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Stability and Convergence of Recurrence Equations

Shaun Wang Harry Panjer

Abstract

This paper discusses the numerical stability of Panjer's recursion (1981) in computing aggregate claim distributions.

Under some regularity conditions, a recurrence equation of order m:

$$g(x) = \sum_{j=1}^{m} A_j(x)\, g(x-j),\ x > k, \tag{1}$$

has a basic set of m solutions $\{g^{(i)}(x),\ i = 1, \cdots, m\}$ such that

- $\lim_{x\to\infty} g^{(1)}(x)/g^{(j)}(x) = \infty, \quad 2 \le j \le m$
- any solution of (1) can be written as:

$$g(x) = c_1\, g^{(1)}(x) + \cdots + c_m\, g^{(m)}(x),$$

where $g(x)$ is **dominant** if $c_1 \neq 0$, and is **subordinate** if $c_1 = 0$.

Since computers can only represent a finite number of digits, round-off errors are inevitable. The error propagation $\varepsilon(x)$, as a disturbance solution, can be written as

$$\varepsilon(x) = \varepsilon_1\, g^{(1)}(x) + \cdots + \varepsilon_m\, g^{(m)}(x).$$

Thus

$$\frac{\varepsilon(x)}{g(x)} = \frac{\varepsilon_1\, g^{(1)}(x) + \cdots + \varepsilon_m\, g^{(m)}(x)}{c_1\, g^{(1)}(x) + \cdots + c_m\, g^{(m)}(x)} \tag{2}$$

remains small if $c_1 \neq 0$, but goes to infinity (blows-up) if $c_1 = 0$.

Generally, recursion (1) is stable in evaluating its dominant solutions, where relative error bounds grow linearly; and recursion (1) is unstable in evaluating its subordinate solutions, where relative error bounds grow exponentially.

It is shown that compound Poisson and compound negative binomial are dominant solutions of Panjer's recursion, so Panjer's recursion is stable in these cases. Furthermore, if r digits are used, the number of significance digits in the computed solution $g(x)$ can be estimated by:

$$\nu(x) \geq r + [\log_{10} 2 - \log_{10}(x+1)]. \tag{3}$$

For binomial claim frequencies, Panjer's recursion is shown to be unstable in both directions. However, interestingly, the unstable range of the forward direction can be covered by the stable range of the backward direction, and vise versa. So a combined usage of both directions can simply deal with this instability.

Convergence and stability are closely related. From (2), one can see that every two dominant solutions converges proportionally. This convergence is exponentially fast in most cases. Thus, for large insurance portfolios (e.g. large Poisson mean λ), where computer underflow ($e^{-\lambda} \approx 0$) may cause a problem, one can evaluate the compound distribution starting from somewhere, say $5\sigma_S$ or $4\sigma_S$ before the aggregate mean μ_S. One can assign arbitrary initial values since later a rescaling multiple can be found by forcing their summation to be 1. For *large* insurance portfolios, this method is very accurate and efficient, it has the advantage of avoiding the underflow problem and having significantly reduced the computing time.