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Approximations of Ruin Probability by Tri-atomic or Tri-exponential Claims

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ABSTRACT

We find the necessary and sufficient conditions for fitting five given moments by a triatomic distribution. We consider three examples drawn from fire (large spread), individual life (medium spread) and group life (small spread) insurance data, fit them with triatomics, and compute the ruin probabilities using well known formulas for discrete and for combination of exponentials claim amounts. We then compare our approximations with the exact values that appeared in the literature. In the fire (large spread) example, we found the triatomic to have 4.8% average relative error, triexponential 3.6%, and the Beekman-Bowers 2.1%. In the medium spread example, the triatomic has maximum relative error of 0.26% and average relative error of 0.03%. In the small spread example, the triatomic has maximum relative error of 0.14% and average relative error of 0.02%. We recommend that for κ^3/σ^3 less than 5, the simple method of triatomic approximant to the claim amount distribution, which produces quick and reliable results, be used.

KEYWORDS

Ruin probability; triatomic distribution; triexponential distribution.

1. INTRODUCTION

In the classical work of Cramér (1955, p.43), the following claim amount distribution was used to represent data from Swedish non-industry fire insurance covering the years 1948-1951:

$$p(x) = 4.897954\, e^{-5.514588\, x} + 4.503\, (x+6)^{-2.75}, \qquad 0 < x < 500. \tag{1}$$

Exact ruin probabilities were computed by numerically solving

$$\psi(u) = \frac{\lambda}{c} \int_0^u [1 - P(y)]\, \psi(u-y)\, dy + \frac{\lambda}{c} \int_u^\infty [1 - P(y)]\, dy\,, \tag{2}$$

which was a nontrivial numerical task then(Cramér 1955, p.45). A modern reference for the above integral equation is Exercise 12.11 in Bowers *et alii* (1986).

A much easier numerical task even now is to approximate (1) by a distribution for which there is a readily executable formula for its ruin probabilities. For the claim amount taking a combination of exponential distributions, there are the Tăcklind (1942) type formulas. See Shiu (1984), Gerber, Goovaerts and Kaas (1987), Dufresne and Gerber (1988) (1989) (1991), and Chan (1990a). For the claim amount taking a discrete distribution (mixture of atomic distributions), there are the Takács (1967) type formulas. See Beekman(1968), Shiu (1989) and Kaas (1991). We considered the special cases of mixture of two atoms (*diatomic*) and of combination of two exponentials (*diexponential*) in Babier and Chan (1991) and here we consider the tri-exponential and tri-atomic claims.

2. RUIN PROBABILITIES FOR TRIATOMIC AND TRIEXPONENTIAL DISTRIBUTIONS

The ruin probability formula for a discrete claim amount distribution has been given by Schmitter (1990). See Kaas (1991, p.136). For similar formulas see Shiu (1989). Proof for the atomic case and a reference to Feller (1971) are found in Shiu (1987). We list the ruin probability formula for triatomic claim amounts with atoms $\{x_1, x_2, x_3\}$ of probabilities $\{p_1, p_2, p_3\}$:

$$\psi(u) = 1 - \frac{\theta}{1+\theta} \sum_{k_1, k_2, k_3} (-z)^{k_1+k_2+k_3} \quad e^z \quad \frac{p_1^{k_1} p_2^{k_2} p_3^{k_2}}{k_1! k_2! k_3!}, \tag{3}$$

$$\text{where } z = \frac{(u - k_1 x_1 - k_2 x_2 - k_3 x_3) +}{(1+\theta)\mu}.$$

The theory of ruin probability for mixture and combination of exponentials is well known. See Tăcklind (1942), Shiu (1984), Dufresne and Gerber (1988), (1989), (1991), and Chan (1990b). Recall that in a compound Poisson surplus process, the ruin probability $\psi(u)$ satisfies

$$\int_0^\infty e^{ru} (-\psi'(u))\, du = \frac{\theta}{1+\theta} \cdot \frac{\frac{M_X(r) - 1}{\mu r}}{1+\theta - \frac{M_X(r) - 1}{\mu r}} \tag{4}$$

where θ is the relative security loading, X is the claim amount random variable, and $\mu = E(X)$.(See Bowers *et alii* (1986), §12.6) When claim amounts are distributed as a mixture of exponentials, i.e.,

$$p(x) = \sum_{i=1}^{n} A_1 \beta_i e^{-\beta_i x} \tag{5}$$

for $x > 0$ where all $A_i > 0$ and $\sum_{i=1}^{n} A_i = 1$, the ruin probability is also a linear combination of exponentials

$$\psi(u) = \sum_{i=1}^{n} C_i e^{-r_i u} \tag{6}$$

where $\{ r_1, \cdots, r_n \}$ are solutions to the adjustment coefficient equation

$$1+\theta = \frac{M_X(r) - 1}{\mu r} \tag{7}$$

and $\{ C_1, \cdots, C_n \}$ are determined by the partial fractions of the right side of (8):

$$\sum_{i=1}^{n} \frac{C_i r_i}{r_i - r} = \frac{\theta}{1+\theta} \cdot \frac{\frac{M_X(r) - 1}{\mu r}}{1+\theta - \frac{M_X(r) - 1}{\mu r}}. \tag{8}$$

This is verified by substituting (6) into the left side of (4) to obtain the left side of (8).

The denominator on the right side of (8) is a rational function with zeros $r_1, \cdots, r_n$ and simple poles $\beta_1, \cdots, \beta_n$. Therefore

$$1+\theta - \frac{M_X(r)-1}{\mu r} = K \frac{\prod_{i=1}^{n}(r_i-r)}{\prod_{i=1}^{n}(\beta_i-r)} = \theta \prod_{i=1}^{n} \frac{\beta_i(r_i-r)}{r_i(\beta_i-r)} \tag{9}$$

where the constant K has been found by evaluating the first equality of (9) at $r=0$ and using

$$\lim_{r\to 0} \frac{M_X(r)-1}{\mu r} = 1$$

The numerator on the right side of (8) can consequently be written as

$$\frac{M_X(r)-1}{\mu r} = 1+\theta - \theta \prod_{i=1}^{n} \frac{\beta_i(r_i-r)}{r_i(\beta_i-r)} \tag{10}$$

With the left side of (8) written in a common denominator and with (9) and (10) into the right side of (8), we transform (8) into an easy partial fraction problem:

$$\frac{\sum_{j=1}^{n} C_j r_j \prod_{\substack{i=1 \\ i\neq j}}^{n}(r_i-r)}{\prod_{i=1}^{n}(r_i-r)} = \prod_{i=1}^{n} \frac{r_i(\beta_i-r)}{\beta_i(r_i-r)} - \frac{\theta}{1+\theta} \tag{11}$$

Multiply both sides by $\prod_{i=1}^{n}(r_i-r)$, we obtain

$$\sum_{j=1}^{n} C_j r_j \prod_{\substack{i=1 \\ i\neq j}}^{n}(r_i-r) = \prod_{i=1}^{n} \frac{r_i(\beta_i-r)}{\beta_i} - \frac{\theta}{1+\theta} \prod_{i=1}^{n}(r_i-r)$$

At last, let $r=r_k$

$$C_k r_k \prod_{\substack{i=1 \\ i\neq k}}^{n}(r_i-r) = \prod_{i=1}^{n} \frac{r_i(\beta_i-r)}{\beta_i}$$

or

$$C_k = \prod_{\substack{i=1 \\ i \neq k}}^{n} \frac{r_i}{r_i - r_k} \prod_{i=1}^{n} \frac{\beta_i - r_k}{\beta_i}. \tag{12}$$

Consider $r \to \beta_k$ in (8), or easier yet in (11) for $k=1, \cdots, n$ to obtain

$$\sum_{i=1}^{n} \frac{C_i r_i}{\beta_k - r_i} = \frac{\theta}{1+\theta} \quad \text{for } k=1, \cdots, n. \tag{13}$$

To solve (13), consider

$$\sum_{i=1}^{n} \frac{C_i r_i}{x - r_i} = \frac{\theta}{1+\theta} - \frac{\theta}{1+\theta} \prod_{i=1}^{n} \frac{(x - \beta_i)}{(x - r_i)}$$

where the two sides are different expressions for the same rational function of (degree $n-1$ / degree n) with simple poles $\{r_1, \cdots, r_n\}$ and takes the value $\frac{\theta}{1+\theta}$ at $x=\beta_1, \cdots, \beta_n$. Multiply by $x - r_k$ and let $x = r_k$ to obtain

$$C_k = \frac{\theta}{1+\theta} \cdot \frac{\prod_{i=1}^{n} (\beta_i - r_k)}{r_k \prod_{\substack{i=1 \\ i \neq k}}^{n} (r_i - r_k)} \tag{14}$$

Comparing (12) and (14), we obtain

$$\prod_{i=1}^{n} \frac{r_i}{\beta_i} = \frac{\theta}{1+\theta} \tag{15}$$

Our derivation of (13) is motivated by Shiu (1984 , p.484, (9)) where he multiply (8) by $r_k - r$ and let $r \to r_k$ to obtain:

$$C_k = \frac{\theta \, p_1}{M'_X(r_k) - (1+\theta) \, p_1}. \tag{16}$$

The expressions for C_k, (49) and (54) in Dufresne and Gerber (1989), arise naturally when a more detailed problem including the severity of ruin is studied. These two expressions can be obtained from summing (9) and (22) in Dufresne and Gerber (1988) over j respectively. In fact, the system in Dufresne and Gerber (1989)

$$\sum_{k=1}^{n} \frac{\beta_i}{\beta_i - r_k} C_k = 1, \quad i=1, \cdots, n, \tag{17}$$

can be solved by the same arguement as our solution to (13). Simply consider

$$\sum_{i=1}^{n} \frac{x}{x-r_i} C_i = 1 - \prod_{i=1}^{n} \frac{r_i (x-\beta_i)}{\beta_i (x-r_i)}$$

where the two sides are different expressions for the same rational function of (degree n / degree n) with simple poles $\{r_1, \cdots, r_n\}$ and takes the value 1 at $x=\beta_1, \cdots, \beta_n$ and the value 0 at $x=0$. Multiply by $x-r_k$ and let $x=r_k$ to obtain (12).

The formulas (12), (14), and (15) and their derivations are still valid even when members involved in the mixture and combination includes some gamma distributions with integral α. To explain how it works, we illustrate by $\alpha = 2$. The only changes needed are that the right side of (8) would have poles of order 2 at β's that come with gamma distributions and that the system (13) would have fewer equations than unknowns. To show (14) is still valid, one needs to perturb the repeated β's to $\beta \pm \epsilon$ and let $\epsilon \to 0$. Gamma(n,β), odd integer $n \geq 3$ would give one real roots and $n-2$ complex roots to the adjustment coefficient equation (7); Gamma(n,β), even integer $n \geq 4$ would give two real roots and $n-2$ complex roots to the adjustment coefficient equation (7). Equation (15) would give real θ because the complex roots in $\{r_i\}$ comes in pairs of r and $\bar{r}$. We encountered complex roots of the adjustment coefficient equation in Gerber, Goovaerts, and Kaas (1987) for example.

3. TRIATOMIC AND TRIEXPONENTIAL AS APPROXIMANTS

In Babier and Chan (1991), we studied three claim amount distirbutions and compute ruin probabilities of approximating diatomic and diexponential with matching first three moments and compare the approximations with the exact values of $\psi(u)$. In the first example (Cramér's fire) the claim amount distribution has a large spread, none of the approximations is very close to the exact value, and there we point out the run-off error problem encountered in the Takács type formulas. In the second example (Reckin, Schwark, and Snyder's individual life) the claim amount distribution has a medium spread, both of the diatomic and diaexponential give good approximations. In the third example (Mereu's group life) the claim amount distribution has a small spread, the diatomic gives an excellent approximation, and the spread is so small that

there is no diexponential fit.

The new contribution here is that we fit triatomics with matching first five moments to all three examples. Cramér's fire has high κ^3/σ^3 that our triatomic approximant works poorly, and Wikstad's (1971) triexponential fares better. The other two examples are discrete distributions with smaller spreads. The triatomic approximants work so excellently for both cases that there is no need to work out the triexponential approximants. In fact, that Mereu's group life has $\sigma^2/\mu^2 = .2508$ would require four exponentials close to a gamma(4, μ^{-1}); but its κ^3 is too small for such a gamma. Thus we have only done the triexponential approximation for the first example.

Example 1: We consider Cramér's fire insurance data, the one mentioned in the introduction. In the following table, the exact values of $\psi(u)$ for $\theta = 0.3$, and the values for the Cramér-Lundberg approximation is from Cramér (1955, p.45). The values for the Beekman-Bowers approximation is from Beekman (1969, p.279). The ruin probability for diatomic claims, (9), encounters convergence problems when u is large. Our experience echoes with that reported in Seah (1990, §4). For values of u close to and above 30 times μ, large numbers are subtracted off each other and we obtain probabilities less than zero or greater than one. In Babier and Chan (1991), these problemic valuse were listed as $**$. This time we use *Mathematica* to 68 digit accuracy to handle $u = 40$, 60, 80, and 100 for the diatomic and triatomic cases. In the table below, the approximating diatomic has atoms { .7657175616, 181.1382584 } and probabilities { .9987011192, .001298880855 }, the approximating diexponential has $\frac{1}{\beta}$ =60.75201696, $\frac{1}{\gamma} = .6552147239$, and $A = .005737165094$; as done in Babier and Chan (1991). The approximating triatomic with atoms { .60220174840013, 77.32991843481469, 371.6249366477063} and probabilities { .995231341384981, .00466023012321588, .0001084284918028845 }. The approximating triexponential is given by Wikstad (1971) with

$$p(x) = (.0039793)(.014631)e^{-.014631x} +$$

$$(.1078392)(.190206)e^{-.190206x} + (.8881815)(5.514588)e^{-5.514588x}$$

and the corresponding ruin probability is found to be

$$\psi(u) = .514735\ e^{-.007381\ u} + .223197\ e^{-.099058\ u} + .031300\ e^{-4.84374\ u}$$

TABLE 1 Cramér's Fire Insurance
$\mu = 1$, $\sigma^2/\mu^2 = 42.20323069$, $\kappa^3/\sigma^3 = 27.69286626$

u	$\psi(u)$	CL	BB	diatom	diexp	triatom	triexp
20	.5039	.4524	.5140	.4133	.4666	.5122	.4749
40	.3985	.3904	.4079	.3841	.4010	.4401	.3874
60	.3280	.3370	.3369	.3535	.3447	.3586	.3311
80	.2757	.2909	.2812	.3214	.2962	.2715	.2853
100	.2346	.2511	.2369	.2877	.2546	.2314	.2461

u	$\psi(u)$	CL/$\psi(u)$	BB/$\psi(u)$	dia/$\psi(u)$	die/$\psi(u)$	tria/$\psi(u)$	triexp/$\psi(u)$
20	.5039	0.898	1.020	0.820	0.926	1.014	0.942
40	.3985	0.980	1.028	0.964	1.006	1.104	0.972
60	.3280	1.027	1.027	1.078	1.051	1.093	1.009
80	.2757	1.055	1.020	1.166	1.074	0.985	1.035
100	.2346	1.070	1.010	1.226	1.085	0.986	1.049

Example 2: In this example, we consider the individual life insurance date from Reckin, Schwark, and Snyder (1984). This is also the claim distribution in Example 3 of Seah (1990). The claim amount X is discrete with support {1,2,3,4,5,7,8,10,12,13,15,16} and probabilities (in order) {.5141, .3099, .0639, .0220, .0194, .0096, .0276, .0036, .0041, .0019, .0013, .0226}. Since the claim amount distribution is more spread out, (i.a) of Proposition 2 in Babier and Chan (1991) is satisfied and we have a diexponential fit.

TABLE 2.1 $\psi(u)$ by Seah for RSS's Individual Life Insurance Data
$\mu = 2.2896$, $\sigma^2/\mu^2 = 1.43257300$, $\kappa^3/\sigma^3 = 3.60560786$

	$\theta = .1$	$\theta = .2$	$\theta = .3$	$\theta = .4$	$\theta = .5$
$u = 0$	.909091	.833333	.769231	.714286	.666667
$u = 10$	.644361	.450722	.334890	.260412	.209732
$u = 20$	.469129	.254324	.152965	.099371	.068466
$u = 30$	.341528	.143813	.070341	.038430	.022840
$u = 40$	.248408	.081101	.032173	.014735	.007526
$u = 50$	.180700	.045752	.014725	.005654	.002482

TABLE 2.2 diatomic approximant/$\psi(u)$ for RSS's Data

The approximating diatomic has atoms { 1.580450117, 12.887964915 } and probabilities { .9372389245, .06276107517} by (3), (4), (5), and (6).

	$\theta = .1$	$\theta = .2$	$\theta = .3$	$\theta = .4$	$\theta = .5$
$u = 0$	1	1	1	1	1
$u = 10$	1.013	1.029	1.045	1.060	1.073
$u = 20$	1.003	1.007	1.012	1.015	1.018
$u = 30$	1.001	1.000	0.996	0.990	0.981
$u = 40$	1.001	0.999	0.992	0.982	0.968
$u = 50$	1.001	0.997	0.988	0.974	0.957

TABLE 2.3 triatomic approximant/$\psi(u)$ for RSS's Data

The approximating diatomic has atoms {1.328358114, 5.718977791, 15.50183293} and probabilities { .848776854065243, .1208354358483337, .03038771008642312}.

	$\theta = .1$	$\theta = .2$	$\theta = .3$	$\theta = .4$	$\theta = .5$
$u = 0$	1	1	1	1	1
$u = 10$	0.99979	0.99987	1.00033	1.00113	0.99744
$u = 20$	1.00006	0.99998	0.99972	0.99924	0.99852
$u = 30$	1.00002	1.00006	1.00009	1.00008	0.99996
$u = 40$	1.00000	1.00001	1.00006	1.00007	0.00027
$u = 50$	1.00001	1.00011	0.99973	1.00000	1.00000

TABLE 2.4 diexponential approximant/$\psi(u)$ for RSS's Data

The approximating diexponential has $\frac{1}{\beta}$ =5.448377581, $\frac{1}{\gamma} = 1.930653556$, and $A = .1020393986$ by (23), and (19).

	$\theta = .1$	$\theta = .2$	$\theta = .3$	$\theta = .4$	$\theta = .5$
$u = 0$	1	1	1	1	1
$u = 10$	0.997	0.984	0.966	0.947	0.928
$u = 20$	0.994	0.985	0.979	0.978	0.984
$u = 30$	0.995	0.991	0.997	1.016	1.047
$u = 40$	0.996	1.000	1.022	1.066	1.132
$u = 50$	0.998	1.009	1.048	1.119	1.224

Example 3: In this example, we consider the group insurance data from Mereu (1972). This is also the claim distribution in Example 2 of Seah (1990). The claim amount X is discrete with support {4,6,8,10,12,14,16,20,25} and probabilities (in order) {.15304533960, .07882237436, .11199119040, .10432698260, .09432769021, .10925807990, .09727308107, .18073466720, .07022059474}.

TABLE 3.1 $\psi(u)$ by Seah for Mereu's Group Life Insurance Data
$\mu = 12.61243786$, $\sigma^2/\mu^2 = 0.25079144$, $\kappa^3/\sigma^3 = 0.30556145$

	$\theta = .25$	$\theta = .5$	$\theta = .75$	$\theta = 1$
$u = 0$	.8	.666667	.571429	.5
$u = 25$	.433995	.232316	.141606	.094198
$u = 50$	.222739	.072766	.030113	.014607
$u = 75$	.114114	.022685	.006349	.002236
$u = 100$	.058463	.007072	.001339	.000342

TABLE 3.2 diatomic approximant/$\psi(u)$ for Mereu's Group Life Insurance Data
The approximating diatomic has atoms { 7.187946466, 19.96691435} and probabilities { .5755141225, .4244858774} by (3), (4), (5), and (6).

	$\theta = .25$	$\theta = .5$	$\theta = .75$	$\theta = 1$
$u = 0$	1	1	1	1
$u = 25$	0.9995	0.9992	0.9986	0.9977
$u = 50$	1.0003	1.0004	0.9988	0.9962
$u = 75$	1.0000	0.9978	0.9929	0.9857
$u = 100$	0.9997	0.9962	0.9888	0.9795

The diatomic approximant is producing excellent values! Since the variance is quite small, there is no diexponential fit as indicated by Proposition 2 A, (v) in Babier and Chan (1991). Note that because the approximating claims distribution has the same mean and variance as the original, the non-ruin probability is overestimated as well.

TABLE 3.3 triatomic approximant/$\psi(u)$ for Mereu's Group Life Insurance Data
The approximating triatomic has atoms { 5.257664493, 13.89153404, 23.08844433} and probabilities { .3461811629837337, .4679100083847454, .185908828631521}.

	$\theta = .25$	$\theta = .5$	$\theta = .75$	$\theta = 1$
$u = 0$	1	1	1	1
$u = 25$	0.99974	0.99939	0.99902	0.99864
$u = 50$	1.00000	1.00000	1.00003	1.00007
$u = 75$	1.00000	1.00000	1.00000	1.00000
$u = 100$	1.00000	1.00000	1.00000	1.00000

4. NECESSARY AND SUFFICIENT CONDITIONS FOR TRIATOMIC FIT

Given three atoms $\{x, y, z\}$ with mean=0 and variance=σ^2, the probabilities are determined to be

$$\{ \frac{\sigma^2 + yz}{(y-x)(z-x)}, \frac{\sigma^2 + zx}{(z-y)(x-y)}, \frac{\sigma^2 + xy}{(x-z)(y-z)} \}.$$

The third to fifth moment equations can be written as:

$$a\sigma^2 + c = \kappa^3 \tag{18.1}$$

$$(a^2 - b)\sigma^2 + ac = f^4 \tag{18.2}$$

$$(a^3 - 2ab + c)\sigma^2 + (a^2 - b)c = v^5 \tag{18.3}$$

where κ^3, f^4, v^5 are the third to fifth central moments and

$$a = x + y + z \tag{19.1}$$

$$b = xy + yz + zx \tag{19.2}$$

$$c = xyz \tag{19.3}$$

Solve (18.1) for a, substitute in (18.2) to solve for b. Substitute both the a and the b obtained into (18.3) to solve for c. Now substitute back for a and b. The solution for (a,b,c) is

$$\frac{\left(\sigma^4\kappa^3+\kappa^3f^4-\sigma^2v^5\ ,\ \sigma^2\kappa^6-\sigma^4f^4+f^8-\kappa^3v^5\ ,\ \kappa^9-2\sigma^2\kappa^3f^4+\sigma^4v^5\right)}{\sigma^6+\kappa^6-\sigma^2f^4}$$

In stead of solving for $x,y,$ and z from (19), we shift to make the a zero and change scale to make $b\ -3/4$. Whatever become of c, write it as $-h$.

$$\widetilde{x}\ +\widetilde{y}\ +\widetilde{z}\ =0 \tag{20.1}$$

$$\widetilde{x}\,\widetilde{y}\ +\widetilde{y}\,\widetilde{z}\ +\widetilde{z}\,\widetilde{x}\ =-\frac{3}{4} \tag{20.2}$$

$$\widetilde{x}\,\widetilde{y}\,\widetilde{z}\ =-h \tag{20.3}$$

Solving (20.1) for $\widetilde{x}$ and substitute into (20.2) and (20.3). Solve the resulting (20.2) which is quadratic in $\widetilde{y}$ for $\widetilde{y}$ and substitute into the processed (20.3). Both solutions for $\widetilde{y}$ give the same equation in $\widetilde{z}$:

$$\widetilde{z}\,(-\tfrac{3}{4}+\widetilde{z}^{\,2})=-h \tag{21}$$

which has solutions

$$\{\ Sin(\frac{Sin^{-1}(4h)}{3})\ ,\ \ -Sin(\frac{Sin^{-1}(4h)}{3}+\tfrac{\pi}{3})\ ,\ Cos(\frac{Sin^{-1}(4h)}{3}+\tfrac{\pi}{6})\ \}$$

This can be verified by using $Sin3A=3SinA-4Sin^3A$. (Thus explains the strange normalization of $-3/4$ in (20.2).) Note that (21) will have three real root if and only if $|h|\leq\frac{1}{4}$ as a plot of the left side of (21) will show. This then translates back to the necessary and sufficient condition for having a triatomic fit.

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