Session 65TS: Statistics and the Valuation Manual

Moderator:

Douglas L Robbins FSA,MAAA

Presenters:

Steven Lane Craighead ASA,MAAA,CERA
Douglas L Robbins FSA,MAAA
Karen K Rudolph FSA,MAAA

2017 Valuation Actuary Symposium

Steven Craighead

VM-20 Statistical Issues

August 29, 2017 Session 65

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VM-20 Statistical Issues Session 65

Steven Craighead
Pacific Life Insurance Company

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Agenda

- Number of scenarios
  - Smallest number based on Law of Large Numbers and CTE Percent
  - Other Rules of Thumb
- Scenario Picking
  - Issues
  - Processes

Number of Scenarios

- Central Limit Theorem.
  - Let X be a random variable with a finite mean μ and finite variance σ^2.
  - Choose n of those randomly and independently.
  - The sample mean S converges in probability and almost surely to μ as n goes to infinity.
  - Or for large enough n, S is near to the normal distribution with mean μ and variance σ^2 / n.

Number of Scenarios and CTE

- Central Limit Theorem.
  - Rule of thumb. The magic number N is 30 (old timers), 50 (Wall Street) or higher
- Say your CTE is 70%. So you need to average the worst 30%.
- If you want your sample CTE to be close to the actual CTE and have a variance/n around that actual CTE, old timers need a minimum of 30 scenarios that fall in the worst 30%, you need .30n>30 or n > 100 scenarios.
- You use the rule that the number of scenarios n need to be bigger than your magic number / (1.00 – CTE%) where CTE% is in decimal form.

CTE

- Say your 100 samples have a sample stdev of 500,000. Now, say your CTE of the 100 samples is $10,503,116. So about 99% of the of the time (3stdev) the “TRUE” CTE would be between
- $10,503,116-3*(500,000)/(30)^.5 = $10,229,255 and $10,503,116+3*(500,000)/(30)^.5 = $10,776,977
- So you have an uncertainty of $547K (6*stdev). If this is not small enough, then increase your number of samples, till you get a range that you can live with.

CTE

- Say the magic number is 51 and so you need 51/.3 = 170 scenarios
- Sample CTE = $10,560,353
- Ranges from $10,560,353 – 3 *(Var(k)/51)^.5 = $10,351,786 to $10,768,920
- 6 Stdev is approximately $417K

CTE

- Say the magic number is 201 and so you need 201/.3 = 670 scenarios
- Your sample CTE and confidence interval will bracket the actual CTE by $205K, 99% of the time.

CTE – Say your upper management want your sample CTE to be within $100K of the actual CTE

- Number of scenarios = (Magic Number)/(.3)

where 6*(var(K)/(Magic Number))^.5 = 100,000

If you run a few hundred and find that Stdev(K) is approximately $1M, then

((6*(1M)/100,000))^2 = Magic Number = 3600

You need to run 12000 scenarios!

Bounded vs Unbounded Options

- The rule of thumb is it usually takes 4 times more scenarios to get an unbounded option to converge than a bounded option.
- To gild the lily, if your company wanted the actual CTE to be within 100K like in our prior example and the CTE is on an unbounded option, now you are at 48K scenarios.
- It is worth your time to think about the embedded options that you are guaranteeing within your product. I would hope that you are not selling something with an unbounded option like a call.

Scenario Classification and Picking

- Usually the idea of picking a representative set of scenarios have two steps. The first is to classify the list of scenarios into a list of k different types of scenarios. The second step is to find a representative scenario within each of those types.
- Usually, you have to develop some type of measurement of distance to classify and may reuse that same measurement of distance to find a central or median scenario within those separate classes.

Problems with distance based on a single key rate

- For instance the Academy picking tool is normally on the 1-year rate.
- What if your business is sensitive to the 10-year rate? Or a mix of several long rates?

Distances that reduce value as the projection period lengthens.

- Some distances apply a discount factor to the distance, so a 10^{th} year yield will have less influence than the 1^{st} year yield.
- What happens if your business is mostly sensitive to the end of your surrender period, when your cash values move toward your fund values and not sooner?

What if you have options that go in the money in extreme situations?

- Back in the 90's very few of us considered interest rates occurring below the guarantee rate. Most of our scenario picking tools wouldn't pick from those scenarios.
- Most picking tools use sum of squared differences as a measure of distance. This distance measure tends to classify scenarios into groups that are more central to the space and when the representatives are chosen in the same fashion, the representative scenarios tend to be more representative of the central possibilities.
- Usually a sum of absolute differences tend to classify more extreme classes and then that same measurement tends to choose good representatives from the actual classes.

Close in the Scenario Space isn't Necessarily Close in the Results Space

- You may have embedded options that go in the money if a scenario value is slightly higher or lower than a specific scenario. So, two scenarios that are measured to be close in regard to classification and representation may actually have wildly different present values.
- Milliman's clustering tool includes the results space in its process of scenario picking.
- If your scenarios have broad enough coverage, you might still be safe, but you probably need to increase your scenario number to safe.

Best state is a complex state

- Use sum of absolute differences.
- Measure the distance on the entire yield curve not one key rate.
- Consider all times in a scenario with equal weight, don't discount the later periods.
- Add a proxy to the results space.
- Results?
  - You get a better set that covers more conditions across time and maturities and products.
  - You will get more severe scenarios that will give you a better understanding of your risks.

The Financial Reporter June 2008 Issue No. 73

- PBA Reserves and Capital Modeling Efficiency: Representative Scenarios and Predictive Modeling
- I did a study using one of Yvonne Chueh's distance formulas that is used in the Academy Scenario Picking tool
- I also used a predictive model to estimate the PBA reserves and compared the results to a 10K set of scenarios and reserves.

Questions?

Milliman

Valuation Actuary Symposium

Session 65TS

Statistics and the Valuation Manual

Karen Rudolph

August 29, 2017

Credibility

- Focus is on Mortality
- VM-20 Requirements for Mortality Credibility Calculations
  - Recent regulatory discussions
- Limited Fluctuation
- Bühlmann
- Advantages/Disadvantages

Mortality Credibility

- VM-20 Mortality requirements are specific
  - Depend on the company having quantified its mortality credibility

- VM-20 has parameters around how credibility is measured for VM-20 purposes

	Industry Table	
	2008 VBT	2015 VBT
Permitted Methods	A method that follows common actuarial practice as published in actuarial literature including but not limited to Limited Fluctuation and Bühlmann Empirical Bayesian	Limited Fluctuation by amount Or Bühlmann Empirical Bayesian by amount
Constraints	None	Limited Fluctuation Method by amount must be calibrated for a minimum probability ≥ **95%** with an error margin of ≤ **5%.** Bühlmann method can use the direct approximation formula for Z provided in VM-20.
Flexibility in Method After First Use	No specific requirements are spelled out if a company using the 2008 VBT as industry table wants to change credibility methods.	A company seeking to change credibility methods must request and subsequently receive the approval of the commissioner. The request must include justification for the change and a demonstration of the rationale in support of the change.

	Industry Table	
	2008 VBT	**2015 VBT**
Permitted Methods		**Limited Fluctuation by amount** **Or** **Bühlmann Empirical Bayesian by amount**
Constraints		**Limited Fluctuation Method by amount must be calibrated for a minimum probability ≥ 95% with an error margin of ≤ 5%.** **Bühlmann method can use the direct approximation formula for Z provided in VM-20.**
Flexibility in Method After First Use		**A company seeking to change credibility methods must request and subsequently receive the approval of the commissioner. The request must include justification for the change and a demonstration of the rationale in support of the change.**

Mortality Credibility

Credibility of data can be evaluated:

- At mortality segment level

  Or

- At more aggregate level, if the mortality for the subclasses (i.e. mortality segments) is determined using an aggregate level of mortality experience.

Mortality Credibility

Accelerated/Innovative/Simplified underwriting techniques

- Policies inforce using Innovative Techniques are few, but gaining mass
- Can experience provided by traditionally underwritten business be used as a predictor for Innovative programs until new programs acquire credibility?
- What demonstrations are regulators willing to accept?
- Are additional margins (above prescribed) to be added?

Mortality Credibility

VM-20 specifies that the mortality used in the modeled reserves be *graded* into industry rates, not blended with industry rates

- Classical credibility methods would blend at all durations in proportion to the partial credibility measurement
- Grading implies that 100% of the company's data is used for a certain number of durations, and then
- Industry data is used to fill in those durations where company data is lacking
- For duration in between, the grading rules move the assumption vector from 100% company to 100% industry.

Mortality Credibility

…continued…..

- Credibility impacts the prescribed margins
- Credibility, together with Sufficient Data Period, controls the grading rules
  - The greater the SDP, the longer the period for which the company can use its own data

Mortality Credibility

Notice that grading rules are not sensitive to Credibility Method used.

Credibility of Company Data	Maximum # of years for data to be considered sufficient	Maximum# of years in which to begin grading after sufficient data no longer exists	Maximum # of years in which the assumption must grade to 100% to an applicable industry table (from the duration where sufficient data no longer exists)*
20%-39%	10	2	8*
40%-59%	20	4	12*
60%-79%	35	7	17*
80%-100%	50	10	25*

Limited to SDP + 15(Credibility %)

Example: SDP = 9 years; Credibility % = 80%, then last column is
Min[25, 9 + 15(80%)] = 21 years

Limited Fluctuation Method

Limited Fluctuation

- A classical statistical method based on confidence intervals
- Premise: Normal Distribution
- Requires only the Company's own data
- Formula below is appropriate if the company chooses 95% and 5% as parameterization, rather than something higher than 95% or lower than 5%

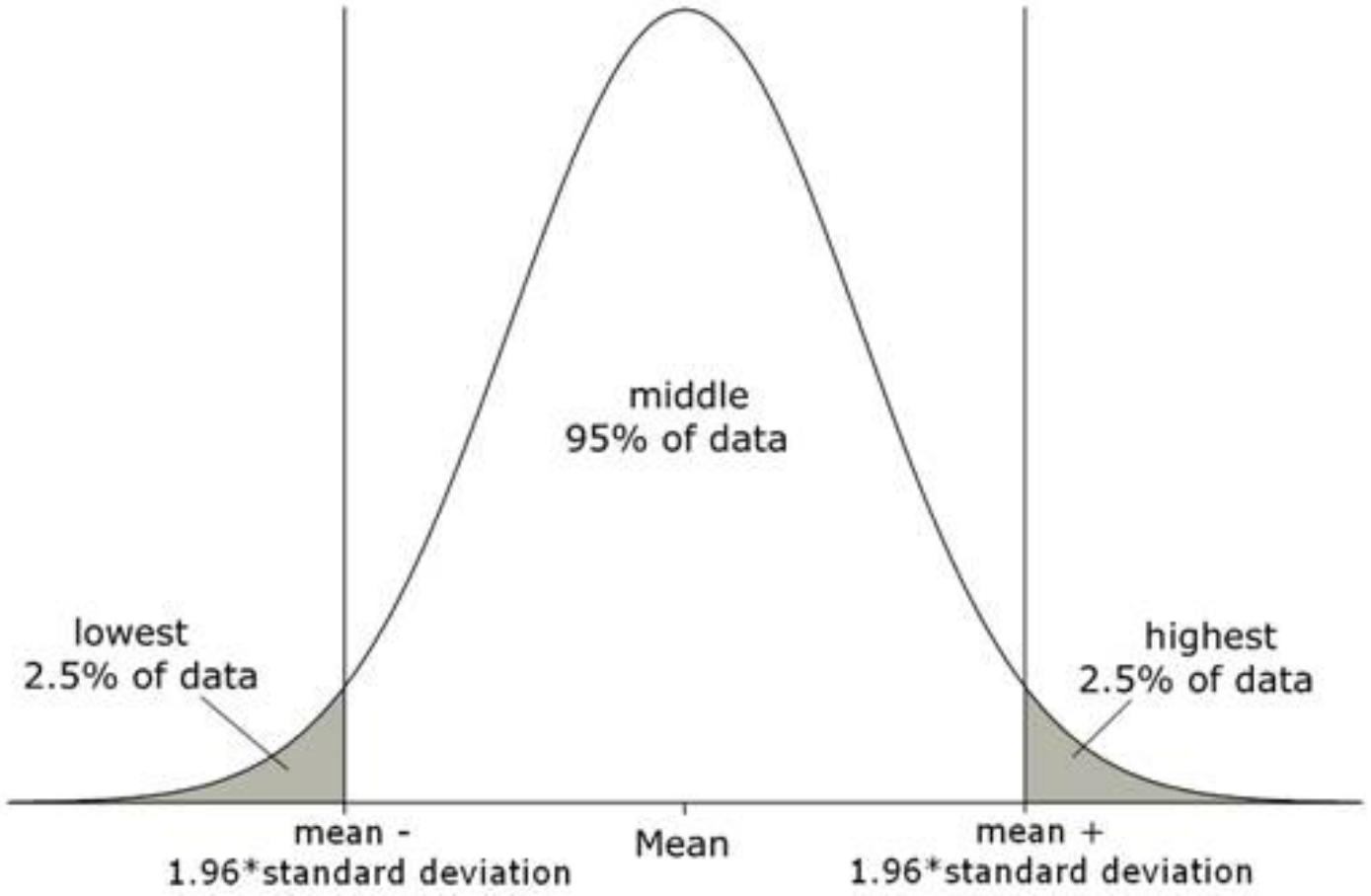
$$\text{Limited Fluctuation } Z = \min\left\{1, \frac{.05 * \hat{m}}{1.96 * \hat{\sigma}}\right\}$$

Limited Fluctuation Method

$$\text{Limited Fluctuation } Z = \min\left\{1, \frac{.05 * \hat{m}}{1.96 * \hat{\sigma}}\right\}$$

$\hat{m}$:	This is the estimator
$\hat{\sigma}$:	This is the standard deviation of the estimate
1.96:	Normal distribution z for the 95th percentile
0.05:	5% margin for error

Limited Fluctuation Method



Limited Fluctuation Method

Taken from "Credibility Theory Practices", December 2009

There are n observed lives with index i

$$A = \sum_{i=1}^{n} b_i d_i$$

$$E = \sum_{i=1}^{n} b_i f_i q_i^s$$

$f_i = Fraction\ of\ year\ for\ which\ the\ life, i, was\ observed$
$b_i = Amount\ of\ insurance$
$d_i = 0\ if\ life\ did\ not\ die; 1\ if\ life\ did\ die$

$$\hat{m} = {}^{A}/_{E}$$

Limited Fluctuation Method

$For\ each\ life\ i, there\ is\ a\ true\ mortality\ rate, q_i,$
$and\ a\ standard\ table\ mortality\ rate, q_i^s$

$We\ assume\ the\ true\ mortality\ is\ a\ multiple\ of\ the\ standard\ table.$

$$q_i = m \cdot q_i^s$$

Limited Fluctuation Method

$$\sigma^2 = Var(\hat{m}) = \frac{\sum_{i=1}^{n} b^2 Var(d_i)}{E^2} = \frac{\sum_{i=1}^{n} b^2 f_i q_i (1 - f_i q_i)}{E^2}$$

$$Substitute: \; q_i = m \cdot q_i^s$$

$$\sigma^2 = \frac{\sum_{i=1}^{n} b_i^2 f_i m q_i^s (1 - f_i m q_i^s)}{E^2}$$

$$and$$

$$\text{Limited Fluctuation } Z = \min\left\{1, \frac{.05 * \hat{m}}{1.96 * \hat{\sigma}}\right\}$$

Bühlmann Method

Bühlmann Empirical Bayesian – also known as Greatest Accuracy Credibility method

$$\text{Bühlmann Z} = \frac{A}{A + \frac{109\% \cdot B - 120.4\% \cdot C}{0.019604 \cdot A}}$$

where

$$A = \sum (amount\ insured)(exposure)(mortality)$$

$$B = \sum (amount\ insured)^2(exposure)(mortality)$$

$$C = \sum (amount\ insured)^2(exposure)^2(mortality)^2$$

Limited Fluctuation	Bühlmann
ADVANTAGES	
Requires data from only one company	Systematic modeling approach with assumptions and optimizations defined
Formulas are easy to implement and interpret. They represent classical statistical credibility	No arbitrary parameters
	Reflects accuracy in both single company and industry data through two variance calculations
DISADVANTAGES	
Only considers the accuracy of the single company's experience and makes no consideration for the accuracy of the industry experience	Calculation process is difficult to interpret and explain
Makes an a priori normal distribution assumption, which may not agree with the true data distribution	Requires the company to rely on statistical agents for the calculation in pure form. VM-20 overcomes this issue by a formulaic approximation

Mortality Credibility

VM-20 Prescribed Mortality Margins by Method for Ages < 45

	Credibility Range								
	53-57%	58-62%	63-67%	68-72%	73-77%	78-82%	83-87%	88-92%	93-100%
LF	8.3%	7.6%	6.9%	6.3%	5.8%	5.3%	4.8%	4.4%	4.0%
B	15.5%	14.6%	13.7%	12.7%	11.6%	10.3%	8.9%	Note (1)	Note (2)

Note (1)
88-89%: 8.0% margin
90-91%: 7.3% margin
92-93%: 6.5% margin

Note(2)
92-93%: 6.5% margin
94-95%: 5.7% margin
96-97%: 4.6% margin
98%: 3.3% margin
99+%: 2.3% margin

Mortality Credibility

For further information and demonstrations on this topic, there is a Credibility module in the PBR Professional Development Series.

www.SOA.org
Sign in with user name and password
Go to Professional Development, then E-Courses
Scroll down to PBR Professional Development Series

If not previously registered, click “Register”

If you have previously registered and you are returning to view another module, log-in and go to your “My SOA” page, scroll down to “Orders and Dues”, then click “Access” under PBR Professional Development Series.

Thank You

Karen Rudolph, FSA, MAAA
Consulting Actuary
Karen.Rudolph@Milliman.com