871,927	-$1,440,437	-$1,989,887	-$2,470,413	-1.1%	-10.3%	-5.1%	-3.9%	-1.9%	-1.4%
	MV of portfolio HSU	$1,172,401	$778,816	$131,020	-$470,221	-$1,013,230	-$1,538,906	-$2,021,103	98.2%	-146.4%	-48.8%	-32.4%	-24.1%	-19.3%
	MV of portfolio HSD	$1,172,401	-$139,901	-$851,545	-$1,496,512	-$2,120,351	-$2,580,251	-$3,123,808	-135.6%	201.6%	62.9%	41.4%	27.3%	24.7%
	MV of portfolio Tilt	$1,172,401	$16,306	-$639,072	-$1,258,223	-$1,817,199	-$2,365,519	-$2,904,862	-95.8%	126.3%	37.0%	21.2%	16.7%	16.0%

		MV of Surplus							Change from Base					
DV01 Hedge		0	1	2	3	4	5	6	1	2	3	4	5	6
Hedge With Cash & 30-year Bonds	MV of portfolio Base	$1,172,401	$392,068	-$284,931	-$923,775	-$1,507,977	-$2,040,560	-$2,523,304	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
	MV of portfolio SU	$1,172,401	$392,466	-$284,186	-$922,728	-$1,506,673	-$2,039,041	-$2,521,609	0.1%	-0.3%	-0.1%	-0.1%	-0.1%	-0.1%
	MV of portfolio SD	$1,172,401	$391,666	-$285,680	-$924,826	-$1,509,286	-$2,042,084	-$2,525,005	-0.1%	0.3%	0.1%	0.1%	0.1%	0.1%
	MV of portfolio HSU	$1,172,401	$410,014	-$233,259	-$842,771	-$1,401,945	-$1,913,554	-$2,379,136	4.6%	-18.1%	-8.8%	-7.0%	-6.2%	-5.7%
	MV of portfolio HSD	$1,172,401	$322,557	-$390,102	-$1,059,908	-$1,670,501	-$2,225,189	-$2,726,027	-17.7%	36.9%	14.7%	10.8%	9.0%	8.0%
	MV of portfolio Tilt	$1,172,401	$214,944	-$448,332	-$1,076,806	-$1,654,065	-$2,183,156	-$2,665,684	-45.2%	57.3%	16.6%	9.7%	7.0%	5.6%

It should be noted that the DV01 hedge strategy we used is not a completely dynamic strategy. It is a strategy in which positions in assets are determined at the run date, given the best estimates of the future yield curve. The same positions apply onward to all six scenarios. In practice, should this strategy be pursued, the positions can be reviewed whenever the company wishes to perform a hedge-assessment exercise. Unless the interest environment has changed dramatically, it is possible that minor adjustments will be needed in order to reflect the company's best estimate assumption.

The right-hand side of Table 4.2.2.2 shows the percentage change in the market value of the portfolio from the base scenario to each of the other five scenarios. It appears that the first strategy with assets in cash and the second strategy with a mixed pool of bonds and mortgages experience similar percentage changes when interest rates move differently, because the assets were not purchased for hedging purposes.

The third strategy with the DV01 hedge clearly mitigates the changes. In fact, it almost completely eliminates volatility under scenario SU and scenario SD, which is expected due to the calculation of DV01. It is not so effective in the other three scenarios. The ineffectiveness under scenarios (HSU) and (HSD) is due to the relatively large size of the interest rate change. DV01 at an initial interest rate of 5% is rather different from the DV01 at an interest rate of 6%. In effect, we have introduced some risk by ignoring asset convexity. The ineffectiveness under scenario (Tilt) arises because the shape of the yield curve tilts, whereas the DV01 hedge works under parallel shifts to the yield curve.

Arguably, the problem of a tilting yield curve is more significant than the problem of convexity. The problem of convexity can be ameliorated if the hedge position is reviewed on a frequent basis. However, the shape of the yield curve can change more rapidly. The liquidity of the long-term bonds is much thinner than that of the short-term bonds. It is often the case that short-term yields have moved while long term yields have not, thus changing the shape of the yield curve. For insurance companies, the long-term yields are perhaps more important than short-term rates due to the long-term nature of the liability.

A technique that could be adopted against change in the shape of the yield curve is known as the Dollar Partial Duration hedge. In the next section, we will look at it in more detail.

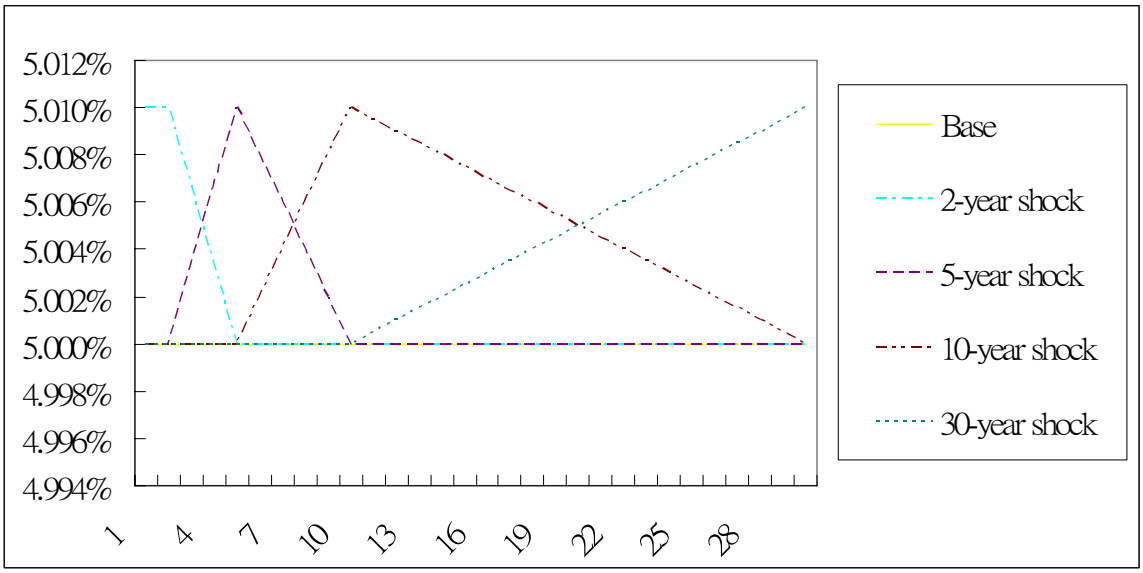
4.3 Dollar Partial Duration

4.3.1 Concepts

The fundamental assumption behind the concept is that the movement of a set of selected time to maturity rates can describe the movement of the entire yield curve. In other words, given the selected points on the yield curve, any other rate on the yield curve can be determined. It should be noted that our analysis is based on shocks to yield curves, and thus the terminology "Partial Duration" is used. Should the shocks be on spot curves, "Key Rate Duration" would be the right terminology. For more details, readers can refer to Reitano[1]. In this paper we select two-, five-, ten-, and thirty-year rates on the yield curve.

By shocking each selected time-to-maturity rate, we obtain four yield curves, based on which we will calculate the Dollar Partial Duration. Each shock affects yields from the term of the previous selected time-to-maturity to the next (or the last term on the yield curve). Specifically, the two-year shock affects yields of term zero to five, the five-year affects yields of term two to 10, the 10-year affects yields of term five to 30, and the 30-year affects yields from 10 on. The impact of each shock is one basis point at its own maturity and declines adjacently to zero at the term of the adjacent selected maturity. To the left of the two-year point and to the right of the 30-year point, the effect remains at one basis point. The shifts are applied to the spot yield curve.

Chart 4.3.1 gives the base yield curve of level 5%, two-year shock, five-year shock, 10-year shock, and 30-year shock.



Dollar Partial Duration is calculated for each shock as the change in price from the base yield curve to the shocked yield curve.

Table 4.3.1 summarizes the Dollar Partial Duration and DV01 of a five-year bond with $100 principal and 5% semiannual coupon at par.

Table 4.3.1 Dollar Partial Duration of Five-year Bond

Year	2-yr Dollar Partial Duration	5-yr Dollar Partial Duration	10-yr Dollar Partial Duration	30-yr Dollar Partial Duration	Sum of Dollar Partial Duration	DV01
1	0.000	0.044	0.000	0.000	0.044	0.044
2	0.012	0.024	0.000	0.000	0.036	0.036
3	0.018	0.009	0.000	0.000	0.028	0.028
4	0.019	0.000	0.000	0.000	0.019	0.019
5	0.010	0.000	0.000	0.000	0.010	0.010

The sum of the Dollar Partial Durations equals DV01, because the pattern of the selected time-to-maturities ensures that the sum of increases at each point on the yield curve equals one basis point.

The goal of Dollar Partial Duration hedging is to find assets that match each Dollar Partial Duration of the liabilities. Obviously, four assets are required to make the strategy work when dealing with four maturity points. Due to the book-value constraint, we will add a fifth asset, cash, as a balancing item.

4.3.2 Dollar Partial Duration Hedging under Deterministic Scenarios

With the same base scenario of a 5% level yield curve, we simulate Dollar Partial Duration hedging using five assets: cash, two-year, five-year, 10-year and 30-year bonds, all with a semiannual coupon rate of 5%.

Table 4.3.2.1 in Appendix 4 captures the hedge positions required for the first 10 projection years.

It is interesting to note that while the DV01 is generally positive, the Dollar Partial Duration can be negative at selected maturities. This has led to a short position in the two-year and five-year bonds. This indicates that Dollar Partial Duration hedging might not be practical for insurance companies. Nonetheless, we shall assume for now that short selling is permitted and revisit this issue later.

Though DV01 of the liability is not the hedge target, the Dollar Partial Duration hedge strategy hedges the DV01 as long as the time-to-maturities are selected so that the sum of the Dollar Partial Duration equals the DV01 of the same asset.

As with the DV01 hedge, we insert the positions determined in the base scenario and test the change in market value of the portfolio under the six artificial deterministic scenarios.

Table 4.3.2.2 shows the results of Dollar Partial Duration hedging together with a recap of results of the other three strategies.

Table 4.3.2.2 Summary of portfolio market value

		MV of Surplus							Change from Base					
Unhedged		0	1	2	3	4	5	6	1	2	3	4	5	6
Cash Assets	MV of portfolio Base	$1,172,401	$392,907	-$282,377	-$918,637	-$1,499,341	-$2,027,470	-$2,504,758	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
	MV of portfolio SU	$1,172,401	$397,540	-$277,470	-$913,510	-$1,494,047	-$2,022,058	-$2,499,276	1.2%	-1.7%	-0.6%	-0.4%	-0.3%	-0.2%
	MV of portfolio SD	$1,172,401	$388,259	-$287,299	-$923,778	-$1,504,648	-$2,032,894	-$2,510,253	-1.2%	1.7%	0.6%	0.4%	0.3%	0.2%
	MV of portfolio HSU	$1,172,401	$791,550	$144,410	-$468,789	-$1,031,416	-$1,546,197	-$2,014,621	101.5%	-151.1%	-49.0%	-31.2%	-23.7%	-19.6%
	MV of portfolio HSD	$1,172,401	-$152,175	-$853,323	-$1,509,730	-$2,105,037	-$2,642,559	-$3,124,350	-138.7%	202.2%	64.3%	40.4%	30.3%	24.7%
	MV of portfolio Tilt	$1,172,401	$19,384	-$618,705	-$1,219,136	-$1,765,647	-$2,261,440	-$2,708,256	-95.1%	119.1%	32.7%	17.8%	11.5%	8.1%

		MV of Surplus							Change from Base					
Unhedged		0	1	2	3	4	5	6	1	2	3	4	5	6
Mixed Pool of Bonds & Mortgages	MV of portfolio Base	$1,172,401	$392,907	-$282,377	-$918,637	-$1,499,341	-$2,027,470	-$2,504,758	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
	MV of portfolio SU	$1,172,401	$397,405	-$304,279	-$934,356	-$1,506,244	-$2,059,617	-$2,492,365	1.1%	7.8%	1.7%	0.5%	1.6%	-0.5%
	MV of portfolio SD	$1,172,401	$388,394	-$253,355	-$871,927	-$1,440,437	-$1,989,887	-$2,470,413	-1.1%	-10.3%	-5.1%	-3.9%	-1.9%	-1.4%
	MV of portfolio HSU	$1,172,401	$778,816	$131,020	-$470,221	-$1,013,230	-$1,538,906	-$2,021,103	98.2%	-146.4%	-48.8%	-32.4%	-24.1%	-19.3%
	MV of portfolio HSD	$1,172,401	-$139,901	-$851,545	-$1,496,512	-$2,120,351	-$2,580,251	-$3,123,808	-135.6%	201.6%	62.9%	41.4%	27.3%	24.7%
	MV of portfolio Tilt	$1,172,401	$16,306	-$639,072	-$1,258,223	-$1,817,199	-$2,365,519	-$2,904,862	-95.8%	126.3%	37.0%	21.2%	16.7%	16.0%

		MV of Surplus							Change from Base					
DV01 Hedge		0	1	2	3	4	5	6	1	2	3	4	5	6
Hedge With Cash & 30-year Bonds	MV of portfolio Base	$1,172,401	$392,068	-$284,931	-$923,775	-$1,507,977	-$2,040,560	-$2,523,304	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
	MV of portfolio SU	$1,172,401	$392,466	-$284,186	-$922,728	-$1,506,673	-$2,039,041	-$2,521,609	0.1%	-0.3%	-0.1%	-0.1%	-0.1%	-0.1%
	MV of portfolio SD	$1,172,401	$391,666	-$285,680	-$924,826	-$1,509,286	-$2,042,084	-$2,525,005	-0.1%	0.3%	0.1%	0.1%	0.1%	0.1%
	MV of portfolio HSU	$1,172,401	$410,014	-$233,259	-$842,771	-$1,401,945	-$1,913,554	-$2,379,136	4.6%	-18.1%	-8.8%	-7.0%	-6.2%	-5.7%
	MV of portfolio HSD	$1,172,401	$322,557	-$390,102	-$1,059,908	-$1,670,501	-$2,225,189	-$2,726,027	-17.7%	36.9%	14.7%	10.8%	9.0%	8.0%
	MV of portfolio Tilt	$1,172,401	$214,944	-$448,332	-$1,076,806	-$1,654,065	-$2,183,156	-$2,665,684	-45.2%	57.3%	16.6%	9.7%	7.0%	5.6%

		MV of Surplus							Change from Base					
Dollar Partial Duration Hedge		0	1	2	3	4	5	6	1	2	3	4	5	6
Hedge With 4 bonds & Cash	MV of portfolio Base	$1,172,401	$415,758	-$229,089	-$843,153	-$1,399,546	-$1,901,227	-$2,352,557	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
	MV of portfolio SU	$1,172,401	$415,859	-$228,949	-$842,944	-$1,399,280	-$1,901,157	-$2,352,280	0.0%	-0.1%	0.0%	0.0%	0.0%	0.0%
	MV of portfolio SD	$1,172,401	$415,655	-$229,232	-$843,366	-$1,399,815	-$1,901,299	-$2,352,839	0.0%	0.1%	0.0%	0.0%	0.0%	0.0%
	MV of portfolio HSU	$1,172,401	$416,971	-$224,462	-$831,904	-$1,382,894	-$1,904,172	-$2,335,514	0.3%	-2.0%	-1.3%	-1.2%	0.2%	-0.7%
	MV of portfolio HSD	$1,172,401	$391,892	-$257,392	-$878,745	-$1,441,117	-$1,923,242	-$2,396,127	-5.7%	12.4%	4.2%	3.0%	1.2%	1.9%
	MV of portfolio Tilt	$1,172,401	$382,013	-$257,260	-$881,783	-$1,460,945	-$1,997,160	-$2,461,256	-8.1%	12.3%	4.6%	4.4%	5.0%	4.6%

The Dollar Partial Duration hedge has done exactly what we expected. It mitigates the volatility of the market value even in the scenario where the yield curve tilts. Therefore, we have shown that the Dollar Partial Duration hedge works well under artificial deterministic scenarios. It remains questionable, however, whether it will also work under more randomly generated stochastic scenarios. We thus perform the same analysis using the 100 stochastic scenarios described in section 3.2.

4.4 DV01 and Dollar Partial Duration Hedge under Stochastic Scenarios

The asset positions are first determined based on the yield curve of December 2005, the initial yield curve of the stochastic scenarios. By doing this, we implicitly assume that the initial yield curve is the best estimate for the future yield curve. This is likely to be inappropriate, but it still serves as a good test of the two hedge strategies, as the results show.

Tables 4.4.1 in Appendix 5 and 4.4.2 in Appendix 6 show the hedge positions for the DV01 hedge and the Dollar Partial Duration hedge for the first 10 years of the projection.

With hedge positions determined, we then run the DV01 hedge and Dollar Partial Duration hedge through the 100 scenarios and compare the results with the strategy of cash only and with a mixed pool of assets.

Given the relatively large number of scenarios, we will show the standard deviation of the portfolio market values over the 100 scenarios instead of values for each scenario individually. Table 4.4.3 captures the standard deviations of the market value of the portfolio for the four strategies over the first six years.

Table 4.4.3 Standard Deviations of Market Value

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Cash Only	$289,131	$506,341	$618,574	$737,305	$769,614	$832,017
Mixed Pool of Assets	$280,666	$469,804	$565,504	$655,525	$676,842	$713,500
DV01 Hedge	$36,320	$47,968	$59,993	$78,516	$98,992	$115,122
Dollar Partial Duration Hedge	$11,152	$25,110	$41,266	$60,690	$48,460	$51,930

The results show declining standard deviation when we move from “Cash Only” to “Dollar Partial Duration Hedge.” We thus conclude that the Dollar Partial Duration hedge effectively mitigates interest-rate risk of the whole-life product.

However, there is still one big problem with the Dollar Partial Duration hedge: Short positions in bonds are not practical for insurance companies. Even if they were allowed, it may not be possible to find a sufficient number of bonds in the market that could be borrowed and sold. We propose to overcome this problem by using bond futures.

In the next section, we describe hedge strategy using futures in more detail and look at the results of the simulation.

5. Hedging with Futures

A bond futures contract always starts with a zero price. As time progresses, the price of the contract depends on the price of the underlying bonds, which moves when interest rates change. Based on this, we can calculate the DV01 and Dollar Partial Duration of a bond futures contract. Futures also provide some desirable features that bonds cannot.

a. Insurance companies can take short positions in futures for hedging purposes.

b. The futures market has relatively good liquidity, and large positions tend to affect the market much less than large positions in bonds.

c. Futures do not affect the book value of assets, so that insurance companies can add assets that maximize their book yield and use futures to hedge against any balancing interest-rate risk.

d. The term of the futures can be selected to correspond with the frequency of hedge review. The term of the futures is the period from the start of the contract to the date when the underlying bond is delivered. The underlying bond can have a much longer term, even if the term of the futures is only 30 days.

If the hedge position is reviewed monthly, the insurance company can add 30-day futures at every review point. This way, futures hedge interest-rate risk only in the next month. This makes hedge planning much simpler. Bonds are of much longer term, and their impact could span as long as 30 years, during which there could be much uncertainty regarding the interest movement and any change in the characteristics of the liabilities or the bond market. At the maturity of the futures, the insurance company can cash settle instead of having to purchase the underlying bond.

However, having the term of the futures the same as hedge-review frequency means that the market value of the futures is always zero at each review date. We thus cannot analyze the impact of the hedging strategy with futures by looking at the movement of the market value. Futures have a maturity cash flow when cash settled, and this cash flow increases the profit of the product when it is positive and reduces the profit when it is negative. In theory, this hedges against the change in profit due to the interest-rate movement. Therefore, we will analyze the effectiveness of futures hedging by looking at the profit.

In the next two sections, we will provide details of futures hedging against DV01 and Dollar Partial Duration, respectively.

5.1 Futures Hedging against DV01

The hedging strategy is to hedge against the DV01 of liabilities with the DV01 of futures contracts. We simulated a quarterly hedge review with a futures contract term of 90 days.

The term of the futures is easy to determine, but the term of the underlying bond is not. As far as DV01 hedging is concerned, futures on a one-year bond and futures on a 30-year bond will work equally well. Futures on a one-year bond have less DV01 and thus

require more in notional amounts to hedge the liabilities. However, Table 4.4.2 in Appendix 5 shows that the liabilities are most sensitive to 30-year shock, with a 30-year Dollar Partial Duration larger than other Dollar Partial Duration in absolute amount. It is perhaps more effective to hedge DV01 using futures on a long-term bond, so that the futures can provide more protection against movement in the long end of the yield curve.

Thus we tested four strategies to compare the hedge effectiveness. More specifically they are:

- Strategy 1: assets in cash only;
- Strategy 2: mixed pool of assets;
- Strategy 3: mixed pool of assets plus futures on one-year bond to back DV01; and,
- Strategy 4: mixed pool of assets plus futures on 30-year bond to back DV01.

Unlike the prior DV01 hedge exercise, we originally planned to assume a mixed pool of assets and also the futures. Our intention was that the futures hedge the balancing DV01 not covered by the mixed pool of assets. During our tests in modeling, we noticed that the mixed pool of assets had a very small DV01 compared to the liabilities. This was consistent with our earlier findings that much larger amounts of bonds than book value of liabilities are required to match the DV01 of the liabilities. Thus, for computational convenience, we have modeled futures to match the DV01 of liabilities instead of the balancing DV01 after the effect of the pool of assets.

Table 5.1 summarizes the statistics of the present value of pretax book profits at a discount rate of 12% over the 100 scenarios for each of the strategies.

Table 5.1: Statistical Measures of Profits over 100 scenarios		
Strategy 1:	Cash Only	
	PV of Pre-Tax Book Profits	Profit Margin
Mean	$670,385	10%
Standard Deviation	$435,070	6%
Minimum	($265,307)	-4%
Maximum	$1,918,077	27%
Quartile 1	$344,741	5%
Median	$656,761	9%
Quartile 3	$940,135	13%
Maximum - Minimum	$2,183,383	31%
Quartile 3 - Quartile 1	$595,394	8%
Strategy 2:	Mixed Pool of Assets	
	PV of Pre-Tax Book Profits	Profit Margin
Mean	$588,777	8%
Standard Deviation	$311,475	4%
Minimum	($102,970)	-1%
Maximum	$1,465,635	21%
Quartile 1	$376,860	5%
Median	$568,412	8%
Quartile 3	$800,302	11%
Maximum - Minimum	$1,568,604	22%
Quartile 3 - Quartile 1	$423,441	6%
Strategy 3:	Futures on 1-year Bond	
	PV of Pre-Tax Book Profits	Profit Margin
Mean	($57,979)	-1%
Standard Deviation	$379,106	5%
Minimum	($777,836)	-11%
Maximum	$1,395,539	20%
Quartile 1	($357,331)	-5%
Median	($58,228)	-1%
Quartile 3	$203,519	3%
Maximum - Minimum	$2,173,376	31%
Quartile 3 - Quartile 1	$560,850	8%
Strategy 4:	Futures on 30-year Bond	
	PV of Pre-Tax Book Profits	Profit Margin
Mean	$120,486	2%
Standard Deviation	$249,768	4%
Minimum	($276,696)	-4%
Maximum	$1,155,384	16%
Quartile 1	($53,961)	-1%
Median	$87,745	1%
Quartile 3	$198,004	3%
Maximum - Minimum	$1,432,079	20%
Quartile 3 - Quartile 1	$251,966	4%

The hedging strategy does not seem to work very effectively. It reduces the average profitability while not reducing the volatility by a large degree. However, the results confirm our conjecture that futures on 30-year bonds are better than futures on one-year bonds. We then turn to the Dollar Partial Duration hedge with futures and expect better results.

5.2 Futures Hedging against Dollar Partial Duration

We still have the mixed pool of assets, and then hedge the balancing Dollar Partial Duration with 90-day futures on a two-year bond, a five-year bond, a 10-year bond, and a 30-year bond.

Again, because the mixed pool of assets has a much smaller Dollar Partial Duration compared with liabilities, and for computational convenience, we have modeled futures to match the Dollar Partial Duration of liabilities instead of the balancing Key Rate 01 after the effect of the pool of assets.

Table 5.2 summarizes the statistics of the present value of pretax book profits at a discount rate of 12% over the 100 scenarios for each of the strategies.

Table 5.2: Statistical Measures of Profits over 100 scenarios		
Strategy 1:	Cash Only	
	PV of Pre-Tax Book Profits	Profit Margin
Mean	$670,385	10%
Standard Deviation	$435,070	6%
Minimum	($265,307)	-4%
Maximum	$1,918,077	27%
Quartile 1	$344,741	5%
Median	$656,761	9%
Quartile 3	$940,135	13%
Maximum - Minimum	$2,183,383	31%
Quartile 3 - Quartile 1	$595,394	8%
Strategy 2:	Mixed Pool of Assets	
	PV of Pre-Tax Book Profits	Profit Margin
Mean	$588,777	8%
Standard Deviation	$311,475	4%
Minimum	($102,970)	-1%
Maximum	$1,465,635	21%
Quartile 1	$376,860	5%
Median	$568,412	8%
Quartile 3	$800,302	11%
Maximum - Minimum	$1,568,604	22%
Quartile 3 - Quartile 1	$423,441	6%
Strategy 3:	Futures on 30-year Bond	
	PV of Pre-Tax Book Profits	Profit Margin
Mean	$120,486	2%
Standard Deviation	$249,768	4%
Minimum	($276,696)	-4%
Maximum	$1,155,384	16%
Quartile 1	($53,961)	-1%
Median	$87,745	1%
Quartile 3	$198,004	3%
Maximum - Minimum	$1,432,079	20%
Quartile 3 - Quartile 1	$251,966	4%
Strategy 4:	Dollar Partial Duration Hedge with Futures	
	PV of Pre-Tax Book Profits	Profit Margin
Mean	$69,146	1%
Standard Deviation	$230,653	3%
Minimum	($365,936)	-5%
Maximum	$1,031,698	15%
Quartile 1	($89,138)	-1%
Median	$44,140	1%
Quartile 3	$157,714	2%
Maximum - Minimum	$1,397,634	20%
Quartile 3 - Quartile 1	$246,852	4%

The results show no significant improvement in hedge effectiveness, either. In fact, it appears that the introduction of futures on two-year, five-year, and 10-year bonds has little impact on the results. This seems to contradict the conclusions in Section 4. We offer an explanation below.

5.3 Overhedge v. Underhedge

Both DV01 and Dollar Partial Duration can be categorized as market-value hedging, because their purpose is to stabilize the market value of the portfolio when interest rates move. We have seen that they work fine in Section 4. Statutory profits, on the other hand, are driven largely by cash flows and changes in statutory reserve.

Because the hedge target represents change in the market value of liabilities instead of change in statutory reserve, the hedge effectiveness is largely reduced. Had the statutory reserve been calculated in a similar manner to the market-value approach, we would expect the hedge effectiveness to be very close to the effectiveness on the market value of the portfolio. The adoption of either principle-based reserves or fair value accounting would make the hedging appear more effective.

However, by adjusting the hedge position, we might still achieve reasonable hedge effectiveness, since it is likely that DV01 and Dollar Partial Duration hedge strategies either overhedge or underhedge the movement in the cash flows. The results in Table 5.2 seem to indicate that overhedge occurs, which means that accumulating too many futures makes hedging appear too costly, due to significant reduction of average profits. Nonetheless, we tested the two strategies again, first assuming that only 80% of the DV01 or Dollar Partial Duration is hedged, and then 120%.

Table 5.3 summarizes the statistics of the present value of pretax book profits at a discount rate of 12% over the 100 scenarios with 80%, 100%, and 120% DV01/Dollar Partial Duration hedged.

Table 5.3 Part 1: Statistical Measures of Profits over 100 scenarios for DV01		
Strategy 1:	80% DV01 Futures on 30-yr Bond	
	PV of Pre-Tax Book Profits	Profit Margin
Mean	$214,144	3%
Standard Deviation	$157,009	2%
Minimum	($60,834)	-1%
Maximum	$903,713	13%
Quartile 1	$115,559	2%
Median	$182,264	3%
Quartile 3	$274,285	4%
Maximum - Minimum	$964,547	14%
Quartile 3 - Quartile 1	$158,726	2%
Strategy 2:	100% DV01 Futures on 30-yr Bond	
	PV of Pre-Tax Book Profits	Profit Margin
Mean	$120,486	2%
Standard Deviation	$249,768	4%
Minimum	($276,696)	-4%
Maximum	$1,155,384	16%
Quartile 1	($53,961)	-1%
Median	$87,745	1%
Quartile 3	$198,004	3%
Maximum - Minimum	$1,432,079	20%
Quartile 3 - Quartile 1	$251,966	4%
Strategy 3:	120 % DV01 Futures on 30-yr Bond	
	PV of Pre-Tax Book Profits	Profit Margin
Mean	$26,827	0%
Standard Deviation	$349,922	5%
Minimum	($492,557)	-7%
Maximum	$1,407,054	20%
Quartile 1	($226,603)	-3%
Median	($22,561)	0%
Quartile 3	$163,010	2%
Maximum - Minimum	$1,899,612	27%
Quartile 3 - Quartile 1	$389,613	6%

Table 5.3 Part 2: Statistical Measures of Profits over 100 scenarios for Dollar Partial Duration		
Strategy 1:	80% Dollar Partial Duration Hedge with Futures	
	PV of Pre-Tax Book Profits	Profit Margin
Mean	$173,072	2%
Standard Deviation	$147,543	2%
Minimum	($132,226)	-2%
Maximum	$804,764	11%
Quartile 1	$81,218	1%
Median	$148,459	2%
Quartile 3	$240,115	3%
Maximum - Minimum	$936,991	13%
Quartile 3 - Quartile 1	$158,897	2%
Strategy 2:	100% Dollar Partial Duration Hedge with Futures	
	PV of Pre-Tax Book Profits	Profit Margin
Mean	$69,146	1%
Standard Deviation	$230,653	3%
Minimum	($365,936)	-5%
Maximum	$1,031,698	15%
Quartile 1	($89,138)	-1%
Median	$44,140	1%
Quartile 3	$157,714	2%
Maximum - Minimum	$1,397,634	20%
Quartile 3 - Quartile 1	$246,852	4%
Strategy 3:	120% Dollar Partial Duration Hedge with Futures	
	PV of Pre-Tax Book Profits	Profit Margin
Mean	($34,780)	0%
Standard Deviation	$323,621	5%
Minimum	($599,646)	-9%
Maximum	$1,258,631	18%
Quartile 1	($257,668)	-4%
Median	($73,170)	-1%
Quartile 3	$80,533	1%
Maximum - Minimum	$1,858,277	26%
Quartile 3 - Quartile 1	$338,201	5%

The results show that with only 80% DV01 or Dollar Partial Duration hedged, the futures contracts provide a better hedge than when 100% hedged, whereas 120% hedged is the worst. This supports our argument that when DV01 and Dollar Partial Duration are sought to hedge against fluctuation in cash flow, or profits, an underhedge position is more effective.

However, it is likely that 80% might not be an optimal position with which to achieve maximum hedge effectiveness. Insurers may have to test more positions to decide what position to take.

Another interesting point is that a Dollar Partial Duration hedge does not improve the effectiveness much, compared to DV01 with futures on a 30-year bond. This is partly because futures on a 30-year bond provide some protection against the 30-year exposure, which affects the liabilities more than the other three maturity point exposures. Also, the rather short hedge-review frequency—quarterly—helps mitigate changes to interest rates.

6. Interest Rate Hedge Analysis of Long-term Care Products

We performed similar analysis for a long-term-care product to show that the hedge strategy described applies to both traditional life and health products. Long-term Care products are usually designed to offer lifetime benefits with an elimination period of 90 days or more. The premium size and pricing assumptions are given in Appendix 7 for reference. Again, the liability profile is assumed to consist of newly issued policies as of December 2005, to males only, at ages 45, 52, 57, 62, 67, and 75. Their sales distribution is as follows:

Age	Sales Distribution
45	8%
52	15%
57	24%
62	23%
67	16%
75	14%

We have included three benefit increase options: increasing at a compound rate of 5%, a simple rate of 5%, or 0%. The distribution is as follows:

Benefit Increase Type	Distribution
Compound at 5%	59%
No Increase	15%
Simple at 5%	26%

As with the whole-life product, the first step is to look at the sensitivity of the profit results when the product is priced over the 100 stochastic interest rate scenarios. The table below shows the NPV of statutory pretax book profits at the discount rate of 12%, as well as the profit margin. As previously mentioned, since the projection period of the Academy scenarios is limited to 30 years, we only captured 30 years of projection results.

Statistical Measures of Profits over 100 scenarios		
	PV of Pre-Tax Book Profits	Profit Margin
Mean	$694,660	14%
Standard Deviation	$304,228	6%
Minimum	$26,490	1%
Maximum	$1,590,854	32%
Quartile 1	$474,427	10%
Median	$678,226	14%
Quartile 3	$903,270	18%
Maximum - Minimum	$1,564,364	32%
Quartile 3 - Quartile 1	$428,843	9%

The results are similar to those we observed with the whole-life product. The range of profit values across different scenarios is quite wide.

With the belief that DV01 and Dollar Partial Duration hedges would reduce volatility of the market value of the LTC product just as they did for whole life, we calculated the standard deviation of the portfolio market values over the 100 scenarios for the first six years just as we did in Table 4.4.3 for whole life. The four strategies captured are the same as those applied to the whole life described in section 4.2.2. The table below displays the results.

Standard Deviations of Market Value						
	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Cash Only	$310,140	$533,844	$653,140	$780,554	$811,246	$876,220
Mixed Pool of Assets	$291,266	$501,194	$613,166	$726,720	$753,600	$804,295
DV01	$32,396	$63,054	$73,908	$96,906	$123,063	$140,055
Dollar Partial Duration Hedge	$10,041	$26,012	$37,258	$57,071	$50,030	$83,268

The problem of having to short sell bonds for Dollar Partial Duration hedging applies to the LTC product, too, leading us to look at the impact of adding futures positions instead. We have observed that an underhedged position helps to mitigate volatility in profits better than an overhedged position. The table below summarizes the statistics of the present value of pretax book profits at a discount rate of 12% over the 100 scenarios with 80%, 100%, and 120% DV01/Dollar Partial Duration hedged.

Statistical Measures of Profits over 100 scenarios for DV01		
Strategy 1:	80% DV01 Futures on 30-yr Bond	
	PV of Pre-Tax Book Profits	Profit Margin
Mean	$228,877	5%
Standard Deviation	$235,513	5%
Minimum	($67,910)	-1%
Maximum	$1,260,335	26%
Quartile 1	$59,654	1%
Median	$188,609	4%
Quartile 3	$281,658	6%
Maximum - Minimum	$1,328,245	27%
Quartile 3 - Quartile 1	$222,004	5%
Strategy 2:	100% DV01 Futures on 30-yr Bond	
	PV of Pre-Tax Book Profits	Profit Margin
Mean	$126,812	3%
Standard Deviation	$338,454	7%
Minimum	($323,497)	-7%
Maximum	$1,535,563	31%
Quartile 1	($131,408)	-3%
Median	$62,494	1%
Quartile 3	$229,482	5%
Maximum - Minimum	$1,859,060	38%
Quartile 3 - Quartile 1	$360,890	7%
Strategy 3:	120 % DV01 Futures on 30-yr Bond	
	PV of Pre-Tax Book Profits	Profit Margin
Mean	$24,746	1%
Standard Deviation	$443,295	9%
Minimum	($580,847)	-12%
Maximum	$1,810,791	37%
Quartile 1	($340,155)	-7%
Median	($45,747)	-1%
Quartile 3	$177,432	4%
Maximum - Minimum	$2,391,637	49%
Quartile 3 - Quartile 1	$517,586	11%

Statistical Measures of Profits over 100 scenarios for Dollar Partial Duration		
Strategy 1:	80% Dollar Partial Duration Hedge with Futures	
	PV of Pre-Tax Book Profits	Profit Margin
Mean	$221,246	5%
Standard Deviation	$219,323	4%
Minimum	($60,839)	-1%
Maximum	$1,193,277	24%
Quartile 1	$80,666	2%
Median	$184,001	4%
Quartile 3	$291,733	6%
Maximum - Minimum	$1,254,115	26%
Quartile 3 - Quartile 1	$211,067	4%
Strategy 2:	100% Dollar Partial Duration Hedge with Futures	
	PV of Pre-Tax Book Profits	Profit Margin
Mean	$117,273	2%
Standard Deviation	$316,232	6%
Minimum	($310,308)	-6%
Maximum	$1,451,740	30%
Quartile 1	($114,520)	-2%
Median	$82,864	2%
Quartile 3	$197,622	4%
Maximum - Minimum	$1,762,048	36%
Quartile 3 - Quartile 1	$312,142	6%
Strategy 3:	120% Dollar Partial Duration Hedge with Futures	
	PV of Pre-Tax Book Profits	Profit Margin
Mean	$13,300	0%
Standard Deviation	$415,501	8%
Minimum	($559,777)	-11%
Maximum	$1,710,203	35%
Quartile 1	($317,798)	-6%
Median	($21,724)	0%
Quartile 3	$139,199	3%
Maximum - Minimum	$2,269,980	46%
Quartile 3 - Quartile 1	$456,997	9%

As expected, the results observed in the LTC product are very similar to those of the whole-life product. Again, DV01 or Dollar Partial Duration hedging works very well in terms of reducing volatility in the market value of the portfolio, but far less so in terms of reducing the pretax statutory profits.

7. Swaps

Though bond futures were modeled, it would be equally effective if interest-rate swaps were employed. Swaps with different terms could offer the same protection against different time-to-maturity exposures. Instead of providing a hedge through maturity cash flows from futures, swaps hedge mainly through the change in the market value of the swap contract. There will also be some cash flows resulting from the difference in the fixed/floating leg coupon payments, but the impact of these cash flows is expected to be much less than the change in the market value.

We chose to model bond futures for the simplicity of modeling and analysis. In reality, swaps are perhaps more popular as hedge instruments due to the more liquid market in the long-term swaps.

8. Summary

Listed below are the key points we learned through this research project.

a. The profitability of both nonpar whole-life and long-term-care products can be highly dependent on the level of interest rates—even before consideration of dynamic policyholder behavior. Pricing with only one interest scenario might significantly understate the interest-rate sensitivity of the product.

b. While duration is a common measure of interest sensitivity of insurance products, it produces results that are difficult to interpret when renewal premiums are deducted from liability cash outflows.

c. For hedging purposes, it is really the dollar change in value relative to interest-rate changes that matters. Therefore, we propose to use DV01 as a measure of interest-rate sensitivity of the insurance product. DV01 also eliminates abnormal results that a duration measure might produce because of renewal premiums.

d. The DV01 hedge strategy works as long as the size of the change in the interest rate is relatively small and the shape of the yield curve is relatively unaltered.

e. Dollar Partial Duration is a measure of interest sensitivity assuming the shape of the yield curve changes. A Dollar Partial Duration hedge mitigates the interest-rate risk even when the shape of the yield curve alters.

f. Bonds, in theory, can be used as hedge instruments when a DV01 or Dollar Partial Duration hedge strategy is pursued. However, the potential short positions in bonds make it impractical for insurance companies.

g. Bond futures provide an alternative to using bonds to hedge. With the term of futures contracts corresponding to the review frequency, an insurance company can follow a true dynamic hedging strategy.

h. Futures on a 30-year bond offer significant 30-year exposure, which affects the liabilities more than other time-to-maturity exposures. Futures on 30-year bonds provide a more effective hedge than futures on one-year bonds.

i. A DV01 hedge with futures on 30-year bonds affords just as effective a hedge as a Dollar Partial Duration hedge with futures on two-year, five-year, 10-year, and 30-year bonds, provided that hedge positions are reviewed on a frequent basis.

j. DV01 and Dollar Partial Duration hedges do effectively hedge volatility in market value, but they overhedge volatility in statutory profits. A change in the reserve basis to a more "market value" basis, such as PBA or fair value option, would make the hedge more effective and attractive.

k. As both DV01 and Dollar Partial Duration are based on a one-basis-point shift in interest, neither strategy manages to eliminate the entire interest-rate risk in practice. More frequent hedge-position review, such as monthly or even weekly, could help mitigate this problem. The insurance company can also consider a second-order change in the value relative to the change in interest. However, an insurance company must strike a balance between the cost of hedging and the effectiveness of hedging. Hedging provides protection at the cost of profit.

REFERENCES

1. Reitano, Robert. *Multivariate Duration Analysis.* Transactions of the Society of Actuaries, XLIII, 1991.

Appendix 1 Survey Form

Interest Rate Hedging on Traditional Health and Life Products

Section 1: General Information

Q1. What percentage of your inforce portfolio are traditional health and life products?

	Reserve
Traditional Life	
Traditional Health	
Other (Int. Sensitive, etc.)	
Total	

Q2. What percentage of this portion of your inforce portfolio was priced using a single projection of interest rates? Does your company price these traditional products currently with a single rate or single set of rates? If not, what method is used?

Q3. What percentage of this portion of your inforce portfolio are recurring premium products? Do you consider the recurring premium in calculating liability durations?

	Premium Paying	Paid Up
Traditional Life		
Traditional Health		
Other		
Total		

Section 2: Details of Hedging Strategy

Q4. Are any of these products hedged directly using specific securities against interest movement?
(If yes, please complete this section. If no, please go to Section 3.)

Q5. What is the target to be hedged?
(e.g., asset liability duration matching, asset liability cash flow matching, providing minimum guarantees, minimizing VaR, etc.)

Q6. What hedging instruments have you used? Which of these is most commonly used?
(e.g., swaps/swaptions, futures/forwards, caps, etc.)

Q7. What is your measure of the effectiveness of hedging? What measures do you use to quantify your hedge effectiveness (effective duration as in $\Delta P/\Delta i$, etc.)? What is the main measure of success?

Q8. How often do you review the hedging strategy?

Q9. Does the hedging strategy cover all the interest-rate risk you intend to hedge? (If your answer is yes, please go to Q12. If your answer is no, please answer Q10 and Q11.)

Q10. Why have you decided not to hedge fully?

Q11. How do you manage the risk that is not hedged?

Q12. How do you think your hedging strategy might be improved?

Section 3: Others

Q13. Did your company consider the impact of interest-rate movements in designing the investment strategy to support these liabilities? If so, how? If not, were there specific reasons to go unhedged?

Q14. Do you monitor and manage unhedged positions on a regular basis?

Q15. How likely are you to pursue an interest-rate hedging strategy in the future?

Appendix 2 Premium Size and Pricing Assumptions of the Whole-life Product

1. Premium Size

Age	Premium per %o Face Amount
25	10
35	15
45	20
55	30
65	50

2. Commission Rate

Year	Commission
1	80%
2-10	10%
11+	25%

3. Expenses

	Premium Relate	Fixed Dollar
Acquisition	15%	200
Maintenance	2%	50

4. Mortality

1990-95 Basic ALB Select and Ultimate

5. Lapse

Year	Lapse
1	15%
2	8%
3+	5%

Appendix 3 Table 4.2.2.1:
DV01 Hedge Summary Report

	0	1	2	3	4	5	6	7	8	9	10
Book Value of Liability											
Other non-ledger liabilities	71,375	811,268	1,439,492	2,027,283	2,553,489	3,021,594	3,433,831	3,792,553	4,100,720	4,361,213	4,571,935
Other non-ledger assets	0	0	0	0	0	0	0	0	0	0	0
Target surplus balance	277,653	261,963	264,828	274,116	281,708	287,713	292,189	295,201	296,836	297,194	296,187
Book Value of Liability	349,028	1,073,231	1,704,320	2,301,399	2,835,196	3,309,307	3,726,020	4,087,754	4,397,556	4,658,408	4,868,122
Book Value of Assets											
Cost basis	349,028	1,073,231	1,704,320	2,301,399	2,835,197	3,309,307	3,726,020	4,087,754	4,397,557	4,658,408	4,868,122
Cost basis (Cash)	-2,437,396	-1,972,967	-1,568,876	-1,170,448	-807,725	-478,895	-183,417	79,356	310,695	511,911	678,763
Cost basis (30-year Bond)	2,786,424	3,046,198	3,273,196	3,471,847	3,642,922	3,788,202	3,909,437	4,008,399	4,086,861	4,146,496	4,189,358
Asset Purchase cash flow											
Purchase cash flow	349,028	724,204	631,089	597,079	533,798	474,110	416,713	361,734	309,802	260,851	209,714
Purchase cash flow (Cash)	-2,437,396	464,430	404,090	398,429	362,723	328,830	295,478	262,773	231,340	201,216	166,852
Purchase cash flow (30-year Bond)	2,786,424	259,774	226,998	198,651	171,075	145,280	121,235	98,961	78,463	59,635	42,862
Asset Notional Amount											
Notional Amount total	349,028	1,073,231	1,704,320	2,301,399	2,835,197	3,309,307	3,726,020	4,087,753	4,397,555	4,658,406	4,868,120
Notional Amount (Cash)	-2,437,396	-1,972,967	-1,568,876	-1,170,447	-807,724	-478,894	-183,416	79,356	310,696	511,912	678,763
Notional Amount (30-year Bond)	2,786,424	3,046,198	3,273,196	3,471,847	3,642,921	3,788,201	3,909,436	4,008,397	4,086,859	4,146,495	4,189,357
Liability DV01 *											
Liability DV01	4,306	4,641	4,914	5,134	5,301	5,418	5,488	5,515	5,499	5,446	5,356
Asset DV01											
Asset DV01	4,306	4,641	4,914	5,134	5,301	5,418	5,488	5,515	5,499	5,446	5,356

Appendix 4 Table 4.3.2.1:
Dollar Partial Duration Hedge Summary Report

	0	1	2	3	4	5	6	7	8	9	10
Book Value of Liability											
Other non-ledger liabilities	71,375	811,268	1,439,492	2,027,283	2,563,489	3,021,594	3,433,831	3,792,553	4,100,720	4,361,213	4,571,935
Other non-ledger assets	0	0	0	0	0	0	0	0	0	0	0
Target surplus balance	277,653	261,963	264,828	274,116	281,708	287,713	292,189	295,201	296,836	297,194	296,187
Book Value of Liability	349,028	1,073,231	1,704,320	2,301,399	2,835,196	3,309,307	3,726,020	4,087,754	4,397,556	4,658,408	4,868,122
Book Value of Assets											
Cost basis	349,028	1,073,231	1,704,320	2,301,399	2,835,196	3,309,307	3,726,020	4,087,753	4,397,557	4,658,408	4,868,122
Cost basis (Cash)	1,019,131	1,677,225	387,418	712,711	1,288,295	1,094,966	1,460,363	1,393,299	1,553,972	1,834,554	1,978,038
Cost basis (2-year Bond)	-2,032,667	-1,527,831	921,483	1,737,004	1,978,695	759,403	432,292	323,437	-140,701	-627,819	-1,384,980
Cost basis (5-year Bond)	-2,073,867	-2,660,458	-3,287,674	-3,875,154	-4,139,860	-2,164,134	-1,662,576	-917,788	-8,619	846,986	1,782,254
Cost basis (10-year Bond)	-370,353	-354,709	-383,061	-462,975	-603,545	-813,995	-1,059,487	-1,391,129	-1,814,642	-2,334,404	-2,582,330
Cost basis (30-year Bond)	3,806,783	3,939,003	4,066,154	4,189,814	4,311,612	4,433,067	4,555,429	4,679,936	4,807,548	4,939,093	5,075,142
Asset Purchase cash flow											
Purchase cash flow	349,028	724,203	-1,401,578	1,101,916	950,444	-279,401	488,463	-164,418	53,550	-11,645	-391,689
Purchase cash flow (Cash)	1,019,131	658,093	-1,289,807	325,293	575,583	-193,329	365,397	-67,063	160,672	280,582	143,484
Purchase cash flow (2-year Bond)	-2,032,667	504,836	416,647	1,320,357	658,338	101,064	331,228	-7,791	-132,910	-494,909	-890,072
Purchase cash flow (5-year Bond)	-2,073,867	-686,591	-627,216	-587,480	-264,706	-98,140	-85,033	117,572	321,688	590,900	837,128
Purchase cash flow (10-year Bond)	-370,353	15,644	-28,353	-79,914	-140,570	-210,450	-245,492	-331,642	-423,513	-519,762	-618,278
Purchase cash flow (30-year Bond)	3,806,783	132,221	127,151	123,659	121,798	121,455	122,363	124,507	127,612	131,545	136,049
Asset Notional Amount											
Notional Amount total	349,028	1,073,231	-328,347	2,326,000	2,756,843	1,511,517	2,557,864	2,224,503	2,457,315	2,556,475	2,228,856
Notional Amount (Cash)	1,019,131	1,677,225	387,418	712,711	1,288,295	1,094,966	1,460,363	1,393,299	1,553,971	1,834,554	1,978,038
Notional Amount (2-year Bond)	-2,032,667	-1,527,831	-1,111,184	1,761,604	1,900,342	1,035,481	1,264	-764,421	-1,343,446	-2,300,614	-3,421,938
Notional Amount (5-year Bond)	-2,073,867	-2,660,458	-3,287,674	-3,875,154	-4,139,860	-4,238,000	-2,399,704	-1,693,182	-746,117	417,848	1,550,298
Notional Amount (10-year Bond)	-370,353	-354,709	-383,061	-462,975	-603,545	-813,995	-1,059,487	-1,391,129	-1,814,642	-2,334,404	-2,952,683
Notional Amount (30-year Bond)	3,806,783	3,939,004	4,066,155	4,189,814	4,311,612	4,433,066	4,555,429	4,679,936	4,807,548	4,939,093	5,075,142
Liability 2-yr Dolla Partial Duration											
Liability 2-year Dolla Partial Duration	-382	-348	-318	-288	-259	-231	-195	-155	-116	-86	-64
Asset 2-yr Dolla Partial Duration											
Asset Dolla Partial Duration 2-year	-382	-348	-318	-288	-259	-231	-195	-155	-116	-86	-64
Liability 5-yr Dolla Partial Duration											
Liability 5-year Dolla Partial Duration	-907	-802	-691	-577	-461	-345	-249	-167	-96	-29	30
Asset 5-yr Dolla Partial Duration											
Asset Dolla Partial Duration 5-year	-907	-802	-691	-577	-461	-345	-249	-167	-96	-29	30
Liability 10-yr Dolla Partial Duration											
Liability 10-year Dolla Partial Duration	-289	89	421	709	954	1,158	1,330	1,469	1,579	1,661	1,719
Asset 10-yr Dolla Partial Duration											
Asset Dolla Partial Duration 10-year	-289	89	421	709	954	1,158	1,330	1,469	1,579	1,661	1,719
Liability 30-yr Dolla Partial Duration											
Liability 30-year Dolla Partial Duration	5,879	5,697	5,498	5,285	5,062	4,832	4,599	4,363	4,128	3,896	3,667
Asset 30-yr Dolla Partial Duration											
Asset Dolla Partial Duration 30-year	5,879	5,697	5,498	5,285	5,062	4,832	4,599	4,363	4,128	3,896	3,667

Appendix 5 Table 4.4.1:
DV01 Hedge Summary Report December 2005

	0	1	2	3	4	5	6	7	8	9	10
Book Value of Liability											
Other non-ledger liabilities	71,375	811,268	1,439,492	2,027,283	2,553,489	3,021,594	3,433,831	3,792,553	4,100,720	4,361,213	4,571,935
Other non-ledger assets	0	0	0	0	0	0	0	0	0	0	0
Target surplus balance	277,653	261,963	264,828	274,116	281,708	287,713	292,189	295,201	296,836	297,194	296,187
Book Value of Liability	349,028	1,073,231	1,704,320	2,301,399	2,835,196	3,309,307	3,726,020	4,087,754	4,397,556	4,658,408	4,868,122
Book Value of Assets											
Cost basis	349,028	1,073,231	1,704,320	2,301,399	2,835,197	3,309,307	3,726,020	4,087,754	4,397,557	4,658,408	4,868,122
Cost basis (Cash)	-2,838,304	-2,363,687	-1,949,537	-1,541,099	-1,168,585	-830,241	-525,591	-254,053	-14,419	194,544	368,534
Cost basis (30-year Bond)	3,187,331	3,436,918	3,653,856	3,842,497	4,003,781	4,139,547	4,251,611	4,341,807	4,411,976	4,463,864	4,499,588
Asset Purchase cash flow											
Purchase cash flow	349,028	724,203	631,089	597,079	533,797	474,110	416,713	361,734	309,803	260,852	209,715
Purchase cash flow (Cash)	-2,838,304	474,617	414,150	408,438	372,514	338,344	304,649	271,538	239,635	208,963	173,990
Purchase cash flow (30-year Bond)	3,187,331	249,587	216,938	188,641	161,283	135,766	112,064	90,196	70,168	51,889	35,724
Asset Notional Amount											
Notional Amount total	349,022	1,073,226	1,704,314	2,301,393	2,835,190	3,309,300	3,726,013	4,087,746	4,397,549	4,658,402	4,868,116
Notional Amount (Cash)	-2,838,304	-2,363,686	-1,949,536	-1,541,098	-1,168,584	-830,240	-525,591	-254,053	-14,418	194,545	368,535
Notional Amount (30-year Bond)	3,187,326	3,436,913	3,653,851	3,842,493	4,003,776	4,139,542	4,251,605	4,341,801	4,411,969	4,463,858	4,499,582
Liability DV01											
*											
Liability DV01	5,290	5,613	5,871	6,070	6,213	6,304	6,345	6,340	6,292	6,204	6,080
Asset DV01											
Asset DV01	5,290	5,613	5,871	6,070	6,213	6,304	6,345	6,340	6,292	6,204	6,080

Appendix 6 Table 4.4.2:
Dollar Partial Duration Hedge Summary Report December 2005

	0	1	2	3	4	5	6	7	8	9	10
Book Value of Liability											
Other non-ledger liabilities	71,375	811,268	1,439,492	2,027,283	2,553,489	3,021,594	3,433,831	3,792,553	4,100,720	4,361,213	4,571,935
Other non-ledger assets	0	0	0	0	0	0	0	0	0	0	0
Target surplus balance	277,653	261,963	264,828	274,116	281,708	287,713	292,189	295,201	296,836	297,194	296,187
Book Value of Liability	349,028	1,073,231	1,704,320	2,301,399	2,835,196	3,309,307	3,726,020	4,087,754	4,397,556	4,658,408	4,868,122
Book Value of Assets											
Cost basis	349,028	1,073,231	1,704,320	2,301,399	2,835,196	3,309,307	3,726,020	4,087,754	4,397,557	4,658,408	4,868,122
Cost basis (Cash)	631,048	1,311,893	-16,788	284,240	827,796	600,603	979,780	987,105	1,184,586	1,446,036	1,651,370
Cost basis (2-year Bond)	-2,080,218	-1,564,182	979,349	1,896,289	2,267,672	1,168,029	814,251	574,568	-29,553	-627,393	-1,418,869
Cost basis (5-year Bond)	-2,097,227	-2,744,284	-3,452,096	-4,139,567	-4,520,915	-2,644,283	-2,117,461	-1,294,654	-250,746	776,815	1,927,054
Cost basis (10-year Bond)	-180,738	-143,079	-150,529	-211,773	-337,311	-538,114	-799,288	-1,155,367	-1,612,750	-2,176,251	-2,667,517
Cost basis (30-year Bond)	4,076,163	4,212,883	4,344,382	4,472,211	4,597,955	4,723,070	4,848,737	4,976,101	5,106,019	5,239,200	5,376,081
Asset Purchase cash flow											
Purchase cash flow	349,028	724,203	-1,449,130	1,113,116	997,110	-190,141	604,352	-12,744	103,248	-26,845	-314,824
Purchase cash flow (Cash)	631,048	680,844	-1,328,680	301,027	543,557	-227,193	379,177	7,325	197,481	261,450	205,334
Purchase cash flow (2-year Bond)	-2,080,218	516,036	463,312	1,432,976	834,696	333,334	480,917	93,651	-123,204	-504,189	-914,679
Purchase cash flow (5-year Bond)	-2,097,227	-647,057	-707,811	-687,472	-381,348	-220,595	-120,235	114,996	356,436	646,213	929,644
Purchase cash flow (10-year Bond)	-180,738	37,659	-7,450	-61,245	-125,538	-200,802	-261,174	-356,080	-457,383	-563,501	-672,004
Purchase cash flow (30-year Bond)	4,076,163	136,720	131,499	127,829	125,744	125,116	125,667	127,364	129,918	133,181	136,881
Asset Notional Amount											
Notional Amount total	349,029	1,073,233	-375,896	2,345,230	2,821,353	1,614,797	2,681,683	2,386,885	2,530,622	2,610,339	2,447,625
Notional Amount (Cash)	631,048	1,311,893	-16,787	284,240	827,796	600,603	979,780	987,105	1,184,586	1,446,036	1,651,370
Notional Amount (2-year Bond)	-2,080,216	-1,564,180	-1,100,868	1,940,116	2,253,823	1,570,741	567,957	-280,368	-1,082,452	-2,168,895	-3,360,295
Notional Amount (5-year Bond)	-2,097,222	-2,744,277	-3,452,087	-4,139,557	-4,520,904	-4,741,498	-2,915,496	-2,140,580	-1,064,776	270,253	1,628,729
Notional Amount (10-year Bond)	-180,738	-143,078	-150,528	-211,773	-337,311	-538,113	-799,286	-1,155,365	-1,612,747	-2,176,247	-2,848,250
Notional Amount (30-year Bond)	4,076,157	4,212,876	4,344,375	4,472,204	4,597,948	4,723,063	4,848,730	4,976,094	5,106,012	5,239,193	5,376,074
Liability 2-yr Dollar Partial Duration											
Liability 2-year Dollar Partial Duration	-395	-359	-327	-295	-264	-235	-197	-155	-115	-83	-60
Asset 2-yr Dollar Partial Duration											
Asset Dollar Partial Duration 2-year	-395	-359	-327	-295	-264	-235	-197	-155	-115	-83	-60
Liability 5-yr Dollar Partial Duration											
Liability 5-year Dollar Partial Duration	-936	-823	-705	-584	-461	-340	-239	-153	-78	-8	54
Asset 5-yr Dollar Partial Duration											
Asset Dollar Partial Duration 5-year	-936	-823	-705	-584	-461	-340	-239	-153	-78	-8	54
Liability 10-yr Dollar Partial Duration											
Liability 10-year Dollar Partial Duration	-145	254	604	907	1,163	1,374	1,551	1,693	1,804	1,884	1,939
Asset 10-yr Dollar Partial Duration											
Asset Dollar Partial Duration 10-year	-145	254	604	907	1,163	1,374	1,551	1,693	1,804	1,884	1,939
Liability 30-yr Dollar Partial Duration											
Liability 30-year Dollar Partial Duration	6,760	6,535	6,292	6,036	5,771	5,499	5,225	4,950	4,676	4,407	4,143
Asset 30-yr Dollar Partial Duration											
Asset Dollar Partial Duration 30-year	6,760	6,535	6,292	6,036	5,771	5,499	5,225	4,950	4,676	4,407	4,143

Appendix 7 Portfolio Mix of the Long-term-care Product

1. Premium Size

$18.43 per unit

2. Commission Rate

Year	Commission
1	93.0%
2+	7.3%

3. Expenses

Year	Premium Related	Fixed Dollar	Claim Related
Acquisition	15%	200	0
Maintenance	0%	60	0
Claim Handling	0%	0	6%

4. Mortality and Morbidity

Mortality: 1983 Table A
Morbidity: Based on some artificial pricing assumption

5. Lapse

Year	Lapse
1	10%
2	6%
3	4%
4	3.5%
5	3.2%
6	2.5%
7	2.0%
8	2.0%
9	2.0%
10+	1.5%