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Report On Pricing Using Market Consistent Embedded Value (MCEV)

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EXECUTIVE SUMMARY

The objective of this report is to explain the concept and construction of market-consistent embedded value (MCEV), and to explore in detail the methodology by which the practitioner might calculate MCEV, or the market-consistent value of new business (MCVNB), in the pricing process. We discuss its usefulness as a profit/risk measure relative to other real-world profit/risk measures commonly used by insurance companies, its attendant risks, and key implementation issues that may arise in its application. The report also compares through case studies how some U.S. products would fare under the MCVNB/risk-neutral paradigm, in contrast to the usual real-world pricing approach.

The main conclusions of this report are that:

- MCVNB explicitly considers hedgeable market risk through risk-neutral valuation, and other risks through the cost of non-hedgeable risk.

- Products whose profits are driven largely by investment performance tend to report lower MCVNB results.

- Products that offer rich embedded options and guarantees tend to report lower MCVNB results.

- Products that are extremely sensitive to policyholder assumptions tend to report lower MCVNB results.

- MCVNB results are typically lower than real-world pricing results because there is a lack of explicit allowance for all risks in the latter.

- Real-world pricing can be adjusted to fully and explicitly reflect all risks and may then produce similar results to MCVNB.

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CAVEATS AND LIMITATIONS

The case studies that are developed in this report, including but not limited to product design, asset/liability assumptions, methodology, and scenario data, are meant to be illustrative. It is certain that different companies will have different product designs and assumptions, and may also apply different methodologies when calculating both real-world and risk-neutral pricing measures. Accordingly, we caution the reader not to interpret the case study results as holding true for all products and in all situations.

The real-world and risk-neutral scenarios used in the case studies correspond to a valuation date of December 31, 2010. This data was decided upon in consultation with the Project Oversight Group (POG). As is discussed in the report, market-consistent pricing is by definition reliant on prevailing economic conditions, and thus it is entirely expected that the results for these product designs will vary over time as economic conditions fluctuate. It is important that the reader take this into account when assimilating the content and conclusions contained in this report.

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BACKGROUND

In recent years, market-consistent embedded value (MCEV) has emerged as an important financial reporting metric in the insurance industry. Already widespread in Europe, MCEV is also calculated by insurance companies in the United States who are subsidiaries of European multinationals. The MCEV approach is one of the most preferred tools for measuring internal performance, and is used for a variety of applications, including setting incentive compensation, determining capital allocation across lines of business, and assessing value of existing business. It is hoped that the adoption of MCEV will lead to improved consistency in reporting across the industry as well as greater transparency for users of the data.

Although companies apply the MCEV approach to the valuation of in-force business, the same market-consistent framework can be considered when measuring the impact of new business, and in particular, as a risk-adjusted measure during the pricing process. In this capacity, the term *market-consistent value of new business* (MCVNB) is employed rather than MCEV, which by convention is usually associated with in-force business only. The MCVNB can be used as an indicator of the market risks inherent in a product and to determine the product's impact to shareholder value.

In this report we define the concept and construction of MCEV, and in particular of MCVNB, and how this profit measure compares with other traditional real-world measures that are commonly used in the pricing of both life insurance and annuity products. We provide case studies in Appendix 1 to illustrate the comparison across different products. We also briefly identify some key challenges that may warrant special consideration when applying a market-consistent approach.

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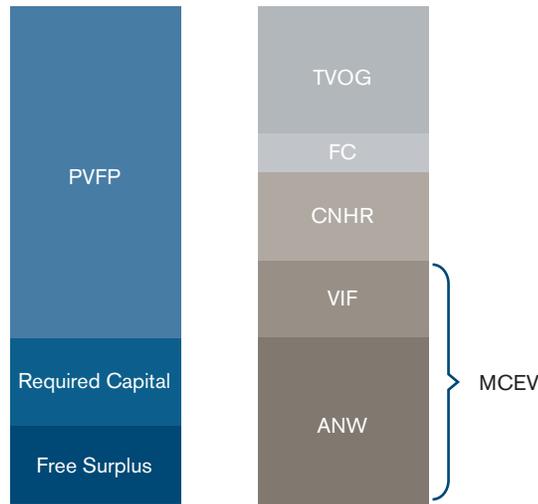
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INTRODUCTION TO MCEV AND MCVNB PRICING

Practitioners are assisted in the calculation of MCEV by the *Market Consistent Embedded Value Principles*, published by the European Insurance CFO Forum (referred to in this report as the *MCEV Principles*).[1] The MCEV Principles document outlines a framework within which to perform the calculation, and it offers guidance rather than enforcing a particular methodology. Formal adoption of these principles was compulsory for CFO Forum members for year-end 2009 reporting.

Conceptually, MCEV is a shareholder's perspective of value, focusing on the present value of all future cash flows available to the shareholder, adjusted for the risks of those cash flows. As is evident by the terminology, MCEV is calculated using market-consistent assumptions, so that cash flows are valued consistently with the prices of similarly traded cash flows in the capital market. From a practical perspective this means calculating cash flows under risk-neutral scenarios that are calibrated to observable market instruments. In particular, the rate at which investment income on reserves and cash flow is earned and the rate at which cash flows are discounted are both assumed to be the risk-free rate. According to the MCEV Principles, a suitable proxy for the risk-free rate of interest is the *reference rate*. The CFO Forum has interpreted the reference rate to generally refer to the swap curve, potentially inclusive of a liquidity premium to reflect any certainty inherent in the liability cash flows.

Figure 1 outlines the components of MCEV, which we describe in more detail below. The breakdown of MCEV into these components is sometimes referred to as the *CFO Forum presentation* approach.

FIGURE 1: CFO FORUM APPROACH

- **Present value of future profits (PVFP)**
  These are post-tax (but pre-cost-of-capital) statutory book profits from the existing business and the assets backing the associated liabilities, developed using local statutory accounting practices. The PVFP typically reflects only the intrinsic value of financial options and guarantees (if those exist in the business), which is essentially the value that the option would have if it were exercised today. In practice, the PVFP component is calculated using a deterministic scenario.

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1 This can be found online at http://www.cfoforum.nl/downloads/MCEV_Principles_and_Guidance_October_2009.pdf

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- **Required capital (RC)**
  This is the market value of assets that can be attributable to the business over and above that which is required to back the liabilities. The RC must be at least equal to that determined by following local regulatory capital requirements, but may also include any amounts required to meet internal risk objectives.

- **Free surplus (FS)**
  This is defined to be the market value of any assets allocated to (but not required to support) the existing business as of the date the MCEV is calculated.

- **Time value of financial options and guarantees (TVOG)**
  This component is only needed if options and guarantees exist in the business. As the name indicates, it captures the time value of the option, which is the portion of the option value that depends on both the time remaining and the potential for the cash-flow components that determine the option price to vary. According to the MCEV Principles, TVOG must be developed using stochastic techniques with due consideration given to discretionary management actions and dynamic policyholder behavior.

- **Frictional costs of required capital (FC)**
  This represents the taxation and investment expenses associated with holding the assets backing the required capital over the lifetime of the underlying risks.

- **Cost of residual non-hedgeable risks (CNHR)**
  This component is meant to encapsulate all non-hedgeable risks, or risks that cannot be hedged using capital market instruments. These include financial and non-financial risks that are not covered in the TVOG and the PVFP for the business that is under consideration. Many of these risks (such as operational or expense risks, for example) are typically not specifically captured in traditional actuarial models. The MCEV Principles do not prescribe a method for calculating the CNHR, but state that it should be represented as an equivalent average cost of capital.

- **Value of in-force (VIF)**
  This is defined as the PVFP less the TVOG, FC, and CNHR components.

For existing business, the MCEV is defined to be the VIF plus the required capital and free surplus components (collectively, the last two items are known as adjusted net worth, or ANW). Importantly, there is no recognition of new business sold in this definition.

For new business, there is no existing required capital and free surplus and so the determination of the market-consistent value of new business (MCVNB) is essentially analogous to the formulation of the VIF, except that the cash flows are those arising from the sale of new business rather than existing business. We will use the terms MCVNB and VIF interchangeably in this report because we are focusing on new business pricing and not in-force valuation. The usual practice is for MCVNB to be calculated at the point of sale, and as a percentage of the present value of (new business) premium. As is the case with the calculation of MCEV for existing business, the noneconomic assumptions used in the projection of cash flows are best estimates.

As discussed earlier, the discount rate used for each of the components of VIF is the reference rate, defined by the CFO Forum to be based on the swap curve, and potentially inclusive of a liquidity premium. Specifically, if the underlying liability cash flows are illiquid, a liquidity premium should be added to the swap curve to reflect the inherent certainty in the cash flows. In practice, the actuary needs to utilize judgment in determining if the inclusion of a liquidity premium is appropriate.

Note that the discounting of cash flows for the components of the VIF should be based on the gross (or unreduced for investment expenses and taxes) reference rate. The rationale behind using the gross reference rate is that investment expenses and taxes are already captured by the frictional cost component of MCVNB.

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BALANCE SHEET APPROACH

As discussed above, MCEV (or MCVNB) can be decomposed into components using the CFO Forum perspective. This approach focuses on calculating the PVFP, TVOG, FC, and CNHR for the business under consideration.

However, there is an equivalent perspective that is commonly referred to as the *balance sheet presentation* that is conceptually easier to understand. This particular approach is shown in Figure 2.

FIGURE 2: COMPONENTS OF MCEV - BALANCE SHEET APPROACH

This approach states that the MCEV (or MCVNB) can be calculated as the difference between the market value of assets less the market value of liabilities.

In this report, we will focus on the CFO Forum approach because we are focused on understanding the components on the liability side. However, it is important to note that the differences between both approaches should be purely presentational, and in particular, the final result should be the same because both approaches are theoretically mathematically equivalent.

Last, we also note that the CFO Forum approach is sometimes referred to as the *indirect* approach because it uses the income statement. In contrast, the balance-sheet approach is referred to as the *direct* approach because it explicitly focuses on calculating the market value of asset cash flows and the market value of liability cash flows.

APPLICATION OF MCVNB

Most companies may apply the rule that the market-consistent value of new business (MCVNB) should never be negative, so that shareholder value is never destroyed. However, it may be that the companies will apply the rule at a more aggregate level, such as at the line of business or even at the company level, so that individual products that have a negative MCVNB (and which would otherwise be discontinued), but which have some diversification benefit to the overall portfolio of products offered by the insurance company, may still be allowed to be sold. Others may allow MCVNB to be negative as long as this can be rationalized and justified given the company strategy and risk appetite.

Companies that report MCEV also report MCVNB on business written during the reporting period, hence pricing and reporting need to be aligned with the models that are used for production and reporting. Some level of *bridging* between the pricing and reporting models needs to be performed and any known

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differences quantified and documented up front to enable crisper analysis and messaging of results. Sensitivity analysis is very important in pricing to better anticipate and understand swings in results that will more naturally occur under MCEV in pricing and reporting. For companies operating in the United States, there needs to be a bridge between real-world and risk-neutral pricing to better understand emergence of earnings and release of margins under both paradigms.

Both MCEV and MCVNB provide a measure of the degree of risk that the company is taking, and if these are managed well, the real-world results will materialize. Both metrics, then, are essentially leading indicators of the degree of market risk based on market valuations. To the extent that management can manage these risks—via product design, investment strategy, hedging, management actions, and so forth—then these margins will be released over time and the real-world results will emerge.

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CALCULATING THE COMPONENTS OF MCVNB

Recall that MCVNB is defined as PVFP less TVOG, CNHR, and FC, and that the discount rate for each component is the stream of discount factors implied by the swap curve (potentially inclusive of an adjusted liquidity premium depending on the product type under consideration), unreduced for investment expenses and taxes. For those components utilizing the stochastic risk-neutral scenarios, the stream of discount factors is path-dependent—that is, they vary by scenario.

- **PVFP (CEQ)**
  The present value of future profits (PVFP) is calculated by discounting the projected future after-tax (but pre-cost-of-capital) statutory profits under the deterministic certainty equivalent (CEQ) scenario. This component of MCVNB is assumed to capture the intrinsic value of options and guarantees only.

  The *CEQ* suffix in the component name distinguishes this from the PVFP that is determined on a stochastic basis for the purposes of calculating the TVOG. Henceforth, if we exclude the *CEQ* suffix when considering PVFP it is implicitly understood that it is the deterministic PVFP that is being discussed.

  In practice, best-estimate assumptions (with no margins for adverse deviation) should be used to calculate the PVFP. Expense assumptions should be fully allocated and include any overhead that exists.

- **TVOG**
  For those products with an embedded option, the time value of options and guarantees (TVOG) should be calculated using stochastic techniques. The following approach is commonly used:

  — Calculate the PVFP over the set of stochastic risk-neutral scenarios, and arithmetically average the result. Denote this by *PVFP (stochastic)*.

  — The TVOG is then PVFP (CEQ) less PVFP (stochastic).

  It is important to note that the overall contribution to MCVNB from both PVFP (CEQ) and TVOG is given by PVFP (CEQ) – TVOG, which (using the above definition for TVOG) reduces to PVFP (stochastic). It is usually the case, therefore, that the PVFP (CEQ) term is only calculated to explicitly illustrate the TVOG component.

  TVOG is a material component of MCVNB for those products that have an embedded option, which generally benefits the policyholder at the expense of the insurance company. These include, but are certainly not limited to, features such as guaranteed minimum death benefits (GMDB), guaranteed minimum living benefits (GMLB), guaranteed minimum crediting rates, and secondary guarantees.

  It is important that the TVOG calculation reflect any dynamic behavior favorable to the policyholder, as well as any management actions that might be carried out (in response to market conditions, for example). The impact of such behavior should be designed to vary on a stochastic basis so as to properly capture the fact that policyholders and/or management are expected to behave differently depending on the prevailing economic conditions. For example, a typical assumption when modeling variable annuities is to reduce lapse rates when the guarantees are higher than the underlying account value.

  Last, we note that there is also a TVOG that could be calculated under the real-world framework. This real-world TVOG is calculated in exactly the same manner, except that the real-world deterministic and stochastic scenarios are used instead. Because real-world scenarios capture risk premiums that are nonexistent in risk-neutral scenarios, one would expect that the real-world TVOG would be smaller than the MCVNB TVOG.

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- **FC**

  The frictional cost (FC) is calculated as the average of discounted investment expenses and tax on investment income on the assets backing required capital over the risk-neutral scenarios.

- **CNHR**

  Cost of non-hedgeable risk (CNHR) can be calculated in different ways, but the MCVNB principles favor the cost-of-capital approach that is also adopted in the Solvency II framework.[2]

  We outline our approach below:

  — The economic capital requirement corresponding to various non-hedgeable risk factors at time zero is determined by applying a variety of shocks to the base model.

  — These non-hedgeable risk factors include shocks to mortality (mortality up 15%), longevity (mortality down 20%), and lapses (lapses up 50% and down 50%), on a stochastic basis; they are assumed to occur throughout the projection. We borrowed the shock sizes from the Solvency II Quantitative Impact Study 5 (QIS5) technical specifications.

  — The market value of net liability cash flows (MVNL) is determined by discounting the cash out-flows and cash in-flows back to the model start date, where the discounting uses the liquidity-adjusted risk-free rates (as used for the calculation of the stochastic PVFP). We define the MVNL as cash out-flows less cash in-flows. Denote the MVNL for each shock by $MVNL^{Mort}$, $MVNL^{Longevity}$, $MVNL^{LapseUp}$, and $MVNL^{LapseDown}$, as well as by $MVNL^{Base}$ for the base stochastic run.

  — The resultant shock matrix is then a vector containing the following entries: the mortality shock ($MVNL^{Mort} - MVNL^{Base}$), the longevity shock ($MVNL^{Longevity} - MVNL^{Base}$), and the lapse shock ($\max(MVNL^{LapseUp}, MVNL^{LapseDown}) - MVNL^{Base}$).

  — The correlation matrix outlined in the Solvency II QIS5 technical specifications is then employed to calculate the economic capital requirement.

FIGURE 3: SOLVENCY II QIS5 CORRELATION MATRIX			
NON HEDGEABLE RISK FACTOR	**MORTALITY SHOCK**	**LONGEVITY SHOCK**	**LAPSE SHOCK**
MORTALITY SHOCK	1	-0.25	0
LONGEVITY SHOCK	-0.25	1	0.25
LAPSE SHOCK	0	0.25	1

The time zero economic capital requirement is calculated as $\sqrt{\sum_{r,c} Corr_{r,c} \times Shock_r \times Shock_c}$, where r and c denote the row and column in the correlation matrix, $Corr_{r,c}$ denotes each entry in the correlation matrix, and both $Shock_r$ and $Shock_c$ denote each entry in the shock matrix.

  — The cost of capital is then calculated by applying a 6% annual charge to the economic capital requirement above. The 6% is also recommended in the Solvency II QIS5, but in reality could vary across companies.

  — The result from the last step is then converted to either a ratio over premiums or a ratio over account values, which can be then used in the pricing projections to determine the projected CNHR.

For simplicity we have used mortality and lapse shocks only to determine the economic requirement. In practice, other risks such as non-hedgeable financial risks, operational risks, and expense risks are often included in the CNHR calculation.

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2 This can be found online at https://eiopa.europa.eu/consultations/qis/quantitative-impact-study-5/technical-specifications/index.html

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We should also note that our approach to CNHR is very much simplified in that we only look at the capital requirement at policy issue. A strict implementation of the cost-of-capital approach requires projections of capital requirements for each non-hedgeable risk factor in the future. The latter is very difficult to model and requires significant run time. However, in reality, many companies do follow the latter approach in their actual MCEV/MCVNB reporting.

It is worth pointing out that on the surface at least there is no real-world counterpart to the CNHR; that is, it exists only in the MCVNB framework. This will be a point of difference in the bridging process from real-world pricing to MCVNB, which we will discuss later in the report.

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REAL-WORLD PRICING MEASURES

Real-world profit measures that are commonly used by pricing actuaries include the following metrics discussed below, which may vary slightly when applied by different actuaries in reality.

We note that, in this report, when we refer to distributable earnings, we mean post-tax and post-cost-of-capital statutory book profits. Under this definition, pre-tax statutory book profits refers to these profits before taxes and cost of capital, and after-tax statutory book profits refers to these profits before cost of capital but with taxes reflected.

These measures are typically calculated using real-world deterministic or stochastic scenarios.

- **Statutory internal rate of return (IRR)**
  This is the solved-for rate of return such that the present value of distributable earnings (at issue) is equal to zero. The objective in a pricing exercise is usually to achieve a rate of return that is in excess of a hurdle rate, which is an agreed-upon minimum acceptable return.

- **Premium margin**
  This is the present value of pre-tax statutory book profits divided by the present value of premiums, where the present value calculation utilizes an assumed discount rate.

- **Return on assets (ROA)**
  This is the present value of pre-tax statutory book profits divided by the present value of projected assets (account values, for example), where the present value calculation utilizes an assumed discount rate.

- *Traditional* **value of new business (TEV)**
  This is the value of new business based on the present value of expected future distributable profits, calculated on a real-world basis. We denote this as the TEV, or *traditional* embedded value of new business.

The pre-tax earned rate is often used as the discount rate for the premium margin and ROA. This rate is a weighted-average net investment earnings rate based on the assets that are supporting the liabilities. If after-tax statutory profits are being utilized to calculate these real-world profit measures (rather than pre-tax statutory profits), then using the after-tax earned rate would be more appropriate. Alternatively, the company's weighted-average cost of capital (WACC) is also commonly used as the discount rate. The WACC is a blend between the cost of equity and cost of debt financing for an insurance company, and is sometimes used as the hurdle rate for the IRR. In general, it is common practice in the industry for practitioners to use the earned rate or the WACC as the basis for discounting the real-world traditional pricing metrics. In some cases, the rate may also be risk-adjusted in order to reflect the inherent riskiness of the cash flows.

The choice of which real-world profit measure to use during a pricing exercise often depends on which product type is under consideration. It is commonly the case to use multiple profit measures in order to better understand the different dimensions associated with the product in question.

For whole life and term products, the IRR and premium margin are likely the most widely used measures. The premium margin directly compares the premiums that are collected to the emerging profit stream. Typical values for the premium margin and IRR for such products range from 10% to 20%, with more bias toward the middle of this range. The ROA is unlikely to be used for such products because the primary source of earnings is premiums and nonforfeiture values are guaranteed.

For fund-based products, such as universal life (with or without secondary guarantees), deferred annuity, and variable annuity products, the ROA and IRR measures are more commonly used. The ROA measure for annuities could vary widely across products and companies but typically tends to be lower than life products, perhaps around 50 bps (on a stochastic basis).

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For TEV, the discount rate can be thought of as essentially a risk-free rate plus a risk premium. In the industry, two approaches have typically been used to determine the discount rate. A *top-down* approach involves using the WACC at the company level, implicitly using a single risk premium for all lines of business. In contrast, a *bottom-up* approach sets the discount rate by differentiating risk premiums on a product-by-product basis based on the specific risk factors for each product.

There are many factors that can influence the WACC that is used for the top-down approach, particularly the cost-of-equity component. Such factors could include the items that are in the company's control, such as the riskiness of the underlying liability cash flows, or the company's investment strategy, dividend policy, or capital risk structure (including the reliance on financial leverage and reinsurance). As a result, the WACC is clearly specific to a particular company, and will vary across the industry.

For simplicity, in this report we will use the pre-tax earned rate for the TEV calculation for the case studies in order to be consistent with how the premium margin and ROA pricing measures are calculated.

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COMPARING REAL-WORLD PRICING RESULTS

AND MCVNB RESULTS

For the purpose of this report, we studied the real-world pricing and the MCVNB results across different product types: variable annuity (VA), equity-indexed annuity (EIA), flexible premium deferred annuity (FPDA), universal life (UL), universal life with secondary guarantee (ULSG), and term (term). Each of these products is discussed in more detail in Appendix 1.

In this section, we will provide a high-level comparison between the pricing results across all products. A comparison between the MCVNB and TEV is presented below for all products. RN stands for risk-neutral and shows the MCVNB results. RW stands for real-world and shows the TEV results. We also show the components of MCVNB and their equivalents in TEV where applicable.

FIGURE 4: MCVNB/TEV COMPONENTS												
	VA		EIA		FPDA		UL		ULSG		TERM	
	% OF PV PREM		% OF PV PREM		% OF PV PREM		% OF PV PREM		% OF PV PREM		% OF PV PREM	
MCVNB / TEV COMPONENTS	RN	RW	RN	RW	RN	RW	RN	RW	RN	RW	RN	RW
PVFP (CEQ) / PVFP (RW DET)	1.8%	5.6%	1.5%	4.8%	0.9%	3.6%	5.7%	6.3%	5.2%	6.1%	11.5%	12.1%
TVOG	-3.2%	-2.3%	-2.7%	-1.8%	-2.2%	-0.7%	-3.0%	-0.3%	-3.1%	-0.9%	0.3%	0.0%
CNHR	-0.9%		-0.1%		-0.6%		-5.4%		-13.9%		-7.9%	
FC	-0.1%	-0.1%	-0.2%	-0.5%	-0.4%	-0.5%	-0.7%	-0.8%	-0.7%	-0.9%	-0.9%	-1.0%
MCVNB / TEV	-2.3%	3.2%	-1.6%	2.4%	-2.1%	2.4%	-3.5%	5.2%	-12.4%	4.3%	3.1%	11.1%

As is evident, MCVNB results are lower than TEV for all products, and negative for products other than term. A closer look at each component reveals that the main difference between MCVNB and TEV comes from three sources:

▪ **Investment spread**
  A comparison between PVFP (CEQ) versus PVFP (RW Det) shows that only the term product has a relatively minor degree of decrease in profits from RW to RN.

  The comparison in this line simply shows how projected profits differ under a deterministic risk-neutral scenario and a deterministic real-world scenario. In other words, volatilities related to the scenarios are ignored and only the expected investment returns are reflected, which means the difference between the RN and RW results in this line is simply the investment spread that is embedded in the real-world framework.

  Products other than term in our study are sensitive to investment assumptions. The VA product relies on the equity-risk premium to provide better returns, and others rely on corporate spreads. Moving from RW to RN removes both the equity-risk premium and the corporate spreads, and therefore would produce worse pricing results.

▪ **Options and guarantees**
  The TVOG line compares the cost of options and guarantees between the risk-neutral measure and the real-world measure. This calculation has to be performed on a stochastic basis, and not only the expected investment returns but also the volatilities would affect the result. Risk-neutral scenarios have lower expected returns and typically higher volatilities. Thus, the cost of options and guarantees will be higher under the risk-neutral framework.

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- **CNHR**

  Best-estimate actuarial assumptions are used in both MCVNB and TEV calculations. However, MCVNB requires an explicit adjustment for the cost of uncertainty in evolving assumptions such as mortality and lapses, but TEV or any other traditional pricing measure does not.

  The CNHR is particularly large for products involving significant lapse guarantees and/or mortality guarantees, hence the large results for the ULSG and term. It might be surprising that the CNHR for the VA is small even though it offers significant mortality/longevity guarantees and is extremely sensitive to the lapse assumption. We believe that the coexistence of guaranteed lifetime withdrawal benefits (GLWB) and GMDB offsets the mortality/longevity risk to a certain degree, and that the dynamic lapse assumption we use in the base pricing is probably already conservative in itself. We did not study the sensitivity to GLWB utilization assumption, and we expect the CNHR to be much higher should we do so.

The three sources of difference also reveal a flaw in a typical real-world pricing exercise. A real-world pricing exercise adopts best-estimate assumptions, both economic and noneconomic, but does not typically reflect the risks associated with these assumptions. The economic assumptions are best-estimate realistic investment returns, which include equity-risk premiums and corporate spreads. However, there is no reflection of the associated equity and credit risks. MCVNB, on the other hand, assumes investment returns are risk-free, thereby reflecting the market risks directly in the economic scenarios. The noneconomic assumptions are also best estimates in the real-world pricing exercise. Sensitivity tests are typically performed, but there may not be an explicit allowance for the cost of uncertainty such as CNHR in MCVNB.

However, MCVNB is not necessarily a pricing measure superior to the real-world pricing measures. One could improve the traditional pricing exercise by explicitly allowing for the risks discussed in the preceding paragraph. One could even achieve similar or even the same pricing results under either MCVNB or real-world pricing, as long as all risks are allowed for explicitly in a similar manner. We will discuss this in more detail in the next section.

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BRIDGING REAL-WORLD PRICING RESULTS AND MCVNB RESULTS

A pricing exercise, in whatever form, is a risk-return analysis. MCVNB seems to be a better measure on this front, as it explicitly recognizes the cost of all risks. However, we could achieve the same objective as long as we can find ways to explicitly allow for all risks in the real-world pricing exercise.

One way to achieve this in the real-world pricing is through the cost of capital. Typically, we only capture regulatory required capital in the real-world pricing exercise. Instead, we could capture an economic capital framework that explicitly covers both market risks and nonmarket risks. In MCVNB, only the CNHR is needed because the market risk is reflected in the risk-neutral scenarios. In real-world pricing, on the other hand, we would need an adjustment for the cost of total risk (CTR). The result would then be a risk-adjusted real-world value of new business (RaRWVNB). Figure 5 compares the components of MCVNB and RaRWVNB.

FIGURE 5: COMPARISON OF COMPONENTS IN MCVNB AND RARWVNB

PVFP (CEQ)	PVFP (DETERMINISTIC RW)
- TVOG	- RW TVOG
- CNHR	- CTR
- FC	- FC
MCVNB	RARWVNB

If the CTR is calibrated to market conditions, then the difference between the CNHR and CTR is, theoretically speaking, the same as the difference between the other components. The MCVNB and RaRWVNB measures may then produce the same results. In reality, the CTR will depend on how a company establishes its own economic capital framework. A company may have a different view on the price of the risk compared to the market's view. Thus, MCVNB and RaRWVNB may differ in reality. MCVNB may then be considered as the minimum acceptable profit target, as that is how much the market is willing to pay to take on the risks related to the product. The RaRRWVNB would be the profit measure that fully reflects the company's own view of the risks related to the product.

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KEY IMPLEMENTATION CHALLENGES

Assuming companies already have the ability to determine real-world distributable income on a stochastic basis, then at least in principle there is not too much additional effort needed to calculate MCVNB. However, in practice there are a number of items that will make both the calculation and the communication of results challenging for the practitioner.

For the calculation itself, the development of appropriate risk-neutral scenarios is likely to involve some effort. Particular consideration needs to be given to the calibration of parameters such that the scenarios that are generated from the economic scenario generator (ESG) fit the initial market conditions and are market-consistent. The calibration of non-observable or minimally observable parameters such as long-term implied volatility in the ESG that is used is particularly challenging. Conceptually, the implied volatility assumption would be a parameter observable from the prices of derivatives that are traded in the market and that could be used directly as an input into the ESG such that the observed market price of the insurance liability can be reproduced. However, in practice there is no liquid secondary market that trades insurance liabilities and, because these liabilities tend to be much longer-dated than most derivatives that are normally traded in the financial markets, there are no comparable financial instruments. As such, it is not possible to find observed implied volatility for a term that would be consistent with the insurance liabilities in question.

A commonly used approach to setting the volatility assumption consists of assuming that the implied volatility (which can be estimated based on quotes from a variety of sources, typically investment banks) linearly grades to an anticipated long-term volatility assumption over some specified period of time. This is the approach that we have taken in this report. However, this is a judgment decision, and other approaches can also be used.

One important challenge with respect to MCVNB is that the estimate will tend to fluctuate over time in a likely greater manner than the comparable real-world pricing measures, which is due to changing capital market conditions. Because the underlying economic environment that is used to generate the risk-neutral scenarios is changing over time, it follows that the associated market-consistent pricing results will also evolve over time. Thus, it is entirely conceivable that products that are priced as of one point in time, and which produce a positive MCVNB, may be priced at a subsequent point in time and result in a negative MCVNB—or vice versa. For instance, pricing on a market-consistent basis as of the time of the financial market crash in late 2008 would very likely result in senior management making very different strategic product decisions as compared to a comparatively more normal economic climate. However, as a less extreme and perhaps more common situation, for most companies there will be a time lag between when a product is conceptualized and initially priced to when it is brought to market and sold. This time lag can become a concern when economic conditions are volatile because the market-consistent pricing results that may have applied at one time may not still be entirely applicable. Because the product is also obviously sold over a period (as opposed to a single point in time), the underlying MCVNB at point of sale for the business that is sold is likely to change over time. This is important for the company to be aware of because a changing economic environment implies that at least some portion of the business that is sold is not necessarily creating the priced-for shareholder value that is expected.

In situations where fluctuation of MCVNB results occurs, it is quite possible that product design tweaks may be necessary in order to ensure that the product is still creating shareholder value, a task that may be particularly difficult if economic conditions are volatile. Moreover, the fluctuation in results may make it difficult to get senior management buy-in with respect to using MCVNB as a viable primary (or even supplemental) pricing measure. In order to alleviate this, it is important to perform movement analysis between successive MCVNB pricing exercises in order to understand and rationalize potential differences in results, especially when communicating results to senior management.

For senior management especially, there is also a challenge to reconcile the MCVNB results to other pricing measures, not only to the statutory-based measures described in this report, but also to the underlying economic and potentially GAAP pricing measures. Further, it may be that despite failing to meet market-consistent pricing criteria (such as the requirement that MCVNB be positive), a product may

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still be brought to market in order to gain a specific segment of market share, gain access to an otherwise inaccessible product distribution channel, or perhaps serve to diversify the existing product portfolio.

Another significant challenge is the calculation of the CNHR. In this report we have calculated the CNHR using a time-zero approach, whereby certain shocks are applied (throughout the projection) to the base model in order to determine the capital requirement as a percentage of the initial premium or account value at time zero. This same risk margin is then applied throughout the projection to determine the CNHR. However, this is at best an approximation that likely understates the true CNHR value—a more accurate approach is to actually project the capital requirements in the future on a nested stochastic basis. This is obviously challenging from a computational and programming perspective, and is beyond the scope of this report.

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SUMMARY

In this report we have summarized the definition and calculation of MCVNB, compared the pricing results between MCVNB and real-world pricing, discussed the merits of each, and presented a way to bridge the two measures.

In general, products with at least one of the following features tend to report lower MCVNB results than real-world results:

- Products whose profits are driven largely by investment performance
- Products that offer rich embedded options and guarantees
- Products that are extremely sensitive to policyholder assumptions

For readers interested in product level details, Appendix 1 presents the product features, assumptions, and comparisons between MCVNB and the real-world pricing results related to each product.

One clear advantage of MCVNB is that it explicitly recognizes all risks in its calculation. However, by calibrating cost of risks on a market-consistent basis, MCVNB results can be very volatile over time, making it challenging for management to make consistent long-term decisions.

MCVNB is certainly a very useful profit measure, but we do not view it as necessarily superior to real-world pricing measures. However, we do believe that the real-world pricing exercise tends to not capture all risks, especially for products with the above features. Perhaps a real-world pricing measure such as the RaRWVNB presented in the last section, which reflects all risks considered in MCVNB and which reflects management's view instead of the market's view of the cost of risks, would be more appropriate for companies in the United States and would allow them to make consistent product decisions.

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APPENDIX 1: CASE STUDIES

GENERAL PRICING APPROACH

In order to compare and contrast the real-world and risk-neutral pricing measures discussed earlier, we have developed a number of case studies in this report. The product types modeled in these case studies consist of the following life insurance and annuity designs:

- Life insurance:
  — Term.
  — Universal life (UL).
  — Universal life with secondary guarantees (ULSG).

- Annuities:
  — Variable deferred annuity (VA).
  — Flexible premium deferred annuity (FPDA).
  — Equity-indexed deferred annuity (EIA).

These product types represent a reasonable cross selection of commonly sold products in the US market place. As discussed in the caveat section of this report, it is important to note that our results are illustrative, and should not be interpreted as holding true for all products and in all situations.

The aim of the pricing analysis in this report is to compare and contrast the real-world profit measures with MCVNB, and specifically to understand and provide some rationale for the differences in results between products. We will present different real-world profit measures but for the purpose of comparing against MCEV we focus on traditional embedded value (TEV).

Our comparison between the TEV and MCVNB starts by comparing the profitability on deterministic scenario projections and then stochastic results where we factor in TVOG, CNHR, and FC.

With this in mind, the steps involved in our pricing analysis can be broadly outlined as follows:

- Real-world

  — Project future distributable earnings using the real-world deterministic scenario.

  — Target an IRR of between approximately 10 to15% based on distributable earnings from the deterministic projection.

  — Calculate additional traditional profit measures such as premium margin and ROA (on a pre-tax basis), as well as TEV. In calculating these measures, discount all cash flows at the pre-tax earned rate. Produce a deterministic source of earnings analysis that highlights key margins and drivers of statutory earnings.

  — Produce the results using stochastic real-world scenarios and calculate the real-world TVOG.

- Risk-neutral

  — Project future distributable earnings using the single CEQ scenario.

  — Discount all cash flows at the reference rate, or swap rate adjusted by the liquidity premium.

  — Produce a deterministic risk-neutral source of earnings analysis.

  — Produce results using stochastic risk-neutral scenarios, and calculate the components of MCVNB, namely PVFP, TVOG, CNHR and FC.

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The projection period varies by case study, but in all cases the length of the projection is consistent with capturing all the material cash flow for the liability type in question. Moreover, all pricing cells are forced to lapse at the end of the projection, allowing for the release of any benefits at that time.

Statutory reserves are calculated for each case study consistent with the applicable reserving actuarial guideline for the product type in question. With the exception of the VA case study, a multiple of RBC-based target surplus is held for the capital requirements in the pricing calculations. Note that in a true MCVNB calculation, capital should be defined as the greater of the statutory capital and the economic capital. However, we are not reflecting economic capital in our MCVNB calculation, and so this is a simplification on our part.

GLOBAL ASSUMPTIONS

This section summarizes those assumptions that are shared by all products. Product-specific assumptions will be discussed in the respective section of each product.

FIGURE 6: SCENARIOS	
STOCHASTIC REAL-WORLD SCENARIOS	1,000 SCENARIOS GENERATED BASED ON HISTORICAL DATA AS OF 12/31/2010
DETERMINISTIC REAL-WORLD SCENARIO	8% LEVEL EQUITY WITH INTEREST RATES BEING AVERAGE OF STOCHASTIC SCENARIOS
STOCHASTIC RISK-NEUTRAL SCENARIOS	1,000 RISK-NEUTRAL SCENARIOS CALIBRATED TO MARKET CONDITIONS OF 12/31/2010 WITH EMBEDDED LIQUIDITY PREMIUM
DETERMINISTIC RISK-NEUTRAL SCENARIO	EQUITY RETURN EQUAL TO RISK FREE RATES AS OF 12/31/2010 PLUS LIQUIDITY PREMIUM

A detailed description of the underlying processes and the Economic Scenario Generators (ESG's) used to develop the scenarios is provided in Appendix 2.

FIGURE 7: REINVESTMENT DISTRIBUTION AND CREDITING ASSUMPTIONS								
REINVESTMENT %	CASH	7-YEAR	10-YEAR	20-YEAR	30-YEAR	ASSET DETAIL	SPREAD (BPS)	DEFAULT RATE (BPS)
TERM			40%	60%		5-YEAR	140	5
UL			50%	50%		7-YEAR	150	10
ULSG			25%	35%	40%	10-YEAR	170	15
VA	100%					20-YEAR	180	20
FPDA		60%	40%			30-YEAR	200	25
EIA		100%						

Figure 7 shows the reinvestment distribution for each case study, where reinvestment is assumed to occur in corporate bonds of varying maturities, with credit spreads and default rates as specified.

An investment expense of 10 basis points is assumed in the asset modeling. Rationalizing the reinvestment distribution table above might be helpful in orienting the reader:

▪ For the VA case study, the single premium is invested in the separate account only. Consequently, we assume that no fixed-income instruments need to be purchased and any positive cash flows are simply reinvested into cash, while any negative positions are supported by borrowing cash at the 90-day rate.

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- For the FPDA and EIA case studies, a medium term reinvestment strategy is employed, approximating the length of the surrender charge period. The EIA product also uses call options that we have simulated in the pricing projection that supports the equity participation.

- For UL, ULSG and term case studies, a longer reinvestment strategy seems appropriate because these liability cash flows are farther out into the tail of the projection.

Credit spreads and default rates are not allowed in the MCVNB calculation and are only applied in the real-world projections.

In the real-world pricing we have assumed a portfolio crediting strategy for those spread based products such as UL, ULSG, FPDA and EIA, where there is a declared credited rate on the account value. Specifically, this crediting rate is the earned rate on the reinvestment asset portfolio supporting the liabilities, less the (product-specific) pricing spread.

In the risk-neutral pricing the same reinvestment distribution is modeled. However, since the risk-neutral valuation assumes all assets are expected to generate the same returns, the investment return is thus expected to be the same under different investment strategies. To simplify, we have modeled the credited rate as a function of the competitor rates (as defined in the product specifications above) with a lag built into the crediting mechanism. Only a certain percentage change from the current credited rate to the competitor rate is reflected in the new credited rate. The percentage change is a function of the average duration of the real-world reinvestment asset portfolio, and is specific to each case study.

We note that under our methodology MCVNB will not change with different asset types, but the different investment strategies will result in different asset durations and thus the change in MCVNB to interest rate changes is expected to be different depending on the asset strategy.

Liquidity Premium
Developing the liquidity premium is an evolving area of practice. In this report, we followed the approach recommended by the CRO Forum in the fifth Quantitative Impact Study (QIS5) technical specifications for Solvency II. It is often referred to as the *50/40 proxy estimate* and is based on a simple transformation of the observed credit spread, such that:

$$\text{Liquidity premium estimate} = \max\,[0,\ 50\% \times (\text{Credit Spread} - 40\text{ bps})]$$

The credit spread in the formula is the iBoxx annual benchmark credit spread adjusted for the swap spread to give a spread over swaps.

For illustrative purposes, we have simply assumed a base liquidity premium of 60 basis points in this report. Each product uses a proportion of the base value of 60 basis points, where the proportion is a reflection of the illiquidity of the product.

We have assumed a proportion of 0% for the VA case study and 75% for the FPDA and term case studies. The UL, ULSG and EIA case studies all utilize a proportion of 50%. A higher proportion is associated with a higher degree of predictability of the underlying liability cash flows. The term and FPDA products tend to have more predictable sets of liability cash flows, in contrast to other products that have embedded guarantees that make such cash flows less predictable over varying economic scenarios.

It is important to note that our assignment of these proportions is illustrative and should not be assumed to hold true for all product types and designs. As with most actuarial functions, there is judgment involved, and it is the responsibility of the actuary to determine the proportion that is most appropriate for the product in question.

For those case studies where it is applicable, the adjusted liquidity premium (that is, the liquidity premium adjusted by the proportion) is added to the starting forward curve in order to generate the liquidity premium adjusted stochastic and CEQ risk-neutral scenarios.

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As part of our modeling, we have assumed that the adjusted liquidity premium is available throughout the projection. In practice a common approach is to assume that the adjusted liquidity premium is graded to zero over a specified number of years, once a certain threshold number of years have elapsed in the projection. For example, the Solvency II QIS5 technical specifications stipulate that for US products, after 30 projection years have elapsed, the adjusted liquidity premium should subsequently be graded to zero over a five-year period.

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TERM CASE STUDY

The term product feature and related pricing assumptions are summarized as below.

FIGURE 8: PRODUCT-SPECIFIC ASSUMPTIONS	
ISSUE AGE DISTRIBUTION	25 (10%), 35 (30%), 45 (40%), 55 (15%), 65 (5%)
GENDER DISTRIBUTION	MALE (60%), FEMALE (40%)
SMOKER DISTRIBUTION	SMOKER (20%), NONSMOKER (40%), PREFERRED NONSMOKER (40%)
AVERAGE SIZE	$500,000
PRODUCT FEATURES	
INSURANCE PERIOD	20 YEARS
PREMIUM PERIOD	LEVEL ANNUAL PREMIUMS FOR 20 YEARS
POLICY FEE	$50
COMMISSIONS	YEAR 1: 120% OF PREMIUM
	YEAR 2+: 3% OF PREMIUM
EXPENSES	
ACQUISITION EXPENSE	$125 PER POLICY + 6% OF INITIAL PREMIUM + 0.8% OF FACE
MAINTENANCE EXPENSE	$100 PER POLICY + 5% OF PREMIUM
DECREMENTS	
MORTALITY	2008 VBT ANB TABLE
LAPSE RATES	7%, 7%, 7%, 6%, 6%, 5%, 5%, 4%, 3%
RESERVES AND SURPLUS	
STATUTORY RESERVE	XXX RESERVE WITH 4% INTEREST RATE AND 2001 CSO MORTALITY
TARGET SURPLUS:	
C1+C3	4.00%
C2	0.20%
C4	3.00%
ECONOMIC ASSUMPTIONS	
LIQUIDITY PREMIUM	45 BPS (ONLY RELEVANT IN MCVNB)

The deterministic real-world profit measures for this product are as follows. We solved for the premium rates that would achieve these targets before we move to the risk-neutral pricing.

- An IRR of 10.1% based on distributable earnings.
- A pre-tax premium margin of 19.0%.
- The ROA is not applicable as there is minimal underlying asset base for a term product (it is possible to use the present value of reserves as a proxy if needed but it would not be as meaningful because the primary source of income is premium).

Figure 9 illustrates, for both the real-world deterministic and risk-neutral deterministic projections, a typical statutory income statement with items expressed as ratios over present value of premiums. As discussed earlier, both real-world and risk-neutral results have been discounted using the pre-tax earned rate (which in the risk-neutral world, is simply the risk free rate inclusive of the adjusted liquidity premium).

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INCOME STATEMENT	% OF PV PREMIUM	
	RN	RW
CASH PREMIUMS	100.0%	100.0%
INVESTMENT INCOME ON RESERVES AND CASH FLOW	21.7%	22.9%
SURRENDER BENEFITS	0.0%	0.0%
COMMISSIONS	-16.1%	-16.2%
ACQUISITION EXPENSES	-3.0%	-3.1%
MAINTENANCE EXPENSES	-10.0%	-10.0%
DEATH BENEFIT	-54.5%	-54.0%
CHANGE IN STATUTORY RESERVE	-20.0%	-20.7%
PRE-TAX STATUTORY PROFIT	18.1%	19.0%
POST-TAX STATUTORY PROFIT / PVFP	11.5%	12.1%
CHANGE IN CAPITAL	-2.7%	-2.8%
INTEREST ON CAPITAL	2.7%	2.8%
TAXES ON INTEREST ON CAPITAL	-0.9%	-1.0%
DISTRIBUTABLE EARNINGS	10.6%	11.1%

It is clear from Figure 9 that the real-world and risk-neutral deterministic results are quite close, both on an individual line item basis, and the overall bottom line profit components. This is not surprising given that the term product has liability cash flows that are not interest-rate sensitive. The slight variation in the liability cash flows line items between the real-world and risk-neutral results is due to the difference in the underlying interest rates used for discounting and investment income - the cash flows themselves are the same between the real-world and risk-neutral projections.

However, when we move from deterministic projections to stochastic projections and factor in required components of MCVNB, results can be very different for the term product. The components of TEV and MCVNB for the term product are outlined below. Note that TEV is essentially the average of the stochastic real-world discounted distributable earnings.

FIGURE 10: MCVNB VS. TEV

	% OF PV PREMIUM	
	RN	RW
PVFP (CEQ) / PVFP (RW DETERMINISTIC)	11.5%	12.1%
TVOG	0.3%	0.0%
CNHR	-7.9%	
FC	-0.9%	-1.0%
MCVNB / TEV	3.1%	11.1%

The PVFP row in Figure 10 above has the present value of after-tax profits corresponding from the deterministic projection.

As one might expect, the TVOG for the term product is very small. This is because the particular term product has no embedded guarantee and is neither equity nor interest-rate sensitive. As a result, the underlying liability cash flows are the same between the deterministic scenario and the stochastic scenarios. However, in practice a very small contribution to TVOG can exist due to the noise produced by the path-dependent interest rates that are used in the projection, which will affect both the discounting of the liability cash flows and the investment earnings on reserves and cash flow.

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The value of CNHR is high (in absolute value), primarily due to the sensitivity of the term product to mortality risk. A shock to mortality has a material impact on the level of death benefits paid, which is a key component of the market value of liabilities. It is also evident that for the term product the bridge between the real-world TEV and the risk-neutral MCVNB is almost entirely provided by the CNHR, since the PVFP (CEQ) is essentially the same in both paradigms. As discussed, the CNHR reflects the explicit allowance for the non-hedgeable risks that cannot be hedged using capital market instruments, such as mortality in this case. The TEV result is based on a best-estimate mortality assumption, and lacks an adjustment for the uncertainty related to the mortality.

Because the pure risk type of insurance design of the term product precludes any reliance on a market risk premium in excess of the risk-free rate, it should be clear that this product type is less likely to be prone to fluctuations in the MCVNB due to changing market conditions.

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UNIVERSAL LIFE CASE STUDY

The universal life (UL) feature and related pricing assumptions are summarized as below.

FIGURE 11: PRODUCT-SPECIFIC ASSUMPTIONS

ISSUE AGE DISTRIBUTION	45 (25%), 55 (50%), 65 (25%)
GENDER DISTRIBUTION	MALE (60%), FEMALE (40%)
SMOKER DISTRIBUTION	PREFERRED NONSMOKER (100%)
AVERAGE SIZE	$250,000

PRODUCT FEATURES

INSURANCE PERIOD	UP TO AGE 121
PREMIUM PERIOD	LEVEL ANNUAL PREMIUMS UP TO AGE 121
EXPENSE LOAD	4% OF PREMIUM + $72 (YEAR 1), $36 (YEARS 2-10) PER POLICY + $0.72 (45), $0.84 (55), $1.20 (65) 0% OF FACE FOR 10 YEARS
DEATH BENEFIT	100% OPTION A DEATH BENEFIT
COIs	CURRENT COI AT 2008 VBT TABLE; GUARANTEED COI AT 2001 CSO
PREMIUM COMMISSIONS	YEAR 1: 115% TO TARGET TO AGE 55, 105% FOR 65 AND 2% EXCESS YEAR 2+: 2%
SURRENDER CHARGE	15 YEARS PER UNIT CHARGE

EXPENSES

ACQUISITION EXPENSE	$125 PER POLICY + 6% OF INITIAL PREMIUM + 0.6% OF FACE
MAINTENANCE EXPENSE	$150 PER POLICY + 3% OF PREMIUM

DECREMENTS

MORTALITY	RR100 VBT
LAPSE RATES	YEARS 1-10: 4%, YEARS 11+: 3%
DYNAMIC LAPSE FACTOR (TO BE MULTIPLIED TO BASE LAPSE)	MAX[0, 0.15% X {MAX(0, COMPETITOR RATE-CURRENT CREDIT RATE)}] COMPETITOR RATE = 10 YEAR TREASURY LESS 150 BPS
PARTIAL WITHDRAWAL	0%

CREDITING STRATEGY

REAL-WORLD CREDITING	PORTFOLIO YIELD LESS 1.5%
RISK-NEUTRAL CREDITING	10 YEAR TREASURY LESS 1.5%
GUARANTEED CREDITING	3%

RESERVES AND SURPLUS

STATUTORY RESERVE	UL MODEL REGULATION WITH 4% INTEREST RATE AND 2001 CSO ULT
TARGET SURPLUS:	
C1+C3	3.50%
C2	0.30%
C4	4.50%

ECONOMIC ASSUMPTIONS

LIQUIDITY PREMIUM	30 BPS (ONLY RELEVANT IN MCVNB)

The premiums of the UL product are paid into the general account and there is no equity indexed participation. There is a minimum interest rate guarantee that is credited to the account value, while front-end loads, surrender charges and cost of insurance (COI) mortality charges are deducted from the account value. The COI charges vary by attained age, gender and underwriting class, but we have only used a single underwriting class in our pricing. A level death benefit equal to the face amount, and adjusted for corridor effects, has been assumed.

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The deterministic real-world profit measures for this product are as follows. We solved for the premium rates that would achieve these targets before we move to the risk-neutral pricing.

- An IRR of 12.1% based on distributable earnings.
- A pre-tax premium margin of 10.1%.
- An ROA of 1.3%, where the denominator is the present value of the projected account value. However, ROA is not a particularly helpful metric for UL products, so this measure may be misleading.

Figure 12 illustrates, for both the real-world deterministic and risk-neutral deterministic projections, a statutory source of earnings analysis with items expressed as ratios over present value of premiums. As discussed earlier, both real-world and risk-neutral results have been discounted using the pre-tax earned rate (which in the risk-neutral world, is simply the risk free rate inclusive of the adjusted liquidity premium).

FIGURE 12: SOURCES OF EARNINGS		
	% OF PV PREMIUM	
SOURCES OF EARNINGS	RN	RW
INVESTMENT INCOME ON RESERVES AND CASH FLOW	31.1%	44.2%
INTEREST CREDITED	-23.9%	-36.3%
COI	41.3%	35.8%
EXPENSE LOAD	7.3%	7.3%
SURRENDER CHARGE	3.2%	2.4%
COMMISSIONS	-11.3%	-10.8%
ACQUISITION EXPENSES	-1.1%	-1.1%
MAINTENANCE EXPENSES	-6.8%	-6.8%
DEATH BENEFIT	-32.4%	-26.5%
CHANGE IN STATUTORY RESERVE OVER AV	1.8%	1.9%
PRE-TAX STATUTORY PROFIT	9.1%	10.1%
POST-TAX STATUTORY PROFIT / PVFP	5.7%	6.3%
CHANGE IN CAPITAL	-1.9%	-2.3%
INTEREST ON CAPITAL	1.8%	2.3%
TAXES ON INTEREST ON CAPITAL	-0.6%	-0.8%
DISTRIBUTABLE EARNINGS	5.0%	5.5%

As one might expect, investment income earned and interest credited are both lower under the risk-neutral paradigm, since there is no credit spread reflected in the risk-neutral projection. The lower interest credited in turn implies a lower account value and a higher net amount at risk, so COI charges are also higher in a risk-neutral environment, which further acts as a drag to the account value growth. This is also consistent with higher excess death benefits paid to the policyholder (over the account value) under a risk-neutral environment. Note that the COI charges exceed the excess death benefits that are paid out, which suggests that (on a deterministic basis at least), the COI assumption is sufficient to cover the mortality exposure.

Figure 13 presents a more intuitive representation of the sources of earnings by defining the following margins:

- An interest margin (investment income earned on the reserve less the interest credited on the account value)

- A mortality margin (the cost of insurance charges less the excess death benefits paid the policyholder over the account value)

- An expense margin (including account value and expense loads net of acquisition expenses, maintenance expenses and commissions)

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- A surrender margin (the account value released on surrender less the surrender benefits paid)

- A reserve margin (the excess of the change in the account value over the change in the reserve), which can be thought of as the statutory expense allowance

FIGURE 13: SOURCES OF EARNINGS

	% OF PV PREMIUM	
SOURCES OF EARNINGS	RN	RW
INTEREST MARGIN	7.2%	8.0%
MORTALITY MARGIN	8.8%	9.3%
EXPENSE MARGIN	-11.9%	-11.4%
SURRENDER MARGIN	3.2%	2.4%
RESERVE MARGIN	1.8%	1.9%
PRE-TAX STATUTORY PROFIT	9.1%	10.1%
POST-TAX STATUTORY PROFIT / PVFP	5.7%	6.3%
DISTRIBUTABLE EARNINGS	5.0%	5.5%

The risk-neutral deterministic result is slightly lower than the real-world deterministic result. However, there is a noticeably wider gap between the risk-neutral and real-world stochastic results. The components of TEV and MCVNB for the UL product are outlined in Figure 14. Note that TEV is essentially the average of the stochastic real-world discounted distributable earnings.

FIGURE 14: MCVNB VS. TEV

	% OF PV PREMIUM	
	RN	RW
PVFP (CEQ) / PVFP (RW DETERMINISTIC)	5.7%	6.3%
TVOG	-3.0%	-0.3%
CNHR	-5.4%	
FC	-0.7%	-0.8%
MCVNB / TEV	-3.5%	5.2%

As shown in Figure 14, the PVFP row ties to the deterministic post-tax profit presented earlier.

We note that the risk-neutral TVOG is considerably larger (in absolute value) than the real-world analog. This is as expected given the fact that under the risk-neutral framework the growth in the account value is lower and so there is a greater net amount at risk. This is probably exacerbated by the fact that the policyholder must be credited at least the guaranteed interest rate (set to 3% for this case study) on the account value, and so on a stochastic basis, there is greater spread compression that is occurring in the risk-neutral projection as compared to the real-world projection.

The value of CNHR is high (in absolute value), primarily due to the sensitivity of the UL product to the mortality risk. There is no real-world counterpart to the CNHR.

We note that the MCVNB of -3.5% indicates that this product is not creating shareholder value, which is in direct contrast to the positive TEV of 5.2% calculated under the real-world approach. The main drivers of this difference are the differences in TVOG and the explicit recognition of the CNHR under the risk-neutral paradigm. Differences in TVOG suggest that UL real-world profitability relies on the additional

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investment spreads in the real-world scenarios. However, the extra risks related to the additional returns are not accounted for explicitly. The CNHR reflects the explicit allowance for the non-hedgeable risks, such as mortality in this case. The TEV result is based on best-estimate mortality assumption, and lacks an adjustment for the uncertainty related to the mortality.

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UNIVERSAL LIFE WITH SECONDARY GUARANTEES CASE STUDY

The universal life with secondary guarantee (ULSG) feature and related pricing assumptions are summarized as below.

FIGURE 15: PRODUCT-SPECIFIC ASSUMPTIONS	
ISSUE AGE DISTRIBUTION	55 (100%)
GENDER DISTRIBUTION	MALE (100%)
SMOKER DISTRIBUTION	NONSMOKER (100%)
AVERAGE SIZE	$1,000,000
PRODUCT FEATURES	
INSURANCE PERIOD	UP TO AGE 121
PREMIUM PERIOD	LEVEL ANNUAL PREMIUMS UP TO AGE 121
EXPENSE LOAD	7.5% OF PREMIUM + $50 PER POLICY + 0.75% OF FACE
DEATH BENEFIT	100% OPTION A DEATH BENEFIT
COIS	60% OF 2001 VBT TABLE
COMMISSIONS	YEAR 1: 100% INITIAL PREMIUM, YEAR 2+: 2% OF PREMIUMS
SURRENDER CHARGE	YEAR 1: $40 PER UNIT YEAR 1, DECLINE BY $1.60 PER YEAR THROUGH YEAR 8, DECLINE BY $2.40 PER YEAR TO $0.00 IN YEAR 20
SHADOW ACCOUNT	
	5% OF PREMIUM LOAD
	COI AT 100% OF 2001 CSO ANB
	$50 LOAD PER POLICY
	7.2% CREDITED RATE
EXPENSES	
ACQUISITION EXPENSE	$125 PER POLICY + 1% OF INITIAL PREMIUM
MAINTENANCE EXPENSE	$100 PER POLICY + 3% OF PREMIUMS
DECREMENTS	
MORTALITY	60% OF THE 2001 VBT ANB TABLE
LAPSE RATES	YEARS 1-3: 4%, YEARS 4-10: 3%, YEARS 11-15: 2%, YEARS 16+: 1% IF ACCOUNT VALUE IS LESS THAN 0, CONTRACT KEEPS IN-FORCE
DYNAMIC LAPSE FACTOR (TO BE MULTIPLIED TO BASE LAPSE)	MAX[0, 0.25% × {MAX(0, COMPETITOR RATE-CURRENT CREDIT RATE)^1.25}] COMPETITOR RATE = 7-YEAR TREASURY - 200 BPS
CREDITING STRATEGY	
REAL-WORLD CREDITING	PORTFOLIO YIELD LESS 2%
RISK-NEUTRAL CREDITING	7 YEAR TREASURY LESS 2%
GUARANTEED CREDITING	1%
RESERVES AND SURPLUS	
STATUTORY RESERVE	AXXX WITH 4% INTEREST RATE AND 2001 CSO ANB S&U
TARGET SURPLUS:	
C1+C3	4.15%
C2	NONE
C4	6.00%
ECONOMIC ASSUMPTIONS	
LIQUIDITY PREMIUM	30 BPS (ONLY RELEVANT IN MCVNB)

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The ULSG product under consideration in this case study (which is unrelated to the UL product considered in the previous section) has a shadow account secondary guarantee provision. Under this structure, the contract will not lapse over the lifetime of the policyholder so long as a shadow account value that is tracked within the product has positive value. This shadow account functions much like the policyholder's account value, except that all the parameters that impact the evolution of the account are on a guaranteed (and hence scheduled) basis. No securitization or reinsurance of the guarantee has been assumed in this case study.

The deterministic real-world profit measures for this product are as follows:

- An IRR of 10.0% based on distributable earnings.
- A pre-tax premium margin of 9.5%.
- An ROA of 1.2%, where the denominator is the present value of the projected account value. However, ROA is not a particularly helpful metric for ULSG products, so this measure may be misleading.

In looking at Figure 16, the sources of earnings components (in both cash flow format and margin format), for this product comprise the same components as the UL product. Please refer to the UL case study for definitions of different margins. Recall that the results below are for the deterministic scenario only. The stochastic results are considered when comparing the components of TEV and MCVNB.

FIGURE 16: SOURCES OF EARNINGS

	% OF PV PREMIUM	
SOURCES OF EARNINGS	**RN**	**RW**
INVESTMENT INCOME ON RESERVES AND CASH FLOW	45.1%	52.8%
INTEREST CREDITED	-24.2%	-32.0%
COI	46.7%	44.1%
EXPENSE LOAD	12.9%	12.9%
SURRENDER CHARGE	4.4%	4.3%
COMMISSIONS	-9.9%	-10.1%
ACQUISITION EXPENSES	-0.2%	-0.2%
MAINTENANCE EXPENSES	-3.9%	-3.9%
DEATH BENEFIT	-55.3%	-51.9%
CHANGE IN STATUTORY RESERVE OVER AV	-7.6%	-6.6%
PRE-TAX STATUTORY PROFIT	8.1%	9.5%
POST-TAX STATUTORY PROFIT / PVFP	5.2%	6.1%
CHANGE IN CAPITAL	-2.4%	-2.7%
INTEREST ON CAPITAL	2.3%	2.7%
TAXES ON INTEREST ON CAPITAL	-0.8%	-1.0%
DISTRIBUTABLE EARNINGS	4.4%	5.2%

	% OF PV PREMIUM	
SOURCES OF EARNINGS	**RN**	**RW**
INTEREST MARGIN	20.8%	20.8%
MORTALITY MARGIN	-8.5%	-7.8%
EXPENSE MARGIN	-1.1%	-1.2%
SURRENDER MARGIN	4.4%	4.3%
RESERVE MARGIN	-7.6%	-6.6%
PRE-TAX STATUTORY PROFIT	8.1%	9.5%
POST-TAX STATUTORY PROFIT / PVFP	5.2%	6.1%
DISTRIBUTABLE EARNINGS	4.4%	5.2%

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As with the UL case study, both the investment income earned and the interest credited are lower under the risk-neutral paradigm on a deterministic basis. However, the interest margin (investment income earned less interest credited) is very similar, which suggests that the minimum interest rate guarantee (3% for this case study) does not seem to come into play, at least on a deterministic basis. Irrespective of this, however, the interest margin is clearly a driver of the overall profitability.

The excess death benefits that are paid out are slightly higher under the risk-neutral projection, which can be attributed to the secondary guarantee feature having a greater impact in this framework than under the real-world framework. Moreover, under both the real-world and risk-neutral projections, the COIs are lower (in absolute value) than the excess death benefits, (resulting in a negative mortality margin) which suggests that the COI assumption used in the pricing may be on the low side.

Consistent with our remarks with the UL case study, we note that the similarity between the risk-neutral and real-world profits is largely because we are focusing here on comparing the deterministic scenario only. The analogous ULSG results based on the stochastic projections should illustrate a wider gap in the profits.

The risk-neutral deterministic result is slightly lower than the real-world deterministic result. However, as shown in Figure 17, there is a noticeably wider gap between the risk-neutral and real-world stochastic results. The components of TEV and MCVNB for the ULSG are outlined below. Note that TEV is essentially the average of the stochastic real-world discounted distributable earnings.

FIGURE 17: MCVNB VS. TEV

	% OF PV PREMIUM	
	RN	RW
PVFP (CEQ) / PVFP (RW DETERMINISTIC)	5.2%	6.1%
TVOG	-3.1%	-0.9%
CNHR	-13.9%	
FC	-0.7%	-0.9%
MCVNB / TEV	-12.4%	4.3%

The PVFP row above ties to the deterministic post-tax profit in the earlier tables. As Figure 17 indicates, the risk-neutral TVOG is materially greater (in absolute value) than the real-world analog. This is due to the fact that the ULSG product is more susceptible to higher spread compression over the stochastic risk-neutral scenarios.

What might be a little surprising is that the TVOG calculated here for the ULSG product is not materially greater (in absolute value) than the UL product. All else being equal, one might expect that the presence of the secondary guarantees in the ULSG would imply a higher TVOG than a UL product, although we note that these are two unrelated designs—the ULSG base product is not the same as the UL product.

Note also that our modeling of the ULSG product assumes the same premium funding pattern over all the economic scenarios. If the funding pattern were assumed to be dynamic with respect to the scenarios then the TVOG (both risk-neutral and real-world) would very likely be greater. However, for simplicity we have assumed that the same premium funding pattern is used. In practice, companies would need to use their own experience to determine the appropriate dynamic funding pattern for the ULSG pricing.

The value of CNHR is very high (in absolute value), primarily due to the sensitivity of the ULSG secondary guarantees to mortality and lapse. Applying mortality and lapse shocks result in changes in the market value of liability cash flows that are primarily driven by the excess death benefits that are paid to the policyholder as a result of the secondary guarantee. For example, lower lapses typically result in slightly higher COIs (since there is more of a policyholder base to which the COI charges can be applied) but

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this is more than offset by the increase in excess death claims arising from the fact that policyholders are choosing to retain their policy and not lapse.

We note that the MCVNB of -12.4% indicates that this product is not creating shareholder value, which is in direct contrast to the positive TEV of 4.3% calculated under the real-world approach. For the same reasons in the UL case study, the main drivers of this difference are the differences in TVOG and the explicit recognition of the CNHR under the risk-neutral paradigm. Again, there might be a question on whether the real-world pricing fully and accurately captures all the risks.

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VARIABLE ANNUITY CASE STUDY

The variable annuity (VA) product feature and related pricing assumptions are summarized as below.

FIGURE 18: PRODUCT-SPECIFIC ASSUMPTIONS	
ISSUE AGE DISTRIBUTION	55 (20%), 65 (50%), 75 (30%)
GENDER DISTRIBUTION	MALE (60%), FEMALE (40%)
PRODUCT FEATURES	
INSURANCE PERIOD	UP TO AGE 115
PREMIUM	$100,000 SINGLE PREMIUM, 60% TO EQUITIES AND 40% TO BONDS
M&E CHARGE	130 BPS
POLICY FEE	$30 PER POLICY WHEN ACCOUNT VALUE IS BELOW $50,000
DEATH BENEFIT	RETURN OF PREMIUM GMDB
COMMISSIONS	5% OF INITIAL PREMIUM + 0.5% OF ACCOUNT VALUE
SURRENDER CHARGE	YEAR 1: 7%, YEAR 2-3: 6%, YEAR 4: 5%, YEAR 5+: 0% OF PREMIUM
INVESTMENT MANAGEMENT FEE	100 BPS
REVENUE SHARING	40 BPS (40% OF INVESTMENT MANAGEMENT FEE PAID TO INSURANCE COMPANY)
GLWB	
	HIGHER OF 5% COMPOUND ROLLUP AND ANNUAL RATCHET, ROLLUP UP TO EARLIER OF YEAR 10 OR FIRST WITHDRAWAL, RATCHET UP TO AGE 80
	GUARANTEED WITHDRAWALS OF 5% GLWB FACE AMOUNT FOR AGE LESS THAN 65, 6% FOR AGE LESS THAN 75, AND 7% OTHERWISE
	100 BPS RIDER CHARGE
EXPENSES	
ACQUISITION EXPENSE	$100 PER POLICY + 1% OF INITIAL PREMIUM
MAINTENANCE EXPENSE	$75 PER POLICY SUBJECT TO 3% INFLATION
DECREMENTS	
MORTALITY	ANNUITY 2000 BASIC TABLE
LAPSE RATES	1.5%, 2.5%, 3%, 3%, 4%, 5%, 8%, 10%, WITH SHOCK LAPSE OF 30% AT END OF SURRENDER CHARGE PERIOD
DYNAMIC LAPSE FACTOR (TO BE MULTIPLIED TO BASE LAPSE)	MIN[1, MAX(0, 1 - 75% X {ITM - 1})] ITM = GMWB/ACCOUNT VALUE
PARTIAL WITHDRAWAL	FULL UTILIZATION OF MAXIMUM GUARANTEE STARTING FROM YEAR 5
RESERVES AND SURPLUS	
STATUTORY RESERVE	AG 43 SUBJECT TO STANDARD SCENARIO FLOOR
CAPITAL	C3P2 SUBJECT TO STANDARD SCENARIO FLOOR
ECONOMIC ASSUMPTIONS	
LIQUIDITY PREMIUM	0 BPS (ONLY RELEVANT IN MCVNB)

The VA product has a single premium, which we have assumed is invested in multiple separate accounts that are segregated from the insurance company's general account. The investment performance of these separate accounts determines the growth in the policyholder's underlying account value. We have assumed that the GLWB rider in this case study has a benefit base that grows using both an annual ratchet and compound rollup design, with systematic withdrawals occurring after a short deferral period. The GLWB allows for a guaranteed source of income that will last the life of the policyholder, irrespective of either market performance and/or whether the underlying account value has been depleted. There is also a return of premium guaranteed minimum

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death benefit on the product, which is assumed to be covered by the mortality and expense charge, and is therefore offered at no additional explicit charge to the policyholder.

The statutory reserves and capital requirements for VA fall under the purview of Actuarial Guideline 43 (AG 43) and C-3 Phase II respectively, both of which involve calculating surplus on a stochastic basis. For this case study, we have generated tables that store reserve and capital factors, and that vary by GLWB in-the-moneyness, age, gender and duration. These factors have been calculated under the CTE Amount/Total Asset Requirement and Standard Scenario approaches that are prescribed under AG 43 and C-3 Phase II. These pre-calculated factors are then used in our real-world and risk-neutral deterministic pricing to determine the reserves and capital. Specifically, at any point in the projection these quantities can be calculated by simply multiplying the projected account value by the appropriate (and likely interpolated) reserve or capital factor.

The deterministic real-world profit measures for this product are as follows:

- An IRR of 10% based on distributable earnings
- A pre-tax premium margin of 8.6%, discounting at the pre-tax earned rate
- An ROA of 1.1%, where the denominator is the present value of the projected account value, discounting at the pre-tax earned rate

Using a single scenario to report VA pricing results ignores the fact that significant claims are unlikely to be generated under a single deterministic scenario, and thus considerably understates the risk. In practice, stochastic scenarios would almost always be used to price a VA. We are simply comparing the deterministic real-world scenario to the risk-neutral CEQ scenario because we are interested in bridging the results, and this bridge is easier constructed using a single scenario. The stochastic pricing results for the VA case study are considered when comparing the components of TEV and MCVNB. Figure 19 illustrates, for both the real-world deterministic and risk-neutral deterministic projections, a statutory source of earnings analysis with items expressed as ratios over present value of premiums. Both real-world and risk-neutral results have been discounted using the pre-tax earned rate (which in the risk-neutral world, is simply the risk free rate inclusive of the adjusted liquidity premium).

FIGURE 19: SOURCES OF EARNINGS

SOURCES OF EARNINGS	% OF PV PREMIUM	
	RN	RW
INVESTMENT INCOME ON RESERVES AND CASH FLOW	7.9%	3.2%
M&E FEE	8.5%	10.6%
GLWB RIDER CHARGE	9.9%	11.0%
NET REVENUE SHARING	2.6%	3.3%
POLICY FEE	1.5%	1.7%
SURRENDER CHARGE	0.4%	0.5%
COMMISSIONS	-7.7%	-8.5%
ACQUISITION EXPENSES	-1.6%	-1.6%
MAINTENANCE EXPENSES	-4.5%	-5.1%
GMDB CLAIMS	-0.8%	0.0%
GMWB CLAIMS	-5.5%	-3.2%
CHANGE IN STATUTORY RESERVE OVER AV	-7.9%	-3.2%
PRE-TAX STATUTORY PROFIT	2.8%	8.6%
POST-TAX STATUTORY PROFIT / PVFP	1.8%	5.6%
CHANGE IN CAPITAL	-0.1%	-0.1%
INTEREST ON CAPITAL	0.1%	0.1%
TAXES ON INTEREST ON CAPITAL	0.0%	0.0%
DISTRIBUTABLE EARNINGS	1.8%	5.5%

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Under the risk-neutral paradigm, the account value tends to be lower than under the real-world because the equity returns are lower. This has several impacts to the risk-neutral results:

▪ Asset based charges, such as the mortality and expense charge, GLWB rider charge and net revenue sharing are correspondingly less.

▪ There is higher investment income on reserves under the risk-neutral projection as compared to the real-world projection because the projected statutory reserves tend to be higher under the former (as a result of the lower account value and hence higher GLWB in-the-moneyness). For similar reasons, the change in reserve line is much higher in the risk-neutral world.

▪ GMDB claims and GLWB claims will be higher due to the higher in-the-moneyness of these guarantees.

The following breakdown is also helpful to understand the different deterministic profit:

FIGURE 20: SOURCES OF EARNINGS		
	% OF PV PREMIUM	
	RN	RW
BASE CONTRACT FEES LESS EXPENSES LESS GMDB CLAIMS	-1.6%	0.8%
INVESTMENT INCOME LESS CHANGE IN RESERVE OVER AV	0.0%	0.0%
GLWB CHARGES LESS GLWB CLAIMS LESS GMDB CLAIMS	4.4%	7.7%
PRE-TAX STATUTORY PROFIT	2.8%	8.6%
POST-TAX STATUTORY PROFIT / PVFP	1.8%	5.6%
DISTRIBUTABLE EARNINGS	1.8%	5.5%

Figure 20 above shows that the base contract cash flows as well as the GLWB rider charges net of claims are both noticeably lower under the risk-neutral paradigm, which is what we would expect given the lower account value and higher net amount at risk.

The components of TEV and MCVNB for the VA product are outlined below, both shown as percentages of present value of premium.

FIGURE 21: MCVNB VS. TEV		
	% OF PV PREMIUM	
	RN	RW
PVFP (CEQ) / PVFP (RW DETERMINISTIC)	1.8%	5.6%
TVOG	-3.2%	-2.3%
CNHR	-0.9%	
FC	-0.1%	-0.1%
MCVNB / TEV	-2.3%	3.2%

As shown in Figure 21, TVOG compares the deterministic and stochastic results. Under stochastic scenarios, there is uncertainty related to the timing and amount of the option claims, thus creating TVOG. TVOG is bigger on a risk-neutral basis, because there is greater impact of the excess GLWB and GMDB claims since account values are expected to be much smaller on a risk-neutral basis.

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The CNHR is relatively small. The mortality shocks primarily impact the GMDB and GLWB claims, and their coexistence offsets the mortality/longevity risk to a certain degree. Lower lapse result in higher GLWB and GMDB claims but also higher expected fees and revenues. The impact of the lapse shock, therefore, is mixed. Also, the dynamic lapse assumption is probably already conservative in itself, making the lapse risk charge smaller. For simplicity, we did not study the sensitivity to GLWB utilization assumption—we might expect the CNHR to be much higher if we were to do so.

It should be noted that the GLWB rider charge of 1% was priced to equal the hedge cost that is implied from the average of the risk-neutral stochastic GLWB claims, and so, on a stochastic risk-neutral basis, the GLWB rider charges and GLWB claims paid line items will nearly offset. Therefore the stochastic PVFP reduces to the residual base contract cash flows only (such as base contract charges, fees and expenses), as well as GMDB claims that need to be paid as well. These items are noticeably lower under the risk-neutral framework.

Thus, the bridge between the real-world TEV and the risk-neutral MCVNB for the VA product can be thought of as comprising mostly the following components:

- Lower base contract cash flows and lower guarantee rider charges net of claims under the risk-neutral paradigm, as reflected in the difference between the real-world and risk-neutral PVFP.

- Recognition of the CNHR under the risk-neutral paradigm, with no comparable such treatment under the real-world.

CNHR captures the cost of uncertainty revolving around mostly the actuarial assumptions, which should be captured and reflected in the real-world pricing too. However, it may be argued that the lower risk-neutral profit due to lower expected returns under risk-neutral scenarios is unreasonable because in reality investments will not be made in risk free assets. This argument has its merit, but it should be coupled with a second argument that real life investments bring in extra risks to the company and there should be an appropriate adjustment to reflect the cost of such risks in the real-world pricing.

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FLEXIBLE PREMIUM DEFERRED ANNUITY

The flexible premium deferred annuity (FPDA) product feature and related pricing assumptions are summarized as below.

FIGURE 22: PRODUCT-SPECIFIC ASSUMPTIONS	
ISSUE AGE DISTRIBUTION	45 (21%), 55 (26%), 65 (32%), 75 (21%)
GENDER DISTRIBUTION	MALE (50%), FEMALE (50%)
PRODUCT FEATURES	
INSURANCE PERIOD	UP TO AGE 100
PREMIUM	$17,500 INITIAL PREMIUM + RENEWAL PREMIUMS STARTING FROM $2,500 DECREASING AT 40% A YEAR
PREMIUM PERIOD	UP TO AGE 90
POLICY FEE	$65 PER POLICY
COMMISSIONS	5% OF PREMIUM
SURRENDER CHARGE	YEAR 1-3: 7%, YEAR 4: 6%, YEAR 5: 5%, YEAR 6: 4%, YEAR 7: 3%, YEAR 8+: 0% OF ACCOUNT VALUE
EXPENSES	
ACQUISITION EXPENSE	$75 PER POLICY
MAINTENANCE EXPENSE	$30 PER POLICY
DECREMENTS	
MORTALITY	ANNUITY 2000 TABLE
LAPSE RATES	3%, 3%, 3%, 4%, 5%, 6%, 7%, 12% WITH 10% SHOCK LAPSE AT END OF SURRENDER CHARGE PERIOD
DYNAMIC LAPSE FACTOR (TO BE MULTIPLIED TO BASE LAPSE)	MAX[0, 0.25% X {MAX(0, COMPETITOR RATE-CURRENT CREDIT RATE)}] COMPETITOR RATE = 7 YEAR TREASURY - 205 BPS
PARTIAL WITHDRAWAL	2.5% OF AV
CREDITING STRATEGY	
REAL-WORLD CREDITING	PORTFOLIO YIELD LESS 2.05%
RISK-NEUTRAL CREDITING	7-YEAR TREASURY LESS 2.05%
GUARANTEED CREDITING	3%
RESERVES AND SURPLUS	
STATUTORY RESERVE	AG 33 WITH 4.75% INTEREST RATE AND ANNUITY 2000 MORTALITY TABLE
TARGET SURPLUS:	
C1+C3	3.33% (SURRENDER CHARGE PERIOD), 6% (AFTERWARDS)
C2	NONE
C4	NONE
ECONOMIC ASSUMPTIONS	
LIQUIDITY PREMIUM	45 BPS (ONLY RELEVANT IN MCVNB)

The FPDA has flexible premiums that are paid into the general account. During the accumulation phase, the account value is credited at least the minimum guaranteed interest rate, although the insurance company will also credit rates in excess of the guaranteed interest rate at its discretion. The latter will depend on a number of factors, including the returns on their general account portfolio and the crediting rates of its competitors in the market. No market value adjustment or bailout provisions are assumed, and only the accumulation phase of the product has been modeled (that is, the payout phase where the policyholder has annuitized their accumulated account value has been ignored). Further, there is no death benefit specified in the contract in excess of either the account value or accumulated premiums net of withdrawals.

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The deterministic real-world profit measures for this product are as follows:

- An IRR of 12.1% based on distributable earnings.
- A pre-tax premium margin of 5.6%.
- An ROA of 0.7%, where the denominator is the present value of the projected account value

The sources of earnings components, in both cash flow format and margin format, for this product comprise similar components to the spread based products discussed earlier (please refer to the UL case study for definitions of different margins). Recall that the results below are for the deterministic scenario only. The stochastic results are considered when comparing the components of TEV and MCVNB.

FIGURE 23: SOURCES OF EARNINGS

	% OF PV PREMIUM	
SOURCES OF EARNINGS	RN	RW
INVESTMENT INCOME ON RESERVES AND CASH FLOW	28.4%	32.5%
INTEREST CREDITED	-24.9%	-24.9%
POLICY FEE	2.3%	2.3%
SURRENDER CHARGE	1.3%	1.1%
COMMISSIONS	-4.9%	-4.9%
ACQUISITION EXPENSES	-0.4%	-0.4%
MAINTENANCE EXPENSES	-1.2%	-1.2%
CHANGE IN STATUTORY RESERVE OVER AV	0.9%	1.1%
PRE-TAX STATUTORY PROFIT	1.5%	5.6%
POST-TAX STATUTORY PROFIT / PVFP	0.9%	3.6%
CHANGE IN CAPITAL	-1.3%	-1.4%
INTEREST ON CAPITAL	1.3%	1.4%
TAXES ON INTEREST ON CAPITAL	-0.4%	-0.5%
DISTRIBUTABLE EARNINGS	0.5%	3.1%

	% OF PV PREMIUM	
SOURCES OF EARNINGS	RN	RW
INTEREST MARGIN	3.5%	7.6%
EXPENSE MARGIN	-4.2%	-4.2%
SURRENDER MARGIN	1.3%	1.1%
RESERVE MARGIN	0.9%	1.1%
PRE-TAX STATUTORY PROFIT	1.5%	5.6%
POST-TAX STATUTORY PROFIT / PVFP	0.9%	3.6%
DISTRIBUTABLE EARNINGS	0.5%	3.1%

As is evident from Figure 23, the margin components are very similar between both the risk-neutral and real-world projections, with the sole exception of the interest margin, which as expected is lower under the risk-neutral framework since no credit spreads are recognized. The similarity of the expense margins is likely due to the commissions, which are dominated by an initial commission that is based on a percentage of premium assumption and not account value (the future renewal premium stream is not affected by the lower projected account value). The overall result is that the deterministic PVFP is lower under the risk-neutral projection, and that is driven by the lower interest margin.

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The components of TEV and MCVNB for the FPDA product are outlined below, both shown as percentages of present value of premium.

FIGURE 24: MCVNB VS. TEV		
	% OF PV PREMIUM	
	RN	**RW**
PVFP (CEQ) / PVFP (RW DETERMINISTIC)	0.9%	3.6%
TVOG	-2.2%	-0.7%
CNHR	-0.6%	
FC	-0.4%	-0.5%
MCVNB / TEV	-2.1%	2.4%

As with the other case studies, the PVFP row in Figure 24 ties to the prior tables showing the deterministic post-tax profit.

The risk-neutral TVOG for the FPDA product is greater (in absolute value) under the risk-neutral projection as compared to the real-world projection. This is as expected given the fact that under the risk-neutral framework the growth in the account value is lower and so there is a greater net amount at risk. This is probably exacerbated by the fact that the policyholder must be credited at least the guaranteed interest rate (set to 3% for this case study) on the account value, and so on a stochastic basis, there is greater spread compression that is occurring in the risk-neutral projection as compared to the real-world projection.

The CNHR is primarily driven by the sensitivity of the FPDA product to the lapse shocks (the mortality and longevity shocks that were produced did not result in very onerous capital requirements). Stated another way, the economic capital for non-market risks are small since for the most part the risks on this product are market related and not actuarial in nature.

We note that the MCVNB of -2.1% indicates that this product is not creating shareholder value, which is in direct contrast to the positive TEV of 2.4% calculated under the real-world approach. The main drivers of this difference are difference in the interest margin between both frameworks and the recognition of the CNHR in the risk-neutral presentation. The difference in interest margin is shown through PVFP line on a deterministic basis, and through TVOG line on a stochastic basis. This difference suggests FPDA real-world profitability relies on the additional investment spreads in the real-world scenarios. However, the extra risks related to the additional returns are not accounted for explicitly. The CNHR reflects the explicit allowance for the non-hedgeable risks, such as lapse in this case. The TEV result is based on best estimate lapse assumption, and lacks an adjustment for the uncertainty related to the lapse.

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EQUITY INDEXED ANNUITY

The equity indexed annuity (EIA) product feature and related pricing assumptions are summarized as below.

FIGURE 25: PRODUCT-SPECIFIC ASSUMPTIONS	
ISSUE AGE DISTRIBUTION	**45 (20%), 55 (30%), 65 (30%), 75 (20%)**
GENDER DISTRIBUTION	**MALE (50%), FEMALE (50%)**
PRODUCT FEATURES	
INSURANCE PERIOD	**20 YEARS**
PREMIUM	**$30,000 SINGLE PREMIUM**
COMMISSIONS	**5% OF PREMIUM**
SURRENDER CHARGE	**YEAR 1: 9%, YEAR 2: 8%, YEAR 3: 7%, YEAR 4: 6%, YEAR 5: 5%, YEAR 6: 4% YEAR 7: 3%, YEAR 8: 0% OF ACCOUNT VALUE**
INDEX BENEFIT DESIGN	**ANNUAL RATCHET**
MINIMUM CASH VALUE	**87.5% OF PREMIUM ACCUMULATED AT 1%**
GUARANTEED MINIMUM CAP	**3% (ACTUAL CAP DYNAMICALLY SOLVED FOR IN THE MODEL)**
PARTICIPATION RATE	**100%**
INDEX HEDGING	**OTC CALL OPTIONS**
DEATH BENEFIT	**ACCOUNT VALUE**
MVA	**NONE**
EXPENSES	
ACQUISITION EXPENSE	**$50 PER POLICY + 2.25% OF PREMIUM**
MAINTENANCE EXPENSE	**$110 PER POLICY**
DECREMENTS	
MORTALITY	**90% OF ANNUITY 2000 TABLE**
LAPSE RATES	**2%, 2.5%, 3%, 4%, 5%, 5.5%, 7%, 25%, 20%, 15%**
DYNAMIC LAPSE FACTOR (TO BE MULTIPLIED TO BASE LAPSE)	**MAX[0, 2% X {MAX(0, COMPETITOR RATE-CURRENT CREDIT RATE)} ^ 1.5] COMPETITOR RATE = 7 YEAR TREASURY - 225 BPS**
PARTIAL WITHDRAWAL	**2.5%**
CREDITING STRATEGY	
REAL-WORLD CREDITING	**PORTFOLIO YIELD LESS 2.25%**
RISK-NEUTRAL CREDITING	**7 YEAR TREASURY LESS 2.25%**
GUARANTEED CREDITING	**1%**
RESERVES AND SURPLUS	
STATUTORY RESERVE	**CARVM WITH MVRM USING BLACK SCHOLES, WITH 4.5% INTEREST RATE AND ANNUITY 2000 MORTALITY TABLE**
TARGET SURPLUS:	
C1	**2.55%**
C2	**NONE**
C3	**3.5% IF SC < 1%** **1.5% IF SC >= 5%** **2.5% OTHERWISE** **% OF STATUTORY RESERVES**
C4	**5% OF PREMIUM**
ECONOMIC ASSUMPTIONS	
LIQUIDITY PREMIUM	**30 BPS (ONLY RELEVANT IN MCVNB)**

An EIA product is essentially a deferred annuity that offers policyholders a return that is a proportion of the appreciation on a specified equity index, such as S&P 500 Index, and subject to a contractual guaranteed

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minimum interest rate. Under this structure, the policyholder benefits from upside equity participation and is simultaneously protected on the downside via the minimum interest rate guarantee. The appreciation on the equity index is also sometimes subject to a cap that limits the potential return to the policyholder.

The insurance company purchases bonds to cover the interest rate guarantee in the liability. Since the latter is usually a low interest rate the cost of buying these bonds is low, and any leftover funds can be used to purchase call options to hedge the exposure to the appreciation in the underlying equity index.

The EIA under consideration in this case study is an annual ratchet product, which provides index benefits based on 100% participation in the performance of the S&P index that are credited on an annual basis on the policy anniversary. A cap rate is dynamically solved each policy year such that the option budget (the funds that are spent on purchasing options) is equal to the asset option cost.

The deterministic real-world profit measures for this product are as follows:

- An IRR of 13.9% based on distributable earnings.
- A pre-tax premium margin of 7.3%.
- An ROA of 1.1%, where the denominator is the present value of the projected account value

The sources of earnings components, in both cash flow format and margin format, for this product comprise similar components to the UL product. Please refer to the UL case study for definitions of different margins. Recall that the results below are for the deterministic scenario only. The stochastic results are considered when comparing the components of TEV and MCVNB.

FIGURE 26: SOURCES OF EARNINGS

	% OF PV PREMIUM	
SOURCES OF EARNINGS	RN	RW
INVESTMENT INCOME ON RESERVES AND CASH FLOW	19.7%	26.5%
OPTION INCOME	5.4%	11.3%
INTEREST CREDITED	-15.7%	-22.2%
SURRENDER CHARGE	2.5%	1.7%
COMMISSIONS	-5.0%	-5.0%
ACQUISITION EXPENSES	-2.4%	-2.4%
MAINTENANCE EXPENSES	-2.7%	-3.1%
CHANGE IN STATUTORY RESERVE OVER AV	0.4%	0.5%
PRE-TAX STATUTORY PROFIT	2.3%	7.3%
POST-TAX STATUTORY PROFIT / PVFP	1.5%	4.8%
CHANGE IN CAPITAL	-1.2%	-1.6%
INTEREST ON CAPITAL	1.2%	1.6%
TAXES ON INTEREST ON CAPITAL	-0.4%	-0.6%
DISTRIBUTABLE EARNINGS	1.1%	4.2%

	% OF PV PREMIUM	
SOURCES OF EARNINGS	RN	RW
INTEREST MARGIN	9.5%	15.6%
EXPENSE MARGIN	-10.1%	-10.5%
SURRENDER MARGIN	2.5%	1.7%
RESERVE MARGIN	0.4%	0.5%
PRE-TAX STATUTORY PROFIT	2.3%	7.3%
POST-TAX STATUTORY PROFIT / PVFP	1.5%	4.8%
DISTRIBUTABLE EARNINGS	1.1%	4.2%

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Figure 26 on page 43 indicates that the main driver of the difference in profits is the interest margin, which is significantly lower under the risk-neutral projection. The interest margin in this case includes both the investment income from fixed assets and income from options, the latter of which should be fully offset by the crediting to the account value. Hence the difference in interest margin should still reflect the corporate spreads in the real-world framework.

The components of TEV and MCVNB for the EIA product are outlined below, both shown as percentages of present value of premium.

FIGURE 27: MCVNB VS. TEV		
	% OF PV PREMIUM	
	RN	RW
PVFP (CEQ) / PVFP (RW DETERMINISTIC)	1.5%	4.8%
TVOG	-2.7%	-1.8%
CNHR	-0.1%	
FC	-0.2%	-0.5%
MCVNB / TEV	-1.6%	2.4%

As with the other case studies, the PVFP row in Figure 27 ties to the prior tables showing the deterministic post-tax profit.

The risk-neutral TVOG for the EIA product is greater (in absolute value) under the risk-neutral projection as compared to the real-world projection. This is largely due to the fact that the policyholder must be credited at least the guaranteed interest rate on the account value, and so on a stochastic basis, there is greater spread compression that is occurring in the risk-neutral projection as compared to the real-world projection.

The CNHR is low since mortality and lapses are not key drivers of statutory earnings for the EIA product. The economic capital for non-market risks is small since for the most part the risks on this product are market related and not actuarial in nature. This is a similar conclusion to that reached for the other annuity products considered in this report (both the variable annuity and the FPDA).

We note that the MCVNB of -1.6% indicates that this product is not creating shareholder value, which is in direct contrast to the positive TEV of 2.4% calculated under the real-world approach. The main drivers of this difference are difference in the interest margin between both frameworks. The difference in interest margin is shown through PVFP line on a deterministic basis, and through TVOG line on a stochastic basis. This difference suggests EIA real-world profitability relies on the additional investment spreads in the real-world scenarios. However, the extra risks related to the additional returns are not accounted for explicitly.

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APPENDIX 2: ECONOMIC SCENARIO GENERATOR

Broadly speaking, an economic scenario generator (ESG) is a model that simulates a random sequence of values corresponding to the various financial assets. These assets might include items such as equity returns, interest rates, bond funds, or real estate for example. The underlying assets are assumed to follow a stochastic process (which may be formulated under either a real-world or risk-neutral paradigm), which is used to generate a distribution of possible future economic conditions pertaining to the future values taken on by these assets. This distribution is referred to in actuarial literature as a set of stochastic scenarios, and is commonly used for a variety of tasks, including (but not limited to) pricing, hedging, reserving and capital assessment.

The task at-hand often determines which particular paradigm (real-world versus risk-neutral) is appropriate. For instance, real-world scenarios are often used in the US for financial reporting purposes. However, risk-neutral scenarios are used to value the options and guarantees that are embedded in insurance liabilities, and for projecting out the value of hedges during a real-world reserve and/or capital calculation.

In general, real-world stochastic scenarios are generated using parameters that are calibrated with historical data. As discussed earlier in this report, risk-neutral scenarios on the other hand need to be calibrated to the initial market conditions as of the time the pricing occurs. Such a calibration involves determining the parameters such that the scenarios could be used to reproduce the observed prices at which derivatives trade in the market.

Key items that need to be considered when developing the risk-neutral scenarios include the following:

- Choice of ESG.
- Model calibration.
- The number of scenarios to be used.

There are a variety of possible ESG's to choose from, each comprising a different underlying mathematical framework. A formal discussion of these is beyond the scope of this report. Whether using different ESG's will give rise to a material difference in results is a complicated question. Generally speaking, this is a function of a number of factors, including (but potentially not limited to): the nature of the different mathematical structures between the ESG's in question, what reference instruments the ESG's are calibrated to, and how the reference instruments compare to the liabilities. It is often the case that the choice of reference instruments used in the calibration dominates. This means that as long as both ESG's use the same reference instruments in the calibration process and both ESG's have been successfully calibrated to reproduce the observed market prices of these reference instruments, it is expected that the corresponding pricing results that are generated using the different ESG's will be similar. However, products where policyholder behavior is closely driven by equity returns or interest rates may produce different liability values even though two ESGs produce similar asset values. For example, one ESG may produce negative interest rates and another may produce all non-negative interest rates but occasionally very large positive rates. The policyholder behavior that would be modeled under each of these is therefore likely to be very different and thus the corresponding liability value would differ.

Running sensitivities using additional risk-neutral scenarios produced by the chosen ESG is an important way to confirm that the number of scenarios that are used in the baseline pricing results is adequate. In general, the greater the number of scenarios that is used, the greater the convergence of the MCVNB estimate. Therefore, if under the sensitivity the MCVNB is materially different than that under the baseline, this would suggest that additional scenarios are needed. Typically 1,000 – 5,000 scenarios are used by most companies. The number of scenarios that are generated should be sufficient enough such that the set of scenarios can be used to reproduce the observed market prices – that is, they satisfy the calibration requirement. In some cases, scientific approaches to scenario compression can be adopted if the number of scenarios that are required is too onerous for the existing resources.

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REAL-WORLD

The Milliman Economic Scenario Generator (ESG) produces stochastic scenarios of *real-world* yield curve movements. It is not an arbitrage-free generator; the objective of the model is not to generate paths that will be able to reproduce market prices of interest rate derivatives. Rather, the goal of the model is to generate scenarios that capture many of the features of yield curve movements observed in the historical data. Among these stylized facts are the following:

- Interest rates are mean reverting, that is, they tend to fluctuate around some long-term target; when rates are very high relative to the target, there is a tendency for them to fall, and vice-versa.

- Points on the yield curve tend to move together.

- On average, investors earn a term premium to compensate them for the incremental risk associated with holding longer term bonds — that is, the yield curve tends to be positively sloped.

- However, the term premium will vary over time. The yield curve may invert occasionally, and there will also be periods of severe steepness.

- As rates rise, the yield curve tends to flatten.

- The short end of the yield curve is more volatile than the long end.

Our model composes the yield curve from three factors, informally termed the level, the slope, and the curvature. These three variables broadly correspond to the first three principal components of the yield curve.

The first step in using the model requires the definition of three points of the yield curve: the short yield (r_s), the mid-term yield (r_m) and the long-term yield (r_l). In many uses of the model, we have defined the short, the mid-term, and the long-term maturities as 1-year, 7-years, and 30-years, respectively.

The short end of the yield curve is generated using a lognormal process, with mean reversion to a long-term mean. The stochastic process for the short rate is defined by the following equation:

$$r_{s,t} = (r_{s,t-1} + A(\mu - r_{s,t-1})) \times \exp(\sigma\varepsilon_1 - \frac{\sigma^2}{2}) \qquad (1)$$

where $r_{s,t}$ and $r_{s,t-1}$ are this period's and the prior period's generated short rate, A represents mean-reversion strength, μ is the target long-term short-rate, σ is short-rate lognormal volatility, and ε is a unit-normal random variable.

In addition we generate the slope and curvature of the yield as additive random walk processes with mean reversion, as well. The slope and curvature variables, combined with the short-term interest rate, determine three points on the yield curve as follows:

$$r_m = r_s + \text{slope}$$

$$r_l = r_m + \text{slope} + \text{curvature}$$

The processes for the slope and curvature variables follow:

$$s_t = (s_{t-1} + A_s(\mu_s - s_{t-1})) + \sigma_s\varepsilon_s \qquad (2)$$

$$c_t = (c_{t-1} + A_c(\mu_c - c_{t-1})) + \sigma_c\varepsilon_c \qquad (3)$$

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where s_t and c_t are the slope and curvature variables at time t, A_s and A_c represent mean-reversion strengths for the two processes, μ_s and μ_c are long-term targets for the two variables, and σ_s and σ_c represent volatilities for the two processes.

In any given period, the rest of the yield curve is deterministically calculated from the three stochastically generated points. The default time-step for the model is monthly.

The model is parameterized using historical interest-rate data. Specifically, the parameters in equation 1 (A, μ, σ) are developed using maximum likelihood, while the parameters in equations 2 and 3 $(A_s, \mu_s, \sigma_s, A_c, \mu_c, \sigma_c)$ are developed using a linear regression model applied to the historical time-series of interest rates.

Lastly, we note that the equity index embedded in the real-world scenarios that have been generated above passes the equity calibration criteria requirements as outlined by the Actuarial Guideline 43 (reserving) and C-3 Phase II (capital) instructions.

RISK-NEUTRAL

For a model in which continuously compounded forward interest rates are normally distributed, the HJM/BGM framework prescribes the following form for generated yield curve changes:

$$F_t(t+T) = F_{t-1}(t+T) + \sum_q [\Lambda_{q,T+1}\Phi_{q,t} + \Lambda_{q,T+1} (-\Lambda_{q,T+1}/2 + \sum_{i=1}^{T+1} \Lambda_{q,i})] \tag{1}$$

Here, $F_t(t+T)$ is the time-t rate T years forward, $F_{t-1}(t+T)$ is the previous rate for this time period, $\Phi_{q,t}$ is a normal random number drawn for the q-th factor at time t, and $\Lambda_{i,j}$ is a matrix element representation of the factors. For a two factor model, $q=1$ or $q=2$. If factors are correlated, then they must be orthogonalized before use in equation (1).

In this framework, the value of a risk free asset is a rolling CD. The value of this asset M_t evolves according to the following equation:

$$M_t = M_{t-1} \exp[F_{t-1}(t-1)] \tag{2}$$

Equities assumed to have log-normally distributed returns evolve as follows:

$$S_t = S_{t-1} \exp[F_{t-1}(t-1) + \sigma_{t-1}\Phi_{t-1}^{(e)} - \sigma_{t-1}^2/2] \tag{3}$$

Here, S_t is the value of the equity at time t, $F_{t-1}(t-1)$ is the realized short rate at the beginning of the period, σ_{t-1} is the forward equity volatility, and $\Phi_{t-1}^{(e)}$ is a normal random number.

Under the two factor Hull White model, the correlated interest rate factors take the following parametric form:

$$\Lambda_{1,T+1} = s_1 [\exp(-a_1 T) - \exp(-a_1(T+1))]/a_1 \tag{4}$$

$$\Lambda_{2,T+1} = s_2/(a_2-a_1) \times [(\exp(-a_1 T) - \exp(-a_1(T+1)))/a_1 - (\exp(-a_2 T) - \exp(-a_2(T+1)))/a_2] \tag{5}$$

In addition to the volatility and mean reversion parameters for each factor, the correlation between factors $\rho_{1,2}$ can be regarded as a fifth parameter in the model.

As (3) indicates, stochastic interest rates contribute to the total return volatility of modeled equity funds. As such, if one wants to be consistent with Black Scholes implied volatilities, the Black Scholes volatilities must be corrected before they can be used in (3).

Expressing the corrected spot variance for maturity T as TV_1 and the Black Scholes spot variance as TV_T^0, the corrected variance is given by $TV_1 = TV_T^0$ for $T = 1$ and

$$TV_T = TV_T^0 + TVC_T - 2\rho_{0,1} \sum_{t=1}^{T-1} \{\sigma_{t-1} CVC_{1,T}\} - 2\rho_{0,2} \sum_{t=1}^{T-1} \{\sigma_{t-1} CVC_{2,T}\} \tag{6}$$

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for $T > 1$. Here, $\rho_{0,1}$ is the correlation between the equity and the first stochastic rate factor, $\rho_{0,2}$ is the correlation between the equity and the second stochastic rate factor, and the vectors VC_T, $CVC_{1,T}$ and $CVC_{2,T}$ are functions of the two factor Hull White parameters only, and can be calibrated in conjunction with calibration of the Hull White parameters.

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