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Elementary Models of Reserve Fund for OASDI in the USA

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In Dr. Nesbitt's interesting analysis he has twice referred to my 1967 TSA paper on mutual insurance company surplus (see his reference [12]). On seeing an earlier draft of the Nesbitt work, I noted (in a letter to him) that the mathematics reminded me of some I had developed in another context some years ago. I assume that my paper is included among Nesbitt's references because of this rather off-hand comment.

Having now given this matter further thought, I believe that a social security fund is a special case of what I have called "surplus", and that Dr. Nesbitt's mathematics of social security fund development is a special case of the very simple mathematical analysis in [12]. This brief discussion is intended to so demonstrate, and to satisfy reader's curiosity as to why reference [12] appears.

The essence of the argument in [12] is:

(a) Assume a contingency fund F_k at time k, invested so as to earn interest at rate i_{k+1} over the ensuing year.

(b) Assume further that F_k is some multiple c of a moving parameter P_k, and that it is intended that the F/P ratio c be constant as k increases.

(c) If follows that, since F_{k+1} is to be $c \cdot P_{k+1}$, there must be a contribution to the surplus fund (from the outside) of

$$C_{k+1} = c(r_{k+1} - i_{k+1})P_k v_{k+1-s,k+1}$$

where r_{k+1} is the growth rate in P_k over the year k to $k+1$, and s is the portion of the year remaining when the surplus contribution is made.

(d) It is noteworthy that C_{k+1} is directly proportional to c, to P_k, and to $r_{k+1} - i_{k+1}$. Further, C_{k+1} takes the algebraic sign of $r_{k+1} - i_{k+1}$, and hence is negative if the interest rate exceeds the parameter growth rate. When $r_{k+1} = i_{k+1}$, $C_{k+1} = 0$.

(e) It easily follows that , if the surplus contribution is less than, equal to, or greater than the amount indicated as C_{k+1} above, the surplus ratio F/P at year end will be less than, equal to, or greater than c.

Let us now leave [12], and start from Nesbitt's equation 5.1. We rearrange it slightly, and think of the difference between I_k and O_k as the outside contribution to surplus.

$$I_k = O_k + f_{k+1}O_{k+1}v_{k,k+1} - c_k O_k(i_{k+1} - r_{k+1})v_{k,k+1} \qquad (5.1)$$

$$I_k - O_k = c_k(r_{k+1} - i_{k+1})O_k v_{k,k+1} + f_{k+1}(1 + r_{k+1})O_k v_{k,k+1}$$

The first term (on the right hand sign of the last shown form) indicates the contribution needed if the ratio c is to be maintained; the second term, the additional contribution if the surplus ratio is to change by f_{k+1}.

We recognize this result as identical to that coming from [12], with the single exception that O_k is a specific parameter, whereas P is a general one. In a insurance context P might represent total premium, or total assets, or any other parameter that might be appropriate to assess the adequacy of F.

Aside from the demonstration above, I have two further comments about the Nesbitt paper.

I prefer the alternative approach of section 5 to that of sections 2, 3, and 4, though the latter is correct and is presented first. The concept that F represents "benefits paid in advance", (and that the function of I is to replenish the contingency fund as it is diminished by the payment of benefits) seems unnecessarily strained, especially if c is other than an integer. Section 5 recongnizes that I will usually be close to O, and that any difference is the outside source needed to control the F/O ratio. It thereby emphasizes the "current cost" nature of social security financing, even when c is large.

It may be instructive to compare Dr. Nesbitt's n-year roll-forward reserves of part II with the theory developed in the earlier part I. When $n = c = 1$, there is no difference, as Dr. Nesbitt shows us. If n is some larger integer (e.g. 4), is the n-year roll-forward reserve identical to the part I reserve with $c = n$?

I suspect, though I am not entirely sure, that these are identical, but only in the special case where $i_{k+1} = r_{k+1}$, throughout the n year period. Otherwise, there many be a technical difference, but hardly enough to be meaningful. In short, I feel that part II adds little to my understanding, and it clearly adds to the complexity.