REVIEW OF SOCIAL SECURITY FINANCING AND RELATED MATTERS

by

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1. INTRODUCTION

This paper is essentially a report on research for undergraduate experience grants held for 8 weeks by Felicity Messner and Aaron Robertson under the direction of Cecil Nesbitt. Felicity will be a junior in the Fall Term, concentrating in both actuarial mathematics and psychology, while Aaron will be an actuarial mathematics senior. The grants formed part of a National Science Foundation program of Research Experience for Undergraduates (REU). The study began with the reading of a number of introductory materials concerning Social Security. These included:

- Outline of the Provisions of Old-Age, Survivors, and Disability Insurance (OASDI) System in [1, Andrews-Beekman, 1987, Chapter 1].
- Mercer 1991 Guide to Social Security and Medicare [8, Mercer, 1992].
- R. J. Myers, Summary of the Provisions of the OASDI System as of January, 1992 [9, Myers, 1992].
- 1992 Trustees' Reports for OASDI, Hospital Insurance (HI) and Supplementary Medical Insurance (SMI), [2,3,4,1992. Copies were provided promptly by Deputy Chief Actuary Richard Foster]. Some comparisons were made with the 1990 and 1991 Reports.
- Cecil J. Nesbitt, Elementary Models of Reserve Fund for OASDI in the U.S.A. [12, Nesbitt, ARCH 1991.1, p. 63. See also Commentary by C. L. Trowbridge].

This last item introduced the concept of n-year roll-forward reserves as a theoretical discipline for financing social security systems, and benefit programs for state and municipal public employees. In the present paper the concept has been clarified and simplified. The resulting procedures may provide a challenging alternative to classical funding methods for public systems. The latter methods are essentially non-feasible for national programs such as Social Security, and this may be true to a lesser degree for state and municipal systems. The reason that classical advanced-funding methods fail for social

insurance is that there can be a long period of years between the average time of contributions and the average time of benefit disbursements [see 5, Bowers-Hickman-Nesbitt, 1979, pp. 116-117]. Meanwhile, the contributions accumulate under interest for those years and thereby create a massive fund which in the case of OASDI may run into trillions of dollars. It can be stated that n-year roll-forward reserve financing, by appropriate choice of n may lessen the massive build-up but at the same time may fail to answer questions as to whether deferred benefits will be paid when they become due. Appropriate choice of n may achieve a good balance.

Other background materials were [6, Dufresne, 1992], and [10,11, Myers, 1991, 1990]. Also a week was devoted to reading and reporting on [13, Robertson, 1992]. Our purpose here is to explore the applicability of n-year roll-forward reserves for public systems, but those readers interested in a broader range of issues may consult [14,1990].

In Section 2, we shall give formulas and illustrative computations of 1-year roll-forward reserves for OASDI, and in Section 3, the same items for 2-year roll-forward reserves (which end up in the trillions of dollars). In Section 4, we present computations which are theoretically correct for HI but not for SMI. In Section 5, we attempt computations of 5, 10 and 15-year roll-forward reserves on the basis of retrospective data for a large state retirement system, but again encounter problems of theory. Corrective measures are being explored. In Section 6, we develop some mathematical formulations, and consider the case of constant growth of benefit outgo and of taxable payroll. A relation for equilibrium is established. In a final Section 7, we summarize our findings regarding the applicability of n-year roll-forward reserves for social insurance and other large public benefit systems. We conclude that n-year roll-forward reserve funding is a refinement of the current method of financing OASDI with greater emphasis on the income required to support the desired level of reserves. Here 'income' is used in the sense of the OASDI Trustees' Reports, and so excludes interest for the system. For pension funding, one would refer to such income as 'contribution' necessary to support the funding principle.

2. n-YEAR ROLL-FORWARD RESERVES. ONE YEAR CASE FOR OASDI

It will become clear that the approach here to n-year roll-forward reserves is somewhat different than in [12, Nesbitt, 1991]. Both in the former paper, and this one, a starting point is the year-by-year projection of outgo for benefits and administration. This simple sentence covers a vast area of actuarial analysis that is required for the projection of such outgo. Fortunately, the annual Trustees' Reports for OASDI provide much information about projections of outgo, and this has been a ready source for applying our theory.

We also need projections of the interest rates to be earned on the Trust Fund of OASDI. In the 1991 paper we assumed varying effective annual rates. Here, we find it convenient to estimate and utilize forces of interest which may vary for the first decade of the projections.

In the 1991 paper, projection of income was also taken from the Trustees' report. Here, we calculate the annual incomes required to support the roll-forward reserves. Thus, starting with the annual outgoes, and interest rates, we proceed to develop the theoretical required income and fund amounts determined by the n-year roll-forward reserve discipline.

For roll-forward reserves, we shall use the following notations:

k = calendar year

δ_k = estimated force of interest for calendar year k

0_k = outgo for benefits and administration in calendar year k

We assume 0_k is centered at the middle of year k*

${}_nI_k$ = income, excluding interest, centered at the middle of calendar year k, required to maintain n-year roll-forward reserves

${}_nA_{12/31/k}$ = amount of reserve fund at $12/31/k$ under n-year roll-forward reserves.

W_k = taxable payroll for calendar year k**.

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*0_k is tabulated by alternative and by year in[2, Table III.B.4, pp. 183,184] for years 1992-2001, and quinquennially thereafter. Where we had need for year-by-year values, we interpolated geometrically.

**W_k is tabulated by alternative and by year in [2, Table III.B.1, pp. 176-178] for years 1992-2001, and quinquennially thereafter.

For the first decade, we estimated δ_k by means of the formula (2.1) from data in [2, 1992, Table III.B.3]

$$\delta_k = \frac{Actual\ Interest\ Income}{(A_{12/31/k-1} + A_{12/31/k})/2} \tag{2.1}$$

where $A_{12/31/k}$ is the tabulated assets at the end of year k. The results, and extensions thereof, are shown in Table 2.1.

TABLE 2.1

ESTIMATED FORCES OF INTEREST δ_k FOR THE COMBINED OASI AND DI TRUST FUNDS BY ALTERNATIVE AND CALENDAR YEARS 1992-2069*.

Calendar Year	Alternative 1	Alternative II	Alternative III
1992	8.21	8.22	8.26
1993	7.76	7.79	7.91
1994	7.29	7.42	7.78
1995	6.95	7.15	7.84
1996	6.70	6.96	7.85
1997	6.50	6.83	7.77
1998	6.35	6.74	7.65
1999	6.23	6.64	7.49
2000	6.14	6.57	7.37
2001	6.09	6.50	7.23
2002	6.03	6.40	6.95
2003	5.97	6.30	6.67
2004	5.91	6.20	6.40
and thereafter			

*Source: [2, 1992, Table III.B.3] for 1992-2001. As our computations are illustrative only, we extended the δ_k values as shown. Note that for Alternative 2, $A_{1/1/1992} = 327.8 + 291.4 - 338.4 = 280.8$, and that $0_{1992}e^{-(\delta_{1992})/2} = 279.7$, that is, the fund at the beginning of 1992 is approximately equivalent to 0_{1992}.

For 1-year roll-forward reserves and required incomes, we used the formulas

$$ {}_1A_{12/31/k} = 0_{k+1}e^{-\delta_{k+1/2}}, k = 1992, \ldots, 2069 \tag{2.2} $$

$$ = \text{fund required at the end of year } k \text{ under 1-year roll-forward reserves} $$

$$ {}_1I_k = 0_{k+1}e^{-(\delta_k+\delta_{k+1})/2} = {}_1A_{12/31/k}e^{-\delta_{k/2}} \tag{2.3} $$

where ${}_1I_k$ is the required income in year k, excluding interest, in order to maintain 1-year roll-forward reserves

$$ 100({}_1I_k/W_k) = \text{percent of taxable payroll required in year } k \text{ to maintain 1-year roll-forward reserves} \tag{2.4} $$

Such ratios (2.4) could be used quadrennially to set the tax rates for the next quadrennium. In doing so, account could be taken of the surplus or deficit between the actual trust fund at end of the quadrennium, and the appropriate ${}_1A_{12/31/k}$ requirement. Such quadrennial adjustments might be even more appropriate for the case of 2-year roll-forward reserves to be discussed in the next section.

In Tables 2.2.1, 2.2.2, and 2.2.3 values of ${}_10_k, W_k, {}_1A_{12/31/k}, {}_1I_k$ and $100({}_1I_k/W_k)$ are shown for the three respective alternatives, and by year to 2001, and quinquennially from 2005. Values for the first four functions are given to the nearest billion, and the percents of payroll are calculated using a larger number of significant figures.

TABLE 2.2.1

VARIOUS VALUES RELATED TO 1-YEAR ROLL-FORWARD RESERVES FOR OASDI UNDER ALTERNATIVE 1

(IN BILLIONS)

Year k	Outgo 0_k	Taxable Payroll W_k	Fund at End of Year k ${}_1A_{12/31/k}$	Income Req'd in Year k ${}_1I_k$	$100 \times ({}_1I_k/W_k)$
1992	290	2553	292	281	10.99
1993	304	2714	307	295	10.87
1994	318	2894	322	310	10.72
1995	333	3088	338	326	10.56
1996	349	3293	353	342	10.38
1997	365	3508	370	358	10.21
1998	382	3732	388	376	10.06
1999	400	3965	406	394	9.93
2000	419	4208	427	414	9.84
2001	440	4460	449	436	9.77
2005	540	5551	556	540	9.73
2010	725	7215	753	731	10.14
2015	1018	9309	1060	1029	11.06
2020	1446	11,915	1498	1454	12.21
2025	1998	15,237	2053	1994	13.08
2030	2657	19,584	2713	2634	13.45
2035	3414	25,348	3468	3367	13.28
2040	4282	32,854	4352	4226	12.86
2045	5386	42,544	5489	5330	12.53
2050	6868	55,015	7020	6816	12.39
2055	8886	71,165	9090	8826	12.40
2060	11,544	92,117	11,800	11,457	12.44
2065	14,931	119,323	15,257	14,813	12.41
2070	19,284	154,491	———	———	———

TABLE 2.2.2

VARIOUS VALUES RELATED TO 1-YEAR ROLL-FORWARD RESERVES FOR OASDI UNDER ALTERNATIVE 2

(IN BILLIONS)

Year k	Outgo O_k	Taxable Payroll W_k	Fund at End of Year k ${}_1A_{12/31/k}$	Income Req'd in Year k ${}_1I_k$	$100 \times ({}_1I_k/W_k)$
1992	291	2532	295	283	11.19
1993	307	2667	312	300	11.26
1994	324	2825	331	319	11.29
1995	343	3000	351	338	11.28
1996	363	3188	371	358	11.24
1997	384	3386	394	380	11.23
1998	407	3599	417	403	11.20
1999	431	3827	442	428	11.18
2000	457	4067	469	454	11.17
2001	485	4320	498	482	11.15
2005	610	5429	630	611	11.25
2010	836	7171	871	844	11.77
2015	1194	9370	1246	1208	12.89
2020	1725	12,099	1792	1737	14.36
2025	2434	15,550	2511	2434	15.65
2030	3320	20,028	3403	3299	16.47
2035	4385	25,918	4473	4337	16.73
2040	5655	33,531	5767	5591	16.68
2045	7284	43,268	7443	7215	16.68
2050	9471	55,654	9699	9403	16.89
2055	12,454	71,486	12,754	12,365	17.30
2060	16,384	91,820	16,757	16,246	17.69
2065	21,414	118,044	21,880	21,212	17.95
2070	27,846	151,762	———	———	———

TABLE 2.2.3

VARIOUS VALUES RELATED TO 1-YEAR ROLL-FORWARD RESERVES FOR OASDI UNDER ALTERNATIVE 3

(IN BILLIONS)

Year k	Outgo 0_k	Taxable Payroll W_k	Fund at End of Year k ${}_1A_{12/31/k}$	Income Req'd in Year k ${}_1I_k$	$100 \times ({}_1I_k/W_k)$
1992	292	2514	299	287	11.41
1993	311	2659	323	311	11.68
1994	336	2861	351	338	11.80
1995	365	3053	380	365	11.96
1996	395	3179	407	391	12.31
1997	423	3414	436	419	12.28
1998	453	3640	467	450	12.35
1999	485	3881	501	482	12.44
2000	520	4133	537	518	12.53
2001	557	4399	575	554	12.60
2005	724	5595	752	728	13.01
2010	1022	7534	1067	1034	13.72
2015	1493	10,023	1563	1514	15.10
2020	2204	13,137	2298	2226	16.94
2025	3191	17,074	3308	3204	18.77
2030	4487	22,175	4627	4481	20.21
2035	6134	28,836	6297	6099	21.15
2040	8206	37,395	8417	8152	21.80
2045	10,934	48,261	11,226	10,872	22.53
2050	14,636	61,944	15,046	14,572	23.52
2055	19,720	79,206	20,257	19,619	24.77
2060	26,473	101,164	27,142	26,288	25.99
2065	35,199	129,354	36,033	34,898	26.98
2070	46,440	165,435	———	———	———

The projected amounts shown of required fund, and required income are awesome. They range from 281 billion to 36 trillion. Alternative 2 by itself projects a range of 283 billion to 22 trillion. A more comfortable range of 11 percent to 18 percent of taxable payroll appears under alternative 2. This can be compared with the present income rate (excluding interest) of 100(313.4/2532) = 12.38 projected for 1992. [2, 1992, Table III.B.3, p. 182]. It should be noted that the 10 percent to 27 percent required income under 1-year roll-forward reserves could be offset partially by the .19-1.29 percent of taxable payroll estimated to become available from the taxation of OASDI benefits [See 2, 1992, pp. 124-125].

Another view of the financing involved is given in [13, Robertson, 1992, Chapter 7]. The view there should be balanced with the more sanguine concepts of his chapters 22-24.

A graphic summary by Alternative is given in Figures 2.1, 2.2, 2.3 and 2.4.

Figure 2.1
OUTGO vs. TIME

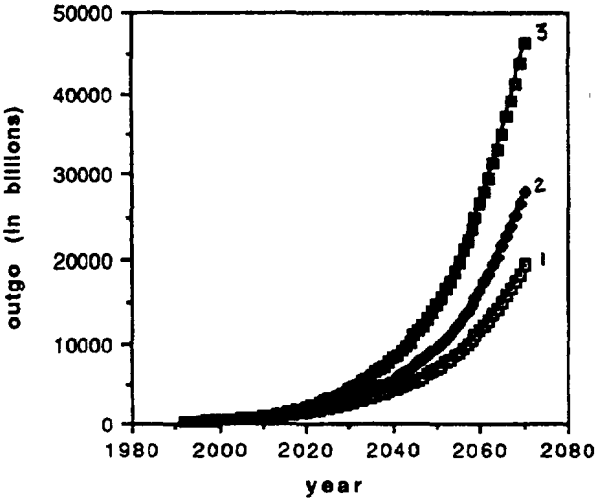


Figure 2.2
TAXABLE PAYROLL vs. TIME

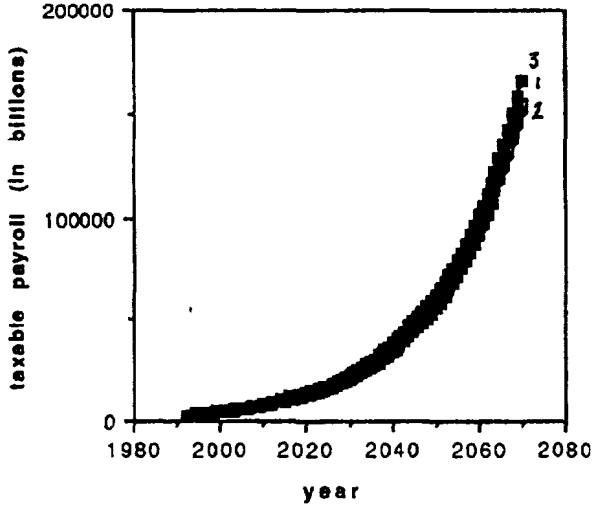


Figure 2.3

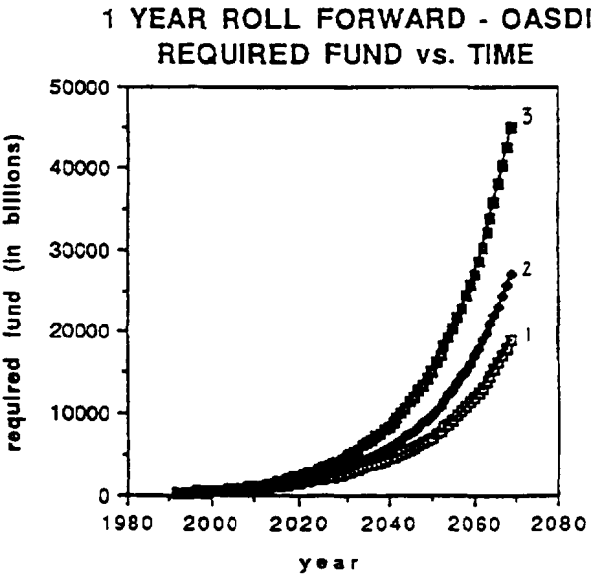
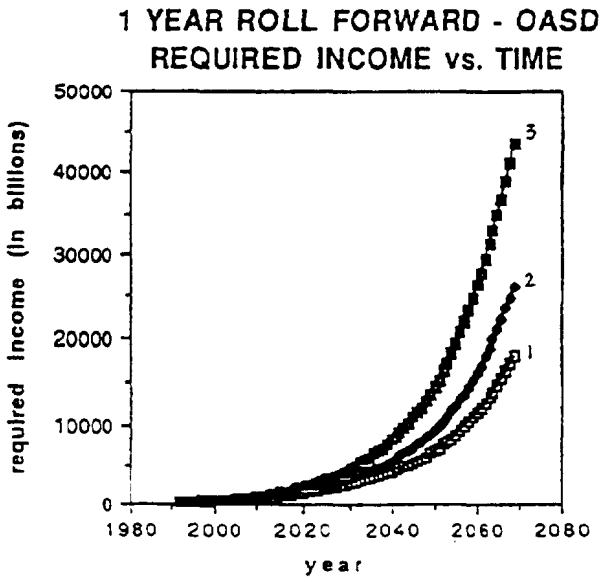


Figure 2.4



3. 2-YEAR ROLL-FORWARD RESERVES FOR OASDI

For the computations in this Section, we used the forces of interest listed in Table 2.1, the outgoes 0_k, and the taxable payrolls W_k previously employed for the 1-year roll-forward reserves (see Section 2). After year 2001, the Trustees' Report gives only quinquennial values of 0_k and W_k, thus we geometrically interpolated the intervening values. For the amount of the reserve fund required at the end of year k, we used the formula:'

$$ {}_2A_{12/31/k} = 0_{k+1}e^{-(\delta_{k+1}/2)} + 0_{k+2}e^{-(2\delta_{k+1}+\delta_{k+2})/2} \tag{3.1} $$

and for the required income in year k, the formula:

$$ {}_2I_k = 0_{k+2}e^{-(\delta_k+2\delta_{k+1}+\delta_{k+2})}/2 \tag{3.2} $$

From (3.2), we see that ${}_2I_k$ is equivalent to the 0_{k+2} that is projected to occur two years later. If this is done each year, and if at the end of year k-2 there was a sufficient reserve fund to provide the outgoes of year k-1 and k, (consider (3.1) with k-2 in place of k), then ${}_2I_{k-1}$ and ${}_2I_k$ can be accumulated to the end of year k to be equivalent on that date to 0_{k+1} and 0_{k+2}, respectively, in the subsequent two years. Continuation of this process provides a reserve at the end of each year $k-1, k, k+2, \ldots$ equivalent to the following two years' outgo.

This is simpler, and somewhat subtly different, from the roll-forward process developed in [12, Nesbitt, 1991]. In that process, actual projected income, taking into account the various sources provided by law, was balanced at the end of each n-year interval with the projected outgo for the following n-year interval. Here, the theoretical required income is determined to be equivalent to the outgo for the n^{th} year thereafter, and no projected income is taken into account. Only projected outgo is needed to determine the required income for this n-year roll-forward reserve process.

For the process to be applicable readily, the fund on hand at the beginning of the process should be at least equivalent to the outgo of the first n years. This frees the incomes of those years for allocation to the subsequent n years.

In Tables 3.1.1, 3.1.2, and 3.1.3, values of $0_k, W_k, {}_2A_{12/31/k}$ and $100({}_2I_k/W_k)$ are shown for the respective Alternatives, year by year to 2001, and quinquennially from 2005. Values for the first four functions are given to the nearest billion, and percents of payroll are calculated using a larger number of significant figures.

Again, it should be observed that the required percents of taxable payroll could be offset partially by the .19 – 1.29 percent estimated to become available from the taxation of OASDI benefits [see 2, 1992, pp. 124, 125]. There may also be some other offset sources, but the bulk of the required income would have to come from payroll taxes.

The following tables are theoretical. In actuality, the projected OASDI funds will not be sufficient to support 2-year roll-forward reserves until 1997, under Alternative 1; until 1999, under Alternative 2; or after 2001, under Alternative 3.

TABLE 3.1.1
VARIOUS VALES RELATED TO 2-YEAR ROLL-FORWARD RESERVES FOR OASDI UNDER ALTERNATIVE 1

(IN BILLIONS)

Year k	Outgo 0_k	Taxable Payroll W_k	Fund at End of Year k ${}_2A_{12/31/k}$	Income Req'd in Year k ${}_2I_k$	$100 \times ({}_2I_k/W_k)$
1992	290	2553	576	272	10.67
1993	304	2714	606	288	10.60
1994	318	2894	636	304	10.49
1995	333	3088	668	319	10.34
1996	349	3293	700	335	10.18
1997	365	3508	734	352	10.04
1998	382	3732	770	370	9.91
1999	400	3965	808	389	9.81
2000	419	4208	850	410	9.74
2001	440	4460	894	432	9.68
2005	540	5551	1113	540	9.73
2010	725	7215	1513	737	10.22
2015	1018	9309	2132	1040	11.18
2020	1446	11,915	3004	1463	12.27
2025	1998	15,237	4102	1989	13.06
2030	2657	19,584	5400	2610	13.33
2035	3414	25,348	6889	3321	13.10
2040	4282	32,854	8647	4170	12.69
2045	5386	42,544	10,922	5274	12.40
2050	6868	55,015	13,988	6764	12.30
2055	8886	71,165	18,120	8767	12.32
2060	11,544	92,117	23,510	11,369	12.34
2065	14,931	119,323	30,394	14,696	12.32

TABLE 3.1.2

VARIOUS VALES RELATED TO 2-YEAR ROLL-FORWARD RESERVES FOR OASDI UNDER ALTERNATIVE 2

(IN BILLIONS)

Year k	Outgo 0_k	Taxable Payroll W_k	Fund at End of Year k ${}_2A_{12/31/k}$	Income Req'd in Year k ${}_2I_k$	$100 \times ({}_2I_k/W_k)$
1992	291	2532	585	277	10.95
1993	307	2667	619	296	11.08
1994	324	2825	657	315	11.13
1995	343	3000	697	334	11.13
1996	363	3188	739	355	11.13
1997	384	3386	783	377	11.12
1998	407	3599	831	400	11.12
1999	431	3827	882	425	11.10
2000	457	4067	936	451	11.08
2001	485	4320	992	479	11.10
2005	610	5429	1261	611	11.26
2010	836	7171	1749	852	11.88
2015	1194	9370	2506	1221	13.04
2020	1725	12,099	3596	1749	14.46
2025	2434	15,550	4720	2288	15.66
2030	3320	20,028	6784	3278	16.37
2035	4385	25,918	8897	4289	16.55
2040	5655	33,531	11,470	5528	16.49
2045	7284	43,268	14,815	7147	16.52
2050	9471	55,654	19,327	9335	16.77
2055	12,454	71,486	25,418	12,277	17.17
2060	16,384	91,820	33,374	16,109	17.54
2065	21,414	118,044	43,554	21,012	17.80

TABLE 3.1.3

VARIOUS VALES RELATED TO 2-YEAR ROLL-FORWARD RESERVES FOR OASDI UNDER ALTERNATIVE 3

(IN BILLIONS)

Year k	Outgo 0_k	Taxable Payroll W_k	Fund at End of Year k ${}_2A_{12/31/k}$	Income Req'd in Year k ${}_2I_k$	$100 \times ({}_2I_k/W_k)$
1992	292	2514	598	287	11.40
1993	311	2659	648	312	11.74
1994	336	2861	702	338	11.81
1995	365	3053	756	362	11.85
1996	395	3179	810	388	12.20
1997	423	3414	869	416	12.19
1998	453	3640	932	448	12.30
1999	485	3881	1000	441	12.39
2000	520	4133	1072	515	12.47
2001	557	4399	1148	553	12.56
2005	724	5595	1506	731	13.07
2010	1022	7534	2147	1046	13.89
2015	1493	10,023	3148	1535	15.32
2020	2204	13,137	4620	2249	17.12
2025	3191	17,074	6631	3218	18.85
2030	4487	22,175	9222	4450	20.07
2035	6134	28,836	12,557	6063	21.03
2040	8206	37,395	16,779	8098	21.66
2045	10,934	48,261	22,388	10,811	22.40
2050	14,636	61,944	30,026	14,509	23.42
2055	19,720	79,206	40,412	19,520	24.64
2060	26,473	101,164	54,095	26,103	25.80
2065	35,199	129,354	71,760	34,601	26.75

One result of maintaining 2-year rather than 1-year reserves would be a doubling of the interest income. This would shift some of the financing of the system from payroll taxes to general revenue*. There are proponents for and against build-up of a large reserve fund, and thereby shifting the financing as indicated. There is, however, need to maintain confidence in the financial capacity of the OASDI system, and a reasonable level of reserve assets is needed for that purpose.

A final comment about reserves for OASDI is that the 1-year level has already been surpassed [see Reference 2, 1992, Table III.B.3, pp. 182], and a rapid build-up is projected to continue until well into the next century. There have been calls to smooth out the "roller-coaster" financing currently projected. Roll-forward reserve theory could provide a disciplined approach to this necessary development.

Some graphical summaries are given in Figures 3.1, 3.2, 3.3 and 3.4 for all three Alternatives.

* There is a subtle point here. Under roll-forward reserve financing, all assumed interest income is dedicated to maintaining the reserve fund from which outgo would be paid. Maintenance of the reserve fund also requires "new money" in the form of ${}_nI_k = 0_{k+n}e^{-\delta n}$ which may be greater or less than 0_k, depending on how much $e^{-\delta n}$ offsets outgo growth $(0_{k+n}/0_k)$. Over the long term, OASDI outgo is projected to grow at a rate only slightly less than the rate of interest, and $100({}_nI_k/W_k)$ may be not much less than $100(0_k/W_k)$.

Figure 3.1
2-YEAR ROLL- FORWARD – OASDI
REQUIRED FUND vs. TIME

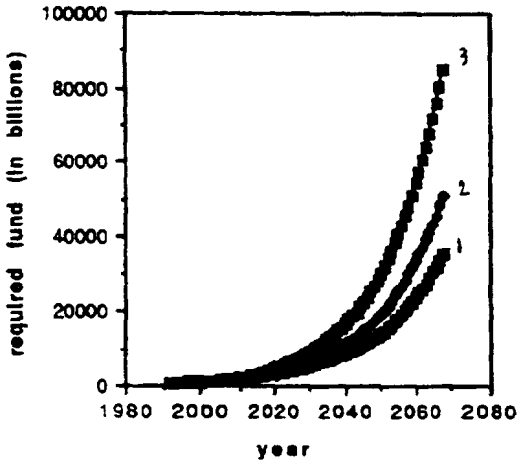
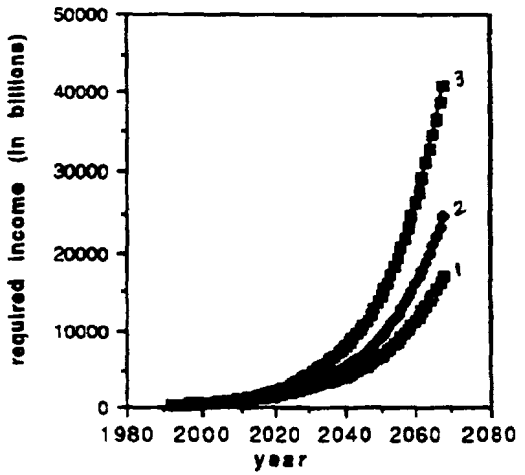


Figure 3.2
2-YEAR ROLL- FORWARD – OASDI
REQUIRED INCOME vs. TIME



On comparing the table for 1-year roll-forward reserves with those for the 2-year roll-forward, we note that the required reserves for the latter, being equivalent to two years' outgo, are approximately double those for the 1-year roll-forward case. The required annual incomes for the 2-year case generally

are slightly lower than for the 1-year case. The relation depends on the offsetting factors: rate of growth of outgo versus the rate of discount on future amounts.

Figure 3.3
1-YEAR ROLL- FORWARD -- OASDI
REQ'D INCOME RATIO vs. TIME

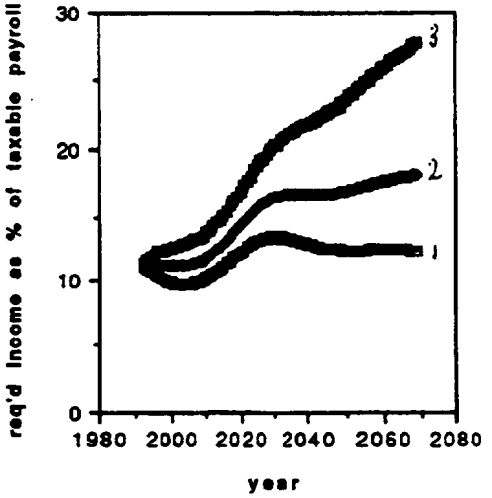
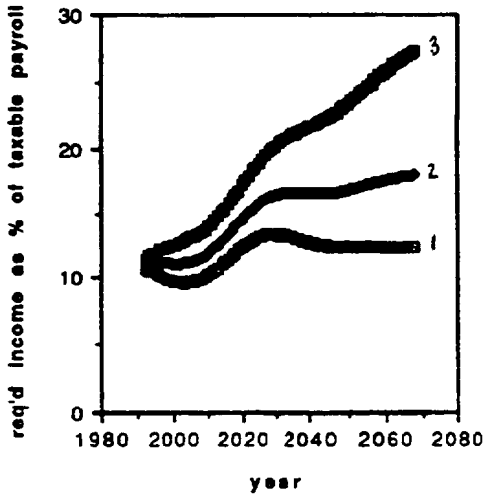


Figure 3.4
2-YEAR ROLL- FORWARD -- OASDI
REQ'D INCOME RATIO vs. TIME



4. 1-YEAR ROLL-FORWARD RESERVES FOR HI AND SMI

In Table 4.1 values of HI outgo for years 1992–2001 are shown for the three Alternatives [2, 1992, Table III.B.4, pp. 183, 184], and required reserve funds at ends of the year, and required incomes, for the years 1992–2000. Formulas (2.2) and (2.3) were used to calculate the ${}_1A_{12/31/k}$ and ${}_1I_k$ values. Again, note that all that is required is a year-by-year list of HI projected outgo, and of the assumed forces of interest, assuming that the fund at the beginning of the year is equivalent to one year's outgo. We assume present funding proceeds until the fund at the end of the year falls below the fund required by the 1-year roll-forward process. For Alternative 1, the current funding suffices through year 2000 but for Alternative 2, the current funding may need modification from 1998 onward, and for Alternative 3, from 1996 onward. Consequently, in 1998 for Alternative 2 and in 1996 for Alternative 3, we assume a switch is made to 1-year roll-forward reserve funding.

The projected future of HI is somewhat alarming. In [2, 1992, Table III.B.4, p. 184], HI outgo is estimated to near 20 trillion, under Alternative 2.

TABLE 4.1

VALUES RELATED TO 1-YEAR ROLL-FORWARD RESERVES FOR HI BY ALTERNATIVES*

(BILLIONS)

Calendar Year k	Outgo 0_k	HI Taxable Payroll W_k	Req'd. Fund ${}_1A_{12/31/k}$	Req'd. Inc. ${}_1I_k$	$100({}_1I_k/W_k)$
Alternative 1					
1992	79	2800	83	79	2.84
1993	86	2948	92	88	2.99
1994	95	3122	100	97	3.10
1995	104	3315	109	106	3.18
1996	113	3521	118	114	3.24
1997	122	3739	128	124	3.31
1998	132	3971	138	133	3.36
1999	142	4217	147	143	3.39
2000	152	4479	157	152	3.40
2001	162	4756			
Alternative 2					
1992	80	2800	84	80	2.87
1993	87	2948	93	90	3.05
1994	97	3122	103	99	3.19
1995	107	3315	115	111	3.35
1996	119	3521	126	121	3.45
1997	130	3739	138	134	3.57
1998	143	3971	152	147	3.70
1999	157	4217	165	160	3.80
2000	171	4479	180	174	3.89
2001	186	4756			
Alternative 3					
1992	80	2800	87	83	2.97
1993	90	2948	98	94	3.20
1994	102	3122	112	107	3.44
1995	116	3315	124	119	3.60
1996	129	3521	139	134	3.81
1997	145	3739	156	150	4.01

* Values of δ_k are taken from Table 2.1

1998	162	3971	174	168	4.23
1999	181	4217	195	188	4.45
2000	202	4479	216	208	4.65
2001	224	4756			

One notes that we now need new values of W_k since the earnings base for the HI tax was raised to 125,000 in 1991, and to 130,200 in 1992, and is subject to automatic adjustment from that level in the future. From various sources in the HI Report [3, 1992] we estimated W_{1991} as 2,697.4 billion. Taxable payrolls thereafter were estimated by applying rates of increase 3.8%, 5.3%, 5.9%, 6.2% for years 1992, 1993, 1994, 1995-2000 [See 3, 1992, Table 14, p. 49]. It should be noted that the taxable payroll is taken the same for all three Alternatives, since there was little difference for the OASDI payrolls by Alternative (see Figure 2.2).

We made two attempts to apply 1-year roll-forward reserve funding to SMI. For this purpose, we used the data in Table 1.C.5, pp. 13,14 of [4, 1992], which provides cash-basis SMI outgo for years 1991-2001 by Alternative. Our second attempt is presented in Table 4.2 for Alternative 2. Some problems arose because: (1) The initial fund on hand at the end of 1991 is only 32 percent of SMI outgo for 1992; (2) From 1992 to 2001, the average rate of growth is 12.51 percent. Advance funding for outgo growing at such a rate can be dubious. Nevertheless, Table 4.2 may be useful to demonstrate what might happen for a transitional approach to 1-year roll-forward reserve funding under circumstances such as (1) and (2) above. For HI, the situation was different: the reserve funds under current funding exceeded the required roll-forward reserves for some years, and the average rate of growth of outgo was less.

TABLE 4.2
TRANSITIONAL 1-YEAR ROLL-FORWARD RESERVE FUNDING FOR SMI, ALTERNATIVE 2*
(BILLIONS)

Year	Outgo	Force Of Interest	$A_{12/31/k}$	${}_1I_k$
1992	56.2	.0822	22.05	58.71
1993	63.6	.0779	26.89	66.54
1994	71.8	.0742	32.99	75.68
1995	81.4	.0715	40.31	86.11
1996	92.4	.0696	48.63	97.63
1997	104.6	.0683	58.31	110.82
1998	118.6	.0674	69.74	125.52
1999	134.2	.0664	83.27	142.66
2000	152.4	.0657	99.20	162.34
2001	173.3	.0650		

* Calculated by $A_{12/31/k-1}e^{\delta_k} + ({}_1I_k - 0_k)e^{\delta_k/2} = A_{12/31/k}$, starting with $A_{12/31/1991} = 17.9$ billions

$${}_1I_k = 0_{k+1}e^{-(\delta_k+\delta_{k+1})/2}$$

5. n-YEAR ROLL-FORWARD RESERVE FOR STATE SYSTEM HYPOTHETICAL DATA

We here report on an unfinished experiment with some state system data supplied by Alan Sonnanstine, A.S.A. [15, Sonnanstine, 1990]. For one large system, we were given outgo for a 26-year period. However, there were a number of changes in valuation date, the most recent being to the fiscal year ending date of September 30. From the given data, we drew off 26 years of hypothetical data approximately adjusted to September 30 where other valuation dates were used. This hyopthetical data, in millions, is shown in the following Table 5.1. One observes that the growth rate of 0_k from the middle of the first year to the middle of the 26th year is $[\ln(524/26)]/25 = 12.01\%$. In contrast, for interest rate we used $\delta = \ln(1.08) = 7.70\%$, so you can imagine we encountered interpretation problems.

For Table 5.1, k denotes fiscal years ending on September 30; 0_k is the benefit outgo for fiscal year k ; ${}_nA_k$ is the required assets at the end of fiscal year k if n-year roll-forward reserves are being sought; and ${}_nI_k$ is the rquired income for fiscal year k for such reserves to develop.

As expected, ${}_5A_k$ is somewhat less than $5({}_5I_k)$; ${}_{10}A_k$ is less than $10({}_{10}I_k)$; and ${}_{15}A_k$ is less than $15({}_{15}I_k)$

TABLE 5.1

${}_nA_k$ and ${}_nI_k$ FOR STATE SYSTEM HYPOTHETICAL OUTGO DATA

Fiscal Year	Outgo 0_k	${}_5A_k$	${}_5I_k$	${}_{10}A_k$	${}_{10}I_k$	${}_{15}A_k$	${}_{15}I_k$
			(MILLIONS)				
1	26	148	31	342	42	609	56
2	29	167	35	392	50	690	61
3	32	189	40	450	58	780	65
4	36	214	46	515	64	877	70
5	40	248	56	590	73	985	76
6	45	285	61	676	82	1103	83
7	52	330	74	769	89	1231	90
8	59	384	85	868	95	1370	99
9	67	442	94	975	103	1536	121
10	83	502	107	1082	112	1726	148
11	90	575	121	1202	121	1943	165
12	108	645	131	1323	132		
13	125	712	140	1450	145		
14	138	782	151	1607	177		
15	157	852	164	1798	218		
16	178	921	178	2010	243		
17	192	997	195				
18	206	1084	213				
19	222	1211	261				
20	241	1390	320				
21	262	1600	357				
22	286						
23	313						
24	383						
25	470						
26	524						

Observe, however, that generally ${}_5I_k < {}_{10}I_k < {}_{15}I_k$. The reason can be discovered by considering the relation fo the average growth rate (12.01 percent) of 0_k to the 7.70 percent force of interest.

The system from which the 0_k column was derived had assets on September 30, 1976 which were 16.84 times the current annual amount of benefit allowances. On Septermber 30, 1988 this ratio had risen to 22.09. It is reasonable to conclude that by September 30, 1976, the system could have applied 15-year roll-forward reserve funding, perhaps with some transition. Without knowing more about the growth of projected outgo from 1988 on, we hesitate to estimate the reserve required on September 30, 1988 by such funding. The assets on hand then were valued in excess of $12 billion. We intend to follow up on this matter with the system's actuary.

That is as far as we have taken our analysis of state system data. Much further work remains to be done to explore the application of n-year roll-forward reserves for state systems.

6. FORMULAS FOR n-YEAR ROLL-FORWARD RESERVES

Using the notations and concepts of part 2, we present here formulas for n-year roll-forward reserves. Recall that our starting point is a year by year listing of outgo values, 0_k, and assumed forces of discount δ_k. For simplicity, we assume δ_k is constant at force δ, but it is not difficult to extend to the case of variable forces. Corresponding to formulas (2.2) and (3.1) we now have:

$$ {}_nA_{12/31/k} = 0_{k+1}e^{-\delta/2} + 0_{k+2}e^{-3\delta/2} + \ldots + 0_{k+n}e^{-(2n-1)\delta/2} \tag{6.1} $$

If in the n-year interval, 0_{k+j} grows at the constant rate τ, that is,

$$ 0_{k+j} = 0_k e^{\tau j}, j = 1, \ldots, n \tag{6.2} $$

then

$$ \begin{aligned} {}_nA_{12/31/k} &= (1 + e^{\tau-\delta} + \cdots + e^{(\tau-\delta)(n-1)})0_{k+1}e^{-\delta/2} \\ &= [(1 - e^{n(\tau-\delta)})0_{k+1}e^{-\delta/2}]/[1 - e^{\tau-\delta}] \end{aligned} \tag{6.3} $$

If $\tau < \delta$, this may be written as

$$ \begin{aligned} {}_nA_{12/31/k} &= [(1 - e^{-n(\delta-\tau)})0_{k+1}e^{-\delta/2}]/[1 - e^{-(\delta-\tau)}] \\ &= \ddot{a}_{\overline{n}}0_{k+1}e^{-\delta/2} \end{aligned} \tag{6.4} $$

where $\ddot{a}_{\overline{n}}$ is calculated at force $\delta - \tau$. In particular, if $\delta = \tau$, then $\ddot{a}_{\overline{n}} = n$, and the required fund at the end of year k (beginning of year $k+1$) is simply $n \cdot 0_{k+1}$ discounted for one-half year.

The required income for year k is given by

$$ {}_nI_k = 0_{k+n}e^{-n\delta} \tag{6.5} $$

That is, ${}_nI_k$ is simply the present value of the projected outgo for year $k+n$. With outgo growing at a constant force τ, we then have

$$ \begin{aligned} {}_nI_{k+j} &= 0_{k+j+n}e^{-n\delta} \\ &= e^{\tau j}(0_{k+n}e^{-n\delta}) \end{aligned} \tag{6.6} $$

If the taxable payroll, W_{k+j}, is also growing exponentially, at rate σ say, that is,

$$W_{k+j} = e^{\sigma j} W_k, j = 1 \ldots, n \tag{6.7}$$

then

$$\begin{aligned} {}_nI_{k+j}/W_{k+j} &= e^{(\tau-\sigma)j}(0_{k+n}e^{-n\delta})/W_k \\ &= e^{(\tau-\sigma)j}{}_nI_k/W_k \end{aligned} \tag{6.8}$$

If $\tau = \sigma$, then ${}_nI_{k+j}/W_{k+j} = {}_nI_k/W_k$, and is constant for $j = 0, \ldots, n-1$. Also if

$${}_nI_{k+j+1}/W_{k+j+1} = {}_nI_{k+j}/W_{k+j}, j = 0, \ldots, n-2$$

we will have

$$e^{(\tau-\sigma)(j+1)}{}_nI_k/W_k = e^{(\tau-\sigma)j}{}_nI_k/W_k$$

which requires

$$e^{(\tau-\sigma)} = 1, \text{ or } \tau = \sigma \tag{6.9}$$

We conclude that a necessary and sufficient condition for ${}_nI_{k+j}/W_{k+j}, j = 0, \ldots, n-1$, to be constant in the n-year interval, is that $\tau = \sigma$. This rather surprising condition does not seem to depend on the force of interest, but it should be remembered that the condition was derived under the assumption of a constant force of interest.

In interpreting these relations we have been thinking of $\delta \geq \sigma \geq \tau$, but one could consider interpretations for all the possible orders of the three rates, δ, σ, and τ.

One other formula to consider is a recursion formula which can be used, with a constant force of interest , δ, to calculate ${}_nA_{12/31//k}$:

$${}_nA_{12/31/k} = {}_nA_{12/31/k-1}e^{\delta} + ({}_nI_k - 0_k)e^{\delta/2} \tag{6.10}$$

The derivation uses the following:

$${}_nA_{12/31/k-1} = 0_k e^{-\delta/2} + 0_{k+1}e^{-3\delta/2} + \ldots + 0_{k+n-1}e^{-(2n-1)\delta/2}$$

$${}_nA_{12/31/k} = 0_{k+1}e^{-\delta/2} + 0_{k+2}e^{-3\delta/2} + \ldots + 0_{k+n}e^{-(2n-1)\delta/2}$$

$${}_nI_k = 0_{k+n}e^{-n\delta}$$

From these equations it follows that formula (6.10) is correct.

If actual year-end-assets are less than ${}_nA_{12/31/k}$, the recursion formula can be utilized transitionally as in the Section 4 treatment of the SMI fund. For this purpose, actual year-end-assets are substituted into (6.10) for the required n-year roll-forward reserves but ${}_nI_k$ and 0_k are unchanged.

In this paper we have been considering n-year roll-forward reserve funding as an alternative to classical pension funding methods when application to very large, mature, public systems is in question. For a more classical view see [7, Hickman, 1992, section 3]. That paper provides a broad view of historical and economic issues but ends with a similar exponential case for classical pension funding from the point of view of an individual participant. Both Hickman's paper and our own are attempts to deal with maintaining incomes in a society subject to various factors of exponential growth.

7. SUMMARY AND CONCLUSIONS

This section will begin with a review of our preceding work, and then continues with some discussion of the potentialities of roll-forward reserve financing.

7.1 Review

Tables 2.2.1 - 2.2.3 and Figures 2.1 - 2.4 summarize our application of 1-year roll-forward reserves to the funding of projected outgo for OASDI under the three Alternatives. This information is mainly illustrative of what can be done rather than being a definitive projection of required fund and required annual income. However, the tables and figures do have as their starting point the projected annual outgo appearing in [2, 1992].

It can be observed that the required annual income as a percent of taxable payroll varies from a low of 9.73 in 2005 for Alternative 1 to a high of 26.98 in year 2065 for Alternative 3. We have noted that taxation of up to one-half of individual benefits would offset those requirements by .19 percent to a projected 1.29 percent in 2065 [2, 1992, Table II.F.16, pp. 124-5]. Thus taxation of benefits can be a significant contribution to required income, and this would be accentuated if a higher percent of individual benefits were taxable.

Figure 2.1 indicated outgo growth graphically. Figure 2.2 shows little distinction between taxable payrolls under the three Alternatives. Figures 2.3 and 2.4 show the growth of required fund and of required income under 1-year roll-forward financing.

Tables 3.2.1 - 3.2.3 and Figures 3.1 - 3.4 summarize our application of 2-year roll-forward reserves to the projected outgo for OASDI under the three Alternatives. Here, it can be observed that required annual income varies from a low of 9.68 percent of taxable payroll for Alternative 1 to a high of 26.75 percent in year 2065 for Alternative 3.

One might expect that because 2-year roll-forward reserves are approximately double those of 1-year roll-forward reserves, that ${}_2I_k$ would be different from ${}_1I_k$. In fact, however, the ratio ${}_2I_k/{}_1I_k$ remains close to 100 percent in all years through 2065. We have

$$ {}_2I_k = 0_{k+1}e^{-(\delta_k+2\delta_{k+1}+\delta_{k+2})/2} $$

and

$$ {}_1I_k = 0_{k+1}e^{-(\delta_k+\delta_{k+1})/2} $$

Thus,

$$ {}_2I_k/{}_1I_k = (0_{k+2}/0_{k+1})e^{-(\delta_{k+1}+\delta_{k+2})/2} $$

and the interest effect is offset by the ratio (O_{k+2}/O_{k+1}).

In Section 4, we explored the application of 1-year roll-forward reserve funding to the HI and SMI systems until the year 2000. In the case of HI, the initial funds on hand exceed the next year's outgo under Alternative 1 for the whole period, and funding changes would not have to be implemented until after the year 2000. For Alternatives 2 and 3, 1-year roll-forward reserves might require funding changes by 1998 and 1996, respectively. Under SMI, current reserve funding would not reach the 1-year reserve level at any year-end before year 2000, neither would a transition to 1-year roll-forward reserve funding. Under the latter, some progress would be achieved.

In Section 5, we explored 5, 10, and 15-year roll-forward reserves for some 26 years of hypothetical data for a large public employee system. Because the data showed outgo growing at any average rate of 12.01 percent, and we assumed $\delta_k = \ln(1.08) = 7.70$ percent, we did not get far into our analysis. However, it appears that 15-year roll-forward reserve financing might have been feasible.

Finally, in Section 6, we presented some mathematics relations for n-year roll-forward reserve funding. These could be developed further.

7.2 Comparison with Current Funding of Social Security

As we have warned before, our figures are illustrative of what computations can be made, but are not definitive. Our calculations for 1-year roll-forward reserves for OASDI under Alternative 2 show year-end reserves of $295 billion in 1992 rising to $16,757 billion in 2060. The required income as a percent of taxable payroll rises from 11.19 to 17.69. The 2-year roll-forward reserves are roughly double those for the the 1-year roll-forward funding, but the required percents of taxable payroll for the 2-year funding are close to those for 1-year funding (see Figures 3.3 and 3.4). In contrast, the currently legislated funding is projected under Alternative 2 to produce a fund of $5535 billion by 2025, and then to rapidly deplete before 2040.

For information regarding 1-year roll-forward funding for HI and SMI, the reader is referred to our Section 4.

7.3 Advantages of Roll-Forward Funding for OASDI

Roll-forward reserve funding provides an easily understood, concrete answer to the comment one hears so often – will the money be there when I retire?

Funding by 1 or 2-year roll-forward reserves would provide the quadrennial Advisory Councils a disciplined guideline for recommendations on the payroll tax for the next quadrennium. It might suggest replacing the current system of a tax-rate, that remains in effect indefinitely unless Congress changes it, by a more flexible system of rates that might be adjusted quadrennially as experience warrants.

It must be recognized that 2-year roll-forward reserves would develop a very large trust relation between the Federal Government and the nationwide contributors and beneficiaries. One-year roll-forward reserves would halve this responsibility. But they might thereby also decrease confidence in the payment of benefits since the contributions will almost immediately be distributed to the beneficiaries.

7.4 Roll-Forward Reserve Funding for State Public Employee Systems

To really study this matter would require projected outgo year-by-year for a number of such systems. For Social Security, year-by-year projected outgo was readily available in the Trustees' Reports for OASDI, and to a lesser extent for HI and SMI. Such study of public employee systems might reveal significant advantages of using a flexible roll-forward reserve funding based on carefully prepared projections of outgo for a considerable period of years. One advantage is that it would deal with the open group of participants for a definite number of future years. This would be in constrast with classical funding methods which deal with the closed group of present participants for an indefinite period. There might also be some perceived (if not real) disadvantages.

7.5 Conclusion

Most actuaries have been convinced that classical penison funding methods are not appropriate for Federally-sponsored Social Security programs. This led R. J. Myers many years ago to develop financing based on projections of outgo and income. Roll-forward reserves are essentially a refinement of projection financing, and may find a role not only in Social Security funding, but also in the funding of state public employee systems.

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