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CURRENT RESEARCH

ON PENSION ACCOUNTING

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This is an account of the talk I gave at the 27th Actuarial conference, which was held in Iowa City (August 1992). The talk described part of the results of a research project sponsored by the Actuarial Education and Research Fund. The complete report of the project will appear in a subsequent issue of ARCH, under the title *Some Aspects of Statement of Financial Accounting Standards No. 87.*

Introduction

The research project dealt with pension accounting, more precisely with Statement of Financial Accounting Standards No. 87 (or "FAS 87"). FAS 87 has been the employers' pension accounting requirement in the United States since 1986. The goal of the project was to study the variability of pension accounting costs under FAS 87. The variability of pension costs is a function of many parameters, including

- the variability of valuation interest rates, or discount rates as they are called in FAS 87;
- the variability of rates of return on the fund's assets;
- the period used to amortize gains and losses; and
- the size of the "corridor" allowed for amortization of gains and losses.

This corridor will be important in what follows, and will be described shortly.

In this paper I address only one specific point, namely the behaviour of experience gains and losses over time. This is directly related to the corridor approach to amortization of gains and losses. However, what will be said also applies to experience gains and losses in other areas, particularly in pension funding and insurance.

Description of corridor

Let me now describe the so-called corridor and how it is used in the amortization of gains and losses. FAS 87 specifies a minimum amortization requirement. No amortization is required as long as past unrecognized gains and losses do not exceed a certain amount. When past unrecognized gains and losses do exceed that level, a fraction of the excess has to be included in pension cost for that year. The level (or *threshold)* above which some amortization has to take place is an amount equal to ten percent of the greater of the value of the fund's assets, and the actuarial liability.

Let *URL* stand for the cumulative sum of past unrecognized losses (a gain is seen as a negative loss). Also, let T stand for the threshold used, i.e. ten percent of the greater of the fund value and the actuarial liability, and let P stand for the amortization period (e.g. 20 years). The amortization period is a function of the age distribution of members, and is specified in FAS 87. In any year the minimum amortization payment is the excess of URL over T, divided by P, i.e.

$$\text{Minimum amortization} = \begin{cases} 0, & \text{if } |URL| \leq T \\ (|URL| - T)/P, & \text{if } |URL| > T. \end{cases}$$

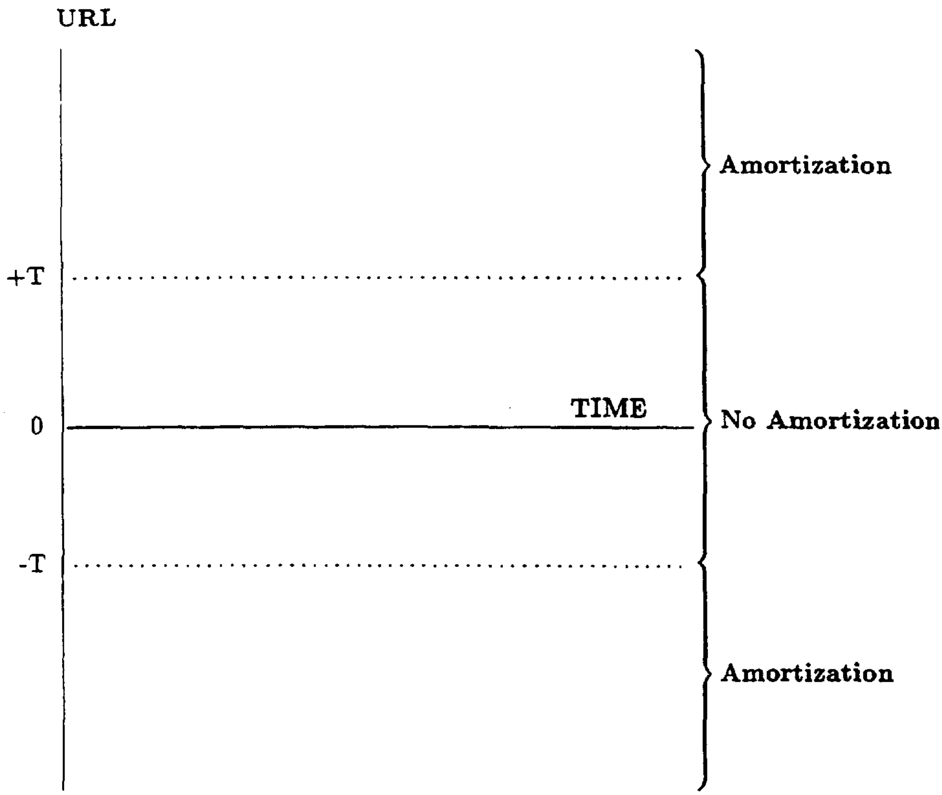
The sign of this amount is then adjusted, that is to say it will be an addition to the other components of the cost, if unrecognized losses are positive, and a subtraction from pension cost if unrecognized losses are negative. Observe that the whole exercise is done anew every year (i.e. no schedule of payments is set up for future years).

Figure 1 describes the situation. A fraction of unrecognized gains or losses has to be brought into expense only when URL escapes from the corridor depicted; when the abolute value of URL is smaller than the threshold amount, the system is left to itself, so to speak. Those familiar with stochastic processes will see that URL has a kind of mean reverting property, though in this case there is more properly a reversion towards the center of the space, rather than towards a single point. (Figure 1 is a simplification, since the threshold will usually change over time.)

Rationale of corridor

The rationale of the corridor appproach to amortization of

FIGURE 1



T = 10% max{Fund, Actuarial Liability}

gains and losses is apparently contained in paragraph 184 of Statement 87:

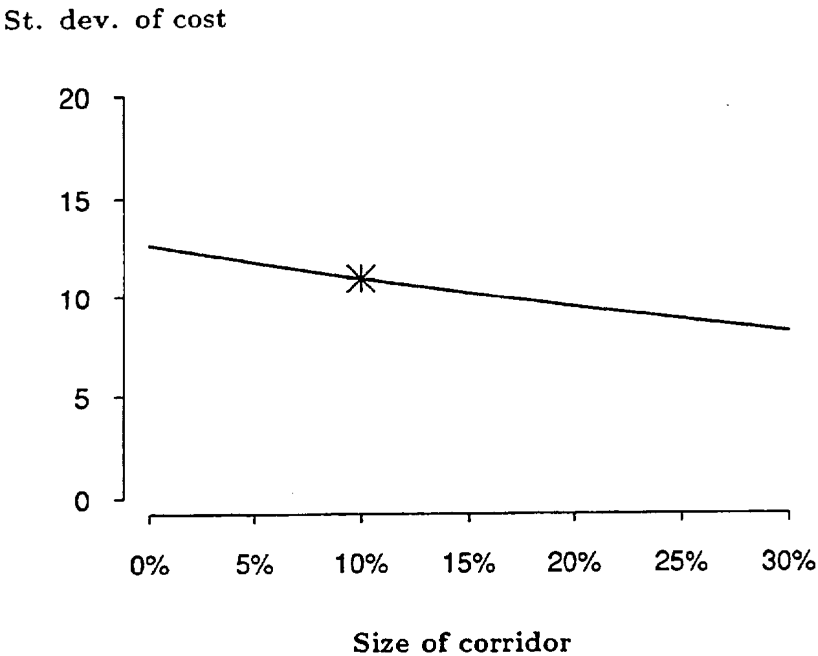
> "The [Financial Accounting Standards] Board noted that, if assumptions prove to be accurate estimates of experience over a number of years, gains or losses in one year will be offset by losses or gains in subsequent periods. In that situation, all gains and losses would be offset over time, and amortization of unrecognized gains and losses would be unnecessary."

I have encountered this belief (that experience gains and losses should "cancel out over time", at least when "actuarial assumptions are accurate") on a number of occasions, and it has always made me feel a little uneasy. We shall come back to this question in a moment, when I try to formulate and analyse it mathematically.

Sensitivity analysis

I now present, very rapidly, some of the numerical results obtained as part of the research project. A simulation model was built, incorporating random valuation rates of interest, random rates of return on assets, etc. Under the model, all actuarial assumptions are correct in the long run, so that experience gains and losses are nil on average. The corridor approach is consistently applied to the amortization of gains and losses. A sensitivity analysis was conducted, with respect to the size of the corridor. A number of possible sizes of the corridor were chosen, ranging from 0% to 30% of the greater of the fund's assets and the actuarial liability.

FIGURE 2



For each fixed percentage, the long-term variability of pension cost was calculated.

The curve obtained is shown in Figure 2. The percentages used for the corridor are on the horizontal axis, while the standard deviation of pension cost is shown on the vertical axis. The point indicated by a star on the curve corresponds to the current requirement, that is to say 10% max{ fund, actuarial liability}. It can be seen that the corridor does not significantly decrease the variability of pension expense. If we were to eliminate the allowed corridor completely, the standard deviation of long-term cost would increase by only about 13%, under the model.

Discussion

The reason why the corridor does not efficiently reduce the volatility of pension cost is that its rationale is incorrect: gains and losses do not offset over time. What actually happens is that, given enough time, cumulative gains and losses become arbitrarily large. Thus, unrecognized losses or gains will, with probability one, escape from any preset corridor.

Let me give a simple example to illustrate what was just said. Suppose a coin is tossed repeatedly, and that you win $1 if head occurs, and lose $1 otherwise. Let

$$\begin{aligned} G_k &= +1 \qquad \text{if } k\text{th result is head} \\ &= -1 \qquad \text{if } k\text{th result is tail} \end{aligned}$$

and

$$S_n = G_1 + \cdots + G_n.$$

G_k is a gain or loss, depending on its sign, and S_n is the cumulative sum of the gains up to time n, a loss being seen as a negative gain. The average gain is zero, and all gains are independent. Do you expect that gains and losses will "offset" in such a way that that S_n will, with a high probability, remain in the corridor (say) [-10,+10]? (Please think about this a little before reading on.)

Here are some known facts concerning the random walk $\{S_n\}$:

1. The Law of Large Numbers says that

$$\frac{1}{n} S_n \to 0 \qquad \text{as} \quad n \to \infty .$$

2. With certainty S_n will eventually return to 0, but this may take "a very long time". (The average time needed for the first return to the origin is infinite.)
3. $\operatorname{Var} S_n = n \operatorname{Var} G_1$; the variance of S_n increases without bounds, which means that its distribution becomes more and more "spread out" as the number of tosses increases.
4. With certainty S_n becomes arbitrarily large, i.e. for any $M > 0$, however how large, there is always an n such that $S_n > M$. (By symmetry there will also be an n' such that $S_{n'} < -M$. It is sometimes said that "in the limit S_n oscillates between plus and minus infinity".)

The first two facts show precisely to what extent gains and losses really do offset over time. The *average* gain $\frac{1}{n} S_n$ converges to zero, but not the *sum* of the gains S_n. Furthermore, there will be a time when S_n returns to zero; but this time is very uncertain and, moreover, nothing says that S_n will remain in the vicinity of zero afterwards. The last two facts say that no matter how wide the corridor, S_n will eventually escape from it.

Let us see what the corridor approach could mean in the context of this simple example. Suppose that it is agreed that an "adjustment" (ADJ) will be made to your wealth after each toss. The adjustment will be nil as long your wealth (S_n) is within (say) the corridor [-10, +10]. If S_n is outside that corridor, then either (1) it is decreased by (say) 25% of its excess over $+10$, if $S_n > 10$, or (2) it is increased by 25% of the amount by which it falls short of -10, if $S_n < -10$.

Remark. The adjustment acts as a tax on wealth (as opposed to income), with a tax refund made to those "in the red". Its overall effect is to decrease the chance of accumulating large sums, as well as the chance of being deeply in debt. The adjustment is a negative feedback control applied to the process $\{S_n\}$, and it can be shown that the controlled process has a much more "stable" behaviour than the uncontrolled one (its variance is bounded over time, etc.). So much for the virtues of taxes. □

ADJ will be nil as long as $|S_n| \leq 10$. But S_n will sooner or later escape from the corridor, causing fluctuations in ADJ. There is a built-in tendency for S_n to return towards the region [-10,+10]. However, when S_n reenters that interval the adjustment vanishes, and S_n once again tends to escape from it. Thus, the variability of the adjustment will be low in the beginning, when it is unlikely that S_n will have left the corridor; over a longer period the variability of ADJ will be comparable to what it would have been without the corridor. It will be slightly smaller, because a fraction of the time $ADJ = 0$, when S_n is within the corridor.

Now let us return to pension accounting. Let

$$U_n = \text{unfunded liability at time } n$$
$$I_n = \text{information known at time } n.$$

Then the experience loss is the "unexpected increase of the unfunded liability during the year":

$$\begin{aligned} L_n &= \text{experience loss during } n\text{th period} \\ &= U_n - (U_n | I_{n-1}). \end{aligned}$$

Let

$$S_n = \text{cumulative sum of experience losses up to time } n.$$

Of course gains and losses are not independent, as they were in the preceding example, but it can be verified that

(1) The $\{L_n\}$ are uncorrelated $\Longrightarrow$

$$\operatorname{Var} S_n = \operatorname{Var} L_1 + \cdots + \operatorname{Var} L_n\,.$$

(2) $\{L_n\}$ is a martingale.

The behaviour of S_n is similar here to what we have seen in the simple coin-tossing example, that is to say the sum of experience gains and losses does not approach 0 as time passes. It stands to reason that, other things remaining equal, allowing a corridor for the amortization of gains and losses will decrease the variability of pension costs. But the fact that S_n always tend to escape from the corridor implies that the reduction in variability may not be very large. It is not possible to give a general formula for the variance of pension costs; all we can do is rely on computer simulations. The results obtained so far show that FASB's corridor brings only a small reduction in the volatility of pension costs. The reduction is the same as if the amortization period had been slightly increased.

Remark. Observe that, more generally, the above analysis applies nearly word for word to experience gains/losses in insurance and pension funding.

Reference

Dufresne, D. (1992). *Some Aspects of Statement of Financial Accounting Standards No. 87.* Report of AERF sponsored research project. (Available from author.)

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