A STEP BEYOND THE "GRAETZ PREPAYMENT ANALYSIS"

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There are errors in the so-called "Graetz prepayment analysis."[1] Those errors result in an understatement of the tax burden borne by mutual life insurance companies. The Graetz prepayment analysis was advanced as a justification for repealing Section 809 of the Internal Revenue Code. Because Section 809 imputes income to a mutual life insurance company by limiting deductions such a company may claim for policyholder dividends it distributes during the taxable year, correcting the errors in the Graetz prepayment analysis strengthens the argument for repeal.

There are two crucially important errors. One is exposed in the language used by Professor Graetz to describe the central point of his analysis and the other is reflected by the numbers supplied in the primary example he used to illustrate the central point of his analysis.

At the core of Professor Graetz' analysis was the following contention:

> If stock and mutual companies are subject to the same constant tax rate, both underwriting and investment income are taxed, and one assumes -- as does the 1984 legislation -- that equity-type returns are identical in the stock and mutual segments of the industry, the tax imposed by including redundant premiums in mutual company income and allowing deduction of policyholder dividends will burden mutual companies in a manner identical *in present value* to the tax burden of stock companies, which exclude shareholder contributions of capital from income but are not allowed any deduction for shareholder dividends.[2]

Five underlying assumptions are explicitly identified by this statement of Professor Graetz' central contention: (1) the same rate of tax is imposed on the

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[1] See *Life Insurance Company Taxation: An Overview of The Mutual-Stock Differential.* It might be noted that Professor Graetz' paper is one of several included in the published proceedings of a conference in March of 1986 that was co-sponsored by the Yale Law School Center for Studies in Law, Economics and Public Policy and by the Metropolitan Life Insurance Company, a mutual life insurer.

[2] *Ibid.*, at page 1-9.

income of a mutual life insurance company as is imposed on the income of a stockholder-owned life insurance company; (2) tax is imposed on both underwriting gain (*i.e.*, the sum of premiums collected) and investment earnings; (3) investments held by a mutual life insurance company yield earnings at the same rate as investments held by a stockholder-owned life insurance company; (4) policyholder dividends are entirely deductible (*i.e.*, Section 809 does not exist to limit the deductibility of policyholder dividends distributed by a mutual life insurance company) while shareholder dividends are entirely nondeductible; and (5) shareholder contributions of capital are excluded from the income of a stockholder-owned life insurance company. On the basis of those assumptions, the core contention can be divided into two parts and restated, as follows:

(I) the amount of tax determined by multiplying the applicable rate of tax times the sum collected from a policyholder by a mutual life insurance company as a redundancy factor incorporated in the gross premium charged is equal, in present value terms, to the amount of tax determined by multiplying the same rate of tax times the sum collected from a shareholder by a stockholder-owned life insurance company as a shareholder contribution of capital; and

(II) the amount available for the mutual life insurance company described here to distribute as a policyholder dividend after payment of its tax is equal, in present value terms, to the amount available for the stockholder-owned life insurance company described here to distribute as a shareholder dividend.

The error exposed by this statement of Professor Graetz' core contention is a misperception that the amount of capital which a mutual life insurance company acquires when it incorporates a redundancy factor in its gross premium equals the amount of the factor so incorporated.

Later in this discussion, an alternative model of the capital acquisition process associated with the incorporation of a redundancy factor in the gross premium is examined. For the moment, however, some of the shortcomings of Professor Gratez' perception should be noted. Professor Graetz himself pointed out that policyholder dividends are more or less universally conceded to be theoretically separable into three elements: one portion reflecting a return on equity; another reflecting "interest" (*i.e.*, a return on *debt*); and the third, a return *of* equity (*i.e.*, a "price rebate" in the form of a return of the redundancy factor or "overcharge" itself). Obviously, if the redundancy factor is "equity capital," it cannot also simultaneously be "debt." It follows that a return on the sum (*i.e.*, a policyholder dividend) could not simultaneously be both "interest" in part and a return on "equity" in part.

Although Professor Graetz did not explicitly confront this point, he may have had it in mind at one point in his analysis. He needed to explain why the entire amount which, after the payment of its taxes, a mutual life insurance company would have available for distribution as policyholder dividends could not in fact be distributed at the end of the same year in which the redundnacy factor was collected. This is the amount comprised of the redundancy factor plus

earnings thereon and alleged to be equal, in present value terms, to the amount available for distribution by a stockholder-owned life insurance company as shareholder dividends. If the entire amount of such available funds were distributed, the company would be left with no addition to capital at all. Presumably recognizing as much, Professor Graetz asserted a need to retain at least some part of the redundancy factor collected in any one year for a sufficiently long time to make it resemble an infusion of equity capital, or, in the Professor's words, to render it "'equity'-like." In effect assuming that equity capital is only temporarily (not permanently) acquired through the medium of premium redundancy, Professor Graetz claimed that:

> The analysis does assume that redundant premiums are "equity-type" contributions of capital and are therefore not returned to policyholders in the same year they are received. If the contributions of capital were repaid to policyholders as dividends by the end of the year in which they were received, the dividend deduction would fully offset the burden of including redundant premiums in income. It does not, however, seem appropriate to assume such an early return of capital in analyzing an "equity-like" return to policyholders. To be "equity-like," the contributed capital should be held at the corporate [sic] level for a considerable period of time and become part of the working capital of the company.

In short, although the redundancy factor might not be equity capital *per se*, so long as it is *temporarily* held by a mutual company, it would nevertheless supposedly be "equity-like."

Such a solution of the underlying dilemma is not entirely satisfactory, of course. It concedes a need ultimately to return the full amount of each redundancy factor to the policyholder. It should be emphasized that what Professor Graetz had in mind was clearly a temporary, *not* an *indefinite*, holding period. In other words, the redundancy factor was not to be retained until the company ceased doing business. Instead, each was to be returned under a particular contract while that contract was still in force. Professor Graetz made no attempt to explain how a mutual life insurance company would go about making determinations of when the moment had arrived to return all the redundancy factors collected under particular policies before they could mature or the insureds under them could die. Apart from the virtual impossibility of making any such determinations accurately, an additional problem remains. If the entire amount of the redundancy factor were truly equity capital, then clearly neither it nor the earnings thereon could justify characterizing a portion of each policyholder dividend as "interest." To resolve the dilemma, one must assume that Professor Graetz perceived the redundancy factor not only as being in some sense "equity-like" (rather than *true* equity) but also as somehow being both "equity-like" and "debt-like" simultaneously. However, no explicit acknowledgement of this point appears in the Graetz analysis.

An additional shortcoming of Professor Graetz' assumption that the "equity-like" sum collected by a mutual life insurance company in the form of a redundancy factor must eventually be returned in its entirety to the policyholder

before the policy matures or the insured dies is that the shareholder contribution with which it is compared does in fact reflect a "permanent" increase in the stockholder-owned life insurance company's capital and surplus account. Such a transparent distinction between the two sums goes far to undermine the alleged equivalence on which the persuasiveness of the Graetz analysis hinges.

The alternative model of the relevant capital acquisition process presented later in this discussion is not subject to any of the shortcomings which burden Professor Graetz' perception of that process. Before turning to that alternative model, however, examination of errors implicit in the primary example supplied by Professor Graetz affords some additional insights which may illuminate a more accurate understanding of some of the issues at hand.

To illustrate his core contention, Professor Graetz supplied the following primary example:

> EXAMPLE A. Assume that both mutual and stock life insurance companies are subject to tax at a 35% rate on both underwriting and investment income. Each raises $100X of "equity" capital, the mutual company through redundant premiums and the stock company through shareholder contributions to capital. Each earns a 20 percent return on the capital invested, which it pays as dividends to policyholders and shareholders respectively.
>
> *Case 1: A Mutual Company:* The mutual company receives $100X of taxable premium income, pays taxes of $35X, leaving $65X after tax which earns an annual return of $13X, all of which can be paid to policyholders as deductible dividends.
>
> *Case 2: A Stock Company:* The stock company receives $100X in excludable shareholder contributions to capital, which earns $20X annually. Since shareholder dividends are not deductible by the company, this return will be subject to tax of $7X (35 percent of $20X) when distributed and shareholders will receive $13X in dividends, an amount identical to that received by the mutual company's policyholders.[3]

In the initial paragraph describing the assumptions made for purposes of the example, Professor Graetz claimed that the mutual company "raise[d] $100X or 'equity' capital .. through redundant premiums." However, in the paragraph labeled "Case 1," he asserted that the $100X collected by the mutual company was reduced by "taxes of $35X, leaving $65X after tax which earns [investment income]." Clearly, collecting the redundant premium increased the mutual company's capital and surplus account (i.e., the sum of assets on which investment income could be earned) by $65X, *not $100X*. That is, while the stockholder-owned company in the example did indeed increase the size of its capital and surplus account by $100X worth of additional equity capital as a

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3   *Op. cit.*, at pages 1-9 and 1-10.

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result of collecting the shareholder contribution, the mutual company only acquired $65X worth of additional equity capital as a result of having collected the redundant premium. In order to have increased the sum of its equity capital by the same amount which the stockholder-owned company added to its equity capital, the mutual company would have had to collect a redundant premium equal to approximately $153.85X. Clearly, there is something wrong either with the characterization of the $100X as equity capital or with Professor Graetz' perception of how capital is acquired through the medium of premium redundancy.

This point can be approached from a different perspective. The assumed rate of tax is applied for a particular taxable year. Hence, it is in fact a rate *per centum per annum*. Likewise, the rate at which investment income is earned on either company's equity capital (*i.e.*, its capital and surplus account) is applied to determine an amount of income in a particular year and, hence, it too is a rate *per centum per annum*. Because Professor Graetz subtracted the $35X amount of the tax payment *before* calculating earnings on the remaining $65X and because $65X invested at the specified rate of 20 *per centum per annum* can only earn $13X if it is invested for an *entire* year and since taxes are not collected at the *beginning* of a taxable year, it is clear that Professor Graetz was not comparing the tax burden borne by the mutual company for *a* taxable year to that borne by the stockholder-owned company for that *same* taxable year. The $100X amount of premium redundancy and the $35X tax thereon must have been collected and paid, respectively, *before the instant taxable year began*. Thus, Professor Graetz was comparing the tax burden borne by the mutual company for *two* taxable years (*i.e.*, the instant year and the preceding one) to the tax burden borne by the stockholder-owned company for *one* taxable year (*i.e.*, the instant one only). If Professor Graetz had compared the mutual company's combined tax burden for the year during which it collected the $100X plus that for the ensuing year with the stockholder-owned company's combined tax burden for the first two years during which it had use of its $100X shareholder contribution, then a greater burden would clearly have been borne by the stockholder-owned company since it would have earned two years' worth of investment earnings subject to tax. Likewise, after paying its taxes, the stockholder-owned company would have been able to distribute more to its shareholders for the two years than the mutual company would have been able to distribute to its policyholders after payment of its taxes for those same two years.

An additional, though perhaps less troublesome, concern posed by Professor Graetz' analysis is that it relies on an assumption that the so-called "social discount rate" is always exactly equal to the average rate of earnings collected on all assets held by stockholder-owned life insurance companies. While that assumption seems farfetched, for the sake of easy comparison with the Graetz analysis it is also made with respect to the model capital acquisition process described later in this discussion.

Mistaken views regarding *borrowed* funds also appear in the Graetz paper. In the part of the paper captioned "Company-Level Treatment of the Interest-Type Component of Policyholder Dividends," Professor Graetz asserted that:

With respect to the component of mutual policyholders' dividends that is asserted to be analogous to an interest-type return, most analysts suggest that there is no mutual company advantage because both the interest payments to bondholders by stock companies and any interest component of mutual policyholder dividends would be fully deductible in the absence of a §809-type addition to income for mutuals. However, a prepayment analysis, similar to that of the prior section, reveals a company-level problem. Redundant premiums charged policyholders by mutual companies will be included in their taxable income, but loans to stock companies from bondholders will be excluded from the company's income. Thus, even with full deductibility of the "interest-type" component of policyholder dividends, mutual companies will experience a company-level disadvantage (vis a vis stock companies) when they borrow from policyholders by charging redundant premiums and pay policyholders interest through policyholder dividends.

This argument is clearly overstated. It again assumes that the redundancy factor included in the gross premium of a participating life insurance contract ineluctably *must*, in its "debt-like" capacity, be "lent" by the policyholder to the insurer for a period in excess of a year.

Under the present-value analysis set out later in this discussion, a policyholder dividend is depicted as what remains of the investment income of a seller of participating life insurance contracts at the end of a taxable year after necessary additions to reserves have been funded, surplus charges have been retained, and income taxes have been paid. As is explained below, perceiving *surplus charges* retained at the end of the period as contributions of equity by policyholders to the insurer is what allows part of the policyholder dividend to be characterized as a return on equity. That return on equity reflects the price the insurer pays to the policyholder for use of extracted capital. As also explained below, its amount can theoretically be calculated by multiplying the sum of surplus charges allocable to a particular contract times the mean rate of interest actually earned during the accounting period on all assets held by the insurer. In order to calculate the amount of an additional part of the policyholder dividend to be characterized as "interest" paid by the insurer to the policyholder for the use of some amount *lent* by the latter to the former, it can again be assumed that the rate at which relevant "interest" must be paid also equals the mean rate of interest actually earned during the accounting period on all assets held by the insurer. Such an assumption allows the sum of the "interest" and "price rebate" components of the policyholder dividend to be viewed as equal to the sum of an amount borrowed plus interest thereon. Having assumed that the appropriate rate at which the "interest" component should be deemed to have been earned for the year in question equals the rate of interest earned on all assets held by the insurer during the year thus allows the amounts of both components to be deduced mathematically. That is, R, the amount of the "price rebate" component of the policyholder dividend, equals the product of the sum remaining after the "return on equity" component has been subtracted from the amount of the policyholder dividend multiplied times the inverse of one plus I_y, the mean rate of interest actually earned during the year

on all assets held by the insurer. To reflect a genuine price rebate, the amount concerned must come into the hands of the insurer as part of the "price" collected from the policyholder at the beginning of the accounting period. Of course a redundancy factor incorporated in the amount of the gross premium satisfies that requirement.

Since it is subsequently returned to the policyholder at the end of some particular year, it can be viewed as having been temporarily (and conditionally) "lent" to the insurer for use, if necessary (*i.e.*, in the event of unfavorable interest-rate or mortality experience), to defray the cost of coverage during the span of time over which it is held. Since a "price rebate" is deductible under §162 as an ordinary and necessary business expense, the net after-tax consequence of collecting the sum in question as part of the gross premium at the beginning of the year and then rebating it to the policyholder at the end of the *same* year is identical to the tax treatment accorded funds borrowed by a corporation through the issuance of bonds (*i.e.*, since the inclusion of the amount as part of premium income is entirely offset by the deduction of the amount as a business expense, the amount "borrowed" is, in effect, excluded from the borrower's tax base). Hence, contrary to Professor Graetz' assertion quoted in the excerpt above, mutual companies do *not necessarily* experience any "company-level disadvantage (*vis a vis* stock companies) when they borrow from policyholders by charging redundant premiums and pay policyholders interest through policyholder dividends."

For exactly the same reason, an argument sometimes made that the technique of so-called "upstream debt financing"[4] evidently used on occasion by certain affiliated groups of stockholder-owned life insurance companies is comparatively tax-favored over borrowing from policyholders through the medium of premium redundancy can also be overstated. The discussion which follows, however, concentrates exclusively on the inaccuracy of Professor Graetz' model of the capital acquisition process.

One final observation regarding Professor Graetz' statement of his core assertion should be made. His explanation for the supposed present-value equivalence of tax burdens allegedly illustrated in his primary example was that "mutual companies in effect will -- at the time redundant premiums are received

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[4]   This technique involves borrowing by an intermediate holding company which conveys loan proceeds by gift to subsidiaries that invest such proceeds and use earnings thereon to pay 100% dividends to the holding company which in turn uses those dividends to pay the interest under the loan. If the loan agreement specifies a floating rate (*e.g., prime plus a specified number of basis points)* to be redetermined periodically, then, during a period of inflation, any time lag between an increase in the rate of earnings on the invested loan proceeds and a redetermination of the rate of interest payable under the loan agreement would give rise to arbitrage from which real increments to the capital and surplus accounts of the subsidiaries could be extracted. Returns filed on a consolidated basis allow deductions to the entire group for interest paid under the loan agreement and increments to gross income only occur when capital is in fact extracted by the subsidiaries.

-- 'prepay' the corporate tax that stock companies bear through nondeductibility of shareholder dividends." Obviously, however, taxes cannot legitimately be described as having been "prepaid" simply by pretending that they were collected at the beginning of the year rather than at the time(s) specified by law.

A More Accurate Model Depicting How Capital Is Acquired When A Redundancy Factor Is Included In The Gross Premium

At the time a redundancy factor incorporated in the gross premium charged for a participating life insurance contract is collected, the company collecting cannot know with certainty whether adverse interest-rate or mortality experience during the year might require using some of the "cushion" afforded by that factor (and earnings received during the year on the factor) to defray actual costs of coverage. Only at the end of the year can the company know how much of the factor (and earnings thereon) really was "redundant" in retrospect. Consequently, it seems intuitively obvious that the amount from which capital can be acquired through the medium of premium redundancy must be a dynamic sum which fluctuates from year to year with fluctuations in actual interest-rate and mortality experience. It also seems intuitively obvious that, in order to reflect a genuine addition to a company's capital and surplus account, any capital which is acquired through the medium of premium redundancy must be *permanently retained* by the company. A model of the process by which capital is acquired through the medium of premium redundancy that is compatable with both of these intuitively obvious requirements might be based on the description of how capital is acquired by mutual life insurance companies that was published some years ago by the Canadian actuary Robin Leckie.[5]

Assuming the strategy a life insurance company adopts for preserving itself in the long-run is to attempt to maintain some constant surplus ratio,[6] it is a relatively simple matter to trace fluctuations in the retrospectively determined actual extent of redundancy of the gross premium it charges for coverage under either participating contracts or nominally nonparticipating contracts which nevertheless distribute the equivalent of policyholder dividends. Obviously, management discretion influences the selection of the constant that serves as the target surplus ratio. Likewise, management discretion can influence how the company's assets and actuarial liabilities are evaluated. In other words, the underlying definition of "surplus" itself can be influenced by management discretion.[7]

With respect to a particular contract, the total amount of assets in excess of reserve requirements that is on hand at the end of a taxable year obviously

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[5]   See *Some Actuarial Considerations For Mutual Companies*, Robin B. Leckie, *Transactions of the Society of Actuaries (TSA)* 31 (1979).

[6]   That is, the ratio of some measure of the company's surplus to some measure of its liabilities.

[7]   See *Leckie, ibid.*, at page 190.

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includes assets accountable for as the redundancy factor incorporated in the gross premium plus earnings collected during the year as a result of the investment of that redundancy factor. This amount (in effect, the sum of "retained earnings" for the year) is disposed of in three ways. Part is used to pay income taxes. Another part (in the aggregate, so-called "divisible surplus") is distributed as policyholder dividends. The third part (the so-called "surplus charge" or "policy contribution to surplus") is the sum which is added to the company's capital and surplus account. The focus of both Section 809 and the Graetz prepayment analysis is obviously on the allocation of the total amount of assets in excess of reserve requirements into the three parts just described. In effect, Section 809 regulates the division of after-tax "profit" between a company share and a policyholders' share. The aim of the Graetz prepayment analysis was to demonstrate that, if Section 809 did not exist, the tax treatment of the allocation of profit between a mutual life insurance company and its policyholders would guarantee that the company bore a tax burden identical in present value to that borne by a stockholder-owned life insurance company as a result of receiving a shareholder contribution equal in amount to the sum of equity capital the mutual company acquired as a result of incorporating in its gross premium a redundancy factor equal in amount to the shareholder contribution.

For the sake of simplicity, the discussion which follows immediately below draws a relevant present-value comparison for a *single year's* addition to capital.

In order to make a present-value comparison of the kind Professor Graetz sought to make, it is necessary to estimate appreciation in the value of the amount of tax initially collected from a life insurance company when it retains surplus charges. The base amount of tax initially collected from the company equals the product of the rate of tax (t) multiplied times the sum of surplus charges in question, which will, as of the time they are retained, forever thereafter remain a constant (C). This product, $(t)(C)$, likewise forever thereafter remains a constant. In any particular subsequent year, appreciation in the implicit value of the tax collected will equal the product of the constant $(t)(C)$ plus any theretofore accumulated appreciation multiplied times the social discount rate[8] for the year (S_y).

On the side of the comparison where capital is acquired through shareholder contribution, taxes are collected on the full amount of current-year earnings. It is assumed here that the entire sum of after-tax earnings is distributed annually as shareholder dividends.[9] Thus, the amount of tax

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[8]   For a discussion of what the social discount rate measures, see the chapter captioned "On the Discount Rate For Public Projects" by William J. Baumol, included in *Public Expenditure and Policy Analysis*, Haveman and Margolis, Rand McNally College Publishing Company, Chicago, 2d Edition.

[9]   This assumption is arbitrary, to be sure. However, if any earnings are permanently retained, they would add to the operating assets of the company in the same fashion retained surplus charges do. In other words, an inequality in the amount of capital would arise and would render the instant comparison

collected in any year must equal the product of the amount of capital initially received through shareholder contribution (a constant equal to the amount acquired through the medium of premium redundancy by the mutual company at the end of the preceding year as surplus charges, C) multiplied times the mean interest rate earned during the current year (I_y) and also multiplied times the rate of tax (t). In addition to the nominal value of the tax collected during the current year, appreciation in the value of taxes collected in previous years must also be taken into account. Once again, that appreciation would be measured through use of the social discount rate. An algebraic illustration may be helpful.

CHART A. Annual comparisons between the cumulative appreciation in the social value of the initial tax collected from a company upon its extraction of surplus charges and the current tax paid by a company on earnings on capital received as a shareholder contribution plus the cumulative appreciation in the social value of taxes collected in previous years from it, stated in generalized algebraic terms.

Year	Company Extracting Surplus Charges	Company Receiving Shareholder Contributions
1	$(t)(C)$	0
2	$[(t)(C)] (1+S_2)$	$(t)[(C)I_2)]$
3	$[(t)(C)] (1+S_2)(1+S_3)$	$(t)[(C)(I_3)] + [(t)(C)(I_2)](1+S_3)$
4	$[(t)(C)] (1+S_2)(1+S_3)(1+S_4)$	$(t)[(C)(I_4)] +$ $[(t)(C)(I_2)](1+S_3)(1+S_4) +$ $[(t)(C)(I_3)](1+S_4)$
*	*	*
*	*	*
*	*	*
y	$[(t)(C)] \prod(1+S_2)(1+S_3) ...$ $(1+S_{y-1})(1+S_y)$	$\sum \{(t)[(C)(I_y)] + [(t)(C)(I_2)]$ $\prod(1+S_3)(1+S_4) ... (1+S_y) +$ $[(t)(C)(I_3)]$ $\prod(1+S_4)(1+S_5)...(1+S_y) + ...$ $[(t)(C)(I_{y-1})](1+S_y)\}$

Subtracting the amount identified in the case of the company acquiring capital as shareholder contributions from the amount identified in the case of the company acquiring capital through extraction of surplus charges invariably leaves the amount $(t)(C)$ remaining, regardless of whether interest rates remain stable, increase, or decline during the span of time over which the comparison

useless for purposes of evaluating the equity of retaining a restriction like that imposed under Section 809.

is made. In other words, the difference between the two amounts is always equal to the amount of the tax initially imposed on the company extracting surplus charges at the time it extracts those charges.[10] Thus, rather than the identity of burden posited by Professor Graetz, the tax burden of a company acquiring capital through the medium of premium redundancy is higher than that borne by a company acquiring the same amount of capital as a shareholder contribution.

It should be recalled that the foregoing comparison of the present values of taxes collected from companies employing the two alternative means of capital acquisition only involves *one year's* addition to capital. For every year in which a further addition to capital was made, another increment to the disparity in comparative tax burdens would result.

Going One Step Beyond The Graetz Prepayment Analysis

The preceding discussion examined the comparative tax burdens associated with two different methods of capital acquisition. However, it is clear that not only mutual life insurance companies but also stockholder-owned life insurance companies acquire capital through the medium of premium redundancy. Mutual companies do so using true participating contracts while stockholder-owned companies may do so using nominally nonparticipating contracts under which the equivalent of policyholder dividends are distributed. The tables which follow in the remainder of this discussion compare pairs of such contracts under the following various interest rate assumptions:

      (A) a stable rate slightly higher than the one used for pricing and reserve purposes;

      (B) a stable rate equal to the one used for pricing and reserve purposes;

      (C) a uniformly but gradually increasing rate, beginning at the rate assumed for pricing and reserve purposes;

      (D) a uniformly but gradually decreasing rate, again beginning at the rate assumed for pricing and reserve purposes;

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    [10]   For example, at the end of Year 3 the mutual company has accumulated an amount equal to $tC + tCS_2 + tCS_3 + tCS_2S_3$ and the stock, an amount equal to $tCI_2 + tCI_3 + tCI_2S_3$. If it is assumed that for any year (y) I_y always equals S_y, then subtracting the latter from the former leaves an amount equal to tC remaining.

(E) an oscillating rate uniformly moving between one full percentage point above and one full percentage point below the rate used for pricing and reserve purposes;[11] and

(F) another oscillating rate which crosses the axis set by the rate assumed for pricing and reserve purposes the same number of times as the rate in the previous table but which uniformly moves between *three* full percentage points above and *three* full percentage points below the rate used for pricing and reserve purposes.[12]

The following general assumptions also apply to all twelve of the examples:

(1) the amount of the benefit payable by the insurer at maturity or upon the death of the insured under the terms of the contract is $10,000X;

(2) the statutory rate of interest used for purposes of calculating required reserves is 6% per year;

(3) the contract is expected to, and does, remain in effect for thirty years;

(4) the net premium determined on the basis of the preceding assumptions is $119.33X per year;

(5) the gross premium charged by a hypothetical insurer selling only participating contracts is $120X (reflecting the net premium of $119.33X plus a redundancy factor of $0.67X) and the gross premium charged by a hypothetical insurer selling only nonparticipating contracts is $119.75X (reflecting the net premium of $119.33X plus a redundancy factor of $0.42X);

(6) agents' commissions, any other acquisition costs, administrative costs, adjustments to accommodate the risk of lapse or any similar risk, and all overhead costs of any kind are ignored and all assets held by the company are assumed to be "admitted" assets; and

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[11] Thus, the mean rate over the contract's thirty-year span equals the rate used for pricing and reserve purposes. This oscillating rate is used in order to demonstrate that there are what might be described as "frequency" effects on the respective financial outcomes generated by the contracts compared. In other words, the amounts of capital acquired and policyholder dividends paid are not equal to those associated with a stable rate of interest.

[12] In this case also, the mean rate over the thirty year span equals the rate used for pricing and reserve purposes. This oscillating rate is used in order to demonstrate the existence of what might be described as "wavelength" effects on the respective financial outcomes generated by the contracts compared. In other words, wider swings in the interest rate influence the amounts of capital extracted and policyholder dividends paid.

(7) in each year, the insurer selling only participating contracts retains as much of any excess of assets generated by investments during the current year over the required addition to reserves for the current year as is necessary in order to maintain, or to resume maintaining, a constant target ratio of surplus to liabilities of 40 basis points and then, after the payment of income taxes, rebates any remainder of that excess as policyholder dividends;

(8) in each year, the insurer selling only nonparticipating contracts also retains as much of any excess of assets generated by investments during the current year over the required addition to reserves for the current year as is necessary in order to maintain, or to resume maintaining, a constant target ratio of surplus to liabilities of 40 basis points and then, after the payment of income taxes, it divides any remainder of the excess assets between policyholders and shareholders by allotting to shareholders 60% of any such excess and allotting to policyholders 40% thereof; and

(9) the strategies of both types of insurers with respect to allocating investment earnings are subject to an ordering rule under which first priority is given to the satisfaction of reserve requirements, then to payment of any taxes due, then to extraction of capital, and finally to distribution of dividends (in the case of the seller of nonparticipating policies, shareholder dividends are paid before the equivalent of policyholder dividends are paid and, furthermore, if there are excess assets but the amount thereof is smaller than desired for a surplus charge, a charge as large as is consistent with paying the required tax thereon is extracted and no excess assets are used for either shareholder or policyholder dividends).

COMPARISONS OF PARTICIPATING AND NONPARTICIPATING CONTRACTS AS VEHICLES FOR THE ACQUISITION OF CAPITAL THROUGH THE EXTRACTION OF SURPLUS CHARGES FROM EXCESS ASSETS RESULTING FROM PREMIUM REDUNDANCY UNDER VARIOUS INTEREST RATE ASSUMPTIONS

Table 1-P. *Participating* contract: flat interest rate (equal to 6.5%).

year	assets (beginning of year)	I_y (actual rate of interest for year)	investment income	gross income	needed addition to required reserve	excess assets	desired surplus charge	actual surplus charge	tax	divisible surplus	assets (end of year)
1	120.00	.065	7.80	127.80	126.48	1.32	0.51	0.51	0.18	0.63	126.99
2	246.99	.065	16.05	136.05	134.09	1.96	0.54	0.54	0.19	1.24	261.61
3	381.61	.065	24.80	144.80	142.11	2.69	0.57	0.57	0.20	1.92	404.29
4	524.29	.065	34.08	154.08	150.65	3.43	0.60	0.60	0.21	2.62	555.54
6	675.54	.065	43.91	163.91	159.70	4.21	0.64	0.64	0.22	3.35	715.88
6	835.88	.065	54.33	174.33	169.27	5.06	0.68	0.68	0.24	4.14	885.83
7	1 005.83	.065	65.38	185.38	179.43	5.95	0.72	0.72	0.25	4.98	1 065 98
8	1 185.98	.065	77.09	197.09	190.19	6.90	0.76	0.76	0.27	5.87	1 256.93
9	1 376.93	.065	89.50	209.50	201.60	7.90	0.81	0.81	0.28	6.82	1 459.33
10	1 579.33	.065	102.66	222.66	213.70	8.96	0.85	0.85	0.30	7.80	1 673.89
11	1 793.89	.065	116.60	236.60	226.52	10.08	0.91	0.91	0.32	8.85	1 901.32
12	2 021.32	.065	131.39	251.39	240.12	11.27	0.96	0.96	0.34	9.97	2 142.40
13	2 262.40	.065	147.06	267.06	254.52	12.54	1.02	1.02	0.35	11.18	2 397.93
14	2 517.93	.065	163.67	283.67	269.80	13.87	1.08	1.08	0.38	12.41	2 668.81
15	2 788.81	.065	181.27	301.27	285.98	15.29	1.14	1.14	0.40	13.74	2 955.94
16	3 075.94	.065	199.94	319.94	303.13	16.81	1.21	1.21	0.42	15.18	3 260.28
17	3 380.28	.065	219.72	339.72	321.34	18.38	1.29	1.29	0.45	16.65	3 582.90
18	3 702.90	.065	240.69	360.69	340.60	20.09	1.36	1.36	0.48	18.24	3 924.87

19	4 044.87	.065	262.92	882.92	361.04	21.86	1.44	1.44	0.50	19.94	4 287.85
20	4 407.85	.065	286.48	406.48	382.70	23.78	1.53	1.53	0.54	21.71	4 671.58
21	4 791.58	.065	311.45	431.45	405.68	25.77	1.62	1.62	0.57	23.58	5 078.88
22	5 198.88	.065	337.93	457.93	430.00	27.93	1.72	1.72	0.60	25.61	5 510.60
23	5 630.60	.065	365.99	485.99	455.82	30.17	1.82	1.82	0.64	27.70	5 968.25
24	6 088.25	.065	395.74	515.74	483.15	32.59	1.93	1.93	0.68	29.98	6 453.33
25	6 573.33	.065	427.27	547.27	512.15	35.12	2.05	2.05	0.72	32.35	6 967.53
26	7 087.53	.065	460.69	580.69	542.88	37.81	2.17	2.17	0.76	34.88	7 512.58
27	7 632.58	.065	496.12	616.12	575.44	40.68	2.30	2.30	0.81	37.57	8 090.32
28	8 210.32	.065	533.67	653.67	609.97	43.70	2.44	2.44	0.85	40.41	8 702.73
29	8 822.73	.065	573.48	693.48	646.58	46.90	2.59	2.59	0.91	43.40	9 351.90
30	9 471.90	.065	615.67	735.67	685.37	50.30	2.74	2.74	0.96	46.60	10 040.01

Note: The amount of assets on hand as of the end of the last year the contract was assumed to remain in force exceeds the face amount of the death benefit. It might be assumed that such an excess might be disposed of by dividing it into two parts: a company share in the form of a surplus charge for the final year of the contract and a termination dividend payable to the policyholder.

Table 1-N. *Nonparticipating* contract: flat interest rate (equal to 6.5%).

year	assets (beginning of year)	L (actual rate of interest for the year)	investment income	gross income	needed addition to required reserve	excess assets	desired surplus charge	actual surplus charge	tax	shareholder divid-ends	policyholder divid-ends	assets (end of year)
1	119.75	0.065	7.78	127.53	126.46	1.05	0.51	0.51	0.16	0.22	0.14	126.99
2	246.74	0.065	16.04	135.79	134.09	1.70	0.54	0.54	0.19	0.58	0.39	261.62
3	381.37	0.065	24.79	144.54	142.11	2.43	0.57	0.57	0.20	1.00	0.66	404.30
4	524.05	0.065	34.06	153.81	150.65	3.16	0.60	0.60	0.21	1.41	0.94	555.65
5	675.30	0.065	43.89	163.64	159.70	3.94	0.64	0.64	0.22	1.85	1.23	715.89
6	835.64	0.065	54.32	174.07	169.27	4.80	0.68	0.68	0.24	2.33	1.55	885.84

7	1 005.59	0.065	65.36	185.11	179.43	5.66	0.72	0.72	0.25	2.83	1.88	1 065.99
8	1 185.74	0.065	77.07	196.82	190.19	6.63	0.76	0.76	0.27	3.36	2.24	1 256.94
9	1 376.69	0.065	89.46	209.23	201.60	7.63	0.81	0.81	0.28	3.92	2.62	1 459.35
10	1 579.10	0.065	102.64	222.39	213.70	8.69	0.85	0.85	0.30	4.52	3.02	1 673.90
11	1 793.65	0.065	116.59	236.34	226.52	9.82	0.91	0.91	0.32	5.15	3.44	1 901.33
12	2 021.08	0.065	131.37	251.12	240.12	11.00	0.96	0.96	0.34	5.82	3.88	2 142.41
13	2 262.16	0.065	147.04	266.79	254.52	12.27	1.02	1.02	0.36	6.53	4.36	2 397.95
14	2 517.70	0.065	163.65	283.40	269.80	13.60	1.08	1.08	0.38	7.28	4.86	2 668.83
15	2 788.58	0.065	181.26	301.01	285.98	15.03	1.14	1.14	0.40	8.09	5.40	2 955.95
16	3 075.70	0.065	199.92	319.67	303.13	16.54	1.21	1.21	0.42	8.95	5.96	3 260.29
17	3 380.04	0.065	219.70	339.45	321.34	18.11	1.29	1.29	0.45	9.82	6.55	3 582.92
18	3 702.67	0.065	240.67	360.42	340.60	19.82	1.36	1.36	0.48	10.79	7.19	3 924.88
19	4 044.63	0.065	262.90	382.65	361.04	21.61	1.44	1.44	0.50	11.80	7.87	4 287.36
20	4 407.11	0.065	286.46	406.21	382.70	23.51	1.53	1.53	0.54	12.86	8.58	4 671.59
21	4 791.34	0.065	311.44	431.19	405.68	25.51	1.62	1.62	0.57	13.99	9.33	5 078.89
22	5 198.64	0.065	337.91	457.66	430.00	27.66	1.72	1.72	0.60	15.20	10.14	5 510.61
23	5 630.36	0.065	365.97	485.72	455.82	29.90	1.82	1.82	0.64	16.48	10.98	5 968.25
24	6 088.00	0.065	395.72	515.47	483.15	32.32	1.93	1.93	0.68	17.83	11.88	6 453.33
25	6 573.08	0.065	427.25	547.00	512.15	34.85	2.05	2.05	0.72	19.25	12.83	6 967.53
26	7 087.28	0.065	460.67	580.42	542.88	37.54	2.17	2.17	0.76	20.77	13.84	7 512.58
27	7 632.33	0.065	496.10	615.85	575.44	40.41	2.30	2.30	0.81	22.38	14.92	8 090.52
28	8 210.07	0.065	533.65	653.40	609.97	43.43	2.44	2.44	0.85	24.08	16.06	8 702.73
29	8 822.48	0.065	573.46	693.21	646.58	46.63	2.59	2.59	0.91	25.88	17.25	9 351.90
30	9 471.65	0.065	615.66	735.41	686.37	50.04	2.74	2.74	0.96	27.80	18.54	10 040.01

Note: The amount of assets on hand as of the end of the last year the contract was assumed to remain in force exceeds the face amount of the death benefit. It might be assumed that such an excess might be disposed of by dividing it into two parts: a company share in the form of a surplus charge for the final year of the contract and a termination dividend payable to the policyholder.

Table 2-P. *Participating* contract: flat interest rate (equal to 6%).

year	assets (beginning of year)	i (actual rate of interest for year)	investment income	gross income	needed addition to required reserve	excess assets	desired surplus charge	actual surplus charge	tax	divisible surplus	assets (end of year)
1	120.00	.06	7.20	127.20	126.48	0.72	0.51	0.51	0.18	0.03	126.99
2	246.99	.06	14.82	134.82	134.09	0.73	0.54	0.54	0.19	0.00^B	261.62
3	381.62	.06	22.90	142.90	142.11	0.79	0.57	0.57	0.20	0.02	404.30
4	524.30	.06	31.46	151.46	150.65	0.81	0.60	0.60	0.21	0.00^B	555.55
5	675.55	.06	40.53	160.53	159.70	0.83	0.64	0.61^A	0.21	0.00^B	715.86
6	835.86	.06	50.15	170.15	169.27	0.88	0.71^C	0.65^A	0.23	0.00^B	885.78
7	1 005.78	.06	60.35	180.35	179.43	0.92	0.78^C	0.68^A	0.24	0.00^B	1 065.89
8	1 185.89	.06	71.15	191.15	190.19	0.96	0.86^C	0.71^A	0.25	0.00^B	1 256.79
9	1 376.79	.06	82.61	202.61	201.60	1.01	0.96^C	0.75^A	0.26	0.00^B	1 459.14
10	1 579.14	.06	94.75	214.75	213.70	1.05	1.06^C	0.78^A	0.27	0.00^B	1 675.62
11	1 793.62	.06	107.62	227.62	226.52	1.10	1.19^C	0.81^A	0.28	0.00^B	1 900.95
12	2 020.95	.06	121.26	241.26	240.12	1.14	1.34^C	0.84^A	0.29	0.00^B	2 141.91
13	2 261.91	.06	135.71	255.71	254.52	1.19	1.52^C	0.88^A	0.31	0.00^B	2 397.31
14	2 517.31	.06	151.04	271.04	269.80	1.24	1.72^C	0.92^A	0.32	0.00^B	2 668.03
15	2 788.03	.06	167.28	287.28	285.98	1.30	1.94^C	0.99^A	0.34	0.00^B	2 954.97
16	3 074.97	.06	184.50	304.50	303.13	1.37	2.19^C	1.01^A	0.35	0.00^B	3 259.11
17	3 379.11	.06	202.75	322.75	321.34	1.41	2.47^C	1.04^A	0.36	0.00^B	3 581.50
18	3 701.49	.06	222.09	342.09	340.60	1.49	2.79^C	1.10^A	0.39	0.00^B	3 923.19
19	4 043.19	.06	242.59	362.59	361.04	1.55	3.13^C	1.15^A	0.40	0.00^B	4 285.88
20	4 405.88	.06	264.32	384.32	382.70	1.62	3.51^C	1.20^A	0.42	0.00^B	4 669.28
21	4 789.28	.06	287.36	407.36	405.68	1.68	3.93^C	1.24^A	0.43	0.00^B	5 076.20
22	5 196.20	.06	311.77	431.77	430.00	1.77	4.41^C	1.31^A	0.46	0.00^B	5 507.51

23	5 627.51	.06	337.65	457.65	455.82	1.83	4.92^C	1.36^A	0.46	0.00^B	5 964.69
24	6 084.69	.06	365.08	485.08	483.15	1.93	5.49^C	1.43^A	0.50	0.00^B	6 449.27
25	6 569.27	.06	394.16	514.16	512.15	2.01	6.11^C	1.49^A	0.52	0.00^B	6 962.91
26	7 082.91	.06	424.97	544.97	542.88	2.09	6.79^C	1.55^A	0.54	0.00^B	7 507.34
27	7 627.34	.06	457.64	577.64	575.44	2.20	7.54^C	1.63^A	0.57	0.00^B	8 084.41
28	8 204.41	.06	492.26	612.26	609.97	2.29	8.36^C	1.70^A	0.60	0.00^B	8 696.08
29	8 816.08	.06	528.96	648.96	646.58	2.38	9.24^C	1.76^A	0.62	0.00^B	9 344.42
30	9 464.42	.06	567.87	687.87	685.37	2.50	10.22^C	1.85^A	0.65	0.00^B	10 031.64^D

A   *Excess assets are insufficient to allow both the extraction of a surplus charge equal in amount to the desired sum needed to maintain the target surplus ratio and the payment of tax at a rate of 35 percent thereon. Consequently, the maximum amount of surplus charge which can be extracted must be deduced mathematically. Since the amount plus 35 percent thereof cannot exceed the sum of excess assets available, the relevant amount equals the sum of excess assets divided by a factor of 1.35.*

B   *Because of the insufficiency of excess assets, the ordering of priorities rules specified under the ninth of the general assumptions set out above mandates extracting as large a surplus charge as is consistent with paying taxes equal to 35 percent thereof. A subsidiary consequence of applying the priority rules in this manner is that there is no divisible surplus available for distribution as policyholder dividends.*

C   *The amount of surplus charge which would otherwise be desired is increased by the amount by which the surplus charges actually extracted in the preceding year fell short of the amount which was desired in that year.*

D   *The amount of assets on hand as of the end of the last year the contract was assumed to remain in force exceeds the face amount of the death benefit. It might be assumed that such an excess might be disposed of by dividing it into two parts: a company share in the form of a surplus charge for the final year of the contract and a termination dividend payable to the policyholder.*

Table 2-N.   *Nonparticipating* contract: flat interest rate (6%).

year	assets (beginning of year)	I_y (actual rate of interest for the year)	invest-ment income	gross income	needed addition to required reserve	excess assets	desired surplus charge	actual surplus charge	tax	share-holder divi-dends	policy-holder divi-dends (divisible surplus)	assets (end of year)
1	119.75	0.06	7.19	126.94	126.48	0.46	0.51	0.34^A	0.12	0.00^B	0.00^B	126.82
2	246.57	0.06	14.79	134.54	134.09	0.45	0.71^C	0.53^A	0.12	0.00^B	0.00^B	261.24
3	380.99	0.06	22.86	142.61	142.11	0.50	0.95^C	0.37^A	0.13	0.00^B	0.00^B	403.72
4	523.47	0.06	31.41	151.16	150.66	0.51	1.18^C	0.38^A	0.13	0.00^B	0.00^B	554.75
5	674.50	0.06	40.47	160.22	159.70	0.52	1.44^C	0.39^A	0.13	0.00^B	0.00^B	714.83

6	834.58	0.06	60.07	169.82	169.27	0.55	1.78^C	0.41^A	3.14	0.00^B	0.00^B	884.51
7	1 004.26	0.06	60.28	180.01	179.43	0.58	2.04^C	0.43^A	0.15	0.00^B	0.00^B	1 064.37
8	1 184.12	0.06	71.05	190.80	190.19	0.61	2.37^C	0.46^A	0.16	0.00^B	0.00^B	1 255.01
9	1 374.76	0.06	82.49	202.24	201.60	0.64	2.73^C	0.47^A	0.16	0.00^B	0.00^B	1 457.09
10	1 576.84	0.06	94.61	214.36	213.70	0.66	3.11^C	0.49^A	0.17	0.00^B	0.00^B	1 671.26
11	1 791.03	0.06	107.46	227.21	226.52	0.69	3.53^C	0.51^A	0.18	0.00^B	0.00^B	1 898.31
12	2 018.06	0.06	121.08	240.83	240.12	0.71	3.98^C	0.53^A	0.19	0.00^B	0.00^B	2 138.95
13	2 258.70	0.06	135.52	255.27	254.52	0.75	4.47^C	0.55^A	0.20	0.00^B	0.00^B	2 394.02
14	2 513.77	0.06	150.83	270.58	269.80	0.78	4.99^C	0.58^A	0.20	0.00^B	0.00^B	2 664.40
15	2 784.16	0.06	167.05	286.80	285.98	0.82	5.55^C	0.61^A	0.21	0.00^B	0.00^B	2 950.99
16	3 070.74	0.06	184.24	303.99	303.13	0.86	6.15^C	0.64^A	0.22	0.00^B	0.00^B	3 254.76
17	3 374.51	0.06	202.47	322.22	321.34	0.88	6.80^C	0.65^A	0.23	0.00^B	0.00^B	3 576.76
18	3 696.50	0.06	221.79	341.54	340.60	0.94	7.51^C	0.70^A	0.25	0.01^B	0.00^B	3 918.04
19	4 037.79	0.06	242.27	362.02	361.04	0.98	8.26^C	0.73^A	0.26	0.00^B	0.00^B	4 279.80
20	4 399.55	0.06	263.97	383.72	382.70	1.02	9.05^C	0.76^A	0.27	0.01^B	0.00^B	4 663.25
21	4 783.00	0.06	286.98	406.73	405.68	1.05	9.91^C	0.78^A	0.27	0.00^B	0.00^B	5 069.71
22	5 189.46	0.06	311.37	431.12	430.00	1.12	10.85^C	0.83^A	0.29	0.00^B	0.00^B	5 500.54
23	5 620.29	0.06	337.22	456.97	455.82	1.15	11.84^C	0.85^A	0.30	0.00^B	0.00^B	5 957.21
24	6 076.96	0.06	364.62	484.37	483.15	1.22	12.92^C	0.90^A	0.32	0.00^B	0.00^B	6 441.26
25	6 561.01	0.06	393.66	513.41	512.15	1.26	14.07^C	0.93^A	0.33	0.00^B	0.00^B	6 954.34
26	7 074.09	0.06	424.45	544.20	542.88	1.32	16.31_C	0.98^A	0.34	0.00^B	0.00^B	7 498.20
27	7 617.95	0.06	457.08	576.83	575.44	1.39	16.63^C	1.03^A	0.36	0.00^B	0.00^D	8 074.67
28	8 194.42	0.06	491.67	611.42	609.97	1.45	18.04^C	1.07^A	0.37	0.00^B	0.00^B	8 685.72
29	8 805.47	0.06	528.33	648.08	646.58	1.50	19.55^C	1.11^A	0.39	0.00^B	0.00^B	9 333.41
30	9 453.16	0.06	567.19	686.94	685.37	1.57	21.19^C	1.16^A	0.41	0.00^B	0.00^B	$10\ 019.94^D$

A   *Excess assets are insufficient to allow both the extraction of a surplus charge equal in amount to the desired sum needed to maintain the target surplus ratio and the payment of tax at a rate of 35 percent thereon. Consequently, the maximum amount of surplus charge which can be extracted must be deduced mathematically. Since the amount plus 35 percent thereof cannot exceed the sum of excess assets available, the relevant amount equals the sum of excess assets divided by a factor of 1.35.*

B   Because of the insufficiency of excess assets, the ordering of priorities rules specified under the ninth of the general assumptions set out above mandates extracting as large a surplus charge as is consistent with paying taxes equal to 35 percent thereof. A subsidiary consequence of applying the priority rules in this manner is that there is no divisible surplus available for distribution as policyholder dividends.

C   The amount of surplus charge which would otherwise be desired is increased by the amount by which the surplus charges actually extracted in the preceding year fell short of the amount which was desired in that year.

D   The amount of assets on hand as of the end of the last year the contract was assumed to remain in force exceeds the face amount of the death benefit. It might be assumed that such an excess might be disposed of by dividing it into three parts: a company share in the form of a surplus charge for the final year of the contract, an amount to distribute as a shareholder dividend, and a termination dividend payable to the policyholder.

Table 3-P. *Participating* contract: ascending interest rates.

year	assets (beginning of year)	I_y (actual rate of interest)	investment income	gross income	needed addition to required reserve	excess assets	desired surplus charge	actual surplus charge	tax	divisible surplus	assets (end of year)
1	120.00	.06	7.20	127.20	126.48	0.72	0.51	0.51	0.18	0.03	126.99
2	246.99	.0601	14.84	134.84	134.09	0.75	0.54	0.54	0.19	0.02	261.62
3	381.62	.0602	22.97	142.97	142.11	0.86	0.57	0.57	0.20	0.09	404.30
4	524.30	.0603	31.62	151.62	150.65	0.97	0.60	0.60	0.21	0.16	555.55
5	675.55	.0604	40.80	160.80	159.70	1.10	0.64	0.64	0.22	0.24	715.89
6	835.89	.0605	50.57	170.57	169.27	1.30	0.68	0.68	0.24	0.38	885.84
7	1 005.84	.0606	60.95	180.95	179.43	1.52	0.72	0.72	0.25	0.55	1 055.99
8	1 185.99	.0607	71.99	191.99	190.19	1.80	0.76	0.76	0.27	0.77	1 256.94
9	1 376.94	.0608	83.72	203.72	201.60	2.12	0.81	0.81	0.28	1.03	1 469.85
10	1 579.35	.0609	96.18	216.18	213.70	2.48	0.85	0.85	0.30	1.33	1 673.90
11	1 793.90	.0610	109.43	229.43	226.52	2.91	0.91	0.91	0.32	1.68	1 901.33
12	2 021.33	.0611	123.50	243.50	240.12	3.38	0.96	0.96	0.34	2.08	2 142.41
13	2 262.41	.0612	138.46	258.46	254.52	3.94	1.02	1.02	0.36	2.56	2 397.95
14	2 517.95	.0613	154.35	274.35	269.80	4.55	1.08	1.08	0.38	3.09	2 668.83
15	2 788.83	.0614	171.23	291.23	285.98	5.25	1.14	1.14	0.40	3.71	2 955.95
16	3 075.95	.0615	189.17	309.17	303.13	6.04	1.21	1.21	0.42	4.41	3 260.29
17	3 380.29	.0616	208.23	328.23	321.34	6.89	1.29	1.29	0.45	5.15	3 582.92
18	3 702.92	.0617	228.47	348.47	340.60	7.87	1.36	1.36	0.48	6.03	3 924.88
19	4 044.88	.0618	249.97	369.97	361.04	8.93	1.44	1.44	0.50	6.99	4 287.36
20	4 407.36	.0619	272.82	392.82	382.70	10.12	1.53	1.53	0.54	8.05	4 671.59
21	4 791.59	.0620	297.08	417.08	405.68	11.40	1.62	1.62	0.57	9.21	5 078.89
22	5 198.89	.0621	322.85	442.85	430.00	12.85	1.72	1.72	0.60	10.53	5 510.61
23	5 650.61	.0622	350.22	470.22	455.82	14.40	1.82	1.82	0.64	11.94	5 968.25

24	6 088.26	.0623	379.30	499.30	483.15	16.15	1.93	1.93	0.68	13.64	6 463.33
25	6 573.33	.0624	410.18	530.18	512.15	18.03	2.05	2.05	0.72	15.26	6 967.53
26	7 087.53	.0625	442.97	562.97	542.88	20.09	2.17	2.17	0.76	17.16	7 512.58
27	7 632.58	.0626	477.80	597.80	575.44	22.36	2.30	2.30	0.81	19.25	8 090.32
28	8 210.32	.0627	514.79	634.79	609.97	24.82	2.44	2.44	0.85	21.53	8 702.73
29	8 822.73	.0628	554.07	674.07	646.58	27.49	2.59	2.59	0.91	23.99	9 351.90
30	9 471.90	.0629	595.78	715.78	685.37	30.41	2.74	2.74	0.96	26.71	10 040.01

Note: The amount of assets on hand as of the end of the last year the contract was assumed to remain in force exceeds the face amount of the death benefit. It might be assumed that such an excess might be disposed of by dividing it into two parts: a company share in the form of a surplus charge for the final year of the contract and a termination dividend payable to the policyholder.

Table 3-N. *Nonparticipating* contract: ascending interest rates.

year	assets (beginning of year)	i_y (actual rate of interest)	investment income	gross income	needed addition to required reserve	excess assets	desired surplus charge	actual surplus charge	tax	shareholder dividends	policyholder dividends	assets (end of year)
1	119.75	.06	7.19	126.94	126.48	0.46	0.51	0.34^A	0.12	0.00^B	0.00^B	126.82
2	246.57	.0601	14.82	134.57	134.09	0.48	0.71^C	0.36^A	0.13	0.00^B	0.00^B	261.26
3	381.01	.0602	22.94	142.69	142.11	0.58	0.92^C	0.43^A	0.15	0.00^B	0.00^B	403.80
4	523.56	.0603	31.57	151.32	150.65	0.67	1.09^C	0.50^A	0.18	0.00^B	0.00^B	554.94
5	674.69	.0604	40.75	160.50	159.70	0.80	1.23^C	0.59^A	0.21	0.00^B	0.00^B	715.23
6	834.98	.0605	50.52	170.27	169.27	1.00	1.32^C	0.74^A	0.26	0.00^B	0.00^B	885.24
7	1 004.99	.0606	60.90	180.65	179.43	1.22	1.30^C	0.90^A	0.32	0.00^B	0.00^B	1 065.57
8	1 185.32	.0607	71.95	191.70	190.19	1.51	1.15^C	1.12^A	0.39	0.00^B	0.00^B	1 256.88
9	1 376.63	.0608	83.70	203.45	201.60	1.85	0.85^C	0.85	0.30	0.42	0.28	1 459.33
10	1 579.08	.0609	96.17	215.92	213.70	2.22	0.85	0.85	0.30	0.64	0.43	1 678.88
11	1 793.63	.0610	109.41	229.16	226.52	2.64	0.91	0.91	0.32	0.85	0.56	1 901.31
12	2 021.06	.0611	123.49	243.24	240.12	3.12	0.96	0.96	0.34	1.09	0.73	2 142.59

13	2 262.14	.0612	136.44	256.19	254.52	3.67	1.02	1.02	0.36	1.37	0.92	2 397.93
14	2 517.66	.0613	154.33	274.08	269.80	4.28	1.08	1.08	0.38	1.69	1.13	2 668.81
16	2 788.56	.0614	171.22	290.97	286.98	4.99	1.14	1.14	0.40	2.07	1.38	2 955.93
16	3 075.68	.0615	189.15	308.90	303.13	5.77	1.21	1.21	0.42	2.48	1.66	3 260.27
17	3 380.02	.0616	208.21	327.96	321.34	6.62	1.29	1.29	0.45	2.93	1.95	3 582.90
18	3 702.55	.0617	228.45	348.20	340.60	7.60	1.36	1.36	0.48	3.46	2.30	3 924.86
19	4 044.61	.0618	249.96	369.71	361.04	8.67	1.44	1.44	0.50	4.04	2.69	4 287.34
20	4 407.09	.0619	272.80	392.55	382.70	9.85	1.53	1.53	0.54	4.67	3.11	4 671.67
21	4 791.32	.0620	297.06	416.81	405.68	11.13	1.62	1.62	0.57	5.36	3.58	5 078.87
22	5 198.62	.0621	322.83	442.58	430.00	12.58	1.72	1.72	0.60	6.16	4.10	5 510.59
23	5 630.84	.0622	350.21	469.96	455.82	14.14	1.82	1.82	0.64	7.01	4.67	5 968.23
24	6 087.95	.0623	379.28	499.03	483.15	15.88	1.93	1.93	0.68	7.96	5.31	6 453.31
25	6 573.06	.0624	410.16	529.91	512.15	17.76	2.05	2.05	0.72	8.99	6.00	6 967.51
26	7 087.26	.0625	442.95	562.70	542.88	19.82	2.17	2.17	0.76	10.13	6.76	7 512.56
27	7 632.31	.0626	477.78	597.53	575.44	22.09	2.30	2.30	0.81	11.39	7.59	8 090.30
28	8 210.05	.0627	514.77	634.52	609.97	24.55	2.44	2.44	0.85	12.76	8.50	8 702.71
29	8 822.46	.0628	554.05	673.80	646.58	27.22	2.59	2.59	0.91	14.23	9.49	9 351.88
30	9 471.63	.0629	595.77	715.52	685.37	30.15	2.74	2.74	0.96	15.87	10.58	10 039.99[D]

A   *Excess assets are insufficient to allow both the extraction of a surplus charge equal in amount to the desired sum needed to maintain the target surplus ratio and the payment of tax at a rate of 35 percent thereon. Consequently, the maximum amount of surplus charge which can be extracted must be deduced mathematically. Since the amount plus 35 percent thereof cannot exceed the sum of excess assets available, the relevant amount equals the sum of excess assets divided by a factor of 1.35.*

B   *Because of the insufficiency of excess assets, the ordering of priorities rules specified under the ninth of the general assumptions set out above mandates extracting as large a surplus charge as is consistent with paying taxes equal to 35 percent thereof. A subsidiary consequence of applying the priority rules in this manner is that there is no divisible surplus available for distribution as policyholder dividends.*

C   *The amount of surplus charge which would otherwise be desired is increased by the amount by which the surplus charges actually extracted in the preceding year fell short of the amount which was desired in that year.*

D   *The amount of assets on hand as of the end of the last year the contract was assumed to remain in force exceeds the face amount of the death benefit. It might be assumed that such an excess might be disposed of by dividing it into two parts: a company share in the form of a surplus charge for the final year of the contract and a termination dividend payable to the policyholder.*

Table 4-P.   *Participating* contract: descending interest rates.

year	assets (beginning of year)	I_y (actual rate of interest for the year)	investment income	gross income	needed addition to required reserve	excess assets	desired surplus charge	actual surplus charge	tax	divisible surplus	assets (end of year)
1	120.00	.06	7.20	127.20	126.48	0.72	0.51	0.51	0.18	0.03	126.99
2	246.99	.0599	14.79	134.79	134.09	0.70	0.54	0.52^A	0.18	0.00^B	261.60
3	381.60	.0598	22.82	142.82	142.11	0.71	0.59^C	0.53^A	0.18	0.00^D	404.24
4	524.24	.0597	31.30	151.30	150.65	0.65	0.66^C	0.48^A	0.17	0.00^B	555.37
5	675.37	.0596	40.25	160.25	159.70	0.55	0.82^C	0.41^A	0.14	0.00^D	716.48
6	835.48	.0595	49.71	169.71	169.27	0.44	1.09^C	0.33^A	0.11	0.00^B	885.08
7	1 005.08	.0594	59.70	179.70	179.43	0.27	1.48^C	0.20^A	0.07	0.00^D	1 064.71
8	1 184.71	.0593	70.25	190.25	190.19	0.06	2.04^C	0.04^A	0.02	0.00^B	1 254.94
9	1 374.94	.0592	81.40	201.40	201.60	$(0.20)^D$	2.81^C	0.00^E	0.00^F	0.00^D	1 456.14^G
10	1 576.14	.0591	93.16	213.16	213.70	$(0.55)^D$	3.66^C	0.00^E	0.00^F	0.00^D	1 669.29^G
11	1 789.29	.0590	105.57	225.57	226.52	$(0.95)^D$	4.57^C	0.00^E	0.00^F	0.00^D	1 894.86^G
12	2 014.86	.0589	118.68	238.68	240.12	$(1.44)^D$	5.53^C	0.00^E	0.00^F	0.00^B	2 133.54^G
13	2 253.54	.0588	132.51	252.51	254.52	$(2.01)^D$	6.55^C	0.00^E	0.00^F	0.00^D	2 386.05^G
14	2 506.05	.0587	147.11	267.11	269.80	$(2.69)^D$	7.63^C	0.00^E	0.00^F	0.00^B	2 653.16^G
15	2 773.16	.0586	162.51	282.51	285.98	$(3.47)^D$	8.77^C	0.00^E	0.00^F	0.00^D	2 935.67^G
16	3 055.67	.0585	178.76	298.76	303.13	$(4.37)^D$	9.98^C	0.00^E	0.00^F	0.00^D	3 234.43^G
17	3 354.43	.0584	195.90	315.90	321.34	$(5.44)^D$	11.27^C	0.00^E	0.00^F	0.00^B	3 550.33^G
18	3 670.33	.0583	213.98	333.98	340.60	$(6.62)^D$	12.63^C	0.00^E	0.00^F	0.00^D	3 884.31^G
19	4 004.31	.0582	233.05	353.05	361.04	$(7.99)^D$	14.07^C	0.00^E	0.00^F	0.00^D	4 237.36^G
20	4 357.36	.0581	253.16	373.16	382.70	$(9.54)^D$	15.60^C	0.00^E	0.00^F	0.00^D	4 610.52^G
21	4 730.52	.0580	274.37	394.37	405.68	$(11.31)^D$	17.22^C	0.00^E	0.00^F	0.00^B	5 004.89^G
22	5 124.89	.0579	296.73	416.73	430.00	$(13.27)^D$	18.94^C	0.00^E	0.00^F	0.00^B	5 421.62^G

23	5 541.52	.0575	320.51	440.31	455.52	$(15.51)^D$	20.76^C	0.00^E	0.00^F	0.00^B	$5\ 861.93^G$
24	5 981.93	.0577	345.16	465.16	483.15	$(17.99)^D$	22.69^C	0.00^E	0.00^F	0.00^B	$6\ 327.09^G$
25	6 447.09	.0576	371.35	491.35	512.15	$(20.50)^D$	24.74^C	0.00^E	0.00^F	0.00^B	$6\ 818.44^G$
26	6 988.44	.0575	398.96	518.96	542.88	$(23.92)^D$	26.91^C	0.00^E	0.00^F	0.00^B	$7\ 387.40^G$
27	7 457.40	.0574	428.05	548.05	575.44	$(27.39)^D$	29.21^C	0.00^E	0.00^F	0.00^B	$7\ 885.46^G$
28	8 005.45	.0573	458.71	578.71	609.97	$(31.26)^D$	31.65^C	0.00^E	0.00^F	0.00^B	$8\ 464.16^G$
29	8 564.16	.0572	491.01	611.01	646.58	$(35.57)^D$	34.24^C	0.00^E	0.00^F	0.00^B	$9\ 075.17^G$
30	9 195.17	.0571	525.04	645.04	685.37	$(40.33)^D$	36.95^C	0.00^E	0.00^F	0.00^B	$9\ 720.21^H$

A   *Excess assets are insufficient to allow both the extraction of a surplus charge equal in amount to the desired sum needed to maintain the target surplus ratio and the payment of tax at a rate of 35 percent thereon. Consequently, the maximum amount of surplus charge which can be extracted must be deduced mathematically. Since the amount plus 35 percent thereof cannot exceed the sum of excess assets available, the relevant amount equals the sum of excess assets divided by a factor of 1.35.*

B   *Because of the insufficiency of excess assets, the ordering of priorities rules specified under the ninth of the general assumptions set out above mandates extracting as large a surplus charge as is consistent with paying taxes equal to 35 percent thereof. A subsidiary consequence of applying the priority rules in this manner is that there is no divisible surplus available for distribution as policyholder dividends.*

C   *The amount of surplus charge which would otherwise be desired is increased by the amount by which the surplus charges actually extracted in the preceding year fell short of the amount which was desired in that year.*

D   *Gross income for the year falls short of the amount needed to cover the required addition to reserves by the sum specified in parentheses here.*

E   *Because there were no excess assets for the year, no surplus charges were extracted.*

F   *Because no surplus charges were extracted, the company had no taxable income and thus no tax liability for the year.*

G   *Because gross income for the year was insufficient to cover the required addition to reserves, assets on hand at the end of the year equalled the sum of assets on hand at the beginning of the year (i.e., the amount in Column B) plus gross income for the year (i.e., the amount in Column E), less the amount by which gross income was deficient (i.e., the amount in parentheses in Column G).*

H   *This figure is obviously lower than the face amount of the death benefit specified under the contract. The nominal sum of the amounts extracted as surplus charges over the course of the years the contract was assumed to have remained in force is $3.02X. Even adding the nominal sum of surplus charges and the amount of assets on hand in support of the reserve under the contract as of the end of the last year the contract was assumed to have remained in force only yields a total of $9,723.23X which is still $276.77X less than the face amount of the death benefit.*

TABLE 4-N. *Nonparticipating* contract: descending interest rates.

year	assets (beginning of year)	I_y (actual rate of interest for the year)	investment income	gross income	needed addition to required reserve	excess assets	desired surplus charge	actual surplus charge	tax	shareholder dividends	policyholder dividends (divisible surplus)	assets (end of year)
1	119.75	.06	7.19	126.94	126.48	0.46	0.51	0.34^A	0.12	0.00^B	0.00^B	126.82
2	246.57	.0599	14.77	134.52	134.09	0.43	0.71^C	0.32^A	0.11	0.00^B	0.00^B	261.23
3	380.98	.0598	22.78	142.53	142.11	0.42	0.96^C	0.31^A	0.11	0.00^B	0.00^B	403.65
4	523.40	.0597	31.25	151.00	150.65	0.35	1.25^C	0.26^A	0.09	0.00^B	0.00^B	554.56
5	674.31	.0596	40.19	159.94	159.70	0.24	1.63^C	0.18^A	0.06	0.00^B	0.00^B	714.44
6	854.19	.0595	49.63	169.38	169.27	0.11	2.13^C	0.08^A	0.03	0.00^B	0.00^B	883.79
7	1 003.54	.0594	59.61	179.36	179.43	$(0.07)^D$	2.77^C	0.00^E	0.00^F	0.00^B	0.00^B	$1\ 053.08^G$
8	1 182.83	.0593	70.14	189.89	190.19	$(0.30)^D$	3.53^C	0.00^E	0.00^F	0.00^B	0.00^B	$1\ 252.67^G$
9	1 372.42	.0592	81.25	201.00	201.60	$(0.60)^D$	4.34^C	0.00^E	0.00^F	0.00^B	0.00^B	$1\ 453.07^G$
10	1 572.82	.0591	92.95	212.70	213.70	$(1.00)^D$	5.19^C	0.00^E	0.00^F	0.00^B	0.00^B	$1\ 664.77^G$
11	1 784.52	.0590	105.29	225.04	226.52	$(1.48)^D$	6.10^C	0.00^E	0.00^F	0.00^B	0.00^B	$1\ 888.33^G$
12	2 008.03	.0589	118.26	238.03	240.12	$(2.09)^D$	7.06^C	0.00^E	0.00^F	0.00^B	0.00^B	$2\ 124.27^G$
13	2 244.02	.0588	131.95	251.70	254.52	$(2.82)^D$	8.08^C	0.00^E	0.00^F	0.00^B	0.00^B	$2\ 373.15^G$
14	2 492.90	.0587	146.33	266.08	269.80	$(3.72)^D$	9.16^C	0.00^E	0.00^F	0.00^B	0.00^B	$2\ 635.51^G$
15	2 755.26	.0586	161.46	281.21	285.98	$(4.77)^D$	10.30^C	0.00^E	0.00^F	0.00^B	0.00^B	$2\ 911.95^G$
16	3 031.70	.0585	177.35	297.10	303.13	$(6.03)^D$	11.51^C	0.00^E	0.00^F	0.00^B	0.00^B	$3\ 203.02^G$
17	3 322.77	.0584	194.05	313.80	321.34	$(7.54)^D$	12.80^C	0.00^E	0.00^F	0.00^B	0.00^B	$3\ 509.28^G$
18	3 629.03	.0583	211.57	331.32	340.60	$(9.28)^D$	14.16^C	0.00^E	0.00^F	0.00^B	0.00^B	$3\ 831.32^G$
19	3 951.07	.0582	229.95	349.70	361.04	$(11.34)^D$	15.60^C	0.00^E	0.00^F	0.00^B	0.00^B	$4\ 169.68^G$
20	4 289.43	.0581	249.22	368.97	382.70	$(13.73)^D$	17.13^C	0.00^E	0.00^F	0.00^B	0.00^B	$4\ 524.92^G$
21	4 644.67	.0580	269.39	389.14	405.68	$(16.54)^D$	18.75^C	0.00^E	0.00^F	0.00^B	0.00^B	$4\ 897.52^G$
22	5 017.27	.0579	290.50	410.25	430.00	$(19.75)^D$	20.47^C	0.00^E	0.00^F	0.00^B	0.00^B	$5\ 288.02^G$

23	5 407.77	.0578	312.57	432.32	455.82	$(23.50)^D$	22.29^C	0.00^E	0.00^F	0.00^B	0.00^B	$5\ 696.84^G$
24	5 816.59	.0577	335.62	455.37	483.15	$(27.78)^D$	24.22^C	0.00^E	0.00^F	0.00^B	0.00^B	$6\ 124.43^G$
25	6 244.18	.0576	359.66	479.41	512.15	$(32.74)^D$	26.27^C	0.00^E	0.00^F	0.00^B	0.00^B	$6\ 571.10^G$
26	6 690.85	.0575	384.72	504.47	542.88	$(38.41)^D$	28.44^C	0.00^E	0.00^F	0.00^B	0.00^B	$7\ 037.16^G$
27	7 156.91	.0574	410.81	530.56	575.44	$(44.88)^D$	30.74^C	0.00^E	0.00^F	0.00^B	0.00^B	$7\ 522.84^G$
28	7 642.59	.0573	437.92	557.67	609.97	$(52.30)^D$	33.18^C	0.00^E	0.00^F	0.00^B	0.00^B	$8\ 028.21^G$
29	8 147.96	.0572	466.06	585.81	646.58	$(60.77)^D$	35.77^C	0.00^E	0.00^F	0.00^B	0.00^B	$8\ 553.26^G$
30	8 673.00	.0571	495.23	614.98	685.37	$(70.39)^D$	38.51^C	0.00^E	0.00^F	0.00^B	0.00^B	$9\ 097.84^H$

A   Excess assets are insufficient to allow both the extraction of a surplus charge equal in amount to the desired sum needed to maintain the target surplus ratio and the payment of tax at a rate of 35 percent thereon. Consequently, the maximum amount of surplus charge which can be extracted must be deduced mathematically. Since the amount plus 35 percent thereof cannot exceed the sum of excess assets available, the relevant amount equals the sum of excess assets divided by a factor of 1.35.

B   Because of the insufficiency of excess assets, the ordering of priorities rules specified under the ninth of the general assumptions set out above mandates extracting as large a surplus charge as is consistent with paying taxes equal to 35 percent thereof. A subsidiary consequence of applying the priority rules in this manner is that no excess assets are available for division between shareholder and policyholder dividends.

C   Gross income for the year falls short of the amount needed to cover the required addition to reserves by the sum specified in parentheses here.

D   The amount of surplus charge which would otherwise be desired is increased by the amount by which the surplus charges actually extracted in the preceding year fell short of the amount which was desired in that year.

E   Because there were no excess assets for the year, no surplus charges were extracted.

F   Because no surplus charges were extracted, the company had no taxable income and thus no tax liability for the year.

G   Because gross income for the year was insufficient to cover the required addition to reserves, assets on hand at the end of the year equalled the sum of assets on hand at the beginning of the year (i.e., the amount in Column B) plus gross income for the year (i.e., the amount in Column E), less the amount by which gross income was deficient (i.e., the amount in parentheses in Column G).

H   This figure is obviously lower than the face amount of the death benefit specified under the contract. The nominal sum of the amounts extracted as surplus charges over the course of the years the contract was assumed to have remained in force is $1.49X. Even adding the nominal sum of surplus charges and the amount of assets on hand in support of the reserve under the contract as of the end of the last year the contract was assumed to have remained in force only yields a total of $9,099.33X which is still $900.67X less than the face amount of the death benefit.

Table 5-P. *Participating* contract: oscillating interest rate ("frequency" effect).

year	assets (beginning of year)	I_y (actual rate of interest for the year)	investment income	gross income	needed addition to required reserve	excess assets	desired surplus charge	actual surplus charge	tax	divisible surplus	assets (end of year)
1	120.00	.0675	8.10	128.10	126.48	1.62	0.51	0.51	0.18	0.93	126.99
2	246.99	.07	17.29	137.29	134.09	3.20	0.64	0.64	0.19	2.47	261.62
3	381.62	.07	26.71	146.71	142.11	4.60	0.57	0.57	0.20	3.83	404.30
4	624.30	.0675	35.39	166.39	160.66	4.74	0.60	0.60	0.21	3 93	555.55
5	675.55	.06	40.53	160.53	159.70	0.83	0.64	0.61^A	0.21	0.00^B	716.86^G
6	835.86	.0525	43.88	163.88	169.27	$(5.39)^C$	0.71^D	0.00^E	0.00^F	0.00^B	874.36^G
7	994.35	.05	49.72	169.72	179.43	$(9.71)^C$	1.43^D	0.00^E	0.00^F	0.00^B	$1\,034.36^G$
8	1 154.36	.05	57.72	177.72	190.19	$(12.47)^C$	2.19^D	0.00^E	0.00^F	0.00^B	$1\,199.61^G$
9	1 319.61	.0525	69.28	189.28	201.60	$(12.32)^C$	3.00^D	0.00^E	0.00^F	0.00^B	$1\,376.57^G$
10	1 496.57	.06	89.79	209.79	213.70	$(3.91)^C$	3.85^D	0.00^E	0.00^F	0.00^B	$1\,582.46^G$
11	1 702.46	.0675	114.92	234.92	226.52	8.40	4.76^D	4.76	1.67	1.97	1 813.73
12	1 933.73	.07	135.36	255.36	240.12	15.24	0.96	0.96	0.34	13 94	2 054.81
13	2 174.81	.07	152.24	272.24	254.52	17.72	1.02	1.02	0.36	16.34	2 310.35
14	2 430.35	.0675	164.05	284.05	269.80	14.25	1.08	1.08	0.38	12.79	2 581.23
15	2 701.23	.06	162.07	282.07	285.98	$(3.91)^C$	1.14	0.00^E	0.00^F	0.00^B	$2\,859.39^G$
16	2 979.89	.0525	156.42	276.42	303.13	$(26.71)^C$	2.36^D	0.00^E	0.00^F	0.00^B	$3\,109.10^G$
17	3 229.10	.05	161.46	281.46	321.34	$(39.88)^C$	3.64^D	0.00^E	0.00^F	0.00^B	$3\,350.68^G$
18	3 470.68	.05	173.53	293.53	340.60	$(47.07)^C$	5.00^D	0.00^E	0.00^F	0.00^B	$3\,597.14^G$
19	3 717.14	.0525	195.15	315.15	361.04	$(45.89)^C$	6.44^D	0.00^E	0.00^F	0.00^B	$3\,866.40^G$
20	3 986.40	.06	239.18	359.18	382.70	$(23.52)^C$	7.97^D	0.00^E	0.00^F	0.00^B	$4\,202.06^G$
21	4 322.06	.0675	291.74	411.74	405.68	6.06	9.59^D	4.49^A	1.57	0.00^B	4 612.23

year	assets (beginning of year)	I_y rate	investment income	gross income	needed addition to required reserve	excess assets	desired surplus charge	actual surplus charge	tax	policyholder dividends	assets (end of year)
22	4 732.23	.07	331.26	451.26	430.00	21.26	6.82^D	6.82	2.39	12.05	5 049.05
23	5 169.05	.07	361.83	481.83	455.82	26.01	1.82	1.82	0.64	23.55	5 506.59
24	5 626.69	.0675	379.80	499.80	483.15	16.65	1.93	1.93	0.68	14.04	5 991.77
25	6 111.77	.06	366.71	486.71	512.15	(25.44)^C	2.05	0.00^E	0.00^F	0.00^B	6 453.04^G
26	6 573.04	.0525	345.08	465.08	542.88	(77.80)^C	4.22^D	0.00^E	0.00^F	0.00^B	6 840.32^G
27	6 960.32	.05	348.02	468.02	575.44	(107.42)^C	6.52^D	0.00^E	0.00^F	0.00^B	7 200.92^G
28	7 320.92	.05	366.05	486.05	609.97	(123.92)^C	8.96^D	0.00^E	0.00^F	0.00^B	7 563.06^G
29	7 683.05	.0525	403.36	523.36	646.58	(123.22)^C	11.55^D	0.00^E	0.00^F	0.00^B	7 963.19^G
30	8 083.19	.06	484.99	604.99	685.87	(80.88)^C	14.29^D	0.00^E	0.00^F	0.00^B	8 487.50^H

A   Excess assets are insufficient to allow both the extraction of a surplus charge equal in amount to the desired sum needed to maintain the target surplus ratio and the payment of tax at a rate of 35 percent thereon. Consequently, the maximum amount of surplus charge which can be extracted must be deduced mathematically. Since the amount plus 35 percent thereof cannot exceed the sum of excess assets available, the relevant amount equals the sum of excess assets divided by a factor of 1.35.

B   Because of the insufficiency of excess assets, the ordering of priorities rules specified under the ninth of the general assumptions set out above mandates extracting as large a surplus charge as is consistent with paying taxes equal to 35 percent thereof. A subsidiary consequence of applying the priority rules in this manner is that no excess assets are available for distribution to policyholders.

C   Gross income for the year falls short of the amount needed to cover the required addition to reserves by the sum specified in parentheses here.

D   The amount of surplus charge which would otherwise be desired is increased by the amount by which the surplus charges actually extracted in the preceding year fell short of the amount which was desired in that year.

E   Because there were no excess assets for the year, no surplus charges were extracted.

F   Because no surplus charges were extracted, the company had no taxable income and thus no tax liability for the year.

G   Because gross income for the year was insufficient to cover the required addition to reserves, assets on hand at the end of the year equalled the sum of assets on hand at the beginning of the year (i.e., the amount in Column B) plus gross income for the year (i.e., the amount in Column E), less the amount by which gross income was deficient (i.e., the amount in parentheses in Column G).

H   This figure is obviously lower than the face amount of the death benefit specified under the contract. The nominal sum of the amounts extracted as surplus charges over the course of the years the contract was assumed to have remained in force is $1.49X. Even adding the nominal sum of surplus charges and the amount of assets on hand in support of the reserve under the contract as of the end of the last year the contract was assumed to have remained in force only yields a total of $9,099.33X which is still $900.67X less than the face amount of the death benefit.

TABLE 5-N. *Nonparticipating* contract: oscillating interest rates: "frequency" effect).

year	assets (beginning of year)	I_y (actual rate of interest for the year)	investment income	gross income	needed addition to required reserve	excess assets	desired surplus charge	actual surplus charge	tax	shareholder dividends	policyholder dividends (divisible surplus)	assets (end of year)
1	119.75	.0675	8.03	127.83	126.48	1.35	0.51	0.51	0.18	0.40	0.26	126.99
2	246.74	.07	17.27	137.02	134.09	2.93	0.54	0.54	0.19	1.82	0.88	261.62
3	381.37	.07	26.70	146.45	142.11	4.34	0.57	0.57	0.20	2.14	1.43	404.30

4	524.05	.0675	35.37	155.12	150.65	4.47	0.60	0.60	0.21	2.20	1.46	555.55
5	675.30	.0525	36.45	155.20	159.70	$(4.60)^C$	0.64	0.00^E	0.00^F	0.00^B	0.00^B	706.25^G
6	826.00	.0525	43.57	163.12	169.27	$(6.15)^C$	1.52^D	0.00^E	0.00^F	0.00^B	0.00^B	863.22^G
7	982.97	.05	49.15	168.90	179.43	$(10.53)^C$	2.04^D	0.00^E	0.00^F	0.00^B	0.00^B	$1\ 021.59^G$
8	1 141.34	.05	57.07	176.82	190.19	$(13.57)^C$	2.80^D	0.00^E	0.00^F	0.00^B	0.00^B	$1\ 185.04^G$
9	1 304.79	.0525	68.50	188.25	201.60	$(13.35)^C$	3.61^D	0.00^E	0.00^F	0.00^B	0.00^B	$1\ 359.94^G$
10	1 479.69	.06	88.78	208.53	213.70	$(5.17)^C$	4.46^D	0.00^E	0.00^F	0.00^B	0.00^B	$1\ 553.30^G$
11	1 683.05	.0675	113.61	233.36	226.52	6.84	5.37^D	5.07^A	1.77	0.00^B	0.00^B	1 794.89
12	1 914.64	.07	134.02	253.77	240.12	13.65	1.26^D	1.26	0.44	7.17	4.78	2 036.27
13	2 155.02	.07	150.92	270.67	254.52	16.15	1.02	1.02	0.36	8.86	5.91	2 291.81
14	2 411.56	.0675	162.78	282.53	269.80	12.73	1.08	1.08	0.38	6.76	4.51	2 562.69
15	2 682.44	.06	160.95	280.70	285.98	$(5.28)^C$	1.14	0.00^E	0.00^F	0.00^B	0.00^B	$2\ 838.11^G$
16	2 957.86	.0525	155.29	275.04	303.13	$(28.09)^C$	2.36^D	0.00^E	0.00^F	0.00^B	0.00^B	$3\ 085.06^G$
17	3 204.8	.06	160.24	279.99	321.34	$(41.35)^C$	3.64^D	0.00^E	0.00^F	0.00^B	0.00^B	$3\ 323.70^G$
18	3 443.45	.05	172.17	291.92	340.60	$(48.68)^C$	5.00^D	0.00^E	0.00^F	0.00^B	0.00^B	$3\ 566.94^G$
19	3 686.69	.0525	193.55	313.30	361.04	$(47.74)^C$	6.44^D	0.00^E	0.00^F	0.00^B	0.00^B	$3\ 832.60^G$
20	3 952.25	.06	237.14	356.89	382.70	$(25.81)^C$	7.97^D	0.00^E	0.00^F	0.00^B	0.00^B	$4\ 163.58^G$
21	4 283.33	.0675	289.12	408.87	405.68	3.19	9.69^D	2.36^A	0.83	0.00^B	0.00^B	4 571.62
22	4 691.37	.07	328.40	448.15	430.00	18.15	8.95^D	8.95	3.13	3.64	2.43	5 010.57
23	5 130.32	.07	359.12	478.87	455.82	23.05	1.82	1.82	0.64	12.35	8.24	5 468.21
24	5 587.96	.0675	377.19	496.94	483.15	13.79	1.93	1.93	0.68	6.71	4.47	5 953.29
25	6 073.04	.06	364.36	484.13	512.15	$(28.02)^C$	2.05	0.00^E	0.00^F	0.00^B	0.00^B	$6\ 409.40^G$
26	6 529.16	.0525	342.78	462.53	542.88	$(80.35)^C$	4.22^D	0.00^E	0.00^F	0.00^B	0.00^B	$6\ 791.58^G$
27	6 911.33	.05	345.57	465.32	575.44	$(110.12)^C$	6.52^D	0.00^E	0.00^F	0.00^B	0.00^B	$7\ 146.78^G$
28	7 266.53	.05	363.33	483.08	609.97	$(126.89)^C$	8.96^D	0.00^E	0.00^F	0.00^B	0.00^B	$7\ 502.97^G$
29	7 622.72	.0525	400.19	519.94	646.58	$(126.64)^C$	11.55^D	0.00^E	0.00^F	0.00^B	0.00^B	$7\ 896.27^G$
30	8 016.02	.06	480.96	600.71	685.57	$(84.68)^C$	14.29^D	0.00^E	0.00^F	0.00^B	0.00^B	$8\ 412.32^H$

A   *Excess assets are insufficient to allow both the extraction of a surplus charge equal in amount to the desired sum needed to maintain the target surplus ratio and the payment of tax at a rate of 35 percent thereon. Consequently, the maximum amount of surplus charge which can be extracted must be deduced mathematically. Since the amount plus 35 percent thereof cannot exceed the sum of excess assets available, the relevant amount equals the sum of excess assets divided by a factor of 1.35.*

B   *Because of the insufficiency of excess assets, the ordering of priorities rules specified under the ninth of the general assumptions set out above mandates extracting as large a surplus charge as is consistent with paying taxes equal to 35 percent thereof. A subsidiary consequence of applying the priority rules in this manner is that no excess assets are available for division between shareholder and policyholder dividends.*

C   *Gross income for the year falls short of the amount needed to cover the required addition to reserves by the sum specified in parentheses here.*

D   *The amount of surplus charge which would otherwise be desired is increased by the amount by which the surplus charges actually extracted in the preceding year fell short of the amount which was desired in that year.*

E   *Because there were no excess assets for the year, no surplus charges were extracted.*

F   *Because no surplus charges were extracted, the company had no taxable income and thus no tax liability for the year.*

G   *Because gross income for the year was insufficient to cover the required addition to reserves, assets on hand at the end of the year equalled the sum of assets on hand at the beginning of the year (i.e., the amount in Column B) plus gross income for the year (i.e., the amount in Column E), less the amount by which gross income was deficient (i.e., the amount in parentheses in Column G).*

H   *This figure is obviously lower than the face amount of the death benefit specified under the contract. The nominal sum of the amounts extracted as surplus charges over the course of the years the contract was assumed to have remained in force is $1.49X. Even adding the nominal sum of surplus charges and the amount of assets on hand in support of the reserve under the contract as of the end of the last year the contract was assumed to have remained in force only yields a total of $9,099.33X which is still $900.67X less than the face amount of the death benefit.*

TABLE 6-P. *Participating* contract: oscillating interest rate ("wavelength" effect).

year	assets (beginning of year)	I_y (actual rate of interest for the year)	invest-ment income	gross income	needed addition to required reserve	excess assets	desired surplus charge	actual surplus charge	tax	divisible surplus	assets (end of year)
1	120.00	0.08	9.60	129.60	126.48	3.12	0.51	0.51	0.18	2.43	126.99
2	246.99	0.09	22.23	142.23	134.09	8.14	0.54	0.54	0.19	7.41	261.62
3	381.62	0.09	34.35	154.35	142.11	12.24	0.67	0.67	0.20	11.47	404.30
4	524.30	0.08	41.94	161.94	150.65	11.29	0.60	0.60	0.21	10.48	555.55
5	575.55	0.06	40.53	160.53	159.70	0.83	0.64	0.61^A	0.22	0.00^B	716.86
6	835.86	0.04	33.43	153.43	169.27	$(15.84)^C$	0.71^D	0.00^E	0.00^F	0.00^B	853.45^G
7	973.45	0.03	29.20	149.20	179.43	$(30.23)^C$	1.43^D	0.00^E	0.00^F	0.00^B	972.42^G
8	1 092.42	0.03	32.77	152.77	190.19	$(37.42)^C$	2.19^D	0.00^E	0.00^F	0.00^B	$1 087.77^G$
9	1 207.77	0.04	48.31	168.31	201.60	$(33.29)^C$	3.00^D	0.00^E	0.00^F	0.00^B	$1 222.79^G$
10	1 342.79	0.06	80.57	200.57	213.70	$(13.13)^C$	3.85^D	0.00^E	0.00^F	0.00^B	$1 410.23^G$
11	1 530.23	0.08	122.42	242.42	226.52	15.90	4.76^D	4.76	1.67	9.47	1 641.51

12	1 761.51	0.09	158.54	278.54	240.12	38.42	0.96	0.96	0.84	37.12	1 882.59
13	2 002.59	0.09	180.23	300.23	254.52	45.71	1.02	1.02	0.36	44.33	2 135.13
14	2 258.13	0.08	180.65	300.65	269.80	30.85	1.08	1.08	0.38	29.39	2 409.01
15	2 529.01	0.06	151.74	271.74	285.98	$(14.24)^C$	1.14	0.00^E	0.00^F	0.00^B	$2\ 666.51^G$
16	2 766.51	0.04	111.46	231.46	303.13	$(71.67)^C$	2.36^D	0.00^E	0.00^F	0.00^B	$2\ 826.30^G$
17	2 946.30	0.03	88.39	208.39	321.34	$(112.95)^C$	3.64^D	0.00^E	0.00^F	0.00^B	$2\ 921.74^G$
18	3 041.74	0.03	91.25	211.25	340.60	$(129.35)^C$	5.00^D	0.00^E	0.00^F	0.00^B	$3\ 003.64^G$
19	3 123.64	0.04	124.95	244.95	361.04	$(116.09)^C$	6.44^D	0.00^E	0.00^F	0.00^B	$3\ 132.50^G$
20	3 252.50	0.06	195.15	315.15	382.70	$(67.55)^C$	7.97^D	0.00^E	0.00^F	0.00^B	$3\ 380.10^G$
21	3 500.10	0.08	280.01	400.01	405.68	$(5.67)^C$	9.59^D	0.00^E	0.00^F	0.00^B	$3\ 774.44^G$
22	3 894.44	0.09	350.50	470.50	430.00	40.50	11.31^D	11.31	3.96	25.23	4 215.75
23	4 335.75	0.09	390.22	510.22	455.82	54.40	1.82	1.82	0.64	51.94	4 673.39
24	4 793.39	0.08	383.47	503.47	483.15	20.32	1.93	1.93	0.68	17.71	5 158.47
25	5 278.47	0.06	316.71	436.71	512.15	$(75.44)^C$	2.05	0.00^E	0.00^F	0.00^B	$5\ 519.74^G$
26	5 559.74	0.04	226.59	346.59	542.88	$(197.29)^C$	4.22^D	0.00^E	0.00^F	$0.00B$	$5\ 663.04^G$
27	5 788.04	0.03	173.64	293.64	575.44	$(281.80)^C$	6.52^D	0.00^E	0.00^F	0.00^B	$5\ 679.88^G$
28	5 799.88	0.03	174.00	294.00	609.97	$(315.97)^C$	8.96^D	0.00^E	0.00^F	0.00^B	$5\ 657.91^G$
29	5 777.91	0.04	231.12	351.12	646.58	$(295.46)^C$	11.55^D	0.00^E	0.00^F	0.00^B	$5\ 713.57^G$
30	5 833.57	0.06	350.01	470.01	685.37	$(215.36)^C$	14.29^D	0.00^E	0.00^F	0.00^B	$5\ 968.22^H$

A   *Excess assets are insufficient to allow both the extraction of a surplus charge equal in amount to the desired sum needed to maintain the target surplus ratio and the payment of tax at a rate of 35 percent thereon. Consequently, the maximum amount of surplus charge which can be extracted must be deduced mathematically. Since the amount plus 35 percent thereof cannot exceed the sum of excess assets available, the relevant amount equals the sum of excess assets divided by a factor of 1.35.*

B   *Because of the insufficiency of excess assets, the ordering of priorities rules specified under the ninth of the general assumptions set out above mandates extracting as large a surplus charge as is consistent with paying taxes equal to 35 percent thereof. A subsidiary consequence of applying the priority rules in this manner is that no excess assets are available for distribution to policyholders.*

C   *Gross income for the year falls short of the amount needed to cover the required addition to reserves by the sum specified in parentheses here.*

D   *The amount of surplus charge which would otherwise be desired is increased by the amount by which the surplus charges actually extracted in the preceding year fell short of the amount which was desired in that year.*

E   *Because there were no excess assets for the year, no surplus charges were extracted.*

F   *Because no surplus charges were extracted, the company had no taxable income and thus no tax liability for the year.*

G   *Because gross income for the year was insufficient to cover the required addition to reserves, assets on hand at the end of the year equalled the sum of assets on hand at the beginning of the year (i.e., the amount in Column B) plus gross income for the year (i.e., the amount in Column E), less the amount by which gross income was deficient (i.e., the amount in parentheses in Column G).*

H   *This figure is obviously lower than the face amount of the death benefit specified under the contract. The nominal sum of the amounts extracted as surplus charges over the course of the years the contract was assumed to have remained in force is $1.49X. Even adding the nominal sum of surplus charges and the amount of assets on hand in support of the reserve under the contract as of the end of the last year the contract was assumed to have remained in force only yields a total of $9,099.33X which is still $900.67X less than the face amount of the death benefit.*

TABLE 6-N. *Nonparticipating* contract: oscillating interest rate ("wavelength" effect).

year	assets (beginning of year)	i_y (actual rate of interest for the year)	investment income	gross income	needed addition to required reserve	excess assets	desired surplus charge	actual surplus charge	tax	shareholder dividends	policyholder dividends (divisible surplus)	assets (end of year)
1	119.75	.08	9.58	129.33	126.48	2.85	0.51	0.51	0.18	1.30	0.86	126.99
2	246.74	.09	22.21	141.96	134.09	7.87	0.54	0.54	0.19	4.28	2.86	261.62
3	881.37	.09	34.52	154.07	142.11	11.96	0.57	0.57	0.20	6.71	4.48	404.50
4	524.05	.08	41.92	161.67	150.65	11.02	0.60	0.60	0.21	6.13	4.08	555.55
5	575.50	.08	40.52	160.27	159.70	0.57	0.54	0.42^A	0.15	0.00^B	0.00^B	715.67
6	635.42	.04	33.42	153.17	169.27	$(16.10)^C$	0.90^D	0.00^E	0.00^F	0.00^B	0.00^B	852.74^G
7	972.49	.03	29.17	148.92	179.43	$(30.51)^C$	1.52^D	0.00^E	0.00^F	0.00^B	0.00^B	971.16^G
8	1 090.90	.03	32.73	152.48	190.19	$(37.71)^C$	2.38^D	0.00^E	0.00^F	0.00^B	0.00^B	$1 085.92^G$
9	1 205.67	.04	48.23	167.98	201.60	$(53.62)^C$	3.19^D	0.00^E	0.00^F	0.00^B	0.00^B	$1 220.28^G$
10	1 340.03	.06	80.40	200.15	213.70	$(13.55)^C$	4.04^D	0.00^E	0.00^F	0.00^B	0.00^B	1 406.88
11	1 526.63	.08	122.13	241.88	226.52	15.36	4.95^D	4.95	1.73	5.21	3.47	1 664.12
12	1 783.87	.09	160.55	280.30	240.12	40.18	0.96	0.96	0.34	23.33	15.55	1 906.20
13	2 024.95	.09	182.25	302.00	254.52	47.48	1.02	1.02	0.36	27.66	18.44	2 160.74
14	2 280.49	.08	182.44	302.19	269.80	32.39	1.08	1.08	0.38	18.56	12.37	2 431.62
15	2 551.37	.06	153.08	272.83	285.98	$(13.15)^C$	1.14	0.00^E	0.00^F	0.00^B	0.00^B	$2 691.30^G$
16	2 811.05	.04	112.44	232.19	303.13	$(70.94)^C$	2.35^D	0.00^E	0.00^F	0.00^B	0.00^B	$2 852.55^G$
17	2 972.30	.03	89.17	208.92	321.34	$(112.42)^C$	3.64^D	0.00^E	0.00^F	0.00^B	0.00^B	$2 949.05^G$
18	3 068.80	.03	92.06	211.81	340.60	$(128.79)^C$	5.00^D	0.00^E	0.00^F	0.00^B	0.00^B	$3 032.07^G$
19	3 151.82	.04	126.07	245.82	351.04	$(115.22)^C$	6.44^D	0.00^E	0.00^F	0.00^B	0.00^B	$3 162.67^G$

20	3 262.42	.06	196.95	316.70	382.70	$(66.00)^C$	7.97^D	0.00^E	0.00^F	0.00^B	0.00^B	$3\ 413.87^G$
21	3 533.12	.06	282.65	402.40	405.68	$(3.28)^C$	9.59^D	0.00^E	0.00^F	0.00^B	0.00^B	$3\ 812.40^G$
22	3 932.24	.09	355.90	475.65	430.00	43.65	11.31^D	11.31	3.96	17.03	11.35	4 253.80
23	4 373.55	.09	393.62	513.37	455.82	57.55	1.82	1.82	0.64	33.05	22.04	4 711.44
24	4 831.19	.08	386.50	506.25	483.15	23.10	1.93	1.93	0.68	12.29	8.20	5 196.52
25	5 316.27	.06	318.98	438.73	512.15	$(73.42)^C$	2.05	0.00^E	0.00^F	0.00^B	0.00^B	$5\ 561.83^G$
26	5 681.58	.04	227.26	347.01	542.88	$(195.87)^C$	4.22^D	0.00^E	0.00^F	0.00^B	0.00^B	$5\ 712.97^G$
27	5 852.72	.03	174.98	294.73	575.44	$(280.71)^C$	6.52^D	0.00^E	0.00^F	0.00^B	0.00^B	$5\ 726.99^G$
28	5 846.74	.03	175.40	295.15	609.97	$(314.82)^C$	8.96^D	0.00^E	0.00^F	0.00^B	0.00^B	$5\ 707.32^G$
29	5 827.07	.04	233.08	352.83	646.58	$(293.75)^C$	11.55^D	0.00^E	0.00^F	0.00^B	0.00^B	$5\ 766.40^G$
30	5 886.15	.06	353.17	472.92	685.37	$(212.45)^C$	14.29^D	0.00^E	0.00^F	0.00^B	0.00^B	$6\ 026.87^H$

A   Excess assets are insufficient to allow both the extraction of a surplus charge equal in amount to the desired sum needed to maintain the target surplus ratio and the payment of tax at a rate of 35 percent thereon. Consequently, the maximum amount of surplus charge which can be extracted must be deduced mathematically. Since the amount plus 35 percent thereof cannot exceed the sum of excess assets available, the relevant amount equals the sum of excess assets divided by a factor of 1.35.

B   Because of the insufficiency of excess assets, the ordering of priorities rules specified under the ninth of the general assumptions set out above mandates extracting as large a surplus charge as is consistent with paying taxes equal to 35 percent thereof. A subsidiary consequence of applying the priority rules in this manner is that no excess assets are available for division between shareholder and policyholder dividends.

C   Gross income for the year falls short of the amount needed to cover the required addition to reserves by the sum specified in parentheses here.

D   The amount of surplus charge which would otherwise be desired is increased by the amount by which the surplus charges actually extracted in the preceding year fell short of the amount which was desired in that year.

E   Because there were no excess assets for the year, no surplus charges were extracted.

F   Because no surplus charges were extracted, the company had no taxable income and thus no tax liability for the year.

G   Because gross income for the year was insufficient to cover the required addition to reserves, assets on hand at the end of the year equalled the sum of assets on hand at the beginning of the year (i.e., the amount in Column B) plus gross income for the year (i.e., the amount in Column E), less the amount by which gross income was deficient (i.e., the amount in parentheses in Column G).

H   This figure is obviously lower than the face amount of the death benefit specified under the contract. The nominal sum of the amounts extracted as surplus charges over the course of the years the contract was assumed to have remained in force is $1.49X. Even adding the nominal sum of surplus charges and the amount of assets on hand in support of the reserve under the contract as of the end of the last year the contract was assumed to have remained in force only yields a total of $9,099.33X which is still $900.67X less than the face amount of the death benefit.

It seems appropriate to compare certain totals derived from the foregoing tables. The totals in question are set out in tabular form below.

Table 7.   Summary of selected results drawn from prior tables.

table	total redundant premiums	total surplus charges	aggregate additions to required reserves funded by contract[A]	total policy-holder dividends	total share-holder dividends	total distri-butions	net gain (loss) of capital[B]
1-P	20.1	40.00	10 000.01	529.32	NA	529.32	40.01
1-N	12.6	40.00	10 000.01	208.53	312.75	521.28	40.01
2-P	20.1	31.64	10 000.01	0.15	NA	0.15	31.65
2-N	12.6	19.97	10 000.01	0.00	0.00	0.00	19.98
3-P	20.1	40.00	10 000.01	217.47	NA	217.47	40.01
3-N	12.6	40.00	10 000.01	88.72	125.57	214.29	40.01
4-P	20.1	3.02	9 717.39	0.03	NA	0.03	(279.59)
4-N	12.6	1.49	9 548.18	0.00	0.00	0.00	(450.33)
6-P	20.1	25.71	9 231.05	105.84	NA	105.84	(743.24)
5-N	12.6	25.71	9 193.51	34.37	51.55	85.92	(780.78)
6-P	20.1	25.71	7 971.26	246.98	NA	246.98	(2 003.03)
6-N	12.6	25.71	7 960.20	95.90	143.87	239.77	(2 014.09)

A   This aggregation equals the sum of additions to required reserves in years when an excess of assets existed plus the full amounts of gross income in years when no relevant excess existed.

B   This amount was computed by adding together the amounts of nominal total surplus charges plus aggregate additions to required reserves that were funded by the contracts in question and then subtracting away the amount of the death benefit payable under the policy ($10,000). In any case where the sum of assets on hand as of the end of the final year the contract is assumed to remain in force exceeds the face amount of the benefit payable under the contract, it is assumed that half of the excess is extracted as a final addition to the insurer's capital and surplus account and the other half is distributed as a termination dividend to the policyholder.

It should be noted that *in no instance* was the amount of capital acquired by a seller of either participating or nonparticipating contracts simply equal to the product of thirty years multiplied times the redundancy factor included per year in the premium charged for coverage. Thus, in effect, the inaccuracy of the model implied by the Graetz prepayment analysis has been demonstrated in every example supplied.

In the first pair of examples, identified as 1-P and 1-N, the actual rate of interest experienced remained constant at 6.5 percent. Obviously, the mean rate of interest over the 30 years during which the contracts remained in force was likewise 6.5 percent and was somewhat *above* the rate assumed for pricing purposes. Consequently, both insurers were able to maintain the target surplus

ratio. Since its redundancy factor was comparatively larger, the seller of participating contracts collected a comparatively larger amount of excess assets. Hence, its distributions of policyholder dividends exceeded the combined total of the shareholder and policyholder dividends distributed by the seller of nominally nonparticipating contracts. Thus, as noted earlier to be typically so, the sum of gross premiums, less year-end policyholder dividends, charged under the participating contract was lower than the sum thereof charged under the nonparticipating contract for a like amount of coverage.

In the pair of examples identified as 2-P and 2-N, the actual rate of interest experienced again remained constant, but at 6 percent. Thus, the mean rate of interest over the 30 years during which the contracts remained in force was likewise 6 percent (*i.e.*, *exactly equal to* the rate assumed for purposes of pricing coverage under both contracts). At this rate of interest, both contracts generated sufficient earnings to satisfy reserve requirements. However, neither generated sufficient "excess" earnings to maintain the target surplus ratio. In the case of the seller of participating contracts, once again because the redundancy factor incorporated in its gross premium was comparatively larger, more than 75% of the amount of surplus charges needed to maintain the target ratio was extracted and a small amount of policyholder dividends were distributed. By contrast, in the case of the seller of nonparticipating contracts, less than 50% of the amount of surplus charges needed to maintain the target surplus ratio was extracted and no shareholder or policyholder dividends were distributed.

In the pair of examples identified as 3-P and 3-N, the actual rate of interest experienced rose steadily from 6% to 6.29% during the 30 years for which the contracts remained in force. Thus, the mean rate over the 30-year span concerned was 6.145% (somewhat *above* the rate assumed for pricing purposes). In this context, both contracts generated sufficient "excess" earnings to maintain the target surplus ratio. Again because the redundancy factor incorporated in its gross premium was comparatively higher, the seller of participating contracts also had a substantial amount of "excess" earnings remaining after it had extracted surplus charges and it distributed that amount as policyholder dividends. The seller of nonparticipating contracts distributed its comparatively smaller, albeit substantial, amount of "excess" earnings, dividing it between shareholder dividends and policyholder dividends.

In the pair of examples identified as 4-P and 4-N, the actual rate of interest experienced fell steadily from 6% to 5.71% during the 30 years for which the contracts remained in force. Thus, the mean rate over the 30-year span was 5.855% (somewhat *below* the rate assumed for pricing purposes). Neither contract generated sufficient relevantly "excess" earnings to allow the target surplus ratio to be satisfied. In both cases, a small amount of surplus charges were extracted. Put another way, both contracts generated capital losses in this context. However, because the redundancy factor incorporated in its gross premium was comparatively larger, the seller of participating contracts suffered a significantly smaller loss. In the case of the participating contract, the size of the redundancy factor incorporated in the gross premium was large enough so that, in the first year the contract was in force, a very small amount of "excess"

earnings remained after a surplus charge large enough to maintain the target surplus ratio had been extracted. That very small amount was distributed as a policyholder dividend at the end of the first year. In the case of the nonparticipating contract, there were never sufficient "excess" earnings to allow for any distributions to either shareholders or policyholders.

In the pair of examples identified as 5-P and 5-N, the actual rate of interest oscillated uniformly between one percentage point above and one percentage point below the 6% rate which had been assumed for pricing purposes. The mean rate over the 30-year span was 6% (*i.e.*, the *same* as the rate assumed for pricing purposes). Both contracts again lost capital and, yet again because the redundancy factor incorporated in its gross premium was comparatively larger, the participating contract lost less capital than did the nonparticipating one. Since the upswings of the actual interest rate (*i.e.*, divergences of the actual rate *above* the mean rate) generated currently "excess" earnings, distributions were made under both contracts: the sum of policyholder dividends distributed under the participating contract again exceeding the combined sum of shareholder and policyholder dividends distributed under the nonparticipating contract.

Since the mean rate of interest earned over the 30 year span during which the respective contracts were assumed to have remained in force was the same in both the 2-P/2-N and 5-P/5-N pairs of examples, it seems clear that what might be characterized as a "frequency" effect must occur. That is, the more frequently actual interest rates cross the axis determined by the mean rate experienced over the entire span of years during which a block of business remains in force, the more adverse the impact on capital acquisition. However, as demonstrated in the 5-P/5-N pair of examples, this "frequency" effect does not affect the comparatively superior performance of the participating contract.

In the pair of examples identified as 6-P and 6-N, the actual rate of interest oscillated uniformly between *three* percentage points *above* and *three* percentage points *below* the 6% rate which had been assumed for pricing purposes. The actual rate crossed the mean rate the same number of times (*i.e.*, with the same "frequency") and at the same times as did the oscillating rate in the pair of examples identified as 5-P and 5-N. The mean rate over the 30-year span was, once again, equal to 6% (*i.e.*, the *same* as the rate assumed for pricing purposes and the same as the mean rate experienced in the 5-P/5-N pair of examples). However, the divergence from the mean was more extreme (*i.e.*, the swings or "waves" were wider) than in the 5-P/5-N pair of examples. Since the amount of capital lost in the 6-P/6-N pair of examples was larger than the amount lost in the 5-P/5-N pair, it seems clear that what might be described as a "wavelength" effect must occur.

Both effects (identified here as a "frequency" effect and a "wavelength" effect) can be explained fairly simply as characteristics of annualized tax accounting. The "actual" rate of interest earned on investments is determined at the end of the tax-accounting period (*i.e.*, at the end of the calendar year in the case of a taxpayer which is a life insurance company). Thus, a rate which oscillates uniformly from year to year implies that, as of the end of one particular year (an "up" year), the "actual" rate of interest earned during that

year (*i.e.*, the mean rate *for the year in question*) will *exceed* the mean rate for the entire span of years the contract remains in force by some *constant* and, as of the end of some subsequent year (a "down" year), the "actual" rate for that subsequent year will be *less than* the mean rate for the entire life of the contract by that *same constant*. Every time the cycle is on an upswing, the comparatively high "actual" rate of interest as of the end of that year is obviously accounted for by a comparatively larger amount of assets on hand as of the end of that year. Some of those assets are distributed (to policyholders alone in the case of a participating contract sold by a life insurance company that has no shareholders and to both policyholders and shareholders in the case of the equivalent of a participating contract sold by a life insurance company that does have shareholders). Neither type of contract affords any mechanism under which, once distributed, such assets can be recalled to offset a subsequent shortfall in earnings that occurs on the downswing of the cycle. Consequently, each upswing increases the aggregate amount of distributions over the total span of years the contract remains in force but downswings have no effect at all on that total. Obviously, the sum of distributions under a particular contract will be larger whenever upswings are either more frequent in number or larger in size.

It is also clear that both upswings *and downswings* must be taken into account in determining how much capital is either acquired or lost over the entire span of years a contract remains in force. Capital is "acquired" as of the end of a particular year when assets on hand at that time are retained and are *accounted for* as an increase in the size of the capital and surplus account. Capital is "lost" as of the end of a particular year when all assets on hand must be treated as held in support of the required reserve under the contract and are accounted for by a corresponding decrease in the size of the capital and surplus account. Hence, in the context of a year-to-year oscillating rate of interest, both the frequency and the size of upswings *and downswings* affect the total amount of capital ultimately acquired (lost) under a particular contract. Since, as noted, once distributed as either policyholder or shareholder dividends, assets cannot be recalled, fewer assets are on hand in the midst of a downswing to earn the lower rate and, as a result, the lower rate is amplified by prior shrinkage of the asset base occasioned by distributions in earlier years. A larger number of upswings and downswings (*i.e.*, a higher "frequency" of oscillation around an axis) increases both the respective amounts of capital lost and the amounts distributed but does not alter the comparative superiority of either type of contract in terms of minimizing capital loss or maximizing distributions (*i.e.*, maximizing relative competitiveness *vis a vis* rewarding suppliers of capital). By contrast, the 6-P/6-N pair of examples demonstrates that, the greater the amplitude of divergence of the "actual" rate from the contract-life mean rate, the greater the relative extent to which each downswing offsets each corresponding upswing. In other words, not only is more capital lost under both types of contracts as the rate of interest swings further from the contract-life mean but also the less the comparative superiority of the participating contract over the nonparticipating contract as such divergence increases. This effect is illustrated in the examples. The ratio of the net loss of capital incurred by the participating contract illustrated in Table 5-P to the net loss of capital incurred by the nonparticipating contract illustrated in Table 5-N is 0.9519198 while that

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of the net loss illustrated in Table 6-P to the net loss illustrated in Table 6-N is 0.9945086.

CONCLUSIONS

It was, of course, the errors implicit in the Graetz prepayment analysis which led to the incorrect conclusion that relevant tax burdens were identical in time-value terms. By contrast, the present-value analysis outlined in this discussion reveals that, even if section 809 of the Internal Revenue Code were repealed, an insurer acquiring capital through the medium of premium redundancy incurs a higher tax burden than another insurer acquiring capital through the medium of shareholder contributions of like amounts collected at like times. Despite that higher tax burden, however, it does not follow that shareholder contribution is the preferred mechanism for capital acquisition. Obviously, in the case of a shareholder contribution, one important additional cost for the use of capital that is incurred is an obligation to pay competitive shareholder dividends. No such obligation is incurred *by a mutual company* when capital is acquired through the medium of premium redundancy. Thus, despite the higher tax burden made clear under the foregoing analysis, a mutual company's total cost of acquiring capital through the medium of premium redundancy (*i.e.*, the cost reflected by the comparatively higher tax burden plus whatever administrative cost is associated therewith) may well nonetheless be lower than a stockholder-owned company's total cost of acquiring capital through the medium of shareholder contribution (*i.e.*, the tax burden, administrative cost associated therewith, the administrative cost of soliciting and collecting a shareholder contribution, and the cost of an *indefinitely ongoing* obligation to pay competitive shareholder dividends).

Assuming that the total cost of capital acquisition through the medium of premium redundancy is indeed lower than the total cost of capital acquisition through the medium of shareholder contribution and, consequently, also implicitly assuming that both mutual and stockholder-owned companies consider the medium of premium redundancy to be the preferred medium for capital acquisition, the subsequent examples illustrated in the various pairs of tables above have demonstrated that, in all of the interest rate scenarios tested, the participating contract (both in terms of the amount of capital acquired and in terms of amounts distributed as policyholder dividends) is never inferior to a nominally nonparticipating contract under which both shareholder dividends and the equivalent of policyholder dividends must be paid. Consequently, to the extent a stockholder-owned life insurance company might suffer a competitive disadvantage should Section 809 be repealed or judicially invalidated, that disadvantage would derive not from any underlying unfairness inherent in the Internal Revenue Code.

It is conceivable that, to the extent the discrimination against insurance companies having no shareholders which is demonstrated by the foregoing present-value analysis (*i.e.*, the discrimination against capital acquisition through the medium of premium redundancy as compared to shareholder contribution) is exacerbated by Section 809, the exacerbation might prove

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constitutionally infirm. To be specific, Section 809 conceivably might not survive scrutiny under the *Equal Protection* standards implicit in the Fifth Amendment's guarantee of *Due Process*.[13] After all, even "rational basis" scrutiny (the least demanding equal-protection test) requires a showing that a statutory classification is not utterly arbitrary but serves (even indirectly or downright obliquely) *some* legitimate governmental interest (even one of only minor significance). The present-value analysis outlined in this discussion undercuts any claim that Section 809 serves a governmental interest in eliminating bias otherwise inherent in the Code's treatment of the divergent capital acquisition techniques characteristic of the two forms of business organization used by life insurance companies. Indeed, the present-value analysis presented in this discussion has demonstrated that, without Section 809, the Code discriminates against, not in favor of, the participating contract (both when the comparison is with capital acquisition through shareholder contributions and when it is with borrowing funds through issuance of corporate bonds or through upstream debt financing). That such discrimination, standing alone, may be insufficient to overcome the demonstrated superiority of the participating contract over a nominally nonparticipating contract as a vehicle for capital acquisition obviously does not justify injecting *additional* discriminatory tax treatment into the Code. No other governmental interest justifying discrimination against participating contracts has to date been identified.

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[13]   Whether (or at least the precise extent to which) federal courts will entertain equal-protection challenges to classifications made in the tax context remains somewhat uncertain and, in any event, is beyond the scope of this discussion.