MR. EDUARDO MELINSKY (*)
REPUBLICA ARGENTINA

(*) Doctor on Economics - Actuarial Sciencies

Ordinary Adjunct Professor of Actuarial Mathematics
University of Buenos Aires

Professor of Mathematics of Finance
National University of Comahue

Actuary-Adviser of the General Manager
Bank of the State of Buenos Aires

MR. EDUARDO MELINSKY
Pumacahua N° 1230
1406 Buenos Aires
Argentina
FAX 54.1. 322-8977

ISSUE AND ANALYSIS OF BONDS IN INFLATIONARY CONDITIONS

Introduction

The object of this paper is to present several aspects on the Valuation of Bonds, which are not usual in general textbooks, but very important to consider in countries with inflation.

My experience is related to my country where for many years we have had high and variable rates of inflation and also many stabilization plans have been applied.

Inflation has many consequencies and it is necessary to consider its effects over usual financial instruments as well as particular aspects of specific securities developped in order to deal with inflationary conditions. We must take into account the real and nominal rates of interest and to remove the money veil of the cash flows. Also, changes in the expected level of the rates of inflation must be analysed.

Firstly we are to consider the conditions of issue and afterwords the analysis of Bonds, due to different ways of determining the monetary values of coupons (cash flow).

1. The Issue of Bonds

1.1. General Elements and Formulae

We consider the issue of bonds where each one is to be redeemed in periodic installments with bond interest being paid only on the outstanding face value.

At the date of issue we have the following general equation of equivalence between the basic elements:

$$VN(0) = \sum_{p=1}^{p=n} \frac{c(p)}{1 + i(o,p)} \cdot$$

where,

VN(0): the initial face value
n: time to maturity or number of equal periods of time
c(p): amount of each coupon, for p = 1, 2, ..., n, being paid at the end of period "p"

$$1 + i(o,p) = \prod_{t=1}^{t=p} [1 + i(t-1,t)]$$

and i(t-1,t) is the coupon rate of interest for the period "p"

Also,

$$c(p) = I(p-1,p) + m(p)$$
$$VN(p) = VN(p-1) - m(p)$$
$$I(p-1,p) = i(p-1,p) \cdot VN(p-1)$$

m(p): amortization of face value
VN(p): outstanding face value
I(p-1,p): interest paid in coupon "P"

To simplify the analysis we consider non callable bonds, no default risk and no tax status at all.

1.2. Monetary Values

In order to determine monetary values we must define: Currency, Interest Rate Clause, Index Clause and the Schedule of Amortization of the face value. So we have to choose between the following specific alternatives:

a) on Currency:
- Local
- Foreign (generally: U.S. dollars, Swiss Francs, D-marks or Sterling Pounds)

b) on Interest Rate:
- Fixed
- Floating (variable)

c) Index - Clause:
- nominal values without indexation
- on a Price Index
- on a Financial Index
- on Foreign Currency Exchange Rate

Then it is usual to consider:

A. Bonds using local currency:
- With floating rate of interest
- with a Price Index Clause and fixed rate of interest
- with a Financial Index Clause and fixed rate of interest
- with Foreign Currency Index Clause and fixed or floating rate of interest

B. Bonds using foreign currency:
- with a fixed rate of interest
- with a floating rate of interest

A financial index clause means that indexation employs an index being made using rates of interest of the money market in ordinary local currency (nominal values), so periodically (e.g. daily) the index is computed using the recursive formula:

$$Y(t+s) = Y(t) \cdot \left[1 + i(t,t+s)\right]$$

where

Y(t): is the index at date "t"
i(t,t+s): is the rate of interest for the period between "t" and "t+s"

It is important to note that in order to perform any analysis when this clause is applied we have to estimate the components of the rate of interest such as the level of inflation and real rates of interest of the money market.

2. The Analysis of Bonds

2.1. General Topics

The basic elements to determine the price of a bond at any time after the date of issue are:

a) issue conditions: the length of time before the bond matures, the coupon rates of interest, amortization schedule of the face value (outstanding), index clause (retrospectively)

b) the expected evolution of the index clause

c) the terms structure of real interest rate to be used in the valuation process ("iv")

d) the expected rate of inflation ("we")

These element give information not only about a reasonable price but also about the structure of the schedule of amortization of the outstanding face value in real terms (also about the Price), and the yield to maturity in nominal and real terms.

2.2. Bonds in local currency without indexation

Considering the basic elements, the formula for the price of one bond at time "p+f" (f<1) from date of issue is:

$$P(p+f) = \sum_{t=1}^{n-p} \frac{c(p+t)}{[1 + iv(p+f,p+t)] \cdot [1 + we(p+f,p+t)]}$$

Note that the volatility is highly dependent on the values of the expected rates of inflation and on the nature of the interest rate clause, because:

a) The ouststanding face value is a decreasing function of the inflation rate.

b) Interest payments include amortization of face value in real terms, in the former periods as an increasing function of inflation rate.

c) The level of the expected rate of inflation modifies in real terms the structure of the coupon payments (cash flow).

If we consider a fixed coupon rate of interest the price of a bond is necessarily a decreasing function of the expected rate of inflation.

If we consider a bond with a floating coupon rate of interest, further analysis is to be made, and if this is a constant rate of interest in real terms and the rates of interest used in the valuation are higher than that one, then the price of the bond will be an increasing function of the expected inflation rate.

The following examples ilustrate over those considerations.

EXAMPLE 2.2.1.

Face value-VN(0)-: 1000
Coupon rate of interest-i(p-1,p)-: 11,10%
Number of periods -n-: 4
Amortization Schedule and Coupons:

p	I(p-1,p)	m(p)	c(p)	VN(p)
0				1000,00
1	111,00		111,00	1000,00
2	111,00		111,00	1000,00
3	111,00		111,00	1000,00
4	111,00	1000,00	1111,00	0,00

Expected rate of inflation-we(p-1,p)-: 10,00%
Real rate of interest of the Bond-ir(p-1,p)-: 1,0000%

Amortization Schedule in Real Terms:

p	I(0,p)	Ir(p-1,p)	mr(p)	cr(p)	VNr(p)
0	1				1000,00
1	1,1	10,00	90,91	100,91	909,09
2	1,21	9,09	82,64	91,74	826,45
3	1,331	8,26	75,13	83,40	751,31
4	1,4641	7,51	751,31	758,83	0,00

Valuation of the bond if "iv(p-1,p)" = 0,50%
Price at the issue date-P(0)-: 1017,23
Amortization Schedule of the Price in Real Terms:

p	Ir'(p-1,p)	mr'(p)	cr(p)	Pr(p)	
0				1017,23	100,00%
1	5,09	95,82	100,91	921,41	90,58%
2	4,61	87,13	91,74	834,28	82,01%
3	4,17	79,22	83,40	755,05	74,23%
4	3,78	755,05	758,83	0,00	

EXAMPLE 2.2.2.

Face value-VN(0)-: 1000
Coupon rate of interest-i(p-1,p)-: 11,10%
Number of periods -n-: 4
Amortization Schedule and Coupons:

p	I(p-1,p)	m(p)	c(p)	VN(p)
0				1000,00
1	111,00		111,00	1000,00
2	111,00		111,00	1000,00
3	111,00		111,00	1000,00
4	111,00	1000,00	1111,00	0,00

Expected rate of inflation-we(p-1,p)-: 10,00%
Real rate of interest of the Bond-ir(p-1,p)-: 1,0000%

Amortization Schedule in Real Terms:

p	1+w(o,p)	Ir(p-1,p)	mr(p)	cr(p)	VNr(p)
0	1				1000,00
1	1,1	10,00	90,91	100,91	909,09
2	1,21	9,09	82,64	91,74	826,45
3	1,331	8,26	75,13	83,40	751,31
4	1,4641	7,51	751,31	758,83	0,00

Valuation of the bond if "iv(p-1,p)" = 2,00%
Price at the issue date-P(0)-: 966,73
Amortization Schedule of the Price in Real Terms:

p	Ir'(p-1,p)	mr'(p)	cr(p)	Pr(p)	
0				966,73	100,00%
1	19,33	81,57	100,91	885,15	91,56%
2	17,70	74,03	91,74	811,12	83,90%
3	16,22	67,17	83,40	743,95	76,96%
4	14,88	743,95	758,83	0,00	

EXAMPLE 2.2.3.

Face value-VN(0)-: 1000
Coupon rate of interest-i(p-1,p)-: 11,10%
Number of periods -n-: 4
Amortization Schedule and Coupons:

p	I(p-1,p)	m(p)	c(p)	VN(p)
0				1000,00
1	111,00		111,00	1000,00
2	111,00		111,00	1000,00
3	111,00		111,00	1000,00
4	111,00	1000,00	1111,00	0,00

Expected rate of inflation-we(p-1,p)-: 12,00%
Real rate of interest of the Bond-ir(p-1,p)-: -0,8036%

Amortization Schedule in Real Terms:

p	1+w(o,p)	Ir(p-1,p)	mr(p)	cr(p)	VNr(p)
0	1				1000,00
1	1,12	-8,04	107,14	99,11	892,86
2	1,2544	-7,17	95,66	88,49	797,19
3	1,404928	-6,41	85,41	79,01	711,78
4	1,5735193	-5,72	711,78	706,06	0,00

Valuation of the bond if "iv(p-1,p)" = 2,00%
Price at the issue date-P(0)-: 908,96
Amortization Schedule of the Price in Real Terms:

p	Ir'(p-1,p)	mr'(p)	cr(p)	Pr(p)	
0				908,96	100,00%
1	18,18	80,93	99,11	828,03	91,10%
2	16,56	71,93	88,49	756,10	83,18%
3	15,12	63,89	79,01	692,22	76,15%
4	13,84	692,22	706,06	0,00	

EXAMPLE 2.2.4.

Face value-VN(0)-: 1000
Coupon rate of interest-i(p-1,p)-: 21,20%
Number of periods -n-: 4
Amortization Schedule and Coupons:

p	I(p-1,p)	m(p)	c(p)	VN(p)
0				1000,00
1	212,00		212,00	1000,00
2	212,00		212,00	1000,00
3	212,00		212,00	1000,00
4	212,00	1000,00	1212,00	0,00

Expected rate of inflation-we(p-1,p)-: 20,00%
Real rate of interest of the Bond-ir(p-1,p)-: 1,0000%

Amortization Schedule in Real Terms:

p	1+w(o,p)	Ir(p-1,p)	mr(p)	cr(p)	VNr(p)
0	1				1000,00
1	1,2	10,00	166,67	176,67	833,33
2	1,44	8,33	138,89	147,22	694,44
3	1,728	6,94	115,74	122,69	578,70
4	2,0736	5,79	578,70	584,49	0,00

Valuation of the bond if "iv(p-1,p)" = 3,00%
Price at the issue date-P(0)-: 970,30
Amortization Schedule of the Price in Real Terms:

p	Ir'(p-1,p)	mr'(p)	cr(p)	Pr(p)	
0				970,30	100,00%
1	19,41	157,26	176,67	813,04	83,79%
2	16,26	130,96	147,22	682,07	70,30%
3	13,64	109,04	122,69	573,03	59,06%
4	11,46	573,03	584,49	0,00	

EXAMPLE 2.2.5.

Face value-VN(0)-: 1000
Coupon rate of interest-i(p-1,p)-: 21,20%
Number of periods -n-: 4
Amortization Schedule and Coupons:

p	I(p-1,p)	m(p)	c(p)	VN(p)
0				1000,00
1	212,00		212,00	1000,00
2	212,00		212,00	1000,00
3	212,00		212,00	1000,00
4	212,00	1000,00	1212,00	0,00

Expected rate of inflation-we(p-1,p)-: 22,18%
Real rate of interest of the Bond-ir(p-1,p)-: -0,8036%

Amortization Schedule in Real Terms:

p	1+w(o,p)	Ir(p-1,p)	mr(p)	cr(p)	VNr(p)
0	1				1000,00
1	1,2218181	-8,04	181,55	173,51	818,45
2	1,4928396	-6,58	148,59	142,01	669,86
3	1,8239786	-5,38	121,61	116,23	548,25
4	2,2285702	-4,41	548,25	543,85	0,00

Valuation of the bond if "iv(p-1,p)" = 2,00%
Price at the issue date-P(0)-: 918,56
Amortization Schedule of the Price in Real Terms:

p	Ir'(p-1,p)	mr'(p)	cr(p)	Pr(p)	
0				918,56	100,00%
1	18,37	155,14	173,51	763,42	83,11%
2	15,27	126,74	142,01	636,68	69,31%
3	12,73	103,50	116,23	533,18	58,05%
4	10,66	533,18	543,85	0,00	

2.3. Bonds in Foreign Currency

In this case is necessary to consider the effects of the Government Monetary Policy, such as Control of rates of exchange for foreign currency and Financing the Fiscal Deficit by demanding deposits in the money market offering high level of interest rates in local currency in real terms.

Also the main market for the bonds considered is the country of issue, then to determine prices we have to take into account the above situation and use a term structure of interest rates for the foreign currency different from the which one of the international markets, because there is an opportunity cost with the local currency, and this generally results in a dawnward sloping forward rate structure and in this case the yield to maturity of the bonds have higher values than those for similar bonds being issued and traded in other markets.

Experience shows for Argentina, estimated yields to maturity of more or less three times the rate for eurodollar deposits in the London Interbank Market, during 1988. The following example shows this situation.

EXAMPLE 2.3.1.
BONDS IN FOREIGN CURRENCY
EXTERNAL BONDS OF THE ARGENTINE REPUBLIC - 1980

Currency U.S. dollars
Valuation Date: March 17th, 1988
Market price in u$s-%: 82,00%
Annual interest rate next coupon: 7,5625%
Expected future interest rate: 7,0000%

Amortization Schedule for an outstanding face value of: 375

Date	Period p	Interest I(p-1,p)	Amortiz. m(p)	Coupon c(p)	Outstanding.F.V. VN(p)	%
17,0388					375,00	100,00%
27,0588	15	14,18		14,18	375,00	100,00%
27,1188	16	13,13	125,00	138,13	250,00	66,67%
27,0589	17	8,75		8,75	250,00	66,67%
27,1189	18	8,75	125,00	133,75	125,00	33,33%
27,0590	19	4,38		4,38	125,00	33,33%
27,1190	20	4,38	125,00	129,38	0,00	0,00%

Price in U$S for 375 is : 307,50

Expected annual yield to maturity: 22,68%
Amortization Schedule of the Price:

Date	Period p	Interest I(p-1,p)	Amortiz. m(p)	Coupon c(p)	Outstanding Price P(p)	%
17,0388					307,50	100,00%
27,0588	15	13,16	1,02	14,18	306,48	99,67%
27,1188	16	34,75	103,37	138,13	203,11	66,05%
27,0589	17	23,03	-14,28	8,75	217,39	70,70%
27,1189	18	24,65	109,10	133,75	108,29	35,22%
27,0590	19	12,28	-7,91	4,38	116,20	37,79%
27,1190	20	13,18	116,20	129,38	0,00	0,00%

2.4. Bonds in Local Currency with Index Clause

In this case monetary values of the bonds are linked to the evolution of a chosen Index and for practical purposes it is used a fixed time lag.

When the rates of inflation are variable or stabilization plans are put into practice, specific problems arise in determining prices or representative yields to maturity.

Time lag is necessary when dealing with any Index, but its lengh depends on the nature of it. When using general Prices Indexes it is usual to consider a two months time lag, but with daily financial indexes or foreign currency exchange rates only five days is enough.

When a stabilization plan is applied inflation stops, at least for several months, and time lag used for the application of a Price Index causes:

a) The bond will charge indexation according to past rates during a period with smaller rates of inflation.

b) The bond may register quotations over its parity values (according to issue conditions) and it would be obtained negative yields to maturity.

Under these circumstances it is necessary to estimate the acquired nominal value of future coupons due to the immediate past inflation experienced before the stabilization plan and compute yield to maturity, real or nominal, according to the future expected inflation.

The following examples illustrate about the general analysis of this kind of bonds.

EXAMPLE 2.4.1
BONDS WITH INDEX CLAUSE

Face value-VN(0)-: 1000
Coupon rate of interest-i(p-1,p)-: 4,00%
Number of periods -n-: 4
Amortization Schedule and Coupons real terms:

p	Ir(p-1,p)	mr(p)	cr(p)	VNr(p)
0				1000,00
1	40,00		40,00	1000,00
2	40,00		40,00	1000,00
3	40,00		40,00	1000,00
4	40,00	1000,00	1040,00	0,00

Expected rate of inflation-we(p-1,p)-: 10,00%

Amortization Schedule in Nominal Terms:

p	1+w(o,p)	I(p-1,p)	m(p)	c(p)	VN(p)
0	1				1000,00
1	1,1	44,00	0,00	44,00	1100,00
2	1,21	48,40	0,00	48,40	1210,00
3	1,331	53,24	0,00	53,24	1331,00
4	1,4641	58,56	1464,10	1522,66	0,00

Valuation of the bond if "iv(p-1,p)" = 6,00%
Price at the issue date-P(0)-: 930,70
Amortization Schedule of the Price in Real Terms:

p	Ir'(p-1,p)	mr'(p)	cr(p)	Pr(p)	
0				930,70	100,00%
1	55,84	-15,84	40,00	946,54	101,70%
2	56,79	-16,79	40,00	963,33	103,51%
3	57,80	-17,80	40,00	981,13	105,42%
4	58,87	981,13	1040,00	0,00	

EXAMPLE 2.4.2.
BONDS WITH INDEX CLAUSE

Face value-VN(0)-: 1000
Coupon rate of interest-i(p-1,p)-: 4,00%
Number of periods -n-: 4
Amortization Schedule and Coupons real terms:

p	Ir(p-1,p)	mr(p)	cr(p)	VNr(p)
0				1000,00
1	40,00		40,00	1000,00
2	40,00	500,00	540,00	500,00
3	20,00		20,00	500,00
4	20,00	500,00	520,00	0,00

Expected rate of inflation-we(p-1,p)-: 10,00%

Amortization Schedule in Nominal Terms:

p	1+w(o,p)	I(p-1,p)	m(p)	c(p)	VN(p)
0	1				1000,00
1	1,1	44,00	0,00	44,00	1100,00
2	1,21	48,40	605,00	653,40	605,00
3	1,331	26,62	0,00	26,62	665,50
4	1,4641	29,28	732,05	761,33	0,00

Valuation of the bond if "iv(p-1,p)" = 6,00%
Price at the issue date-P(0)-: 947,02
Amortization Schedule of the Price in Real Terms:

p	Ir'(p-1,p)	mr'(p)	cr(p)	Pr(p)	
0				947,02	100,00%
1	56,82	-16,82	40,00	963,84	101,78%
2	57,83	482,17	540,00	481,67	50,86%
3	28,90	-8,90	20,00	490,57	51,80%
4	29,43	490,57	520,00	0,00	

3. Final Remarks

We have seen several specific aspects of the Valuation of Bonds in inflationary conditions which are necessary to plan any financial strategy.

It is important to remark that the analysis of Bonds must consider the economic background, the consequencies of the monetary policy and the retrospective and prospective evolution of the particular index of each bond.