ACTUARIAL RESEARCH CLEARING HOUSE
1992 VOL. 2

THE APPLICATION OF FUZZY SETS TO GROUP HEALTH UNDERWRITING

by Virginia R. Hosler

Fellowship Credit Research Paper 92-1

Approved by Society of Actuaries
Education and Examination Research Paper Committee

Effective March 31, 1992

THE APPLICATION OF FUZZY SETS TO GROUP HEALTH UNDERWRITING

Abstract

Fuzzy sets are used to model the process of selection in group health insurance. In general, fuzzy set theory is implemented to describe collections of objects whose boundaries are not precisely defined, as in the judgment of what constitutes a good risk group. First, single-plan underwriting is considered, then the work is extended to multiple-option plans.

TABLE OF CONTENTS

1. INTRODUCTION

In this paper, fuzzy sets are used to model the selection process in group health insurance. First, the case of an employer group 's choosing a single plan of health insurance is considered. From that point, the work is extended to group selection in a multi-choice environment. As more employers offer multiple plans to their employees, an underwriting scheme for such situations becomes important.

Fuzzy sets are used to describe collections of objects whose boundaries are not precisely defined. Indeed, a fuzzy set is a mapping f from the universe of discourse X to the unit interval I = [0, 1]. The value f(x) represents the degree to which x can be considered a member of the fuzzy set given by f. This definition is a generalization of the identification of a crisp set with its characteristic function:

$$f: X \to \{0,1\}.$$

For example, consider the statement: There should be a minimum percentage participation in the group health plan. Often this is translated into a requirement of 75% to 85% participation. One way to represent this rule as a fuzzy set is to define the function:

$$f(x) = \begin{cases} 0, & 0 \le x \le 0.75, \\ 10x - 7.5, & 0.75 \le x \le 0.85, \\ 1, & 0.85 \le x \le 1.00, \end{cases}$$

in which f(x) is the possibility of accepting the group that has x proportion of participation. This function can be refined to include variations for group size, for participation of employees with dependents, or for both.

Group selection criteria for both single- and multiple-option plans are described in Section 2. Such rules are given in several Study Notes published by the Society of Actuaries [5; pp 2, 10-18], [8; pp 48-52], and [10; pp 4-9].

An introduction to fuzzy sets is found in Section 3. References to this topic include [1], [3], [4], [9], [11], [12], and [13].

Lemaire [6] gives a model of underwriting for individual life insurance using fuzzy sets. His lead is followed by creating models of underwriting for group health insurance. In Section 4, single-option plans are examined; in Section 5, multiple-option plans. The major problem to be addressed is the determination of the interaction of the variables used in underwriting. Mailander [7] lists factors that influence selection in multiple-option plans. These include the plan of benefits, access to care, employee costs, and the age, sex, and marital status of the individuals in the group. Fuzzy sets are defined to measure the possibility of accepting the group based upon such considerations. This measure of possibility can also be the foundation for developing underwriting loads for various benefit plans.

To conclude, a summary of the paper and suggestions for areas of future research are presented in Section 6. Some topics for research include the application of fuzzy sets to trend analysis and to credibility theory.

The author wishes to acknowledge the assistance of several colleagues: Joseph W. Michel, for direction as the sponsor of this work; David W. Pray, for suggestions concerning the structure and content of this paper; and Wendell L. Holt, for comments abouts the underwriting rules presented herein.

2. GROUP SELECTION CRITERIA

2.1 *Single-option plans*

In using fuzzy sets to model the process of group selection for health insurance, the case of an employer offering a single plan of insurance to his employees is first considered. In this portion of the paper, therefore, desirable characteristics of such a group are outlined [5; pp 2, 10-18].

A. Obtaining insurance is incidental to the purpose of the group. An employer-employee group usually satisfies this criterion.

B. Eligibility is based upon employment status. For example, an employee may be required to be actively at work at least 30 hours per week in order to receive benefits, and only the spouse and dependent children of such an employee are eligible for dependent coverage. An actively-at-work rule is used because a certain level of health is needed for one to be able to work.

C. There is a minimum number of persons in the group. For example, some companies require that the group have a minimum size of five.

D. Benefits are determined automatically. For example, this criterion is met in the case of a single-option health plan in which benefits are determined by class of employment.

E. There is a constant flow of young lives into the group and old lives out. Such a requirement helps to ensure stable morbidity. A way of verifying this rule is to check whether the age/sex factor has been relatively stable and the group size has not fluctuated greatly during the past few years.

F. There is a minimum participation in the plan. One hundred percent enrollment of those who have no insurance and who are eligible for the given plan is usually required for a non-contributory (employer-pay-all) plan. Since there is no reason for an employee not to join such a plan, attention in this paper is restricted to contributory ones. A typical requirement in this case is one of 100% participation for groups of five or less grading to 75% or 85% for groups of ten or more.

   In addition, it is desirable that of those covered employees eligible for family coverage a minimum percentage of them choose to receive it. For example, suppose there is a group of twenty eligible employees; then, using a 75% participation basis, fifteen employees are needed to elect coverage. If sixteen of the twenty are enrolled and twelve of them are eligible for family coverage, then at least nine must choose it.

G. The employer pays some or all of the cost of insurance; contributions of one-fourth of the premium is often used as a minimum.

H. There is a strong, central administrative function to handle billing, enrollment, and certification of eligibility.

I. Industries such as agriculture and entertainment should be avoided. Either a company may use a list of industries it considers uninsurable, or it may apply a load to the medical manual rates. Be aware that the size of the load may be restricted for small groups due to state regulation or law.

J. The policyholder has a good credit rating that helps ensure that the premium will be paid in a timely manner. The quality of the credit risk can be evaluated through the company's financial statements or through credit reports.

K. On-going claims are not large as a proportion of total claims. For the initial underwriting of a small group (less than twenty-five lives), there are no employees who are health risks as determined by a health questionnaire or possibly by an attending physician's statement or physical exam. In renewal underwriting, information is available from one's own claim file.

L. The claim experience of the group has been good. One possible measure of such a characteristic is the loss ratio. It should be kept in mind that experience for smaller groups fluctuates more than for larger groups; therefore, a high loss ratio for a large group is more likely to indicate that future experience will be bad than would be the case for a small group.

M. The group has a low turnover rate with respect to carriers. A high turnover rate may signify a group with bad experience that frequently shops around for better rates.

2.2 *Multiple-option plans*

In addition to or in modification of the underwriting rules presented above for single-option plans, the following guidelines are used in the case of multiple-option plans [8; pp 48-52], [7; pp 4-9]. Again only an employer-employee group is contemplated.

D1. Benefits within each option are determined automatically.

E1. As a continuation of E, a group may be required to have an average age less than a stated number of years and female participation less than a given percentage. A large proportion of females and older participants leads to high claim costs and may lead to selection against the plan with the greater level of access to care or richer benefits. In order to combat higher expected claims, the manual rates need to be adjusted by an appropriate age/sex factor. Otherwise, using such a factor as a guide, a requisite is that it be sufficiently low as compared to the average of one 's block of business or to the nationwide average determined by the industry of the group.

F1. There is a minimum participation when considering all the benefit options. As in the preceding subsection, one may require 75% to 85% participation in some health plan for groups of twenty-five or more.

In addition, there is a minimum enrollment in the given option. This minimum increases as the size of the group increases; therefore, the minimum may be expressed as a minimum percentage participation.

N. The maximum differential between the employee 's contribution for the HMO or PPO and the indemnity product is limited to, say, twenty dollars on the single rate and fifty dollars on the family rate. This out-of-pocket comparison may also include deductible and coinsurance differences; however, in this work these features are accounted for in the following item.

O. The benefits of one plan are not overly rich in relation to the other(s) particularly with respect to selected benefits, such as prescription drugs and organ transplants. This requirement helps to reduce adverse selection and premium differentials.

The latter two underwriting criteria are factors that influence an employee 's choice of health coverage. Others include the level of access to care and the age, sex, and dependent coverage of the employee. The age of the employee is important in that older employees tend to have established relations with their physicians and may be reluctant to relinquish them for a closed-panel setting. A similar argument can be made for employees with dependent children.

3. FUZZY SETS

3.1 *Definition and examples*

The theory of fuzzy sets is introduced by Zadeh [11] in a paper published in 1965. The research in this area has expanded to the point of having two journals devoted to fuzzy sets, and some of the regions of application are artificial intelligence (including machines with fuzzy controllers such as air conditioners and washing machines [3; p 301], [9; p 37]), linguistics, economics, and decision making. The employment of fuzzy sets in actuarial studies is relatively new and can be marked by a paper of Lemaire [6] published in 1990.

Fuzzy sets are used to describe concepts that are vague or ambiguous. The fuzziness of a set arises from the lack of well-defined boundaries. This lack is due to the equivocal nature of language and to the subjective interpretation of it. For example, let A represent the set of good basketball players among the set of teenage basketball players, X. There are teenagers who clearly do not play basketball well; on the other hand, there are decidedly talented players. Between these two extremes lie players who are borderline cases. In set theory, elements in the given set are assigned a grade of membership of 1; those not in the set are assigned a grade of 0. Generalizing, the borderline cases will have membership values in A between 0 and 1, with the better players having values closer to 1.

Subjectivity comes into play in the assignment of membership values. Indeed, people disagree as to what constitutes a good basketball player; thus, they arrive at dissimilar fuzzy sets to represent A. Also, one person may develop distinct fuzzy sets to represent the same concept at varying times. Context, or the underlying universe of discourse, X, is another important factor in determining the membership grades of elements in a fuzzy set. If X were changed to that of the set of all basketball players thirteen years of age or older, then the fuzzy set of good basketball players in X would necessarily change. For instance, an otherwise proficient sixteen-year-old basketball player will not be considered as skilled when compared to a professional player.

Complexity can also add to the fuzziness of a set. If one attempts to evaluate what comprises a good basketball player, several attributes must be considered: ball handling (dribbling and passing), shooting (field shots and free throws), defensive work (rebounding, guarding, and ability to cause turnovers), physical stamina, and aerobic capacity. In what follows, fuzziness that comes from vague, complex concepts is dealt with, but the change of fuzzy sets from person to person, over time, or from context to context is not considered. In this section, the definition of fuzzy sets and several examples are presented [11; pp 339-340], [12; pp 4-10, 13], [13; pp 199-201].

3.1.1. Definition A fuzzy set A in a universe of discourse X is a function:

$$f_A : X \rightarrow I = [0,1].$$

f_A is called the membership function of A, and for any x in X, $f_A(x)$ in [0,1] represents the grade of membership of x in A.

This definition generalizes the one for a non-fuzzy set or crisp set. Such a set can be given by a characteristic function:

$$f_A : X \rightarrow \{0,1\},$$

in which $f_A(x) = 1$ if x is in A; otherwise, $f_A(x) = 0$.

If the least upper bound of f_A is 1, then A is said to be normal. A subnormal fuzzy set can be normalized by dividing f_A by its least upper bound.

The following examples illustrate the use of fuzzy sets in situations that might be encountered in actuarial practice.

3.1.2. Example Several factors influence health care trend: aging of the insured population, higher expectations of medical care and the consequent increase in utilization, higher standards of living of the insured population, use of more expensive technology, cost shifting from public to private payors, rising cost of medical malpractice insurance, defensive medical practices due to threat of malpractice, leveraging effect of deductibles, mandated benefits, and increased risk due to new diseases, such as AIDS. These components interact in a complex manner; e.g.,one might expect that the costs associated with defensive medical practice would vary directly with the cost of malpractice insurance. As another example, the changes in charges and utilization may vary inversely as physicians attempt to maintain a given standard of living.

Fuzziness exists inherently in these complex relationships. It also arises in the decision of the trend to use in prospective pricing: If the trend chosen is too high, then good risks may leave the insurance company, and an assessment spiral can develop. If the trend is set too low, then profits may be low or even negative due to inadequate pricing. See Example 3.2.13 below for further discussion of this particular topic.

3.1.3. Example Consider risk evaluation of fixed-income securities [2; pp 30-39]. Evaluation of securities in general is very complex; indeed, risk arises from several sources: being forced to sell when interest rates are rising (market risk), reinvesting income when interest rates are falling, timing or call risk, political risk, sector risk, purchasing power or inflation risk, credit risk, currency risk for foreign investments, liquidity risk, and event risk.

To simplify the discussion, look at one part of risk: the risk of call of the security. An investor may feel that there is no possibility of call if market interest rates are at least 10% when the bond has a coupon rate of 9.5% and that the risk will increase to certainty as market rates fall to 9% or less. Let A be the fuzzy set of bonds that might be called:

$$f_A(x) = \begin{cases} 0, & 0.10 \le x, \\ -100x + 10, & 0.09 \le x \le 0.10, \\ 1, & x \le 0.09, \end{cases}$$

in which x is the market interest rate. See Figure 3.1.1.

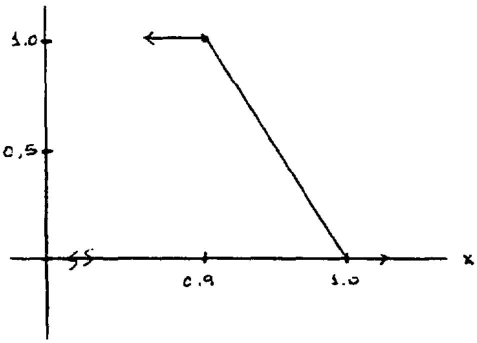


Figure 3.1.1.

Note that randomness is associated with the behavior of market interest rates and that fuzziness arises from the subjective opinion of the investor. The function f could be altered to reflect the desirability of the security based upon the call risk. Other such functions could be developed for the remaining risks, and they could be combined through methods introduced below in order for an investor to make a decision as to whether or not the security is a good investment.

3.2 *Operations on fuzzy sets*

In order to be able to consider several fuzzy conditions simultaneously, the ways in which fuzzy sets can be combined or operated upon are defined in this section [11; pp 340-342], [12; pp 11-14].

3.2.1. Definitions The union, A U B, of two fuzzy sets A and B is given by:

$$f_{A \cup B}(x) \equiv \max(f_A(x), f_B(x)), \quad x \in X,$$

and the intersection, A $\cap$ B, is given by:

$$f_{A \cap B}(x) \equiv \min(f_A(x), f_B(x)), \quad x \in X.$$

The complement, -A, of fuzzy set A is given by:

$$f_{-A}(x) \equiv 1 - f_A(x), \quad x \in X.$$

The above definitions lead to an expression for the difference, A - B, namely:

$$\begin{aligned} f_{A-B}(x) &\equiv f_{A \cap -B}(x) \\ &\equiv \min(f_A(x), f_{-B}(x)) \\ &\equiv \min(f_A(x), 1 - f_B(x)), \quad x \in X. \end{aligned}$$

Note that these definitions degenerate to those used for crisp sets if A and B are nonfuzzy.

3.2.2. Example Let X = {1, 2, 3, 4, 5}, and define fuzzy sets A and B on X by

f_A = {(1, 0.2), (2, 0.5), (3, 1.0), (4, 0.2), (5, 0.0)}, and

f_B = {(1, 0.2), (2, 0.4), (3, 0.6), (4, 0.8), (5, 1.0)}.

The fuzzy set A can be expressed verbally as the collection of small integers in X very close to 3, and B as the set of integers in X close to 5. Then the following are obtained:

$f_{A \cup B}$ = {(1, 0.2), (2, 0.5), (3, 1.0), (4, 0.8), (5, 1.0)},

$f_{A \cap B}$ = {(1, 0.2), (2, 0.4), (3, 0.6), (4, 0.2), (5, 0.0)},

$$f_{-A} = \{(1, 0.8), (2, 0.5), (3, 0.0), (4, 0.8), (5, 1.0)\},$$

$$f_{A-B} = \{(1, 0.2), (2, 0.5), (3, 0.4), (4, 0.2), (5, 0.0)\}.$$

The operation of union acts as an "or" operator; intersection, as "and"; and complement, as "not." Thus, for example, $f_{A \cap B}$ represents the fuzzy set of small integers in X that are very close to 3 and close to 5. Similarly, f_{A-B} portrays the fuzzy set of small integers in X that are very close to 3 but not close to 5. In order to normalize $f_{A \cap B}$ and f_{A-B}, divide the function values by the maximum of each set, 0.6 and 0.5, respectively.

In what follows, fuzzy sets represent characteristics of a group that must be simultaneously "good" in order for the group to be classified as a good risk. It is desirable that the characteristics be allowed to interact when taking combinations of them, especially intersections; however, the above definition of intersection does not satisfy this property. Other definitions of intersection are presented below, and they take into account some or all of the following properties [6; pp 41-42]:

Property 1 (cumulative effects):

$$f_{A \cap B}(x) < \min(f_A(x), f_B(x)),$$
$$\text{if } f_A(x),\ f_B(x) < 1.$$

This property states that if two characteristics are both "less than desirable," then the two taken together are worse than each separately.

Property 2 (interactions between criteria):

The effect of a change of $f_A(x)$ upon $f_{A \cap B}(x)$ may also depend on $f_B(x)$. In other words, the effects of A and B are not independent.

Property 3 (compensation between criteria):

The effect of a decrease of $f_A(x)$ upon $f_{A \cap B}(x)$ may be eliminated by an increase of $f_B(x)$.

Property 4

If A and B are crisp sets, then

$$f_{A \cap B}(x) = \begin{cases} 1, & \text{if } x \in A \text{ and } x \in B, \\ 0, & \text{else.} \end{cases}$$

Note that the minimum operator satisfies only Property 4 of the above. Lemaire [6; p 42] follows the description of these properties with alternate definitions of the intersection.

3.2.3. Definition The algebraic product, AB, is given by:

$$f_{AB}(x) \equiv f_A(x) \cdot f_B(x), \quad x \in X.$$

The algebraic product satisfies all four properties.

3.2.4. Definition The bounded difference, $A \ominus B$, is given by:

$$f_{A \ominus B} \equiv \max[0, f_A(x) + f_B(x) - 1], \quad x \in X.$$

The bounded difference satisfies all but Property 2. Indeed, a change effected upon $A \ominus B$ by $f_A(x)$ is independent of the value of $f_B(x)$ as long as $f_A(x) + f_B(x) - 1$ is greater than 0.

3.2.5. Definition The Hamacher operator, H, which depends on p, is given by:

$$H(A,B;p)(x) \equiv \frac{f_A(x) f_B(x)}{p + (1-p)[f_A(x) + f_B(x) - f_A(x) f_B(x)]}, \quad 0 \le p \le 1.$$

The Hamacher operator satisfies all four properties; in general, $f_{AB} \le f_H^{\,p} \le f_H^{\,q} \le f_{A \cap B}$, if $0 \le q \le p \le 1$. Note that the interaction between A and B depends upon the parameter p; the degree of interaction decreases as p decreases. When $p = 1$, the Hamacher operator reduces to the algebraic product, which is the intersection that provides maximum interaction.

3.2.6. Definition The Yager operator, Y, which depends on p, is given by:

$$Y(A,B;p)(x) \equiv 1 - \min\{1, [(1-f_A(x))^p + (1-f_B(x))^p]^{1/p}\}, \quad p \geq 1.$$

The Yager operator satisfies all four properties as long as $1 < p < \infty$. It reduces to the minimum operator as p goes to infinity and to $A \ominus B$ when $p = 1$.

In order to obtain a better comprehension of the above operators, it is helpful to understand their level curves. For graphs of the level curves of the above operators, see Appendix A.

3.2.7. Example Let A, B, and X be as in Example 3.2.2.

$f_{AB} = \{(1, 0.04), (2, 0.2), (3, 0.6), (4, 0.16), (5, 0.0)\}$,

$f_{A \ominus B} = \{(1, 0.0), (2, 0.0), (3, 0.6), (4, 0.0), (5, 0.0)\}$,

$H(A, B; 0.5) = \{(1, 0.059), (2, 0.235), (3, 0.6), (4, 0.174), (5, 0.0)\}$,

$Y(A, B; 2) = \{(1, 0.0), (2, 0.219), (3, 0.6), (4, 0.175), (5, 0.0)\}$.

Again, the above fuzzy sets can be normalized by dividing each by its least upper bound, or the maximum value in a discrete case.

There are other means of combining fuzzy sets that allow for interaction among criteria [10; p 345], [11; pp 14-15].

3.2.8. Definition Given fuzzy sets $A_1, A_2, \ldots, A_n$, the convex combination, B, is defined by:

$$f_B(x) \equiv w_1(x)\, f_{A_1}(x) + \ldots + w_n(x)\, f_{A_n}(x),$$

in which $\sum_{i=1}^{n} w_i(x) = 1$, and $0 \le w_i(x)$, for all $x \in X$.

A special case of the above is given when $w_i(x) = w_i$, a constant, $i = 1, \ldots, n$. In this case, B is said to be a <u>convex linear combination</u> of the A_i. If $w_i = 1/n$, $i = 1, \ldots, n$, then B is the <u>arithmetic mean</u> of the A_i.

3.2.9. Definition Given fuzzy sets $A_1, A_2, \ldots, A_n$, their <u>geometric mean</u>, B, is defined by:

$$f_B(x) \equiv (f_{A_1}(x) \cdots f_{A_n}(x))^{1/n}, \quad x \in X.$$

If a given criteria A_j is deemed relatively more important than the others, then there are ways in which this can be taken into account in the above definitions of intersection and combination. In the case of the latter, A_j can be weighted with a larger value of w_j. For the former, A_j can be concentrated before application of the chosen version of intersection, where the operation of concentration is defined as below:

3.2.10. Definition The <u>concentration</u>, CON(A), of a fuzzy set A is given by:

$$f_{CON(A;a)}(x) \equiv [f_A(x)]^a, \quad a > 1.$$

Concentration acts to reduce the grade of membership of all elements x, with $f_A(x) < 1$, such that the closer $f_A(x)$ is to 0, the more its grade of membership is reduced. It is most common to set $a = 2$.

The inverse operation of concentration is called dilation and can be used to reduce the importance of a given criteria.

3.2.11. Definition The <u>dilation</u>, DIL(A), of a fuzzy set A is given by:

$$f_{DIL(A;a)}(x) \equiv [f_A(x)]^a, \quad 0 < a < 1.$$

Usually, a = 1/2.

3.2.12. Example Again, let A, B, and X be as in Example 3.2.2.,and let C be given by:

$$f_C = \{(1, 0.6), (2, 0.8), (3, 0.2), (4, 0.1), (5, 0.5)\}.$$

Form the convex combination:

$$D = C \cdot A + (1 - C) \cdot B;$$

$$f_D = \{(1, 0.2), (2, 0.48), (3, 0.68), (4, 0.74), (5, 0.5)\}.$$

Calculate the geometric mean of A and B:

$$f_E = \{(1, 0.2), (2, 0.447), (3, 0.775), (4, 0.4), (5, 0.0)\}.$$

The concentration of A can be thought of as those very small integers in X that are also very, very close to 3:

$$f_{CON(A;\,2)} = \{(1, 0.04), (2, 0.25), (3, 1.0), (4, 0.04), (5, 0.0)\}.$$

Dilation has the opposite effect of that of concentration. The dilation of B is the fuzzy set of integers in X that are somewhat close to 5:

$$f_{DIL(B;\,0.5)} = \{(1, 0.447), (2, 0.633), (3, 0.775), (4, 0.894), (5, 1.0)\}.$$

3.2.13. Example The model developed in this example is based upon one used in the determination of washing time for a particular Japanese washing machine [9; p 37] and continues the presentation in Example 3.1.2. The discussion is limited to the consideration of two rules for the estimation of trend:

1) If the increase in the medical Consumer Price Index (CPI) is high and the increase in the reimbursement of providers under Medicare is low, then the trend will be high.

2) If the increase in the medical CPI is moderate and the increase in the reimbursement of providers under Medicare is moderate, then the trend will be moderate.

Define the following fuzzy set functions:

$$\text{CPI\_High}(x) = \begin{cases} 0, & 0 \le x \le 0.05, \\ 20x-1, & 0.05 \le x \le 0.10, \\ 1, & 0.10 \le x, \end{cases}$$

$$\text{CPI\_Mod}(x) = \begin{cases} 20x, & 0 \le x \le 0.05, \\ -20x+2, & 0.05 \le x \le 0.10, \\ 0, & 0.10 \le x, \end{cases}$$

in which x is the annual increase in the medical CPI.

$$\text{Medicare\_Low}(y) = \begin{cases} -25y+1, & 0 \le y \le 0.04, \\ 0, & 0.04 \le y, \end{cases}$$

$$\text{Medicare\_Mod}(y) = \begin{cases} 40y, & 0 \le y \le 0.025, \\ -40y+2, & 0.025 \le y \le 0.05, \\ 0, & 0.05 \le y, \end{cases}$$

in which y is the annual increase in the reimbursement of providers under Medicare.

$$\text{Trend\_High}(z) = \begin{cases} 0, & z \le 0.10, \\ 10z-1, & 0.10 \le z \le 0.20, \\ 1, & 0.20 \le z \le 0.25, \\ -20z+6, & 0.25 \le z \le 0.30, \\ 0, & 0.30 \le z, \end{cases}$$

$$\text{Trend\_Mod}(z) = \begin{cases} 0, & z \le 0.05, \\ 20z-1, & 0.05 \le z \le 0.10, \\ -10z+2, & 0.10 \le z \le 0.20, \\ 0, & 0.20 \le z, \end{cases}$$

in which z is the annual trend. See the graphical representations of the above fuzzy sets at the end of Section 3 on page 23.

Suppose that x is 0.075 and y is 0.025; then, CPI_High(0.075) = 0.5 and Medicare_Low(0.025) = 0.375. Taking the intersection of these two values through the minimum operator yields 0.375. Truncate the graph of the function Trend_High at 0.375 from above. Similarly, CPI_Mod(0.075) = 0.5 and Medicare_Mod(0.025) = 1.0. The intersection of these two gives 0.5; again, truncate the graph of the function Trend_Mod at 0.5 from above. The union of the two planar regions is formed (see Figure 3.2.1.),and the trend is taken to be the abscissa of the center of gravity of the resulting region. The trend thus determined in this example is 0.1684 = 16.84%.

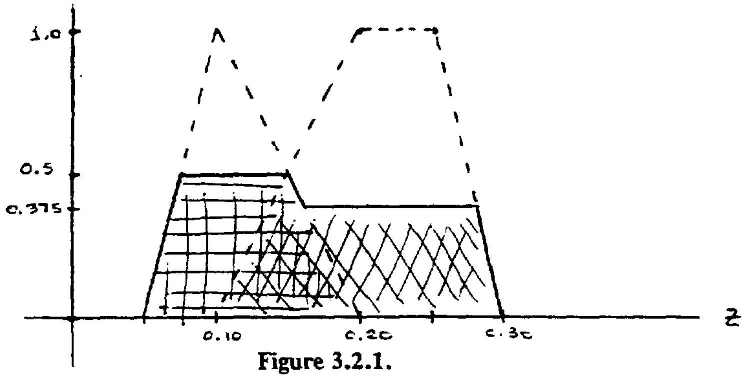


Figure 3.2.1.

3.3 *Fuzzy set theory vs. probability theory*

Fuzzy set theory and probability theory are related in that they both deal with uncertainty. Fuzzy set theory treats the uncertainty that comes from ambiguity and considers to what extent an event occurs. On the other hand, probability theory contemplates the uncertainty that arises from randomness and regards the question of whether or not an event will occur.

After the given event occurs (or not), randomness no longer exists; however, ambiguity does not dissipate with the acquisition of additional information. For example, suppose that one in five men are at least six feet tall; therefore, if a man is selected at random, one would say the probability that the man is at least six feet tall is 0.20. If the actual height of the man is measured and found to be 5' 11", then the uncertainty associated with whether he is at least six feet tall has been eliminated. The ambiguity related to whether or not the man is tall, however, has not been eliminated; for example, one might say that his membership value in the fuzzy set of tall men is 0.80.

In general, fuzziness arises from the overlap between a concept, A, and its opposite, -A:

3.3.1. Example Let A be as in Example 3.2.2.

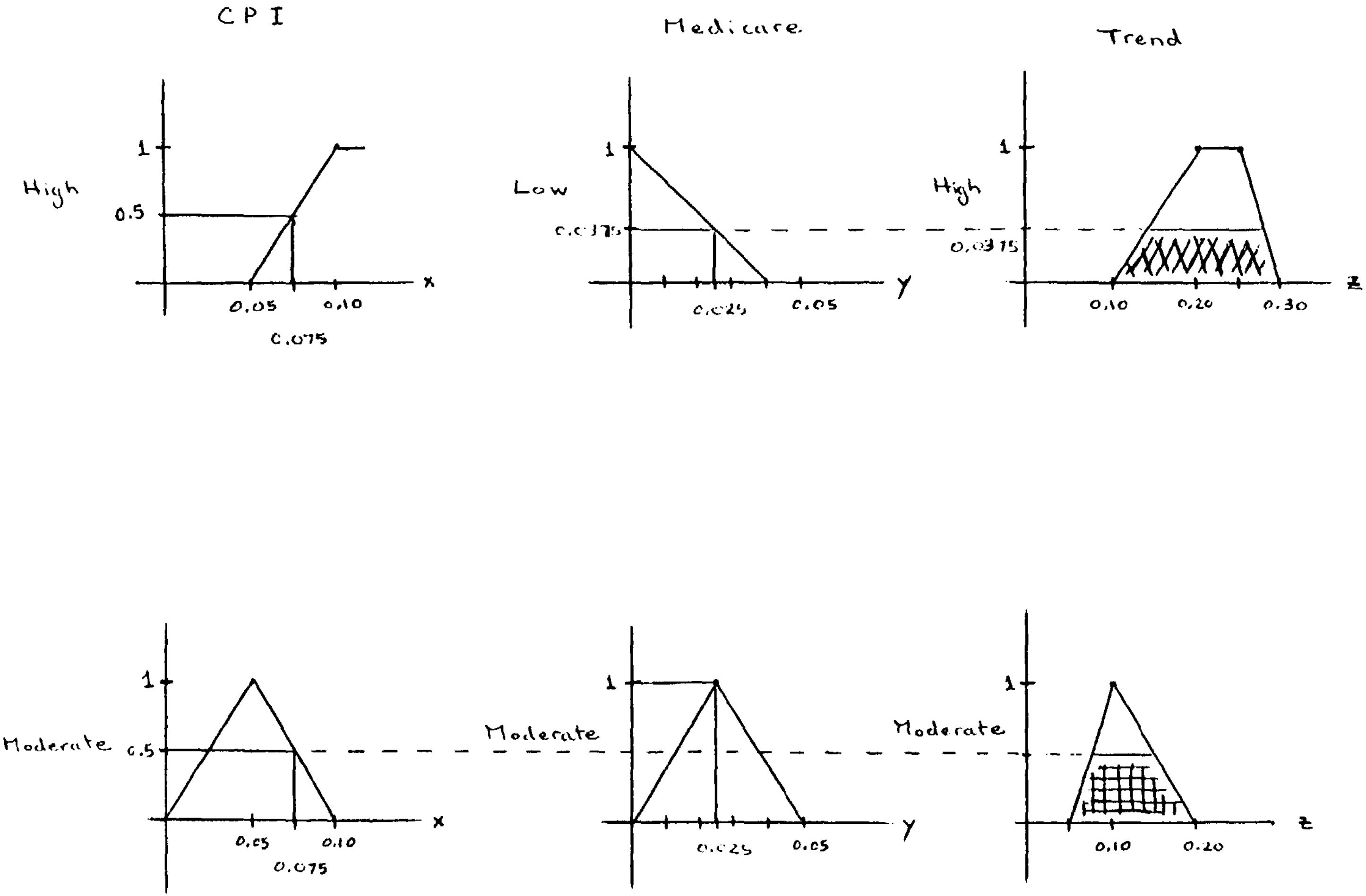
$$f_{A \cap -A} = \{(1, 0.2), (2, 0.5), (3, 0.0), (4, 0.2), (5, 0.0)\}.$$

In probability theory, an event and its opposite cannot both occur, yet in fuzzy set theory, each can occur to some degree. For example, a man can be both tall and not tall to some extent.

In Section 2 and in the sections that follow, the problem of characterizing what constitutes a group that is a good risk with respect to group health insurance is addressed. From the viewpoint of probability theory, a group that is a good risk is one that can be expected to be profitable. From this perspective, uncertainty is connected with claim and expense fluctuation relative to the price charged.

From the standpoint of fuzzy set theory, a group can still be a good risk even if it is unprofitable in a given year. Events beyond the control of a group can lead to unprofitability, such as a severe epidemic attacking members of the group, an automobile accident involving one or more members, higher than expected expenses associated with administration, or inadequate pricing due to competitive pressure or to lack of expertise within the pricing unit of the group health insurance department. The attributes that are used to describe a good risk are those that are outlined in Section 2--characteristics that indicate stability and lack of potential for an assessment spiral to occur. (Note that these qualities are fuzzy in nature themselves.)

For further information concerning the relationship between probability theory and fuzzy set theory, refer to "Fuzziness vs. probability" by Bart Kosko [4].


CPI
Medicare
Trend
High
Low
High
Moderate
Moderate
Moderate
1
0.5
0.0375
0.0375
0.05
0.10
0.075
0.025
0.05
0.10
0.20
0.30
x
Y
z

4. FUZZY SET MODEL FOR UNDERWRITING SINGLE-OPTION PLANS

After the preceding introduction, fuzzy sets are applied below to the underwriting criteria listed in Section 2.1. A few of those requirements can most appropriately be described with non-fuzzy sets, so they are only briefly mentioned now, using the same lettering as in Section 2.1.

A. Obtaining insurance is incidental to the purpose of the group.

B. Eligibility is based upon employment status.

C. There is a minimum number of persons in the group.

D. Benefits are determined automatically.

As to the remainder of the criteria, possible fuzzy set representations are presented for them in the following discussion. One of the major difficulties in fuzzy set theory is the creation of an appropriate function by which to describe the given condition or characteristic. In this paper, the functions are essentially derived by working backwards. Four categories of groups are considered: preferred risk, normal or acceptable risk, substandard risk, and unacceptable or declinable risk. For each individual variable, the function assigns to a preferred risk a membership value of 1.0. Normal risks have values lying between 0.5 and 1.0; substandard risks, between 0.25 and 0.5. Finally, unacceptable risks have membership values ranging from 0.0 to 0.25. In the intersections of the various fuzzy functions, these breakdowns may not be preserved, so it is important to find the resultant corresponding boundaries.

The functions developed are for illustrative purposes only and are not intended to represent any particular company's underwriting guidelines. In practice, the fuzzy set functions should reflect one 's actual underwriting policy. Graphs of the fuzzy set functions in this section and in the next are sketched in Appendix B.

E. There is a constant flow of young lives into the group and of old lives out of it.

Let a/x equal the percentage or relative change in the age/sex factor for the past two years and g/s the percentage change in the group size during the same period:

$$e_1(a/x) = \begin{cases} 1, & a/x \le 0.05, \\ -5a/x+1.25, & 0.05 \le a/x \le 0.25, \\ 0, & else. \end{cases}$$

$$e_2(g/s) = \begin{cases} 1, & -0.05 \le g/s, \\ 5g/s + 1.25, & -0.25 \le g/s \le -0.05, \\ 0, & else. \end{cases}$$

Before moving to the next factor, examine the information contained in the above functions. An increase of up to 5% or any decrease in the age/sex factor describes a preferred risk. A group is normal if the percentage increase lies between 5% and 15% ($0.5 \le e_1 \le 1.0$), while an unacceptable group has an age/sex factor increase of 20% or more ($0.0 \le e_1 \le 0.25$). As to change in group size, the opposite viewpoint is taken; i.e.,decrease of down to 5% or any increase is preferred, while a 5% to 15% decrease is acceptable. The form of these two functions is linear, but for a particular insurance company's underwriting guidelines, one does not necessarily expect the functions to be linear.

One way to combine these two functions while preserving the boundaries of the four classes is the following:

$$e(a/x, g/s) = \sqrt{e_1(a/x) \cdot e_2(g/s)}.$$

F. There is a minimum participation in the plan.

A preferred group is specified as having 90% participation or better. A normal group has 80% to 90% participation, and a substandard group has participation of 75% to 80%. Such a set of partitions could correspond to the requirement that a group have 85% participation with possible bending of the rule down to 75%. Let p be the proportion of employees that select group coverage:

$$f(p) = \begin{cases} 1, & 0.90 \le p, \\ 5p - 3.5, & 0.70 \le p \le 0.90, \\ 0, & else. \end{cases}$$

For small groups it may be desirable to require higher percentages of participation than those given in the above equation. Also the same or a similar function may account for participation of employees who are eligible for dependent coverage, then some intersection of the two would integrate the criteria. One possible intersection is the square root of the product as in E, assuming that the two fuzzy sets have equal importance.

G. The employer pays some or all of the cost of insurance.

A preferred risk is one for which the employer pays all of the employee 's cost and 75% or more of the dependent 's cost. For an acceptable risk, the percentages go down to 75% and 50%, respectively; for a substandard risk, down to 50% and 25%, respectively. Let r_1 equal the proportion of the employee 's premium paid by the employer and r_2 the proportion of the dependent's premium paid by the employer:

$$g_1(r_1) = \begin{cases} 0, & r_1 \le 0.25, \\ r_1 - 0.25, & 0.25 \le r_1 \le 0.75, \\ 2r_1 - 1, & 0.75 \le r_1 \le 1.0. \end{cases}$$

$$g_2(r_2) = \begin{cases} r_2, & r_2 \le 0.50, \\ 2r_2 - 0.50, & 0.50 \le r_2 \le 0.75, \\ 1, & 0.75 \le r_2. \end{cases}$$

As in criterion E, the above functions can be linked by the following:

$$g(r_1, r_2) = \sqrt{g_1(r_1) \cdot g_2(r_2)}.$$

The function g allows trade-offs between r_1 and r_2. For example, if an employer pays all of the employee 's cost but only 40% of the dependent 's cost, then

$$g(1.0, 0.4) = \sqrt{1 \cdot 0.4} \cong 0.6325 \in [0.5, 1.0],$$

from which it can be inferred that the group is an acceptable risk.

One should also take into account the fact that if the necessary employee contribution decreases, then the participation is likely to increase.

H. There is a strong, central administrative function.

Fuzziness arises by the use of the term "strong." Such a word cannot be quantified easily as neither can concepts like "beautiful" or "nice." An underwriter should use his own judgment in assigning a value between 0 and 1 to represent the administrative ability of the group. Some factors to be considered include the size and qualifications of the personnel staff, the function it normally performs, and any knowledge as to previous experience with the staff. Call the developed fuzzy set function h.

I. The industry of the group should be considered.

If the underwriting department has a list of unacceptable industries, then any group in one of those industries would have a membership value of 0 in the fuzzy set of groups in industries that can be underwritten. Similarly, a group in an industry on a list of questionable organizations has a membership value between 0 and 1. Call the corresponding fuzzy set function i.

J. The policyholder has a good credit rating.

Dunn and Bradstreet credit reports or financial statements of the prospect can be examined in order to determine whether or not the prospect is a good credit risk. Based upon the willingness of one 's company to accept risk, develop a fuzzy set function to correspond to this criterion and call it j.

K. On-going claims are not large as a proportion of expected claims.

The number and size of on-going claims that can be tolerated depend upon the group size and the nature of the claims. An acute condition may be resolved in a relatively short time, whereas a chronic complaint will continue much longer. The developed function presupposes the following: The expected payments for on-going claims should be less than 1/2% of the total expected paid claims in order for a group to be preferred. For a normal group, the percentage should lie between 1/2% and 1 1/2%. If the percentage of on-going claims is above 2%, then the group is unacceptable. Let c equal the percentage of on-going claims as a proportion of the total expected claims:

$$k(c) = \begin{cases} 1, & c \le \frac{1}{2}, \\ -\frac{1}{2}c + \frac{5}{4}, & \frac{1}{2} \le c \le 2\frac{1}{2}, \\ 0, & 2\frac{1}{2} \le c. \end{cases}$$

(Note that 100 times the decimal representation of c is used to evaluate k; e.g., if c = 3/4%, then 0.75 is used to determine k.)

L. The past claim experience of the group has been good.

Let LR equal the previous year's loss ratio and s the group size. Here, the loss ratio refers to incurred claims divided by expected claims for the given time period, i.e.,the actual to expected experience. The inclusion of retention in the denominator distorts the loss ratio because retention, as a percentage of gross premium, varies with the size of the premium:

$$L_1(LR) = \begin{cases} 1, & LR \le 0.95, \\ -5\,LR + 5.75, & 0.95 \le LR \le 1.15, \\ 0, & 1.15 \le LR. \end{cases}$$

In this function, a group is preferred if its loss ratio is 95% or less. An acceptable group has a loss ratio between 95% and 105%. If a given company 's underwriting guidelines are looser than the above indicates, then the intervals should be broadened.

The size of the group should be taken into account because the loss ratio is less indicative of future claims for smaller groups than for larger ones:

$$L_2(s) = \begin{cases} \sqrt{\dfrac{s}{500}}, & 0 \le s \le 500, \\ 1, & 500 \le s. \end{cases}$$

Another possible function for L_2 could be derived from a credibility table. The two functions are combined as follows:

$$L(LR, s) = L_1(LR) \cdot L_2(s) + (1.0 - L_2(s)).$$

Note that L(LR, s) is a weighted average of L_1(LR) and 1.0 with L_2(s) as the weight. (The capital "el" is used here to avoid confusion with the symbol for the number one.)

M. The group has a low turnover rate with respect to carriers.

Let n equal the number of carriers the group has had in the past five years:

$$m(n) = \begin{cases} 1, & n = 1, \\ 0.5, & n = 2, \\ 0, & n \ge 3. \end{cases}$$

Again, in practice, the actual function defined should reflect a company's underwriting guidelines.

The remainder of this section deals with the problem of combining the above rules so that a decision can be made as to the acceptability of the group. In other words, a single fuzzy set is created to describe the set of good risks. It is assumed that criteria A, B, C, and D are satisfied from the outset; therefore, they are not considered further in this section.

The first question to contemplate is whether or not a 0 in any one of the categories leads to outright rejection of the group. If not, then it may be appropriate to include the operation of convex combination together with intersection; otherwise, operations of intersection alone should be employed. For the sake of argument, assume that a grade of 0 in any of the following rules does not automatically disqualify a group from being insured:

H. There is a strong, central administrative function.

I. The industry of the group is acceptable.

K. The on-going claims are not large.

The manual rates can be loaded to take into account the effects of criteria I and K, and an extra margin for expenses can be added to compensate for any deficiency concerning criterion H. Create the linear combination:

$$P = \frac{1}{6}h + \frac{1}{3}i + \frac{1}{2}k.$$

By the choice of weights, the amount of on-going claims is viewed as somewhat more important than industry, whereas strong administration is deemed less important than either. P can be intersected with the remaining criteria in many ways as seen in Section 3.2. The fuzzy set used in making the decision of whether or not to accept the group may have the following form:

$$Q = P * e * f * g * j * L * m,$$

in which * is the selected intersection. Two options are presented below:

4.1. Option

$$Q_1 = P \cap e \cap f \cap g \cap j \cap L \cap m,$$

in which $\cap$ denotes the minimum operator. Note that a group which is preferred in each category is preferred in total. Since the minimum operator does not allow for interaction among the variables, the relations among them are missed.

So that the interactions among the criteria might be determined, the following matrix is developed:

Interaction between Criteria	P	e	f	g	j	L	m
P					some		little
Flow of lives, e			some				
Participation, f		some	imp 't	max			
Emp 'r contribution, g			max				
Good credit, j	some						little
Loss ratio, L							
Turnover, m	little				little		

The label of "some " will be represented by the Hamacher operator with p = 0.5; "little, " by the Hamacher operator with p = 0; "max, " by the algebraic product; and no interaction, by the minimum operator. Finally, "imp 't " denotes that the criterion is important and should be given special emphasis.

One possible fuzzy set function representing the above table is the following:

4.2. Option

$$Q_2 = [H(H(P,j;0.5),m;0)]^{1/3} \bigcap [f \cdot g]^{1/2} \bigcap [H(f,e;0.5)]^{1/2} \bigcap [L'],$$

in which a acts to change the importance of the fuzzy set representing the loss ratio. The less important it is, the less the value of a should be.

In order to develop this function, the variables are first grouped according to whether or not there are any interactions among them. The variables are partitioned as follows: {P, j, m}, {e, f}, {f, g}, and {L}. Note that there is not a true partition because f appears in two distinct subsets.

Since P and j have some interaction and each has little interaction with m, P and j are first combined via the Hamacher operator with p = 0.5, then the result is joined with m using the Hamacher operator with parameter 0. The cube root of the outcome is taken in order to negate the effect of the product $P \cdot j \cdot m$ and make the result comparable to other terms that involve fewer than three factors. If, instead, the interaction between P and m had been "some" and not "little," then one could have first combined m and j via the Hamacher operator with p = 0, then joined that result with P using the Hamacher operator with parameter 0.5.

As an aside to the above observation, please note that the Hamacher operator is not associative if the parameter changes, i.e.,

H(H(A,B;p),C;q) is not necessarily equal to H(A,H(B,C;q);p).

This inequality can be seen algebraically by taking $f_B \equiv 1$; in this case, the left-hand side is equal to H(A,C;q), while the right-hand side is equal to H(A,C;p). For a general fuzzy set B, if $p = 0.5$ and $q = 0$, then a qualitative interpretation of the left-hand side is that A and B have some interaction and both have little interaction with C. Similarly, the right-hand side can be construed to mean that B and C have little interaction and both have some interaction with A. From this perspective, there is no reason to expect that the two sides would be equal.

Because f and g have maximum interaction, they are intersected through the algebraic product. The square root of the product is used in order to make the term commensurate with the others.

The function f and e have some interaction; therefore, they are combined via the Hamacher operator with $p = 0.5$. Again, the square root of the result is employed in the end. The importance of f is reflected in its appearance in the above two terms. Alternately, the concentration CON(f; 2) could have been used in either or both of the two terms in place of f, or one could have included an extra term of the form CON(f; 2), in which the importance of f is made explicit. Ambiguity arises in this example because e and g are each related to f but not to each other.

Since L does not interact with any of the other variables, it is a term in the final intersection by itself. Its value, however, can be adjusted through the operations of dilation or concentration in order to reflect the significance of the loss ratio. The degree of dilation or concentration is influenced by any considerations of credibility not taken into account by the variable of group size, e.g., a high turnover rate within the plan.

Finally, the four terms, $H(H(P,j;0.5),m;0)^{(1/3)}$, $(fg)^{(1/2)}$, $H(f,e;0.5)^{(1/2)}$, and L^*, are intersected through the minimum operator. This operator is utilized because the four terms do not interact, except by means of f.

In general, the cutoff point for choosing or not choosing a group depends upon the function selected for Q. An alternative to using a particular number between 0 and 1 is to implement a fuzzy decision scheme. For example, if Q lies between 0.75 and 1.0, then the group is definitely acceptable. If Q is in the range from 0.50 to 0.75, then the group is most likely acceptable, and if Q is less than 0.25, then the group is definitely unacceptable. Otherwise, if Q lies between 0.25 and 0.50, then the discretion of the underwriter is to be relied upon. This outline roughly follows the concepts of preferred, normal, substandard, and unacceptable risks introduced at the beginning of this section.

4.3. Example Consider a group with the following characteristics:

Group size, s	250
Age/sex factor change, a/x	10%
Size change, g/s	-15%
Participation, p	85%
Emp'r contribution, r_1/r_2	100%/40%
Strong administration	h = 0.9
Industry	i = 1.0
Credit rating	j = 0.95
On-going claims, c	0.75%
Loss ratio, LR/a	1.05/0.75

Number of carriers, n 1

Using the fuzzy functions presented above, it follows that:

$$Q_2 = \min[0.8765^{1/3}, 0.4743^{1/2}, 0.4827^{1/2}, 0.6464^{3/4}]$$
$$= 0.6887.$$

Since $Q_2 = 0.6887$ lies between 0.50 and 0.75, the group is most likely acceptable. The employer may be interested in knowing how to improve his group 's acceptability. Such information can be determined by examining the function $\sqrt{(fg)}$ because its value yields the minimum Q_2. The contribution made by the employer to the single rate is 100%, so g_1 cannot be improved. In order to measure the effect of an increase in the participation rate or the employer contribution to the dependent 's cost, consider the following first partial derivatives:

$$\frac{\partial}{\partial p}\sqrt{fg} = \frac{5}{2}\sqrt{\frac{g}{f}} = 2.2958.$$

$$\frac{\partial}{\partial r_2}\sqrt{fg} = \frac{1}{4}\sqrt{\frac{f}{g}} \cdot \sqrt{\frac{g_1}{g_2}} = 0.4305.$$

The greater betterment in $\sqrt{(fg)}$ comes from an increase in participation rate because 2.2958 > 0.4305. On the other hand, it is likely that this change may be obtained most easily through a higher employer contribution. Suppose the variable $r_2 = 0.50$ and, as a result, $p = 0.90$, then:

$$Q_2 = \min[0.8765^{1/3}, 0.7071^{1/2}, 0.6124^{1/2}, 0.6464^{3/4}] = 0.7209.$$

The above process may be continued by considering ways in which the importance of L can be lessened; e.g., implement changes in plan design that will affect utilization or a strong pre-existing condition exclusion.

There is another use for the first partial derivatives inherent in the above discussion, and that application is sensitivity analysis. In addition to calculating Q for various scenarios to determine its appropriateness, the partial derivatives can be evaluated to see if Q's sensitivity to the variables follows the underwriting guidelines it is to embody.

Such evaluation would be important if there were several factors that interact intricately. For instance, suppose there are four variables that represent fuzzy sets, A, B, C, and D. Assume that they interact as follows:

Variable	A	B	C	D
A		some	little	some
B	some		some	little
C	little	some		some
D	some	little	some	

Ambiguity exists because B and D have some interaction with each of A and C, but B and D have little interaction with each other, as do A and C. One possible representation of these relationships is as follows:

$$H(H(A,C;0),H(B,D;0);0.5).$$

The vagueness of this example emphasizes the importance of verifying that the chosen functions characterize the qualities correctly and that the combination of those functions accurately reflects the given underwriting process.

5. FUZZY SET MODEL FOR UNDERWRITING MULTIPLE-OPTION PLANS

The factors that receive the most attention in the underwriting of multiple-option plans are those that affect participation levels. They include the level of access to care, the out-of-pocket cost, the plan of benefits, and the age, sex, and dependent coverage of the employee. These factors are also at work in single-option selection because an employee has a choice of whether or not to take the coverage. Also the existence of a working spouse 's plan and that plan 's benefits, out-of-pocket expense, and access to care influence an employee 's decision. It is nearly impossible to underwrite against shadow plans; therefore, the underlying factors that affect participation in single-option plans are not considered. They are more visible, however, in multiple-option cases, because the underwriter is aware of the competition.

The level of access to care encompasses the size of the provider panel relative to the provider population in the area, the geographic distribution of the panel relative to the location of the employees, the operating hours of the panel, the particular gate-keeper mechanism used, and the ease of obtaining referrals. In general, a staff model or small, closed panel Health Maintenance Organization (HMO) with tight controls has restricted access. Moderate control is found in a large, open panel HMO, for example, and loose control in an HMO that is based upon an Independent Practice Association (IPA). Little or no control describes a Preferred Provider Organization (PPO) or an indemnity plan.

The richness of the plan of benefits should be measured relative to the competing plans and should consider the scope of coverage and any copayments, deductibles, and coinsurance. The out-of-pocket expense of the employee is the contribution required by the employer. This contribution may or may not be tied to the benefits and access to care. For example, the employer may contribute a flat dollar amount for each employee--say, the cost of single coverage for the lowest priced option. In this case, the excess paid by the employee would vary with the benefit design and access to care, assuming that the tighter the control, the lower the premium. On the other hand, the employer may seek to direct employees to a particular option by requiring less contribution from the employees for that option.

In that which follows, the above variables are considered in developing an age factor that denotes possible participation in the plan. In order to simplify the presentation of the age factor table, the categories of restricted access and moderate control are combined into one of limited access; the categories of loose and no control, into one of free access. Also, benefit design is limited to either rich or poor; out-of-pocket cost, to high or low. Only three employee age groups are considered: younger (under forty), middle-aged (forty to fifty-five), and older (over fifty-five).

A number between 0 and 1 is associated with each age group given the level of access to care, the plan design, the out-of-pocket expense, and whether the employee has single or family coverage. The number represents the fuzzy-set possibility of the employee's participating in the described plan. Single and family age factors are obtained by forming the weighted averages of these individual factors where the weights are the proportions of employees in the corresponding single and family age brackets. Sex can be taken into account if the composition of a company's block of business warrants doing so. For example, in some parts of the country, a married female employee is more likely to have coverage through her husband 's plan than vice versa.

Access	Benefits	Cost		Single			Family	
			Young	Middle	Older	Young	Middle	Older
Free	Rich	High	0.2	0.5	0.8	0.3	0.6	0.9
		Low	0.9	0.9	0.9	1.0	1.0	1.0
	Poor	High	0.1	0.4	0.7	0.1	0.5	0.8
		Low	0.5	0.6	0.7	0.6	0.7	0.8
Limited	Rich	High	0.2	0.2	0.2	0.1	0.1	0.1
		Low	0.8	0.5	0.2	0.7	0.4	0.2
	Poor	High	0.1	0.1	0.1	0.1	0.1	0.1
		Low	0.7	0.4	0.1	0.6	0.3	0.1

Free, Rich, and High refer to free access, rich benefits, and high out-of-pocket cost. This convention is followed throughout the age table.

The author wishes to emphasize that the above possibility distributions are theoretical in nature only and not based upon empirical data. The following is a list of assumptions used in creating the table:

1) Older people prefer less control to more control regardless the cost because they have established physician relations and want no restrictions as to providers. Within a given level of access, benefits are more important than out-of-pocket expense.

2) Younger people are more interested in low cost options. Within a particular cost bracket, less control and richer benefits are preferred with the latter taking precedent.

3) An employee with family coverage is more interested in easy access to care than one with single coverage.

4) Middle-aged people seek to balance the preferences of younger and older employees.

The health status of an individual also plays a role in selection: Less healthy employees and dependents favor greater access to care, and more healthy ones look for low out-of-pocket costs. In fact, the tacit assumption above is that health status is related to age. For small groups, the health status of individuals may be known, but for large groups, gaining such information would be administratively costly in initial underwriting.

The relative differences among the factors in the various categories should be adjusted based on the actual variations among the plans. For example, if the average difference in the out-of-pocket expenses between two given plans is $50 and between two other plans is $25, then the relativities between the factors in the low and high out-of-pocket categories for the first case should be much greater than for the second case. Indeed, one may wish to represent the level of access to care, the plan of benefits, and the out-of-pocket expense by fuzzy set functions and to vary the participation factors according to the membership values of those fuzzy sets.

5.1. Example There are two plans from which to choose: Plan 1 with free access, rich benefits, and high out-of-pocket costs, and Plan 2 with limited access, rich benefits, and low out-of-pocket cost. The census for the group is as follows:

	Single	Family
Younger	10	20
Middle-aged	20	35
Older	15	15

The single/family age factors for Plan 1 are calculated below:

	Single			Family		
	Emp 'ees	Factor	Product	Emp 'ees	Factor	Product
Younger	10	0.2	2.0	20	0.3	6.0
Middle-aged	20	0.5	10.0	35	0.6	21.0
Older	15	0.8	12.0	15	0.9	13.5
Total	45		24.0	70		40.5

Single factor = 24/45 = 0.533; family factor = 40.5/70 = 0.579.

Similarly, the factors for Plan 2 are:

Single factor = 21/45 = 0.467; family factor = 31/70 = 0.443.

Plan 1 is the favorite; if neither were clearly preferred, then the single and family factors could be combined to obtain one age factor for each plan. One possible combination is the weighted average with the weight equal to the relative premium size, say 2.7 = (family rate)/(single rate). Following this scheme, the age factor for Plan 1 is:

$$(0.533 + 2.7 * 0.579)/(1 + 2.7) = 0.567,$$

and that for Plan 2 is:

$(0.467 + 2.7 * 0.443)/(1 + 2.7) = 0.450.$

Since there is a twenty-six percent difference between the factors, one might expect roughly 5/9 of the participants to chose Plan 1 and 4/9 Plan 2; however, the desirability of a certain level of participation may be different from plan to plan. A given participation may be considered good for Plan 2 because it expects to attract the younger and supposedly healthier lives, whereas the same participation may be less than adequate for Plan 1 because older and presumably sicker lives will be drawn to it. (Percentage participation in the different options is in reference to those employees selecting some type of coverage, and the corresponding age factors also are based upon that subgroup.)

In order to account for varying acceptable levels of participation for different types of plans, a distinct fuzzy set should be defined for each plan to reflect the corresponding desired level of participation. For instance, the following fuzzy set may be created for Plan 1 in the above example:

$$f_{11}(p_1) = \begin{cases} 1, & 0.6 \le p_1, \\ 5p_1 - 2, & 0.4 \le p_1 \le 0.6, \\ 0, & p_1 \le 0.4, \end{cases}$$

in which p_1 is the expected percentage participation in Plan 1. Similarly, for Plan 2, the following fuzzy set may be appropriate:

$$f_{12}(p_2) = \begin{cases} 1, & 0.5 \le p_2, \\ 4p_2 - 1, & 0.25 \le p_2 \le 0.5, \\ 0, & p_2 \le 0.25, \end{cases}$$

in which p_2 is the expected percentage participation in Plan 2. The variables p_1 and p_2 are calculated based upon the participation age factors of the plan. For example, assume that $p_1 = 5/9$ and $p_2 = 4/9$ for the given group.

Now it is time to review the underwriting criteria in Section 2.2 and to define fuzzy sets for them where appropriate.

D1. Benefits within each option are determined automatically.

Since this rule is non-fuzzy in nature, it is assumed that D1 is satisfied from the beginning.

E1. The group should have an average age less than a stated number of years and female participation less than a given percentage. These two rules can be combined into one by requiring that the claim age/sex factor be lower than a given number.

Such a rule is more important for a plan against which anti-selection is more likely. For example, a plan with freer access to care may attract those with higher expected claim costs, as mentioned above. Different plans, therefore, will need distinct fuzzy sets to represent the above criterion.

Let asf be the age/sex factor of the group. Define a fuzzy set representation for Plan 1 by:

$$e_{11}(asf) = \begin{cases} 1, & asf \le 1.1, \\ -5\,asf + 6.5, & 1.1 \le asf \le 1.3, \\ 0, & 1.3 \le asf. \end{cases}$$

Similarly, define one for Plan 2 by:

$$e_{12}(asf) = \begin{cases} 1, & asf \le 1.3, \\ -2.5\,asf + 4.25, & 1.3 \le asf \le 1.7, \\ 0, & 1.7 \le asf. \end{cases}$$

F1. There is a minimum participation when considering all benefit options. In addition, there is a minimum enrollment in the given option.

The discussion at the beginning of this section expounds on this topic, and the rule is mentioned here for the sake of completeness.

N. The difference between the out-of-pocket expenses of the plans is not too great.

The difference may lead to selection against the higher cost plan. Let oop be the average difference in the employee contribution between Plan 1 and Plan 2, in which the average is one between the single and family costs. For the higher cost plan, Plan 1, the following fuzzy set is defined:

$$n_1(oop) = \begin{cases} 1, & oop \le 25, \\ -0.04\,oop + 2, & 25 \le oop \le 50, \\ 0, & 50 \le oop. \end{cases}$$

For the lower cost plan, Plan 2, the fuzzy set is defined to be identically equal to 1; i.e.,

$$n_2 \equiv 1.$$

Note that it is possible to use the same function for both plans by allowing oop to assume negative values.

O. The benefits of one plan are not overly rich in relation to the other(s).

One measure of the relative richness is the ratio of the manual claims of one plan divided by the other, where differences in provider discounts are not considered. Let rr be this ratio, and define the following fuzzy set:

$$o(rr) = \begin{cases} 1, & rr \le 1.2, \\ -2.5\,rr + 4, & 1.2 \le rr \le 1.6, \\ 0, & 1.6 \le rr. \end{cases}$$

As in Section 4, the following matrix will be used to combine the above rules:

Interaction between Criteria	e_{1i}	f_{1i}	n_i	o	Q_2
Age/sex, e_{1i}		some			some
Participation, f_{1i}	some	imp 't	max	max	some
Out-of-pocket, n_i		max			some
Benefits, o		max			some
Q_2	some	some	some	some	

In order to define a fuzzy set function representing the above table, first the variables associated with an option 's characteristics are grouped according to whether or not there are any interactions among them: $\{e_{1i}, f_{1i}\}$ and $\{f_{1i}, n_i, o\}$. Since e_{1i} and f_{1i} have some interaction, they are merged by means of the Hamacher operator with p = 0.5. The square root is taken in order to make the result comparable to the other term.

Because f_{1i}, n_i, and o have maximum interaction, they are intersected through the algebraic product. The cube root of the product is used so that the two terms are commensurate. The importance of f_{1i} is implicit in its appearance in both of the above terms. As in Section 4, this importance could be made explicit through use of the concentration of f_{1i} either as a substitute for f_{1i} in the two terms or as an extra term.

The two terms are combined through the minimum operator, and the resulting function represents an option 's qualities. This function is then joined with Q_2 by the Yager operator with $p = 2$; the Hamacher operator with parameter 0.5 could also have been used. The resulting combination is as follows:

$$R_i = Y(Q_2, [f_{1i} \cdot n_i \cdot o]^{1/3} \cap [H(f_{1i}, e_{1i}; 0.5)]^{1/2}; 2),$$

in which $i = 1,2$ corresponds to the two plans. Note that R_i generalizes the results of Section 4 by having Q_2, the fuzzy set related to group characteristics, embedded within it. The other four functions, e_{1i}, f_{1i}, n_i, and o, depend upon the plans ' characteristics as well as those of the group.

5.2. Example Assume the group is as given in Example 4.3, and use the plans and single/family age distributions from Example 5.1. Let asf = 1.2, oop = 40, and rr = 1.

$$
\begin{aligned}
R_1 &= Y(0.6887,\min[0.6776,0.6417];2) \\
&= 1 - \min\{1,[(1-0.6887)^2+(1-.6417)^2]^{1/2}\} \\
&= 1 - \min\{1,0.4746\} \\
&= 0.5254.
\end{aligned}
$$

$$
\begin{aligned}
R_2 &= Y(0.6887,\min[0.9196,0.8819];2) \\
&= 1 - \min\{1,[(1-0.6887)^2+(1-.8819)^2]^{1/2}\} \\
&= 1 - \min\{1,0.3329\} \\
&= 0.6671.
\end{aligned}
$$

According to the above results, the sponsor of Plan 2 will be more happy about offering his plan against Plan 1 than vice versa.

6. CONCLUSIONS AND AREAS FOR FURTHER RESEARCH

The fuzzy set approach to underwriting presented in this paper generalizes the linear method of debits and credits: For each group characteristic a given number of points are added or subtracted; the resulting number is used to categorize the group and to determine a load or credit on the manual rates for that group. Fuzzy set theory allows for more interaction between the variables in that multiplication is used as well as addition. The flexibility allowed in creating the fuzzy set functions and the variety of ways in which to combine them also add to the attractiveness of this theory.

The mathematics of fuzzy set theory used in this paper is simple enough to allow its ready application. A PC spreadsheet can be easily created to calculate membership values for all the fuzzy functions needed. Testing of the appropriateness of the particular functions chosen and of their combinations would then be straightforward. Since the foundation of the work in this paper is a theoretical one, it would be particularly interesting if someone who has much empirical data relating to multiple-option plans were to fit fuzzy set functions to that data. Other forms of intersection may be more fitting than those given here. Also more multi-variate functions may prove useful in modelling asymmetrical situations. For example, an increase in age/sex factor may be less harmful if the group is growing than if it is decreasing in size.

One immediate extension of this work is the use of the resulting fuzzy set functions as the basis of underwriting loads. Please keep in mind that if the employee 's out-of-pocket expense increases with the load, then the anti-selection that one is trying to guard against may be more likely to occur. It may be more effective in the long run to change the plan design or to require the employer to increase his contribution than to load the rates in anticipation of anti-selection.

Other areas in actuarial science that lend themselves to fuzzy sets are trend analysis in group benefits and credibility theory. Relations among economic indicators, such as the medical Consumer Price Index, and trend many be interesting to explore from the viewpoint of fuzzy sets. In credibility theory, it is usual to base the credibility factor on the exposure months during the experience period. Other variables may also influence credibility, such as turnover within the group, plan design, or chronic vs. acute on-going claims; fuzzy set functions, then, can be defined to represent those associations simply. One particular problem that would make use of both of these ideas is the determination of the renewal increase for the health benefits of a group. A model for this question could be developed in a similar manner as introduced in Example 3.2.13.

7. REFERENCES

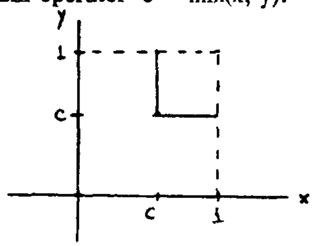
1. Dubois, D. and Prade, H., *Fuzzy Sets and Systems: Theory and Applications*, Academic Press, New York, 1980.
2. Dattatreya, R. E., et al., Risks associated with investing in fixed income securities, in: F. J. Fabozzi et al., eds., *The Handbook of Fixed Income Securities*, 3rd ed., Business One Irwin, Homewood, IL, 1991.
3. Hogan, H., Shades of gray, *The World & I*, (September 1991) 298-303.
4. Kosko, B., Fuzziness vs. probability, *Int. J. General Systems* 17 (1990) 211-240.
5. Lehman, D., Underwriting group insurance, Study Note 320-32-88, Society of Actuaries, 1988.
6. Lemaire, J., Fuzzy insurance, *ASTIN Bulletin* 20 (1) (1990) 34-55.
7. Mailander, E. M., A multioption pricing model, *Health Section News* 21 (1991) 8.
8. Ogden, D. E., Health maintenance organizations, Study Note 320-31-87, Society of Actuaries, 1987.
9. Sangalli, A., Fuzzy logic goes to market, *New Scientist*, (8 February 1992) 36-39.
10. Sutton, H. L., and Nelson, N. F., Underwriting issues for alternative healthcare delivery systems, Study Note 520-38-91, Society of Actuaries, 1991.
11. Zadeh, L. A., Fuzzy sets, *Information and Control* 8 (1965) 338-353.
12. ________, A fuzzy-set-theoretic interpretation of linguistic hedges, *Journal of Cybernetics*, 2:2 (1972) 4-34.

13. ________, The role of fuzzy logic in the management of uncertainty in expert systems, *Fuzzy Sets and Systems* 11 (1983) 199-227.

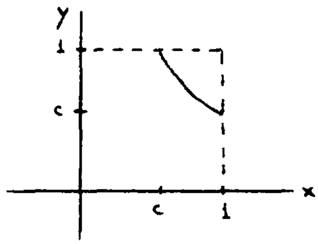
APPENDIX A. LEVEL CURVES OF INTERSECTION OPERATORS

In this section, c will represent a fixed, but arbitrary, number between 0 and 1.

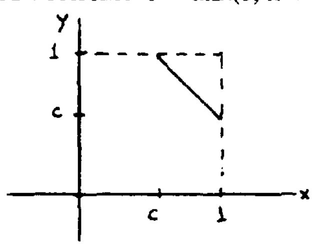
A.1. Minimum operator c = min(x, y).



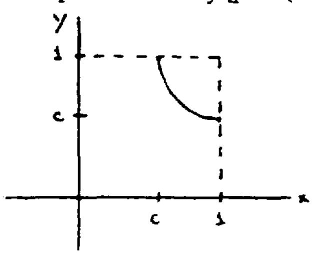
A.2. Algebraic product c = xy.



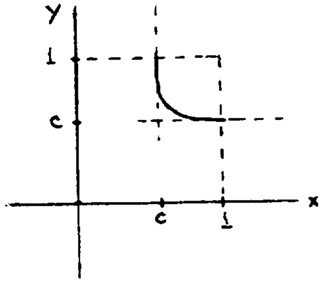
A.3. Bounded difference c = max(0, x + y - 1).



A.4. Hamacher operator c = xy/[p + (1 - p)(x + y - xy)].

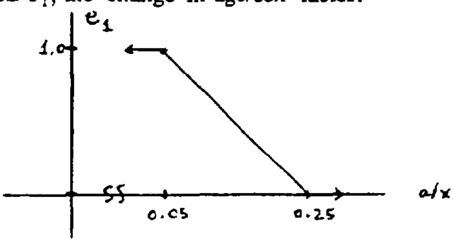


A.5. **Yager operator** $c = 1 - \min(1, [(1-x)^p + (1-y)^p]^{1/p})$.

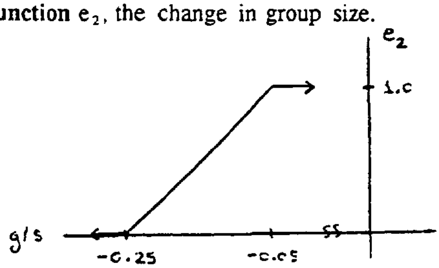


APPENDIX B. GRAPHS OF FUZZY SET FUNCTIONS FOR UNDERWRITING CRITERIA

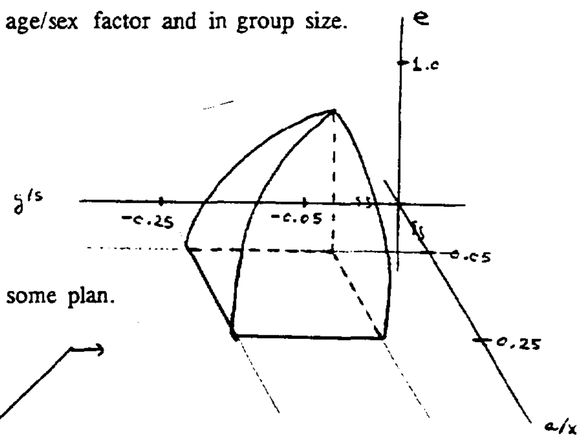
B.1. Function e_1, the change in age/sex factor.



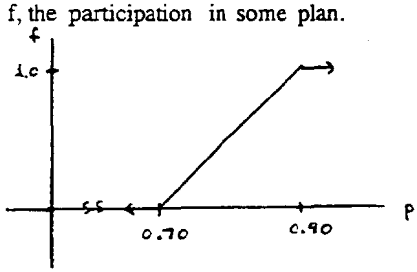
B.2. Function e_2, the change in group size.



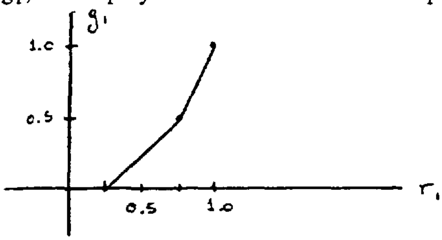
B.3. Function e, both the change in age/sex factor and in group size.



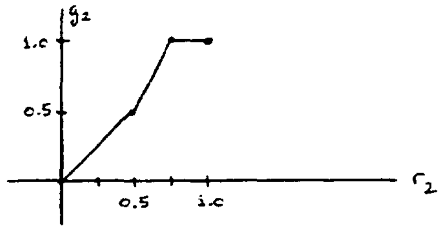
B.4. Function f, the participation in some plan.



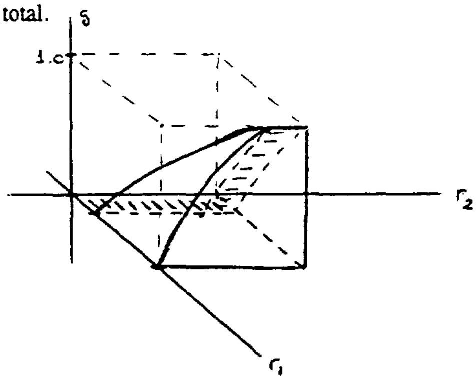
B.5. Function g_1, the employer 's contribution to the employee 's premium.



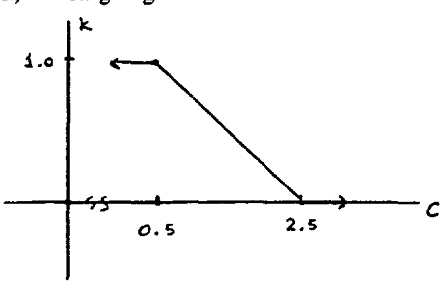
B.6. Function g_2, the employer 's contribution to the dependent 's premium.



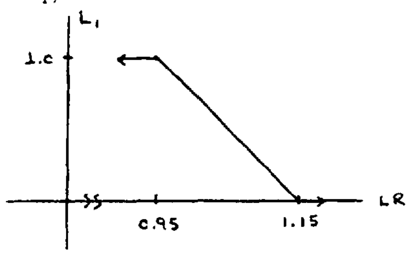
B.7. Function g, the employer 's contribution in total.



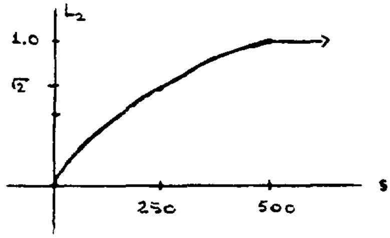
B.8. Function k, the on-going claims.



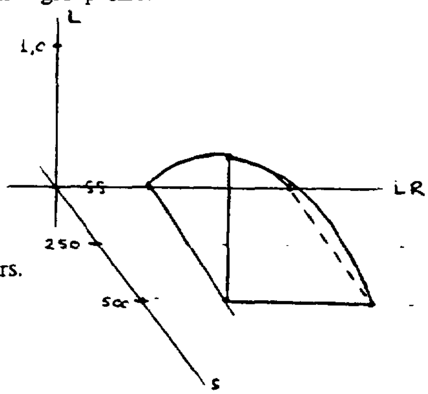
B.9. Function L_1, the loss ratio.



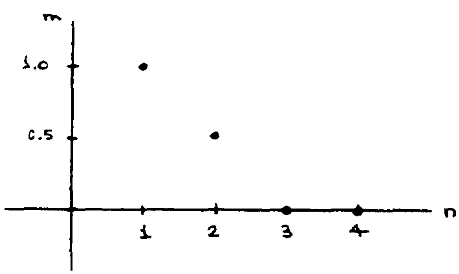
B.10. Function L_2, the group size.



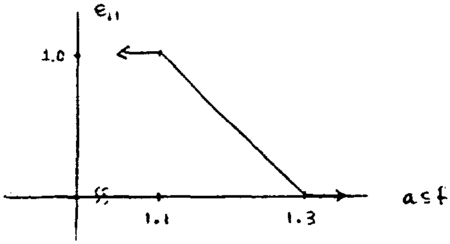
B.11. Function L, the combination of loss ratio and group size.



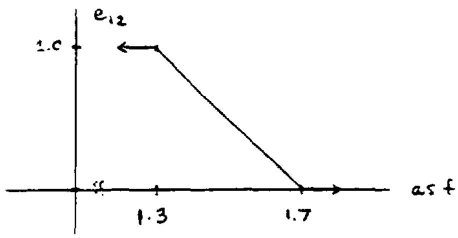
B.12. Function m, the number of previous carriers.



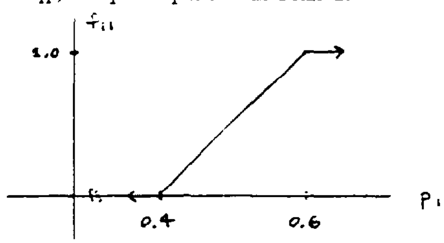
B.13. Function e_{11}, the effect of the age/sex factor on Plan 1.



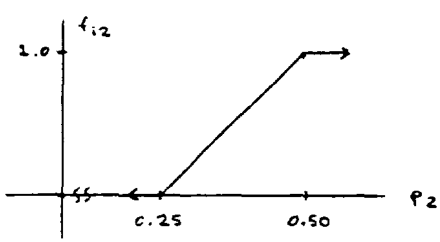
B.14. Function e_{12}, the effect of the age/sex factor on Plan 2.



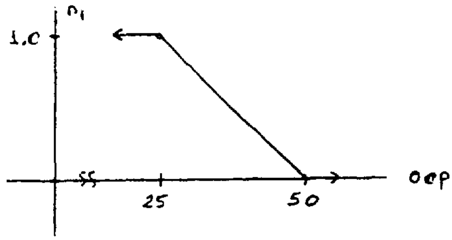
B.15. Function f_{11}, the participation in Plan 1.



B.16. Function f_{12}, the participation in Plan 2.



B.17. **Function** n_1, the out-of-pocket difference.



B.18. **Function** o, the relative richness between plan benefits.