Using Dynamic Reliability in Estimating Mortality at Advanced Ages

Fanny L.F. Lin, Professor Ph.D.

Graduate Institute of Statistics and Actuarial Science
Feng Chia University
P.O. Box 25-181
Taichung 407, Taiwan

Tel: +886-4-24517250 ext. 4013, 4012
Fax: +886-4-24507092
E-mail: lflin@fcu.edu.tw

Abstract

Traditionally, the Gompertz's mortality law has been used in studies that show mortality rates continue to increase exponentially with age. The ultimate mortality rate has no maximum limit. In the field of engineering, the reliability theory has been used to measure the hazard-rate function that is dependent on system reliability. Usually, the hazard rate (H) and the reliability (R) have a strong negative coefficient of correlation. If reliability decreases with increasing hazard rate, this type of hazard rate can be expressed as a power function of failure probability, $1-R$. Many satisfactory results were found in quality control research in industrial engineering. In this research, this concept is applied to human mortality rates. The reliability $R(x)$ is the probability a newborn will attain age x. Assuming the model between $H(x)$ and $R(x)$ is $H(x) = B + C(1 - R(x)^A)^D$, where A represents the survival decaying memory characteristics, B/C is the initial mortality strength, C is the strength of mortality, and D is the survival memory characteristics.

Eight Taiwan Complete Life Tables from 1926 to 1991 were used as the data source. Mortality rates level off at a constant $B+C$ for very high ages in the proposed model but do not follow Gompertz's mortality law to the infinite. In the simulation study, the mean life will be increased as B decreases, when the pair of (A, B/C, D) is fixed. The age x is normalized by dividing the mean life. As a result, different survival functions for variants B, under a fixed pair of (A, B/C, D), will follow the same curve after age normalization. This concept was applied to eight Taiwan Complete Life Tables to obtain the different estimated values of parameters A, B, C, and D. To prolong the human life, the mean life is increased gradually. The pairs of (A, B/C, D) of each table will be observed to check whether the survival functions are the same after age normalization. The mean life is the area under the reliability function. For investigating the pattern of the mean life of each table, the average reliability changing-rates pattern in ages will be observed. The mean life and the corresponding age,

where the maximum value of average reliability changing rate occurred, are highly correlated. Sex comparisons will be made. The results of the proposed model will be compared to Gompertz's law by using Taiwan Complete Life Tables.

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1. INTRODUCTION

1.1 Background

In recent decades, the increasing age for average mortality is viewed as a positive change both for individuals and society but have forced consideration of the public cost for spending on old-age support. How to predict the age for ageing population, will be very important in the coming years. Traditionally, the Gompertz mortality law has been used to show that mortality rates continue to increase exponentially with age. The Gompertz curve has played an influential role in the development of the pattern for the study of mortality. Wilkins (1981) pointed out that the validity of the Gompertz curve is questionable, particularly when studying high ages. Extensive studies on the subject include the following: Vaupel and Yashin, 1985; Yashin et al. 1985; Vaupel et al. 1988; Lee and Carter 1992; Himes et al.1994; Manton and Stallard 1994; Herskind et al. 1996; Wachter and Finch 1997. A more flexible model uses the parametric model to understand the population mortality structure (such as in the work of Tenebein and Vanderhoff 1980; Heligman and Pollard 1980; Wetterstrand 1981; Siler 1983; Carriere 1992). Theses studies provide a good fit for ages before 85, caution is advisable when predicting the mortality for higher ages. A comprehensive review was produced by Tuljapurkar and Boe (1998). Using various mortality models we attempt accurate mortality projection. A good reference paper was proposed by Willets (1999) in which Willets studies international mortality rates and discusses a cohort basis study for estimating future age of mortality improvements.

In the field of engineering, the reliability theory has been used to measure the hazard rate function (H) that is dependent on the system reliability (R). Many satisfactory results were found in quality control in industrial engineering (Wang el al., 1993, 1996, 1997a, 1997b). The bathtub curve is usually adopted to represent the general trend of the hazard rate function. However, many studies have concentrated on the study of the time-dependent relation of *H*. These studies focus on depicting the geometric shape of the bathtub curve with less discussion of the physical meanings. Usually, the hazard rate and reliability have a strong negative coefficient, so using reliability as the dependent variable in non-linear regression is most suitable. The dynamic behavior of system reliability is affected by maintenance activities or by dealing with the improvement in the revision of system design. A system adjusts its performance continuously according to the capability with regard to the following: adaptation between mated subsystems and system with surroundings and resistance to the cumulative damage. The former dominates behavior in the infant mortality stage, the latter dominates in the wear-out stage. Applying these concepts to human life, the health and genetic situations subjugate performance in infant mortality, and the illness and accident events accumulate opportunities for death to increase the death probability. Medicare systems maintain and improve human life scales. However, the human scale is hard to evaluate as a study is not complete until all study participants are deceased. Hence, it is hard to obtain different reliability curves from various methods in improving human life in order to observe the performance of the function *H*. Nowadays there are eight Complete Life Tables of Taiwan constructed between 1926 and 1991. Eight reliability curves can be observed from these tables according to the time factor. According to the yearly changed rate of the reliabilities, Lin (2001) showed that the mean life of newborns is highly correlated to age with respect to the yearly maximum reliability change (male: r=0.973; female: r=0.979; r=coefficient of correlation). In this article, the relationships between reliability curves and hazard rate functions through the time factor will be discussed.

2. ANALYTICAL MODEL

Ageing is the symptom of cumulative damage to the human system, which emphasizes the memory of failures when time passes by, thus it depends on the failure probability, $1-R$. In this section we present a model of hazard rates that is a function of failure probabilities.

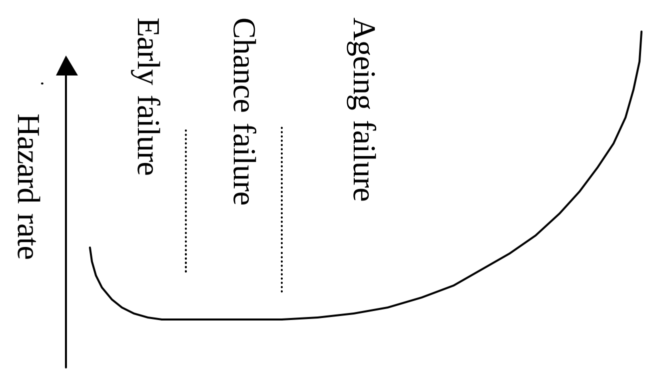
Assume the random variable X is the survival age for the newborns at age zero. Let $f(x)$ and $F(x)$ be the probability density and distribution of X respectively. The survival function, $S(x)$, that is the probability of the newborn who will survive at the time x. *S(x)=Pr(X>x)=1-F(x)=* ${}_x p_0$, $x \geq 0$. Then the hazard rate function $H(x)$ and the survival function $R(x)$ can be shown as following:

$$H(x) = \frac{-S^{'}(x)}{S(x)} = -\frac{d \ln S(x)}{dx} \tag{2.1}$$

In actuarial science and demography, *H(x)* is called the force of mortality, and *S(x)* is called the survival probability function. In reliability theory, the survival probabilities of systems is called the reliabilities *R(x)*, and *H(x)* is called the failure rate or hazard rate or the hazard rate function.

2.1 The Model

There are three patterns of the hazard rate function shown in *Figure 2.1*. (1) The early failure: which is the newborn age with a high hazard rate that decreased quickly until it leveled off. (2) The chance failure: death occurrence is unpredictable during this period. Death can be caused by anything at any time for this group. (3) Ageing failure (or wear-out failure): when people are older the hazard rates will be accumulated without avoidance in this period. There is a big difference in the bathtub curve between human and mechanical systems. Since human life is much longer than machine life, the ageing failure period is much longer than just the bathtub curve.



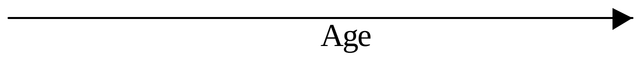


Figure 2.1 Hazard rate bathtub curve for human

In the ageing period, cumulative maturity emphasizes the memory of failures when people are older, so it depends on the failure probability, *1-R(x).* Usually, the hazard rate, $H(x)$, and the reliability, $R(x)$, have a strong negative coefficient of correlation. If the reliability decreases with increasing hazard rate, this type of hazard rate can be expressed as a power function of failure probability, $1-R(x)$. In this article, we will focus on the ageing failure for estimating the mortality for advanced ages. That is, the assuming model between $H(x)$ and $R(x)$ is

$$H(x) = B + C(1 - R(x)^{A})^{D} \tag{2.2}$$

for $x \geq y - t, x \geq 0$, y is the attained age and t is the starting age of the ageing failure period, y>0 and t>0.

Let's define (1) A: the survival decaying memory characteristics, (2) B/C: the initial mortality strength, (3) C: the strength of mortality, and (4) D: the survival memory characteristics.

Since $H(x) = \dfrac{-\dfrac{dR(x)}{dx}}{R(x)}$, then $\dfrac{dR(x)}{dx} = -H(x) * R(x)$. The differential equation of the dynamic reliability function is as follows:

$$\frac{dR(x)}{dx} = -(B + C(1 - R(x)^{A})^{D} * R(x). \tag{2.3}$$

The mean life is as follows:

$$T = \int_0^\infty R(x)dx = \int_0^1 \frac{dR(x)}{B + C(1 - R(x)^{A})^{D}} \tag{2.4}$$

Then we multiply by C on both sides to get the following:

$$CT = \int_0^1 \frac{dR(x)}{\frac{B}{C} + (1 - R(x)^{A})^{D}} = F(\frac{B}{C}, A, D) \tag{2.5}$$

R(x) is the conditional probability for a newborn who will survive at age $x+t$ given that it will attain age t. Let's define *R(0)*=1 and $R(\infty) = 0$. In our model (2.2), when x is zero, *H(x)* will be *B*, and when the age is extreme high, *H(x)* will near a constant $B+C$. This assumption is different from Gompertzs', which defines the hazard rate function increasing exponentially to infinity.

2.2 The Properties of Parameters

In this section, we will discuss the properties of the parameters. When one parameter is given various values and the other parameters are fixed, then equation (2.4) will have a different mean life. To understand the physics of the parameter, we will normalize the age scales (x-axis) by dividing by mean life. For example, fixing *A*=0.06, *B/C*=0.0005, and *D*=1.164, for various *B*=0.002, 0.005, and 0.01, the reliability curves are shown in *Figure 2.2*. The mean life is 40.47, 31.07, and 24.28, respectively. *Figure 2.3* shows the curves after age normalization; all the reliability curves follow the same curve for variants *B* and *C*, while *B/C* is fixed. For a better understanding of the properties of the parameters, we will use the normalized age scale to show the figures in the following sections.

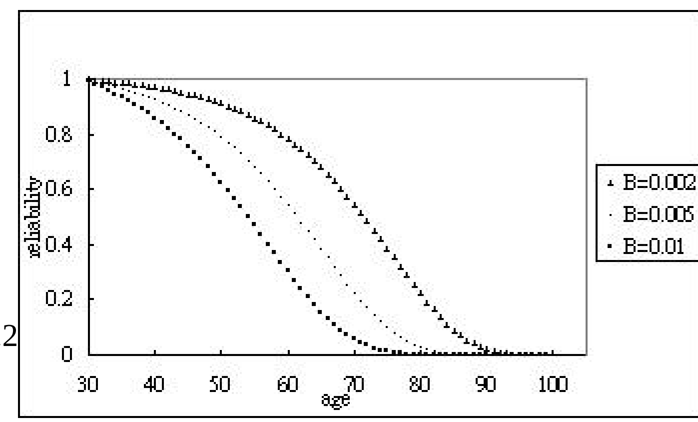


Figure 2.2 … =1.164

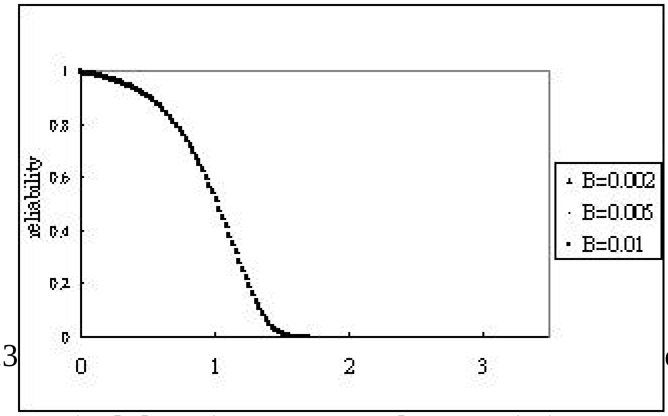


Figure 2.3 d *D*=1.164

2.2.1 Parameter *A*: Survival decaying memory characteristics

If *A* is zero or infinite then the hazard rates will become constant as *B* or *B*+*C*, respectively. For fixing *B*=0.001, *C*=1, and *D*=1.5, assuming *A*=0.5, 1, or 2, the mean life times are 32.89, 23.76, or 15.18, respectively. (*Figure 2.4*, *2.5*)

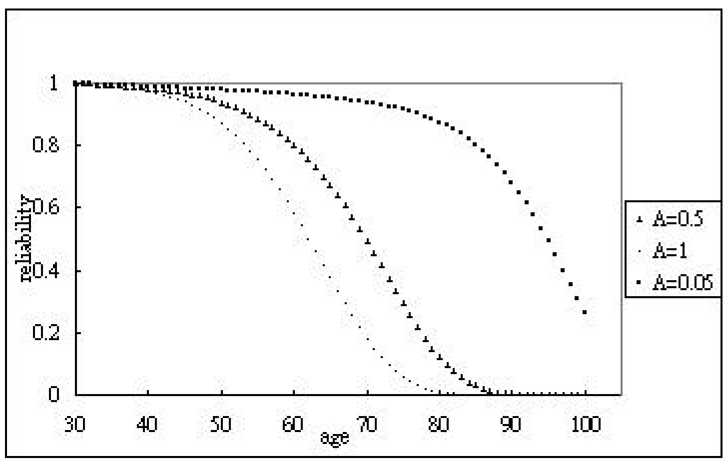


Figure 2.4 The reliability function for variants *A* (*B*=0.001, *C*=1, and *D*=1.5)

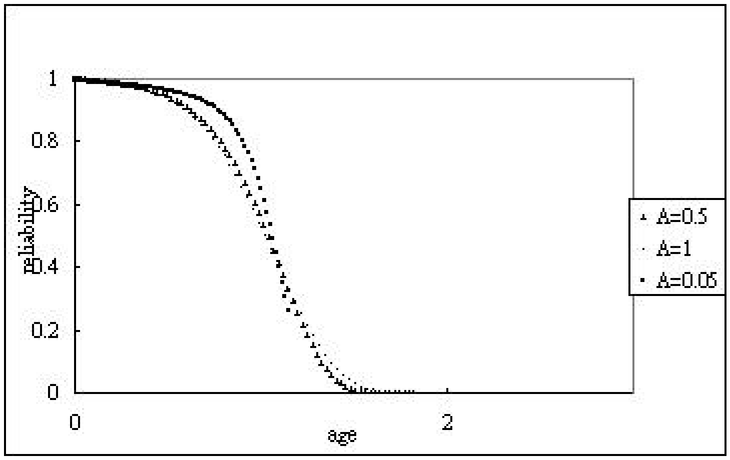


Figure 2.5 The reliability function for variants *A* (*B*=0.001, *C*=1, and *D*=1.5) after age normalization

The survival decaying memory characteristics is represented by *A*. When *A* is small, the reliability curve deccreases slowly at early ages. That is, if *A* is smaller then the survival probability is larger and the mean life is longer.

2.2.2 Parameter *C*: Strength of mortality

Assuming *A*=0.2, *B*=0.001, and *D*=1.5, and *C* is chosen as 2, 1, and 0.5, the mean life times are 38.39, 44.34, and 50.17, respectively.(*Figure 2.6, 2.7*)

Figure 2.6 The reliability function for variants *C* (*A*=0.2, *B*=0.001, and *D*=1.5)

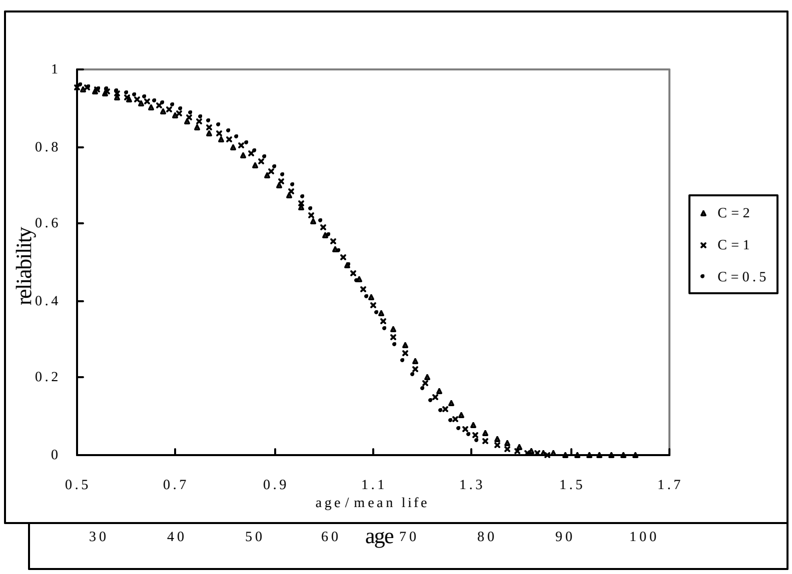


Figure 2.7 The reliability function for variants *C* (*A*=0.2, *B*=0.001, and *D*=1.5) after age normalization

The parameter *C* represents the strength of mortality. When *C* is smaller the survival probability is larger and the mean life is longer. The ultimate hazard rate approaches *B*+*C* when the reliability nears infinity. When *C* is small, it has a small mortality rate.

2.2.3 Parameter *B/C*: The initial mortality strength

Parameters are fixed at *A*=0.2, *C*=1, and *D*=1.5 to observe the influence of the parameter *B/C* with regard to the behavior of the reliability and hazard functions. Assuming *B/C*=0, 0.01, and 0.01, the mean life times are 49.45, 38.40, and 9.40, respectively (*Figure 2.8, 2.9*). When *A*, *D*, and *B* are fixed, the reliability curve will decrease quickly when the value of *B/C* is large.

The initial mortality strength is *B/C*. When *B/C* is small the mortality decreass slowly and the mean life is longer.

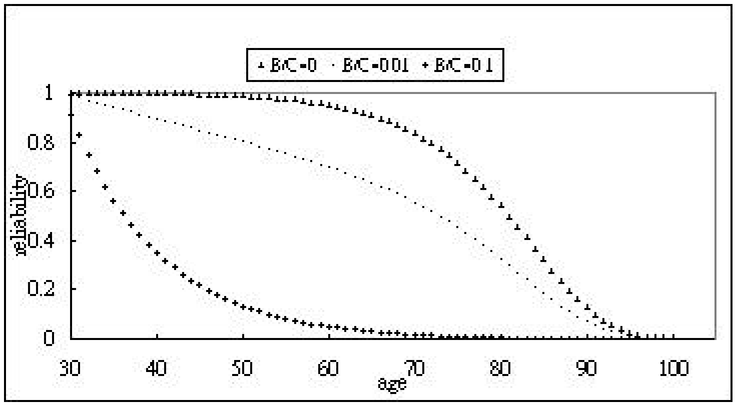


Figure 2.8 The reliability curves for variants *B/C*, when fixed *A*=0.2, *B*=0.001, and *D*=1.5

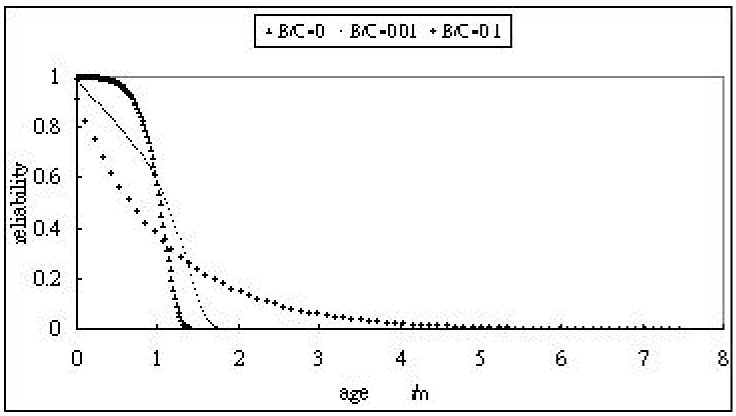


Figure 2.9 The reliability curves for variants *B/C*, when fixed *A*=0.2, *B*=0.001, and *D*=1.5, after age normalization

2.2.4 Parameter *D*: Survival memory characteristics

Assuming *A*=0.2, *B*=0.001, and *C*=1, and *D* is chosen as 2, 1, and 0.5, the mean life times are 52.01, 30.169, and 7.81, respectively. (*Figure 2.10*, *2.11*)

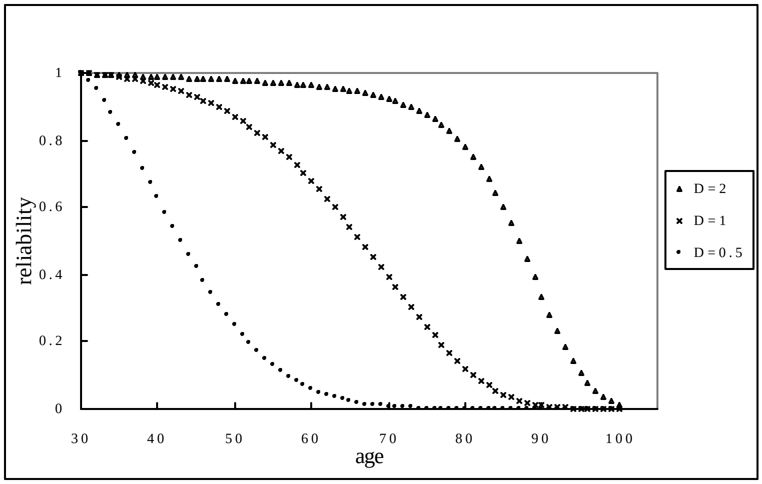


Figure 2.10 The reliability function for variants *D* (*A*=0.2, *B*=0.001, and *C*=1)

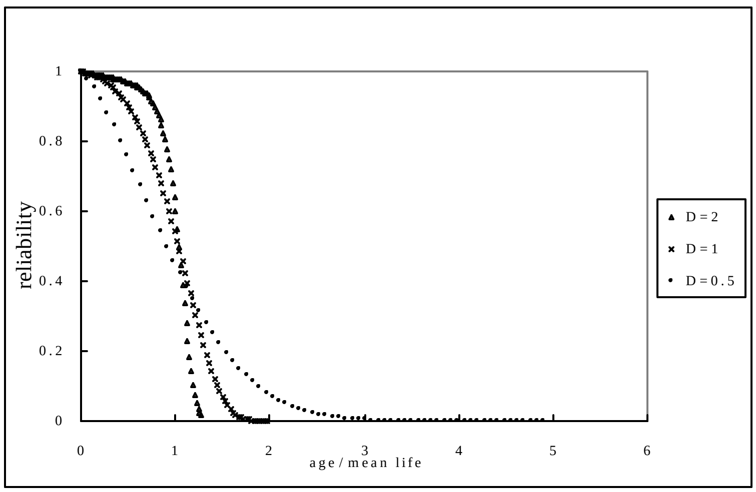


Figure 2.11 The reliability function for variants *D* (*A*=0.2, *B*=0.001, and *C*=1) after age normalization

The survival memory characteristics are represented by *D*. Parameter *D* has similar properties as parameter *A*. The hazard rate approaches a constant when *D* is close to zero or infinity. When *D* is zero, $H(x)=B+C$, and when *D* is approaching infinity, $H(x)$ B. The behavior of the curve with relation to the parameter *D* is the opposite of *A*. When *D* is large, the reliability curve shifts right and decreases slowly. When *D* is larger the survival probability is larger and the mean life is longer.

3. APPLICATION TO EIGHT TAIWAN COMPLETE LIFE TABLES

3.1 Model Fitting

There are eight Taiwan Complete Life Tables constructed between 1926 and 1991. The tables were obtained from the Department of Statistics, Ministries of Interior (1994), and detailed information is shown below:

Table 3.1 Eight Taiwan Complete Life Table constructed years

Released Order	Collected Period	Released Data
1	1926–1930	November,1936
2	1935–1940	June,1947
3	1956–1958	September,1965
4	1966–1967	June,1972
5	1970–1971	September,1977
6	1975–1976	June,1982
7	1980–1981	June,1992
8	1990–1991	December,1994

Lin (1996) showed that the Gompertz model fits better for the ages between 30 and 90. The mean squared errors are all less than 10^{-5} using the Gompertz model for both sexes in these intervals. In order to compare the reliability model with the Gompertz model, the starting age t=30 for the ageing failure period and the attained age y is between 30 and 90. Based on the estimates, the mortality rates will be forecast for the ages above 90. The numerical computations are executed by NLIN procedure of the software SAS using minimum mean squared of errors criteria. The estimates are listed in *Table 3.2* and *Table 3.3*.

Table 3.2 The estimates for 1926–1992 Taiwan male mortality rates

Year	1926–30	1936–40	1956–58	1966–67	1970–71	1975–76	1980–81	1989–91
A(%)	4.13	2.77	3.58	2.07	3.23	2.57	4.35	5.81
B(%)	1.53	1.25	0.41	0.27	0.26	0.37	0.23	0.31
C	3.97	5.98	4.47	5.92	3.93	6.97	4.18	3.89
D	1.35	1.28	1.16	1.05	1.09	1.16	1.17	1.27
B/C(‰)	3.86	2.10	0.92	0.46	0.67	0.53	0.54	0.80

Table 3.3 The estimates for 1926–1992 Taiwan female mortality rates

Year	1926–30	1936–40	1956–58	1966–67	1970–71	1975–76	1980–81	1989–91
A(%)	8.32	6.02	6.69	6.43	4.81	2.20	10.40	6.16
B(%)	0.89	0.74	0.30	0.15	0.08	0.07	0.00	0.11
C	2.00	3.38	2.43	2.23	2.84	6.72	1.69	2.93
D	1.37	1.35	1.18	1.10	1.06	1.07	1.16	1.13
B/C(‰)	4.49	2.19	1.24	0.67	0.27	0.10	0.04	0.39

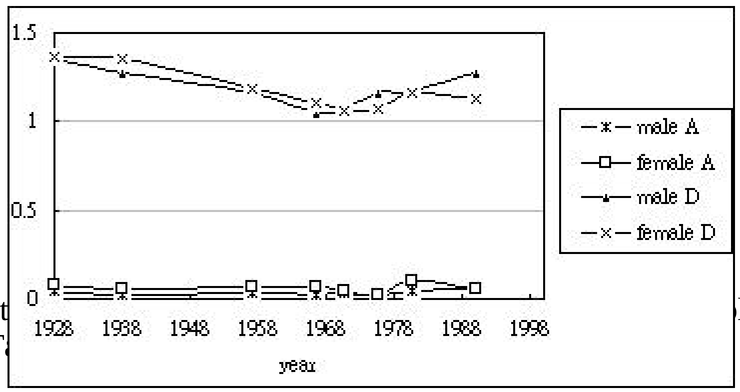


Figure 3.1 The est... lete
Life T...

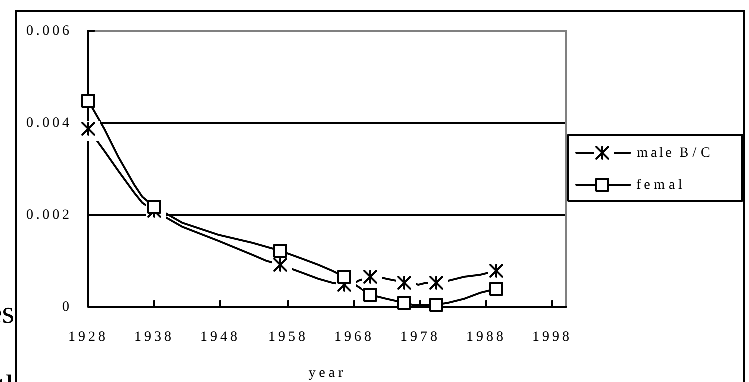


Figure 3.2 The es... Life Tables

In *Figure 3.1*, the estimates for A in all eight tables are relatively small. That is, human life reliabilities remain similar before ages 50–60 and then after decreases quickly, so the survival decaying memory A is very small. In *Table 3.3* and *Figure 3.1*, the estimates for *D* of females are about 1.10 since year 1966, so the survival characteristics are similar. Although, the estimates of D are similar for males, but the estimates have an increasing pattern from 1966 onward (*Table 3.2*). From 1970 onward, *D* value for males is larger than for females, so the mean life is improving much more for males than for females. In *Figure 3.2*, the pattern of the estimates for *B/C* is much more stable for males and is larger than for females from 1966. Also, the *B/C* value for females is smaller than for males and it has slightly increased since 1980. Therefore, the characteristics of prolonging mean life for both sexes are different. That is, the mean life of males has much more rate improvement potential. Both sexes have relatively small *A*, and females' *D* are similar and males' *D* are increasing slightly; the structures of *B/C* are different as well.

3.2 Reliability Improvement Rates

In this section, we would like to analyze the yearly reliability improvement rates for both sexes. In section 3.1, we found that the parameter characteristics have significant differences between males and females since year 1966. It is clear in *Figure 3.3a* and *Figure 3.3b* that the yearly reliability improvement rates are larger for females than those for males at age 70 or above from 1966 to 1976. The difference in improvement rates significantly increases for females at age 70 or above during those years. The improvement rates for males grow much more significantly than those for females between age 60 and 78 from 1975 to1981 (*Figure 3.3c*), but those for females are still much larger than for males at age 80 and above. However, this situation changed significantly from 1980 to 1991 (*Figure 3.3d*), male improvement rates are higher than the female at age 70 and above.

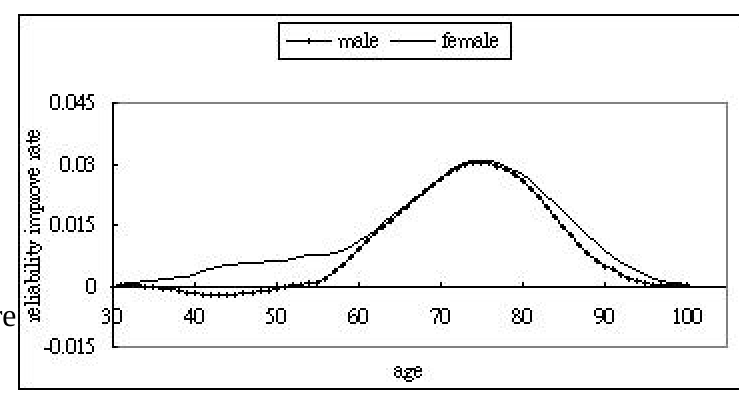


Figure 1971

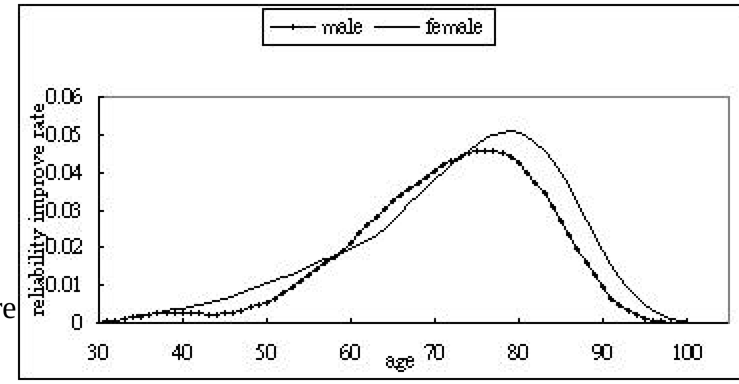


Figure 1976

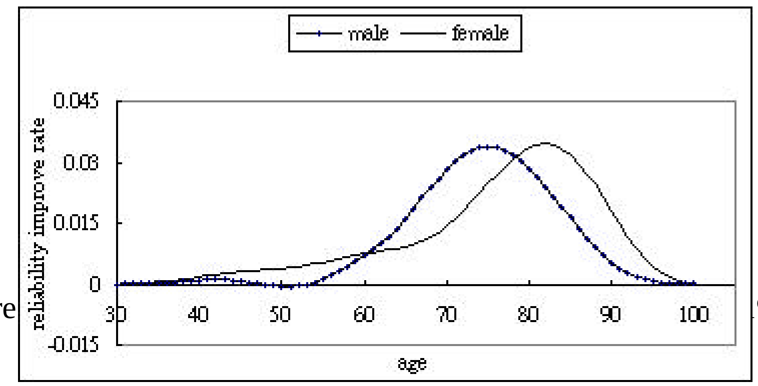


Figure 1981

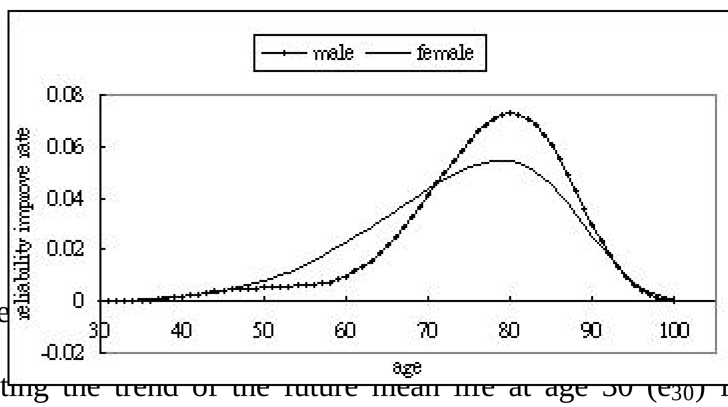


Figure 1991

For investigating the trend of the future mean life at age 30 (e_{30}) for each table, we observe the corresponding age (MA) of the maximum value of average yearly reliability improvement rate occurring between each consecutive life table (*Table 3.4*). The future mean life at age 30 is highly correlated with MA values, which has the coefficient of correlation 0.99 for male and 0.96 for female. For people at age 30, the male's maximum reliability improvement rate occurred at age 80 from 1980 to 1991. The female's MA values are larger than the male's before 1981, but there is a significant decrease between 1980 and 1991. Therefore, we see that the male's mortality improved significantly, and the age of the MA is higher every year. The slope of the regression line, MA regressed on years, is 0.4865 and 0.372 for male and female, respectively. That is, the yearly increase rate of MA is higher for males. Future mean life at age 30 increases yearly, and it is highly correlated with time (correlation of coefficients are 0.99 for both sexes). The future mean life e_{30} is increased 0.2706 years and 0.255 years every year for male and female, respectively. As we can see, the societal effect on ageing is more severe for males now.

Table 3.4 Future mean life e_{30} and MA for each consecutive life table at age 30

Collected Period	Mean life		MA	
	Male	Female	Male	Female
1926-1940	28.89	33.99	24	31
1936-1958	33.35	37.91	33	40
1956-1967	38.28	42.42	42	43
1966-1971	39.86	43.90	45	45
1970-1976	40.79	45.03	46	49
1975-1981	41.75	46.14	45	52
1980-1991	42.90	47.31	50	49

4. COMPARISON OF THE MODELS

The Gompertz's mortality law was proposed by Benjamin Gompertz in which the force of mortality (hazard rate function)

$$H(x) = E * F^{x} \tag{4.1}$$

was modeled as an increasing exponential function in age. It has been used in various studies and it fits observed mortality rates well in the adulthood (Wetterstrand, 1981). In Taiwan, there are eight Complete Life Tables, two of which were collected before World War II. The mortality rates of these two tables (covering years1926–1940) were much larger than the other six tables (covering years 1956–1991) and the ultimate age was assumed to be 100 in most of the tables. Lin (1996) fitted this model to the last six Taiwan Complete Life Tables and only used the data for persons between age 30 to 90 for males and age 25 to 90 for females. We believe that the data for persons over age 90 is unreliable because at age 90 and above, the mortality rates q_x were graded smoothly into a rate of one at age 100. Here we would like to fit these hazard rates to the reliability model proposed in the article and compare it with the Gompertz model. First we break the ages into two groups. To obtain the estimates, we use the data between ages 30 and 90 for both sexes. To verify the prediction validity of these two models, we use the data above age 90. We use minimum mean squared of errors (MSE) to measure the accuracy of models and these two models with the smaller MSE will be shown in *Table 4.1* and *Table 4.2*. *Table 4.1* is the summarized table for showing the model with the smaller MSE in fitting the hazard rates *H(x)* and *Table 4.2* is in fitting q_x, where $q_x = \frac{H(x)}{1+H(x)}$. The comparison results are similar in these two tables. The Gompertz model predicts better after age 90 for female data. However, the reliability model fits better before age 90 since year 1970, and it predicts well after age 90 since 1980 for male data. The model has a significant difference between the sexes before 1980. We can observe that the Gompertz model fits better for female data and the reliability model works well with male data. We use the mortality data from 1990–1991 to compare the results. From *Figure 4.1* and *Figure 4.2*, we can see the performance of the reliability model is similar to that of the Gompertz model, but the reliability model works best for all the ages of the ageing failure.

Table 4.1 Model with smaller MSE in fitting hazard rates

Data Period	Male		Female	
	Age 30–90	Age 91–100	Age 30–90	Age 91–100
1956–58	Gompertz	Gompertz	**Reliability**	Gompertz
1966–67	Gompertz	Gompertz	Gompertz	Gompertz
1970–71	**Reliability**	Gompertz	**Reliability**	Gompertz
1975–76	**Reliability**	**Reliability**	**Reliability**	Gompertz

1980–81	**Reliability**	Gompertz	Gompertz	Gompertz
1990–91	**Reliability**	**Reliability**	**Reliability**	Gompertz

Table 4.2 Model with smaller MSE in fitting q_x

Data Period	Male		Female	
	Age 30–90	Age 91–100	Age 30–90	Age 91–100
1956–58	**Reliability**	Gompertz	**Reliability**	Gompertz
1966–67	Gompertz	Gompertz	Gompertz	Gompertz
1970–71	**Reliability**	Gompertz	**Reliability**	Gompertz
1975–76	**Reliability**	**Reliability**	**Reliability**	Gompertz
1980–81	**Reliability**	Gompertz	Gompertz	Gompertz
1990–91	**Reliability**	**Reliability**	**Reliability**	Gompertz

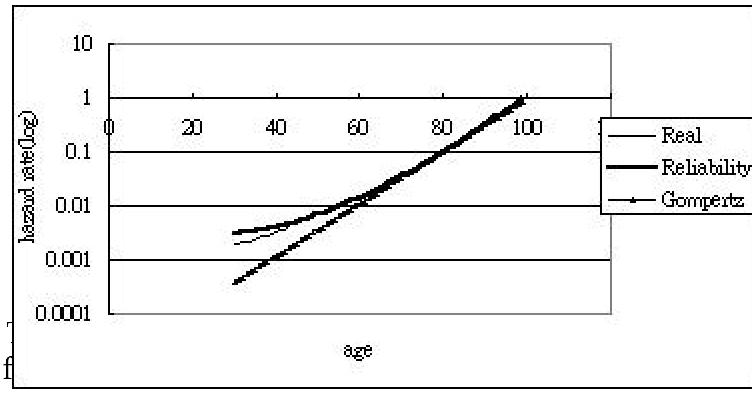


Figure 4.1 T ... d values
f ... 991 (male)

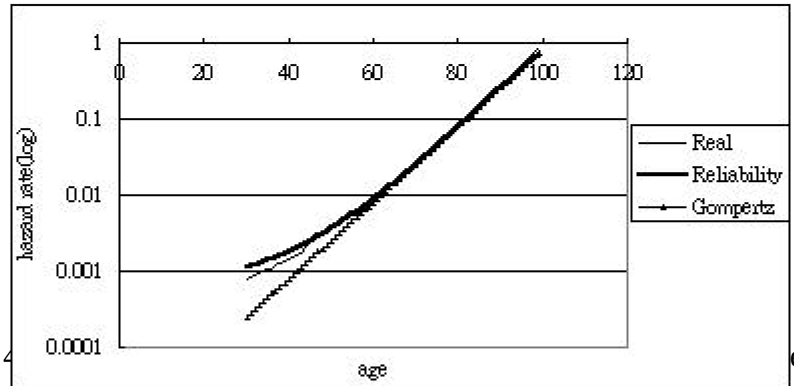


Figure 4 … ed values for the reliability model and Gompertz model for 1990 to 1991 (female)

5. CONCLUSIONS

In this article we presented a general law for hazard rates (mortality rates) as a function of reliabilities to analyze the mortality structure of Taiwan during 1926 to 1991. It is completely different from traditional parametric models as those are age-dependent relations for hazard rates. The results showed that this model could provide a detailed model to understand the physical meanings of each parameter. This model can also give a clear picture of the changing pattern of average reliability improvement rates by year that is also highly correlated with future mean life e_{30}. In addition, the reliability improvement patterns between both sexes had a sizeable change since 1990. Comparing this model to the popular Gompertz model in appropriateness for studying adult mortality rates, we observed that the reliability model provides a fitting explanation for mortality for both sexes after 1990.

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