ELEMENTARY MODELS OF RESERVE FUND FOR THE OLD-AGE, SURVIVORS AND DISABILITY INSURANCE (OASDI) IN THE U.S.A.

by

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This paper began as a mathematical discussion of relations involved in attempting to maintain a fixed level of contingency fund for a social insurance program. There was also some consideration of means to raise or lower such fixed level. Part I of this paper reviews those ideas which were developed in advance of attending the Social Security Financing Seminar [11], on March 9, 1990, sponsored by the Academy of Actuaries and the Society of Actuaries. Since then I have received helpful comments from John A. Beekman, Robert J. Myers, Charles L. Trowbridge and Howard Young, and expressions of interest by others. My thanks are due to Charles Trowbridge for relating my ideas to those of his 1967 TSA paper "Theory of Surplus in a Mutual Insurance Organization" [12].

A budding idea in the original discussion was that of n-year roll-forward reserves, with integer $n \geq 1$. These roll-forward reserves for $n \geq 2$, go beyond what Richard Foster (of the Office of the Actuary) considers as adequate in his May 15, 1990 draft report "Level of OASDI Trust Fund Assets Needed to Handle Adverse Contingencies" [5]. Reserve funds at this level ($n \geq 2$), while possibly justifiable for actuarial reasons, collide with the economic conclusions of the 1984 paper by Alicia H. Munnell and Lynn E. Blais, "Do We Want Large Social Security Surpluses?" [7].

Part II explores these roll-forward reserves, both by algebra, and by preliminary numerical illustrations based on data from [3], [2], and [1]. Some commentary will be offered on the potential usefulness of such reserves in the development and maintenance of social insurance programs.

Part III indicates a number of questions for further study. How to define and manage effective contingency funds? Are reserves (beyond adequate contingency funds) needed, and if so, do four-year or two-year roll-forward reserves suggest the way to go? Is Social Security financing entirely different from that for large public employee retirement systems? If not, what relations should be considered? How independent should our Social Security institutions be? Can further mathematical ideas for Social Security financing be developed? What are actuarial aspects of consolidating, or not, the OASDI and Medicare systems?

Social Security, and our maturing pension funds, provide much new scope for actuarial science.

I. ELEMENTARY CONTINGENCY FUNDS

1. Preliminaries

There has been much discussion of the "roller-coaster" form of financing that level contribution rates (of 6.2 percent of pay by employee and by employer) may develop over the next 50 years. There also has been discussion of pay-as-you-go financing augmented by maintaining a contingency fund of 100 to 150 percent of the current year's outgo for benefits and administration [see 11, 13].

Both financing by level contribution rates and by pay-as-you-go plus a contingency fund introduce some rigidity into the financing process, and from time-to-time adjustments will have to be made under either system. The purpose here is to explore the mathematical implications of maintaining a constant level of the contingency fund ratio, that is, the ratio of the trust fund at the beginning of the year to that year's outgo. More generally, we shall also observe the effect of increasing or decreasing that ratio.

By a somewhat odd chance, a simple mathematical model to incorporate recognition of nuclear holocaust hazard in actuarial mathematics suggests an elementary means of exploring the contingency fund ratio. To see the connection, and also to review some of the information of the 1988 Trustees Report for OASDI, refer to [9].

In the former work, I specified by subscripts the underlying rates affecting a function, and denoted the value of the function at time k by use of (k). In the present study, we consider only the effective rates of interest, and of growth in benefit outgo, and will not specify them in the function notation. This leaves subscripts available to indicate time, and formulas are thereby shortened and made easier to read. I am indebted to Robert J. Myers for this suggestion.

2. Models of the Contingency Fund

The models are discrete and relate to the calendar year spanning the interval $(k, k+1)$. As will be seen in the notations, the models are simplified and for practical application would require considerable numerical refinement to take account of the operational details. Nevertheless, the simple mathematical models provide insights that may not be entirely evident from tables of projection figures.

The notations are:

F_k = the value in current dollars of the OASDI funds at time k, ignoring for simplicity, details such as advance transfers from the Treasury.

I_k = the value in current dollars at time k of projected income for year $(k, k+1)$, exclusive of interest.

O_k = the value in current dollars at time k of projected outgo for year $(k, k+1)$. For simplicity and explicitness, I_k and O_k are values at time k.

i_{k+1} = the effective annual rate of interest for year $(k, k+1)$.

$v_{k,k+1} = 1/(1 + i_{k+1})$ = the value at time k of 1 due at time $k+1$ on the basis of interest at rate i_{k+1}.

r_{k+1} = the effective annual rate of growth of O_k, so that $O_{k+1} = O_k[1 + r_{k+1}]$.

$c_k = F_k/O_k$ = the contingency fund ratio at time k.

$c_k + f_{k+1} = F_{k+1}/O_{k+1}$ = the contingency fund ratio at time $k+1$. If $c_k \geq 1$, then we shall be interested in having $-1 \leq f_{k+1} \leq 1$. In view of the increasing size of O_k, the extreme value of $+1$ for f_{k+1} is unlikely; the value -1 of f_{k+1} might be more likely to occur.

The starting equation for the OASDI trust funds is then

$$(F_k + I_k - O_k)(1 + i_{k+1}) = F_{k+1} \ . \tag{2.1}$$

This may be written as

$$(c_k O_k + I_k - O_k)(1 + i_{k+1}) = (c_k + f_{k+1})O_{k+1} \tag{2.2}$$

or

$$I_k = v_{k,k+1}(c_k + f_{k+1})O_{k+1} - (c_k - 1)O_k \ . \tag{2.3}$$

Rewriting (2.3) as

$$\begin{aligned} I_k &= v_{k,k+1}\Big[c_k O_{k+1} - (c_k - 1)O_k(1 + i_{k+1}) + f_{k+1}O_{k+1}\Big] \\ &= v_{k,k+1}\Big[O_{k+1} + (c_k - 1)O_{k+1} - (c_k - 1)O_k(1 + i_{k+1}) + f_{k+1}O_{k+1}\Big] \ , \end{aligned}$$

and recalling that $O_{k+1} = O_k(1 + r_{k+1})$, we have

$$I_k = v_{k,k+1}\Big[O_{k+1} - (c_k - 1)O_k(i_{k+1} - r_{k+1}) + f_{k+1}O_{k+1}\Big] \ . \tag{2.4}$$

Equation (2.3) has an interesting interpretation. To discover this, consider that from the initial fund $F_k = c_k O_k$, the amount O_k is split off for the current year's outgo. There remains the need to accumulate an amount for next year's initial trust fund, $(c_k + f_{k+1})O_{k+1}$. The current year's remaining initial fund, $(c_k - 1)O_k$, helps to offset this accumulation. This is easily understood if $c_k \geq 1$. If $c_k < 1$, then $(1 - c_k)O_k$ would be borrowed to help meet the outgo O_k, and would require repayment from I_k.

Equation (2.4) is algebraically equivalent to (2.3), and so it holds whether $c_k \gtrless 1$. But its interpretation is more complex and will not be inflicted on the reader. The chief use of the equation is to suggest the various applications in Section 3. These lead to another general approach [represented by equation (5.1)] wherein the current year's outgo O_k is met from I_k, and the whole fund at the beginning of the year is accumulated toward the fund at the end of the year.

3. Applications

If it is desired to maintain the contingency fund at a constant level, c_k, times outgo, then once that level has been attained, one would set $f_{k+1} = 0$ in equation (2.4), and have the guideline

$$I_k = v_{k,k+1}\Big[O_{k+1} - (c_k - 1)O_k(i_{k+1} - r_{k+1})\Big] \,. \tag{3.1}$$

In practical operation, adjustment of the contribution rates would be required from time to time to keep I_k and the contingency fund at the desired levels. A particularly simple case is that for which $c_k = 1$, that is, the trust fund at the beginning of the year is equivalent to the current year's outgo. Then

$$I_k = v_{k,k+1}O_{k+1} \,. \tag{3.2}$$

In effect, the initial fund, $F_k = O_k$, provides the year's outgo, and I_k accumulates to the outgo O_{k+1} for the following year.

Another special case occurs when $i_{k+1} = r_{k+1}$. Then equation (2.4) becomes

$$I_k = v_{k,k+1}(1 + f_{k+1})O_{k+1} \,. \tag{3.3}$$

Here, the remaining fund, $(c_k - 1)O_k$, after O_k is split off for current outgo, does not provide any interest offset in excess of growth required on $(c_k - 1)O_k$. Then I_k must accumulate to more than O_{k+1} in order that the trust fund may reach the level of $(c_k + f_{k+1})$ times the outgo. In fact, at year end

$$\Big[(c_k - 1)O_k + I_k\Big](1 + i_{k+1})$$

$$= (c_k - 1)O_k[1 + i_{k+1}] + (1 + f_{k+1})O_{k+1} \qquad \text{[from (3.3)]}$$

$$= (c_k + f_{k+1})O_{k+1}\ , \text{ since } i_{k+1} = r_{k+1}\ .$$

For another view of what is going on, we return to equation (2.2) and note that $O_{k+1} = O_k(1 + i_{k+1})$ in this case, and the equation becomes

$$c_kO_k + I_k - O_k = (c_k + f_{k+1})O_k\ ,$$

or

$$I_k = (1 + f_{k+1})O_k\ . \tag{3.4}$$

But this is the same as (3.3) since

$$v_{k,k+1}O_{k+1} = \Big[1/(1 + i_{k+1})\Big]O_k(1 + r_{k+1})$$

and $i_{k+1} = r_{k+1}$.

Equation (3.4) permits a second interpretation of the year's financing when $i_{k+1} = r_{k+1}$. In this interpretation, the initial fund c_kO_k will accumulate under interest to $c_kO_k(1+i_{k+1}) = c_kO_{k+1}$; the income I_k will provide the current outgo plus $f_{k+1}O_k$ which will accumulate under interest to $f_{k+1}O_{k+1}$, to bring the trust fund to the $(c_k+f_{k+1})O_{k+1}$ level. This type of analysis, in which the whole initial fund, c_kO_k, is used to accumulate under interest toward the year-end fund, and no portion of it is used for current outgo, will be pursued in Section 5. This alternative approach may be simpler but is not as revealing as the approach followed in Section 2.

A final application is to the extreme case $f_{k+1} = -1$, which cancels the O_{k+1} terms in the right hand side of equation (2.4). For this case to be applicable, c_k should exceed 1. Then, if I_k is to be positive, this case requires r_{k+1} to be greater than i_{k+1}. In other words, outgo is increasing at a rate greater than the interest rate, and it behooves one to lower the fund requirement of equaling c_k times outgo. With $f_{k+1} = -1$, we have from equation (2.4)

$$I_k = v_{k,k+1}(c_k - 1)O_k(r_{k+1} - i_{k+1}) \,. \tag{3.5}$$

One can verify that if O_k is split from the beginning of the year fund, then the remaining fund, $(c_k - 1)O_k$, plus I_k from (3.5), do indeed accumulate under interest to $(c_k - 1)O_{k+1}$.

4. Some Conclusions

a. A simple minimal standard of funding for OASDI would be to have at the beginning of each year a fund equivalent to the current year's outgo, that is, $c_k = 1$, and to obtain income during the year equivalent to next year's outgo. Thereby, the system would be funded ahead for one year. This type of funding could be generalized to funding ahead for a biennium, or a triennium, but large funds would develop thereby. Part II explores this more general funding, whereby after a given plateau level is reached, funding is aimed to maintain that level in the future.

b. If a higher level, $c_k > 1$, of funding is desired, then from equation (3.1) one sees that if the interest rate exceeds the outgo growth rate, the required income would be less than that [see formula (3.2)] for the minimal standard, once the fund had reached the desired level. If the outgo growth rate is larger than the interest rate available on the trust funds, then the required income is higher than the minimal standard would require. This will not surprise pension actuaries.

c. The case $f_{k+1} = 0$, $i_{k+1} = r_{k+1}$, is especially interesting. For this case, from (3.3), $I_k = v_{k,k+1}O_{k+1}$, no matter what the level of c_k is. Whether $c_k = 1$

or $c_k = 3$, to have $F_{k+1} = c_k O_{k+1}$ requires I_k to be equivalent to next year's outgo. Of course, the total income, *including* interest, will be higher in the $c_k = 3$ case than for the $c_k = 1$ case.

d. If the contingency fund ratio is greater than 1, and the outgo growth rate exceeds the interest rate, a unit of step-down in the contingency fund ratio could be accomplished by income at the level indicated in equation (3.5).

e. Smaller step-ups, or step-downs in the contingency fund ratio could be accomplished by arranging for income in accordance with equation (2.4). Depending on the size of c_k, the relation of i_{k+1} and r_{k+1}, and the value of f_{k+1}, the required income might be greater or less than the income required for the minimal standard.

5. An Alternative Approach

Instead of considering O_k being provided by splitting off a portion of $F_k = c_k O_k$, we here think of O_k becoming available from the income I_k, and leaving all of F_k to accumulate. Starting again with equation (2.2), namely,

$$(c_k O_k + I_k - O_k)(1 + i_{k+1}) = (c_k + f_{k+1})O_{k+1}$$

let us rearrange it in the form

$$c_k O_k[1 + r_{k+1}] + c_k O_k[i_{k+1} - r_{k+1}] + (I_k - O_k)(1 + i_{k+1}) = (c_k + f_{k+1})O_{k+1} \ .$$

Cancelling of $c_k O_k(1 + r_{k+1}) = c_k O_{k+1}$ from each side, and rearranging, reduces the equation to

$$I_k = O_k + f_{k+1} O_{k+1} v_{k,k+1} - c_k O_k (i_{k+1} - r_{k+1}) v_{k,k+1} \ . \tag{5.1}$$

An interpretation of equation (5.1) is that the initial fund accumulated with interest provides

$$c_kO_k(1+i_{k+1}) = c_kO_k(1+r_{k+1}) + c_kO_k(i_{k+1}-r_{k+1})$$
$$= c_kO_{k+1} + c_kO_k(i_{k+1}-r_{k+1}) \, .$$

Then I_k must provide the current outgo, O_k, and the rest, $f_{k+1}O_{k+1}$, of the fund $(c_k + f_{k+1})O_{k+1}$ at time $k+1$, but $c_kO_k(i_{k+1}-r_{k+1})$ is available to offset these requirements.

When $i_{k+1} = r_{k+1}$, it is an easy exercise to derive equation (3.3) from equation (5.1).

6. Comments

Those interested in the interaction of Trust Fund financing and the Federal budget should refer to [13, General Accounting Office, 1989], [8, National Academy of Social Insurance, 1990], and [10, American Association of Retired Persons (AARP), 1989]. The presentations by Henry Aaron, Robert Myers and Robert Ball in Session I of the AARP Conference give a broad overview, and some diversity of opinion, on how the financing of Social Security (OASDI) should proceed. As indicated earlier, the purpose of the present paper is to provide some mathematical insights into the implications of whatever policy may emerge from current discussions of Senator Moynihan's proposals, and related ideas.

I agree with Robert Ball's comment that we do not have to settle these questions right away since we are not yet (although close) to an adequate level for the contingency fund. Also, I am somewhat bemused by the idea that if a substantial contingency fund is built up, then some further degree of having general revenue income, as well as employee-employer payroll taxes, to fund OASDI would be realized. It appears that a large fund build-up would be cost-effective in current dollars only if interest rates generally exceed outgo growth rates [11, see J.C. Wilkin, "An Actuarial Perspective on Social Security Financing"]. What the situation would be in terms of constant dollars requires further study. Also, as stated before, practical application of these ideas require recognition of many numerical details.

How c_k might be approximated, for OASDI, if alternative II-B assumptions are realized, is discussed in [11, by Robert J. Myers, "Pay-as-you-go Financing for Social Security is the only Way to Go", and John C. Wilkin, "An Actuarial Perspective on Social Security Financing"]. Related details appear in Table 1 of Part II. The adequacy of such a contingency fund ratio is discussed in [5] and [11].

II. n-year Roll-Forward Reserves

7. General Concept

In this part, we explore in a preliminary way the mathematics and numerical computations for n-year roll-forward reserves. Instead of working with one-year periods, one here works with n-year periods. It is assumed that at the beginning of the first n-year period the fund on hand is equivalent to the value of the outgo (for benefits and administration) during the n-years of that first period. That frees the income of the period, if properly determined, to be accumulated to the end of the period to provide for the outgo of that second period. Thus a reserve is built up in the first n-year period to be ready to meet the outgo of the succeeding period. It is in that sense that the reserve rolls forward from period to period.

Note that this is substituting "plateau" funding for the "roller-coaster" funding developing under present law for OASDI. Once an n-year level of funding is attained, future funding is aimed to maintain that level rather than permit a steep decline as outgo outstrips income.

The concept is not entirely unrelated to pension funding mathematics. I first came across the idea from an economist's report on the reserve needed to sustain the Retirement System of the Government of Puerto Rico and Its Instrumentalities. The idea recurs again in [4, p.

116, formula (70)] where the annual normal cost for a mature, stable fund, under exponential growth conditions, is accumulated for a fixed average term of years before being paid out in benefit outgo. Such term of years would be much longer than we shall consider here for OASDI.

8. Elementary Mathematics of Roll-Forward Reserves

For our purpose here, we supplement the notation of Section 2 by:

$$u_{k,k+n} = \prod_{h=1}^{n} (1 + i_{k+h}) \tag{8.1}$$

= accumulated value at time $k+n$ of 1 invested under interest at time k.

$$v_{k,k+n} = 1/u_{k,k+n} = \prod_{h=1}^{n} v_{k+h-1,k+h} \tag{8.2}$$

= present value at time k of 1 due at time $k+n$.

$$\alpha_{k,k+n} = \sum_{j=0}^{n-1} I_{k+j} u_{k+j,k+n} \tag{8.3}$$

= accumulated value at time $k+n$ of income, exclusive of interest, for the period $(k, k+n)$. Note that I_{k+j} represents the value at time $k+j$ of income for the year $(k+j, k+j+1)$.

$$\beta_{k+n,k+2n} = \sum_{j=0}^{n-1} O_{k+n+j} v_{k+n,k+n+j} \tag{8.4}$$

= present value at time $k+n$ of outgo for the period $(k+n, k+2n)$. Note that O_{k+n+j} represents the value at time $k+n+j$ of outgo for the year $(k+n+j, k+n+j+1)$.

$$F_k^A = \beta_{k,k+n} + \varepsilon_k \tag{8.5}$$

Here F_k^A = actual fund at time k

= present value at time k of outgo for the period $(k, k+n)$ plus an adjustment ε_k to allow for fluctuation from $\beta_{k,k+n}$.

The basic equation for n-year roll-forward reserves is

$$(F_k^A - \beta_{k,k+n})u_{k,k+n} + \alpha_{k,k+n} = F_{k+n}^A = \beta_{k+n,k+2n} + \varepsilon_{k+n} \tag{8.6}$$

or

$$\alpha_{k,k+n} = \beta_{k+n,k+2n} + \varepsilon_{k+n} - \varepsilon_k u_{k,k+n} \; . \tag{8.7}$$

Ideally, planning should be accomplished during the $(k-n, k)$ period (where ε_k is projected to be relatively small) to provide sufficient $\alpha_{k,k+n}$ that ε_{k+n} will be relatively small, and preferably positive. Thus $3n$ years would come under projection scrutiny, covering a total period of $(k-n, k+2n)$ as a short-range testing term.

To aid planning, a useful indicator is

$$\gamma_{k+n} = [\beta_{k+n,k+2n}/\alpha_{k,k+n} - 1] \cdot 100 \tag{8.8}$$

$=$ the percentage by which $\alpha_{k,k+n}$ should be modified if it is to balance $\beta_{k+n,k+2n}$. If γ_{k+n} is negative, a decrease in $\alpha_{k,k+n}$ may be in order; while if γ_{k+n} is positive, an increase may be indicated.

As projected values are developed, a number of cases may unfold and indicate required adjustments.

Case I. $\gamma_{k+n} \leq 0$.

a. $\varepsilon_k \leq 0$. Consider steps to cause ε_{k+n} to approach 0.

b. $\varepsilon_k > 0$. Consider possible decrease in $\alpha_{k,k+n}$.

Case II. $\gamma_{k+n} > 0$.

a. $\varepsilon_k \leq 0$. Consider increase in $\alpha_{k,k+n}$.

b. $\varepsilon_k > 0$. Consider steps to cause ε_{k+n} to approach 0.

In Part I, we considered only projected funds. In contrast, here we consider $\alpha_{k,k+n}, \beta_{k,k+n}$ and approximations to *actual* funds.

9. Preliminary Computations for One-Year Reserves

Some preliminary tests have been made for one-year and four-year roll-forward reserves. One-year contingency funds seem adequate to a number of experienced Social Security actuaries [see 5, 6, 11], and have been explored as an attractive simple mathematical case in Part I. Four-year roll-forward reserves could be geared to the quadrennial Advisory Councils, and may offer more security to contributing members than one-year reserves. Also, they provide more time for Congressional adjustment if Social Security financing encounters difficult circumstances.

Unless otherwise stated, calculations are made on the basis of Alternative II-B assumptions, with some simplification of the interest rate assumptions.

Table 1 is presented in full detail for years 1990 through 2064. The reason for doing so is to give the reader a ready picture of the magnitude and variation in the projected income and outgo flows. The rates of increase of the the income flow is higher than those for outgo, during the years 1990 to 2002, thereafter the outgo rates of increase are higher till 2035, after which the rates of increase are mixed but generally slightly higher for outflow.

Table 1 also exhibits values of $\alpha_{k-1,k}, \beta_{k,k+1}$, and γ_{k+1} which can be used to adjust one-year income and outgo to maintain a close balance. By the end of 1990, the fund will approximate $\beta_{91,92}$ and the negative γ's from 1991 to 2016 indicate possibilities for decreasing income. Thereafter from 2017 onward γ is positive, and indicates there would be a need to increase income gradually by approximately 25 percent to maintain a one-year reserve up to about 2050. Eventually the income may require a 31 percent raise over the amounts presently projected. It must be remembered that not all OASDI income is from payroll taxes but most of it is, so that a 31 percent raise in required income is indicative of a similar raise in payroll taxes, which is consistent with rates shown by Myers and Wilkin in [11] for Senator Moynihan's proposal.

A corresponding projection of III data was made in Table 2 but is shown only to 2012 as the percentage increases required become large. The γ_{t+1} column indicates income increases may need to be scheduled by year 2000 if a one-year outgo fund is to be maintained [cf. 2, p. 54].

TABLE 1

PRELIMINARY PROJECTIONS IF A ONE-YEAR ROLL-FORWARD RESERVE IS TO BE MAINTAINED FOR OASDI[1]

(Dollar amounts in billions)

Year $(k, k+1)$	Income I_k Excluding Interest	$100 \cdot \Delta I_k / I_k$	Outgo O_k	$100 \cdot \Delta O_k / O_k$	$\alpha_{k-1,k}$ $I_{k-1}(1+i_k)$	$\beta_{k,k+1} = O_k$	γ_{k+1} [2,3]
1990	$299.1	6.69%	$253.5	6.82%			-15.4%
1991	319.1	6.33	270.8	6.54	$320.0	$270.8	-15.5
1992	339.3	6.31	288.5	6.34	341.4	288.5	-15.5
1993	360.7	6.27	306.8	6.13	363.1	306.8	-15.6
1994	383.3	6.05	325.6	5.99	385.9	325.6	-15.8
1995	406.5	6.25	345.1	5.74	410.1	345.1	-16.1
1996	431.9	6.16	364.9	5.81	435.0	364.9	-16.4
1997	458.5	6.15	386.1	5.85	462.1	386.1	-16.7
1998	486.7	6.14	408.7	5.97	490.6	408.7	-16.8
1999	516.6	5.96	433.1	5.93	520.8	433.1	-16.2
2000	547.4	6.08	458.8	5.91	547.6	458.8	-17.1
2001	580.7	6.11	485.9	5.91	585.7	485.9	-17.2
2002	616.2	6.10	514.6	6.00	621.3	514.6	-16.5
2003	653.8	6.09	545.5	6.12	653.2	545.5	-16.5
2004	693.6	6.19	578.9	6.29	693.0	578.9	-16.3
2005	736.5	6.12	615.3	6.34	735.2	615.3	-16.2
2006	781.6	6.00	654.3	6.46	780.7	654.3	-15.9
2007	828.5	6.01	696.6	6.78	828.5	696.6	-15.3
2008	877.7	5.89	743.8	7.03	878.2	743.8	-14.4
2009	929.4	5.80	796.1	7.15	930.4	796.1	-13.4
2010	983.3	5.74	853.0	7.32	985.2	853.0	-12.2
2011	1,039.7	5.62	915.4	7.49	1,042.3	915.4	-10.7
2012	1,098.1	5.54	984.0	7.67	1,102.1	984.0	-9.0
2013	1,158.9	5.53	1,059.5	7.78	1,164.0	1,059.5	-7.0
2014	1,223.0	5.49	1,141.9	7.82	1,228.4	1,141.9	-5.0
2015	1,290.1	5.46	1,231.2	7.93	1,296.4	1,231.2	-2.8
2016	1,360.5	5.40	1,328.8	7.92	1,367.5	1,328.8	-0.6
2017	1,433.9	5.38	1,434.1	7.89	1,442.1	1,434.1	1.8
2018	1,511.0	5.37	1,547.3	7.85	1,519.9	1,547.3	4.2
2019	1,592.1	5.36	1,668.7	7.76	1,601.7	1,668.7	6.6

[1] Data from Table F5, 1990 Annual Report of the Board of Trustees of OASDI, Alternative II-B Assumptions.

[2] For this illustration, interest is assumed to be at the annual rate of 7% in the 1990's decade, and at 6% hereafter.

[3] See formulas (8.3), (8.4), (8.8). Here $\gamma_{k+1} = [\beta_{k+1,k+2}/\alpha_{k,k+1} - 1] \cdot 100$. Note that $k = 1990$ in the first row, 1991 is the second row, etc.

TABLE 1
(continued)

Year $(k, k+1)$	Income I_k Excluding Interest	$100 \cdot \Delta I_k / I_k$	Outgo O_k	$100 \cdot \Delta O_k / O_k$	$\alpha_{k-1,k}$ $I_{k-1}(1+i_k)$	$\beta_{k,k+1} = O_k$	γ_{k+1}
2020	$1,677.4	5.34%	$1,798.2	7.60%	$1,687.6	$1,798.2	8.8%
2021	1,767.0	5.31	1,934.9	7.45	1,778.0	1,934.9	11.0
2022	1,860.8	5.30	2,079.0	7.32	1,873.0	2,079.0	13.1
2023	1,959.5	5.32	2,231.2	7.18	1,972.4	2,231.2	15.1
2024	2,063.8	5.33	2,391.4	7.08	2,077.1	2,391.4	17.1
2025	2,173.7	5.34	2,560.7	6.91	2,187.6	2,560.7	18.8
2026	2,289.8	5.32	2,737.7	6.72	2,304.1	2,737.7	20.4
2027	2,411.6	5.38	2,921.7	6.55	2,427.2	2,921.7	21.8
2028	2,541.3	5.39	3,113.2	6.37	2,556.3	3,113.2	22.9
2029	2,678.2	5.38	3,311.5	6.19	2,693.8	3,311.5	23.9
2030	2,822.3	5.40	3,516.4	6.09	2,838.9	3,516.4	24.7
2031	2,974.6	5.43	3,730.4	5.98	2,991.6	3,730.4	25.4
2032	3,136.1	5.44	3,953.5	5.82	3,153.1	3,953.5	25.8
2033	3,306.6	5.42	4,183.6	5.65	3,324.3	4,183.6	26.1
2034	3,485.9	5.39	4,419.9	5.49	3,505.0	4,419.9	26.2
2035	3,673.9	5.37	4,662.7	5.38	3,695.1	4,662.7	26.2
2036	3,871.2	5.37	4,913.5	5.30	3,894.3	4,913.5	26.1
2037	4,079.2	5.38	5,174.1	5.26	4,103.5	5,174.1	26.0
2038	4,298.8	5.37	5,446.5	5.23	4,324.0	5,446.5	25.8
2039	4,529.8	5.34	5,731.2	5.21	4,556.7	5,731.2	25.6
2040	4,771.9	5.33	6,029.7	5.20	4,801.6	6,029.7	25.4
2041	5,026.1	5.31	6,343.4	5.24	5,058.2	6,343.4	25.3
2042	5,292.8	5.33	6,675.6	5.27	5,327.7	6,675.6	25.3
2043	5,574.9	5.30	7,027.5	5.30	5,610.4	7,027.5	25.2
2044	5,870.4	5.29	7,400.0	5.33	5,909.4	7,400.0	25.3
2045	6,181.2	5.28	7,794.5	5.37	6,222.6	7,794.5	25.4
2046	6,507.7	5.28	8,213.2	5.43	6,552.1	8,213.2	25.5
2047	6,851.4	5.28	8,659.3	5.50	6,898.2	8,659.3	25.8
2048	7,213.0	5.26	9,135.7	5.54	7,262.5	9,135.7	26.1
2049	7,592.5	5.27	9,641.9	5.58	7,645.8	9,641.9	26.5
2050	7,992.5	5.28	10,180.4	5.65	8,048.1	10,180.4	27.0
2051	8,414.2	5.29	10,755.8	5.64	8,472.1	10,755.8	27.4
2052	8,859.5	5.30	11,362.5	5.63	8,919.1	11,362.5	27.8
2053	9,329.0	5.31	12,002.3	5.66	9,391.1	12,002.3	28.2
2054	9,824.3	5.31	12,681.7	5.63	9,888.7	12,681.7	28.6
2055	10,345.8	5.32	13,396.2	5.62	10,413.8	13,396.2	29.0
2056	10,896.1	5.33	14,149.6	5.62	10,966.5	14,149.2	29.4
2057	11,476.5	5.32	14,944.5	5.58	11,549.9	14,944.5	29.7
2058	12,087.4	5.33	15,778.4	5.57	12,165.1	15,778.4	30.0
2059	12,731.7	5.33	16,657.4	5.54	12,812.6	16,657.4	30.3

TABLE 1
(continued)

Year $(k, k+1)$	Income I_k Excluding Interest	$100 \cdot \Delta I_k / I_k$	Outgo O_k	$100 \cdot \Delta O_k / O_k$	$\alpha_{k-1,k}$ $I_{k-1}(1+i_k)$	$\beta_{k,k+1} = O_k$	γ_{k+1}
2060	$13,410.2	5.33%	$17,580.5	5.51%	$13,495.6	$17,580.5	30.5%
2061	14,125.0	5.33	18,548.5	5.49	14,214.8	18,548.5	30.7
2062	14,878.1	5.33	19,567.6	5.48	14,972.5	19,567.6	30.9
2063	15,671.5	5.33	20,639.6	5.45	15,770.8	20,639.3	31.0
2064	16,507.1		21,765.3		16,611.8	21,765.3	

TABLE 2
PRELIMINARY PROJECTIONS OF A ONE-YEAR ROLL-FORWARD RESERVE TO BE MAINTAINED FOR HOSPITAL INSURANCE (HI)
(Dollar amounts in billions)

Year $(k, k+1)$	Income I_k Excluding Interest	$100 \cdot \Delta I_k / I_k$	Outgo O_k	$100 \cdot \Delta O_k / O_k$	$\alpha_{k-1,k}$ $I_{k-1}(1+i_k)$	$\beta_{k,k+1} = O_k$	γ_{k+1} [1]
1990	$72.4	6.35%	$64.0	11.09%			-8.3%
1991	77.0	6.36	71.1	10.41	$77.5	$71.1	-4.7
1992	81.9	6.11	78.5	10.45	82.4	78.5	-1.0
1993	86.9	6.10	86.7	10.5	87.6	86.7	3.0
1994	92.2	6.18	95.8	10.33	93.0	95.8	7.1
1995	97.9	6.13	105.7	9.93	98.7	105.7	10.9
1996	103.9	6.06	116.2	9.64	104.8	116.2	14.6
1997	110.2	6.17	127.4	9.50	111.2	127.4	18.3
1998	117.0	6.07	139.5	9.68	117.9	139.5	22.2
1999	124.1	5.80	153.0	9.28	125.2	153.0	25.9
2000	131.3	5.94	167.2	8.49	132.8	167.2	30.3
2001	139.1	6.04	181.4	8.32	139.2	181.4	33.3
2002	147.5	5.97	196.5	8.35	147.4	196.5	36.1
2003	156.3	5.95	212.9	8.22	156.4	212.9	39.0
2004	165.6	6.10	230.4	8.33	165.7	230.4	42.2
2005	175.7	6.09	249.6	8.45	175.5	249.6	45.4
2006	186.4	5.90	270.7	8.72	186.2	270.7	48.9
2007	197.4	5.88	294.3	9.04	197.6	294.3	53.4
2008	209.0	5.79	320.9	8.63	209.2	320.9	57.4
2009	221.1	5.65	348.6	8.03	221.5	348.6	60.7
2010	233.6	5.61	376.6	8.44	234.4	376.6	64.9
2011	246.7	5.43	408.4	8.99	247.6	408.4	70.2
2012	260.1	5.34	445.1	9.19	261.5	445.1	76.3

[1] See footnotes 1,2 and 3 of Table 1. We continued calculations to 2036 where $\gamma_{2037} = 182.1\%$.

10. Preliminary Computations for Four-Year Reserves

This section will give some illustrations of four-year roll-forward reserves. A four-year period was chosen as it coordinates with the quadrennial Advisory Councils, and also because it is probably the maximum period to be considered as it generates a very large fund and corresponding responsibilities.

Some advantages of a four-year reserve are that:

(1) It would help allay doubts as to whether "the money will be there" when the "baby-boomers" reach retirement. There is a widespread public suspicion that a system which transfers current income into current outgo is not to be trusted to continue providing benefits indefinitely. This suspicion increases as required contributions from employees and employers become sizable.

(2) If the economy undergoes severe financial problems, a four-year reserve provides Congress more time to adjust OASDI as may be needed.

(3) It is more consistent with the funding of state and large municipal retirement systems than pay-as-you-go funding with a one-year contingency fund.

(4) By being invested in special federal bonds, and thereby being a component of the National Debt, the fund shifts part of the benefit costs away from the payroll tax to a more progressive and broader tax base. This process needs more publicity and public awareness.

The obvious disadvantage is the huge size of the fund that is generated. This creates many problems including possible demands for new or increased benefits. Our financial systems may not be ready to adapt to such a fund yet. Also, many other public services such as education and national security are provided on a current cost basis, which may argue for like treatment of Social Security, despite its long-range commitments.

At any rate, let us examine the $n = 4$ case as a possible extreme alternative to the $n = 1$ case discussed in Section 9.

In Table 3, preliminary calculations based on Alternative II-B assumptions, are summarized for OASDI four-year roll-forward reserves. For $k = 1994, 1998, \cdots, \alpha_{k-4,k}$ and $\beta_{k,k+4}$ are exhibited, and for $k = 1990, 1994, \cdots$, values of γ_{k+4} are shown.

From [3, Figure 4, p. 81], it appears ε_k is projected to be negative prior to about 2006 that is, the fund is less than the equivalent of 4-years outgo. Fairly soon thereafter γ_{k+4} becomes positive and consideration of increasing $\alpha_{k,k+4}$ would emerge. Increase of the projected income by 20-25 percent is indicated for the period 2022-2050. Such increases do not differ greatly from those indicated by Table 1, and may result from the closeness of interest rates and growth rates of outgo [ef. 11, Wilkin, p. 15].

The \$40.5 trillion shown in 2050 for $\beta_{k,k+4}$ indicates the magnitude of four-year reserves targeted toward $\beta_{k,k+4}$. Even if deflated to \$4 trillion in 1990 dollars, the sum is huge and shows the long-term result of even moderate exponential growth. How to maintain Social Security equilibrium in such a situation is an extraordinary challenge to actuaries.

Table 4 is similar to Table 3 but is based on Alternative II-A assumptions. Under the more favorable assumptions of this alternative, the required increases in $\alpha_{k,k+4}$ are lower than were indicated in Table 3 for Alternative II-B.

Table 5 exhibits calculations for 1980-2012 if four-year reserves were the goal. The positive γ_{1994} in the 1980 row would have been a warning signal. The quick turn around in fund growth after the 1983 Amendments is also evident.

11. Other Public Employee Retirement Systems

This section will be comparatively short as I have not studied it sufficiently to arrive at firm conclusions. The question that has been disturbing my mind is why many actuaries consider Social Security actuarially sound if it operates on a pay-as-you-go basis plus a contingency fund of one year's outgo, while many public employee retirement systems aim at a full funding status, by one of the standard funding methods. Such full founding will often be the equivalent of a roll-forward reserve for a high number of years, such as 20 or more.

One public employee retirement system for which I have a substantial record turned out to have such a spectacular growth, and so many special actions, that I concluded a large fund, such as a State system, would be needed to give more stable data. The matter will be left with that comment but will be referred to again in Part III which outlines some open questions related to this paper.

TABLE 3

PRELIMINARY PROJECTIONS IF A FOUR-YEAR ROLL-FORWARD RESERVE IS TO BE ATTAINED FOR OASDI[1]

(Dollar amounts in billions)

Year $(k, k+1)$	$(k-4, k)$ Income Accumulated to Time k $\alpha_{k-4,k}$	$(k, k+4)$ Outgo Discounted to Time k $\beta_{k,k+4}$	$[\beta_{k+4,k+8}/\alpha_{k,k+4} - 1] \cdot 100 = \gamma_{k+4}$ [2,3]
1990			-17.7%
1994	$1,557.4	$1,282.0	-18.7
1998	1,985.5	1,614.6	-16.8
2002	2,477.8	2,061.1	-15.2
2006	3,116.6	2,641.9	-11.7
2010	3,944.9	3,481.9	- 5.1
2014	4,942.0	4,690.1	+3.5
2018	6,129.1	6,346.5	11.9
2022	7,561.6	8,462.2	18.7
2026	9,306.0	11,045.1	22.8
2030	11,457.4	14,066.8	24.1
2034	14,134.6	17,535.9	23.5
2038	17,450.2	21,545.7	22.9
2042	21,511.6	26,435.6	23.2
2046	26,469.9	32,608.5	24.6
2050	32,528.6	40,517.2	

[1] Calculated from Table F5, 1990 Annual Report of the Board of Trustees of OASDI, Alternative II-B Assumptions.

[2] For this illustration, interest is assumed to be at the annual rate of 7% in the 1990's decade, and at 6% thereafter.

[3] See formulas (8.3), (8.4), (8.8). Note that $k = 1990$ in the first row, 1994 in the second row, etc.

TABLE 4

PRELIMINARY PROJECTIONS IF A FOUR-YEAR ROLL-FORWARD RESERVE IS TO BE ATTAINED FOR OASDI[1]

(Dollar amounts in billions)

Year $(k, k+1)$	$(k-4, k)$ Income Accumulated to Time k $\alpha_{k-4,k}$	$(k, k+4)$ Outgo Discounted to Time k $\beta_{k,k+4}$	$[\beta_{k+4,k+8}/\alpha_{k,k+4} - 1] \cdot 100 = \gamma_{k+4}$ [2,3]
1990			-20.8%
1994	$1,569.5	$1,242.6	-23.9
1998	1,996.3	1,519.3	-22.5
2002	2,446.3	1,895.7	-21.0
2006	3,015.1	2,380.9	-17.5
2010	3,728.5	3,077.5	-11.0
2014	4,563.7	4,063.5	- 2.7
2018	5,530.6	5,382.8	5.2
2022	6,666.5	7,014.3	11.5
2026	8,015.1	8,938.0	15.2
2030	9,642.7	11,104.4	16.2
2034	11,624.1	13,504.1	15.5
2038	14,021.3	16,192.5	14.9
2042	16,890.3	19,405.0	15.2
2046	20,309.2	23,399.8	16.6
2050	24,390.8	28,429.0	

[1] Calculated from Table F5, 1990 Annual Report of the Board of Trustees of OASDI, Alternative II-A Assumptions.
[2] For this illustration, interest is assumed to be at the annual rate of 7% in the 1990's decade, and at 6% thereafter.
[3] See formulas (8.3), (8.4), (8.8). Note that $k = 1990$ in the first row, 1994 in second row, etc.

TABLE 5

CALCULATIONS LEADING TO A FOUR-YEAR ROLL-FORWARD RESERVE FOR OASDI FOR YEARS 1980-2012[1]

(Dollar amounts in billions)

Year $(k, k+1)$	$(k-4, k)$ Income Accumulated to Time k $\alpha_{k-4,k}$	$(k, k+4)$ Outgo Discounted to Time k $\beta_{k,k+4}$	$[\beta_{k+4,k+8}/\alpha_{k,k+4} - 1] \cdot 100 = \gamma_{k+4}$ [2,3]
1980			1.8%
1984	$683.2	$695.7	-12.3
1988	994.3	872.2	-16.4
1992	1,343.6	1,123.1	-18.2
1996	1,738.1	1,422.1	-19.0
2000	2,231.7	1,807.7	-19.0
2004	2,833.9	2,294.1	-17.1
2008	3,592.7	2,980.1	-12.2
2012	4,527.2	3,976.6	

[1] Based on Table 147 of Actuarial Study 103 for years 1980-1997 and Table F5 of the 1990 Annual Report of the Board of Trustees of OASDI, Alternative II-B Assumptions.

[2] For this illustration, interest is assumed to be at the annual rate of 8% in the 1980's, at 7% in the 1990's, and at 6% thereafter.

[3] See formulas (8.3), (8.4), (8.8). Note that $k = 1980$ in the first row, 1984 in the second row, etc.

III. Some Open Questions

In this part, there will be mentioned various questions which are interrelated with matters discussed in this paper.

12. Unanswered Questions

My Michigan colleague, Howard Young, has suggested more than once that there is need for a full discussion of what would be an optimal contingency fund ratio for Social Security. He has in mind that such a ratio would have an acceptable range, with the Social Security tax rates planned so as to keep the ratio within that range, and bring it back to the mid-range level within a reasonable time period. He has written "That would be different than requiring the ratio to return to target (e.g. 100%) each year; the latter arrangement could cause large annual variations in the Social Security tax rate, a result that the contingency fund should mitigate. That is, I think the contingency fund has two purposes: first, to assist ongoing payment of benefits; and second, to reduce the annual variability of the Social Security tax rates".

If as in Part II of this paper, one considers adequate reserve funds as being somewhat larger than adequate contingency funds, one is recognizing the additional extremely important purpose of reassuring the public that funds will be available for the payment of their benefits, and will not be totally subject to the whims of future legislation. For this purpose, four-year roll-forward reserves may not be excessive. Two-year roll-forward reserves, involving planning over at least a six-year span may be more feasible. Such reserves should be publicized strongly, and their role in providing interest income toward benefit outgo emphasized.

Another question that has not been addressed here is the gap between the nature of Social Security funding and the funding of large, mature public employee retirement systems. A career's worth of actuarial study should be devoted thereto.

There are also actuarial aspects to the question of at least partial independence of the Social Security Systems. They involve long-term inter-generational commitments but are not unique in this regard, as education, health, and national security involve long-term interactions. However, actuaries should interpret the Social Security commitments to the public, and should not be overwhelmed by attempts of others to balance the current economy, or to solve deep social problems by means of Social Security.

Another direction which I have not pursued yet is to enlarge the formulas of Part I by following the lead of C.L. Trowbridge and consider growth rates of income, as well as outgo. In Part I, only the growth rate of outgo was designated, and income was the essential balancing factor. If growth rates of both income and outgo are considered, the relations will be more complex but possibly have interesting consequences. In Part II of this paper, only projected growth rates of income and outgo were noted, and mathematical treatment of them was not explored.

Much further study remains. The Hospital Insurance (HI) is barely touched on here, and Supplementary Medical Insurance (SMI) not at all. Some views concerning Medicare are available in [11, see Wilkin and Hustead]. Should OASDI and Medicare (HI and SMI) be consolidated into one Social Security system is a political issue, but has actuarial funding aspects that should be expressed. There is opportunity for the actuarial profession and science to serve the national interest by creative study of how these Social Security systems are developed for present and future generations.

May actuaries continue to study, write, and speak effectively on Social Security. That is my difficult challenge to you.

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