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AGGREGATE CLAIMS DISTRIBUTIONS FOR TWO CORRELATED BENEFITS

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1. INTRODUCTION

This paper was prompted by a group insurance problem that came up in practice. The actuary was asked to price stop loss insurance by employee for two benefits combined.

Available information included a claims probability distribution for each benefit and the correlation coefficient between the benefits.

Let X be the aggregate claims on benefit 1 with a proibability distribution $P_1(x)$ for $x_1, x_2, \ldots x_n$. For benefit 1 the mean claims are given by $E_1(x) = \Sigma x P_1(x)$ and the standard deviation is $SD_1(x)$.

Let Y be the aggregate claims on benefit 2 with a probability distribution $P_2(Y)$ for $y_1, y_2, \ldots y_m$. For benefit 2 the mean claims are given by $E_2(Y)$ and the standard deviation is $SD_2(y)$.

Let $P(x,y)$ be the probability that benefit 1 claims amount to X and benefit 2 claims amount to y. For consistency the following relations must hold.

$$P_1(x) = \Sigma_y P(x,y)$$

$$P_2(y) = \Sigma_x P(x,y)$$

Diagrammatically $P(x,y)$ is a matrix with column totals equal to the vector

$P_1(x)$ and row totals equal to the vector $P_2(y)$.

For a particular matrix $P(x,y)$ the correlation coefficient can be computed from

$$\rho = \frac{E(x,y) - E.(x).E_2(y)}{SD_1(x).SD_2(y)}$$

where $E(x,y) = \Sigma_x \Sigma_y xyP(x,y)$ and $E_1(x), E_2(y), SD_1(x), SD_2(y)$ are determined from the individual probability distributions.

Let $A(x,y)$ be a matrix constructed by letting $A(x,y) = P_1(x).P_2(y)$. Checking for consistency

$$\sum_y A(x,y) = \sum_x P_1(x).P_2(y) = P_1(x)\sum_y P_2(y) = P_1(x)$$

$$\sum_x A(x,y) = \sum_x P_1(x).P_2(y) = P_2(y)\sum_x P_1(x) = P_2(y)$$

Computing the correlation coefficient we get

$$E(xy) = \sum_x \sum_y xyA(x,y) = \sum_x \sum_y xyP_1(x).P_2(y)$$

$$= \sum_x xP_1(x) \sum_y P_2(y) = E_1(x).E_2(y)$$

$$\therefore \rho_a = 0$$

The matrix A is appropriate if benefits 1 and 2 are uncorrelated.

Let $B(x,y)$ be a matrix constructed by letting

$$B(x,y) = \min[P_1(x) - \sum_{z<x} B(y)], [P_2(y) - \sum_{z<y} B(x,z)]$$

See the appendix for a numerical example. As the values of $B(x,y)$ are concentrated close to the main diagonal ρ_B will tend to be close to 1.

Let $C(x,y)$ be a matrix constructed from $C(x,y) = (1-w).\ A(x,y) + wB(x,y)$

$$E_c[xy] = \sum_y \sum_x xyC(x,y)$$

$$= \sum_y \sum_x xy[(1-w)A(x,y) + wB(x,y)]$$

$$= (1-w)E_A(xy) + wE_B(xy)$$

$$\rho_C = \frac{E_c[xy] - E_1[x].E_2[y]}{SD_1(x).SD_2(y)}$$

$$= (1-w)\rho_A + w\rho_B$$

$$= w.\rho_B$$

If ρ_C is to be the desired correlation construct C from A and B with $w = \rho_C/\rho_B$.

From the probability matrix $C(x, y)$ the aggregate claims distribution for the two benefits combined can be determined, and stop loss premiums computed.

The matrix C is not unique. Other matrices which satisfy the conditions of the problem can be constructed, each leading to a different stop loss premium. It would be of interest to do sensitivity testing of the premium.

One approach would be to construct matrices in a random fashion. A random number generator could be used to fill in one row at a time subject to the constraints on row and column totals. This is yet to be explored.

APPENDIX

Given

$$P_1(x) = .1, .2, .3, .4 \text{ for } x = 0, 1, 2, 3$$

and

$$P_2(y) = .2, .3, .3, .2 \text{ for } y = 0, 1, 2, 3$$

$A(x, y)$ is given by

y/x	0	1	2	3
0	.02	.04	.06	.08
1	.03	.06	.09	.12
2	.03	.06	.09	.12
3	.02	.04	.06	.08

$B(x, y)$ is given by

y/x	0	1	2	3
0	.1	.1	0	0
1	0	.1	.2	0
2	0	0	.1	.2
3	0	0	0	.2

$$\rho_A = 0$$

$$\rho_B \approx .9$$