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# AN ANALYSIS OF LOSS RESERVES IN CANADA

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## ABSTRACT

This paper investigates the accuracy of loss reserves. Estimates made from 1975 through 1983 by Canadian property and casualty insurers were compared with results five years later. The variation of results is analyzed based on: size of company, domestic versus foreign companies, direct insurers versus reinsurers, year and company. The ultimate purpose is to provide an estimate of the amount of variability which cannot be explained by the listed factors. The resulting amount of variability gives an indication of the amount required for a provision for adverse deviation (PAD).

## 1. Introduction

The adequacy of loss reserves has received significant attention during the 1980's in both Canada and the United States from both the point of view of solvency as well as from the point of view of taxation. In Canada during the last few years, there has been significant discussion relating to the amount required for a provision for adverse deviation for property and casualty insurance loss reserves.

It has been argued that the loss reserve established by an insurance company should reflect the expected liability with explicit rather than implicit margins for adverse deviations. The practice of not discounting loss reserves is a way of providing an implicit margin for adverse deviation.

One approach to estimating the variability is to analyze the inherent variability associated with commonly used methodologies. These techniques estimate the variability by developing a statistical model and measuring the amount of "noise". This has been done recently by

Verrall (1989) and is used in the comprehensive model of Zehnwirth (1985). Using this approach, the loss reserver can estimate the "unexplained variation" when a statistical model is applied to a particular set of run-off data.

In this paper, we choose to take a more pragmatic approach. We recognize that there are two significant sources of variation. The first is associated with the inherent variability of the claims process. For example, liability lines tend to have higher variability than property lines. The second major source of variation relates to the process of estimating the mean liability. Different actuaries will come up with different estimates of losses because they use different methods or may have different views of the future. An example in times of high interest rates is the variability in estimates of future inflation and the resulting influence on future claim payments. The degree to which this future inflation is not offset by discounting may significantly affect the magnitude of loss reserves.

In this paper, we will estimate the combined effect of these two sources of variation by analyzing the past performance of loss reservers.

## 2. The Data

Data used in this paper come from the annual statements of 174 property and casualty insurers in Canada. The study is restricted to federally chartered companies which were still in business at the end of 1988. We compared the estimated loss reserve for all prior accident years with the corresponding estimate for the same set of accident years after five years of run off. We compared the estimates made in the years 1975 through 1983 with the "results" five years later; that is, as reported in 1980 through 1988 respectively. As an example, we compared the estimated outstanding losses for 1976 and prior years as estimated at the end of 1976 with the corresponding results (paid plus outstanding) for 1976 and prior years as reported in the 1981 annual statement. Although these "results" still contain estimates, they are after at least 5 years of development and should be significantly more accurate than the original estimates. For the purpose of this study these values are treated as exact.

If the estimated outstanding losses in a year differed by more than a factor of 10 from the estimated "exact" losses five years later, the estimates are ignored in the analysis. This can occur, for example, when no reserve is established or when a reserve is established but no future claim costs emerge. This occurs when a company is very small, either just beginning or perhaps winding down their book of business. As a result of this criterion, 223 values of the data set of 1566 values were ignored in the subsequent analysis.

## 3. Previous Studies

In a recent paper, Aiuppa and Trieschmann (1986) do an empirical analysis of IBNR reserves for the 20 largest and the 20 smallest property and casualty insurers in the U.S.A. They cite earlier papers of Forbes (1970), Anderson (1971), Balcarek (1975), Ansley (1978), Smith (1980) and Hafling (1981). These studies generally focus on biases in the loss reserving process either over time or by other factors such as the size of the company.

## 4. Focus of This Paper

In this paper, we study the variability of loss reserving results over a long period of time which includes a period of price controls as well as a period of high interest and inflation rates. In analyzing the variability of loss reserving results, we recognize that it is appropriate to first factor out such temporal biases. Similarly one might expect that smaller companies would tend to over-reserve more than large companies recognizing a higher degree of relative variability. We also consider possible differences between direct insurers and reinsurers as well as between domestic and foreign insurers.

## 5. The Analysis

Let $E_i$ denote the estimate made in year $i$ of outstanding losses in respect of all accident years $i$ and prior. Let $U_i$ denote the estimate made in year $i+5$ of outstanding losses at the end of year $i$ for accident years $i$ and prior. In the analysis in this paper, $U_i$ is treated as the "true" level of outstanding losses at the end of accident year $i$ and $E_i$ is an estimate of this true value. Of course $U_i$ is itself an estimate; but, at year $i+5$, all accident year values are at least 5 years mature. For lines of business that are not too long-tailed, the estimate at year $i+5$ will be reasonably accurate, at least significantly more accurate than the estimate $E_i$ made at year $i$.

The excess/deficiency of the estimate $E_i$ is defined as $(E_i - U_i)/U_i$. It is measured as a fraction of the "true" value. For positive values of $E_i$ and $U_i$, the excess/deficiency only takes on values greater than -1. For the purpose of the statistical analysis described below we transform the excess/deficiency to obtain values taking on all possible values on the real line.

Let $X_i = 100\log(E_i/U_i)$. Then $E_i = U_i e^{X_i/100}$, resulting in a simple multiplicative model for the estimate $E_i$. We now introduce explanatory variables and carry out a statistical analysis of the values of $X_i$ for all 9 years and for all 174 companies in the data. The explanatory (categorical) variables in the analysis are:

| | | |
|---|---|---|
| Year: $y_i$, | $i =$ | 1975,1976,...,1983 |
| Size: $s_j$, | $j =$ | Small, Medium, Large, Reinsurer |
| Type: $t_k$; | $k =$ | Domestic, Foreign |
| Company: $c_l$; | $l =$ | company identifiers. |

Using a standard analysis of variance (ANOVA) procedure we examined the model

$$X_{ijkl} = \mu + y_i + s_j + t_k + \alpha_{ij} + \alpha_{ik} + \alpha_{jk} + \epsilon_{ijkl} \tag{1}$$

where $\mu$ is the overall mean level of $X_{ijkl}$, $y_i$ is the effect of year $i$, $s_j$ is the effect of size

$j$, $t_k$ is the effect of type $k$. The quantities $\alpha_{ij}$, $\alpha_{ik}$ and $\alpha_{jk}$ represent interaction terms of year and size, year and type, and size and type. Finally $\epsilon_{ijkl}$ represents the residual "error" and has mean 0 and variance $\sigma^2$. It represents that part of $X_{ijkl}$ that cannot be explained by the above mentioned factors including two-way interactions between these factors.

The results of the analysis of variance are shown in Table 1. It shows that, at a 5% significance level, each of year, type, and size are statistically significant explanatory variables but that each two-factor interaction is not significant. Hence, it is appropriate to eliminate them from the model. Non-significance of interaction terms means that the factors year, type and size act independently. Using the reduced model

$$X_{ijkl} = \mu + y_i + s_j + t_k + \epsilon_{ijkl} \tag{2}$$

results in the analysis of variance table given in Table 2. From Table 2 it can be seen that the $R^2$ = 14.26%, meaning that only 14.26% of the total variance can be explained by the effects of year, type, and size.

TABLE 1

Analysis of Variance for Excess/Deficiency

| Source of variation | Sum of Squares | % | d.f. | Mean square | F-ratio | Sig. level |
|---|---|---|---|---|---|---|
| MAIN EFFECTS | 213,575.63 | 14.3% | 12 | 17,797.970 | 18.413 | .0000 |
| YEAR | 152,700.32 | | 8 | 19,087.540 | 19.747 | .0000 |
| SIZE | 47,024.15 | | 3 | 15,674.716 | 16.216 | .0000 |
| TYPE | 3,815.23 | | 1 | 3,815.227 | 3.947 | .0472 |
| 2-FACTOR INTERACTIONS | 32,721.78 | 2.2% | 35 | 934.9081 | .967 | .5239 |
| YEAR SIZE | 22,663.21 | | 24 | 944.3005 | .977 | .4944 |
| YEAR TYPE | 5,993.16 | | 8 | 749.1449 | .775 | .6249 |
| SIZE TYPE | 5,082.18 | | 3 | 1,694.0587 | 1.753 | .1544 |
| RESIDUAL | 1,251,749.20 | 83.6% | 1295 | 966.602 | | |
| TOTAL (CORR.) | 1,498,046.70 | 100% | 1342 | | | |

TABLE 2

Analysis of Variance for Excess/Deficiency

| Source of variation | Sum of Squares | % | d.f. | Mean square | F-ratio | Sig. level |
|---|---|---|---|---|---|---|
| MAIN EFFECTS | 213,575.63 | 14.3% | 12 | 17,797.970 | 18.429 | .0000 |
| YEAR | 152,700.32 | | 8 | 19,087.540 | 19.764 | .0000 |
| SIZE | 47,024.15 | | 3 | 15,674.716 | 16.230 | .0000 |
| TYPE | 3,815.23 | | 1 | 3,815.227 | 3.950 | .0471 |
| RESIDUAL | 1,284,471.00 | 85.7% | 1330 | 965.76 | | |
| TOTAL (CORR.) | 1,498,046.70 | 100% | 1342 | | | |

TABLE 3

Summary Statistics of Residuals of ANOVA in Table 2

| | |
|---|---|
| Sample size | 1343 |
| Average | 0.00 |
| Median | -0.01 |
| Standard deviation | 30.94 |
| Minimum | -209.94 |
| Maximum | 172.12 |
| Lower quartile | -14.73 |
| Upper quartile | 16.18 |
| Skewness | -0.27 |
| Kurtosis | 6.10 |

| Percentages | Percentiles |
|---|---|
| 50 | -0.01 |
| 55 | 3.16 |
| 60 | 6.35 |
| 65 | 8.85 |
| 70 | 12.29 |
| 75 | 16.18 |
| 80 | 20.41 |
| 85 | 24.59 |
| 90 | 31.26 |
| 95 | 46.11 |

FIGURE 1

HISTOGRAM OF RESIDUALS

ALL INSURERS

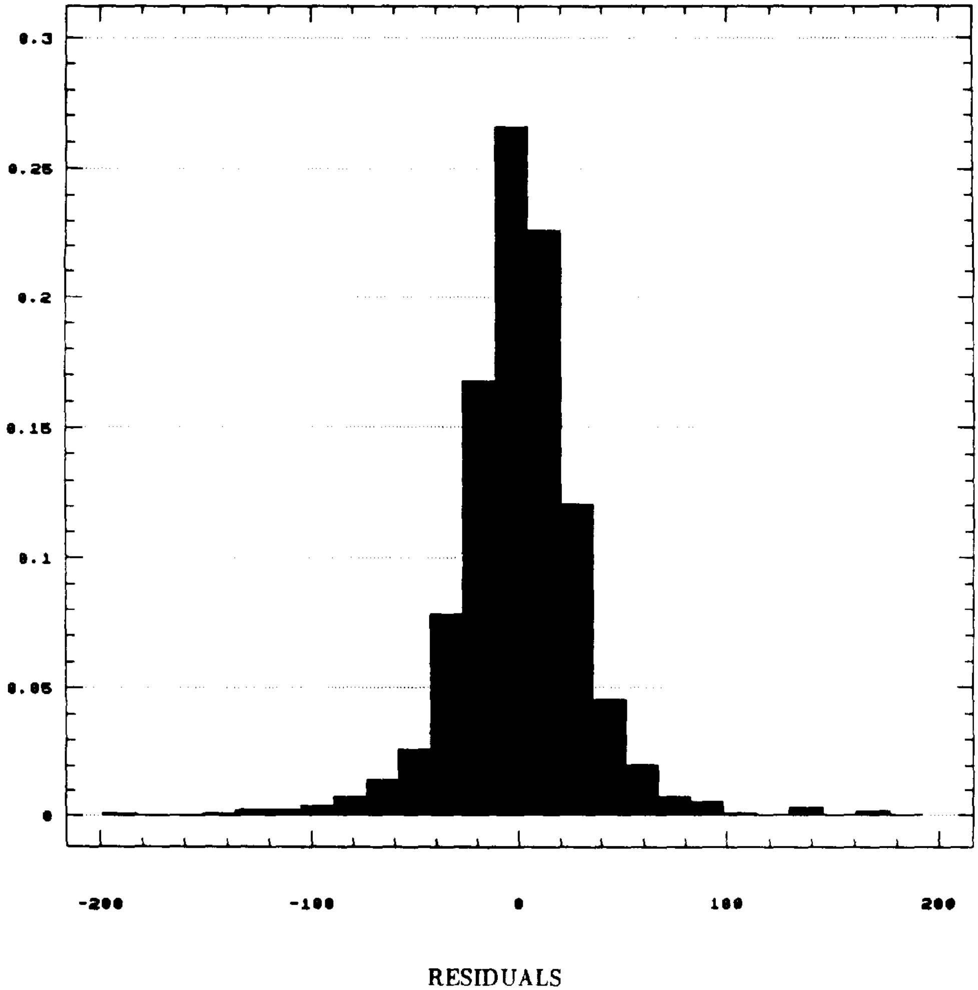


FIGURE 2

HANGING HISTOGRAM FROM A NORMAL DISTRIBUTION

ALL INSURERS

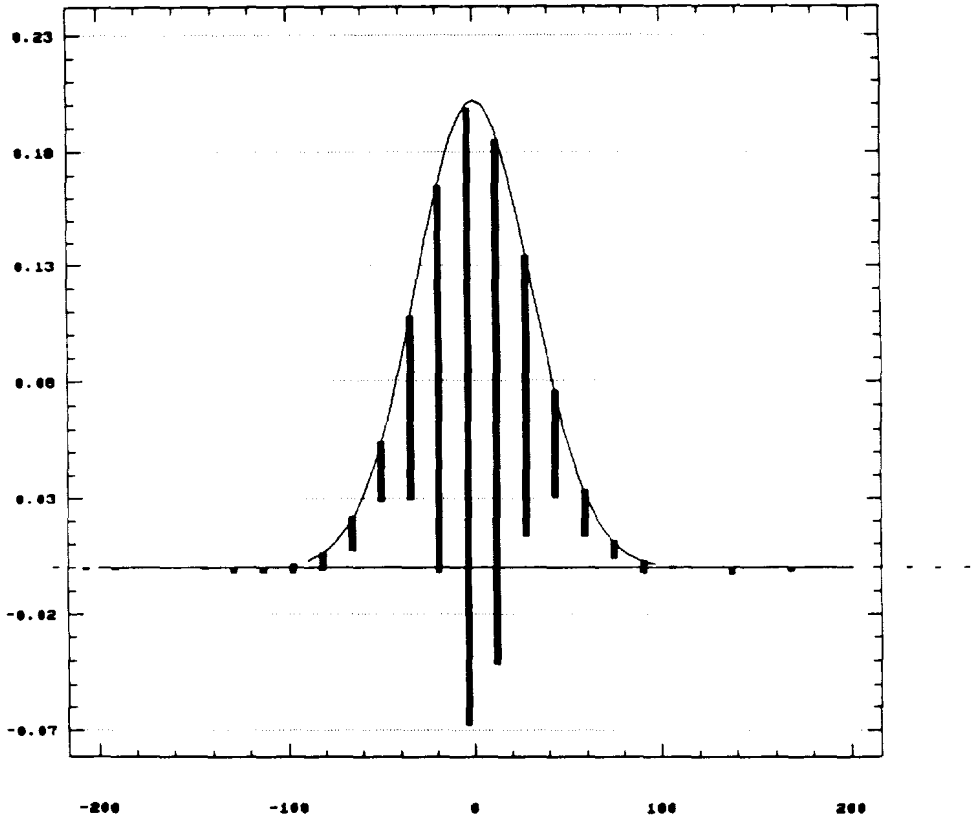


FIGURE 3

95% CI FOR MEANS BY YEAR

ALL INSURERS

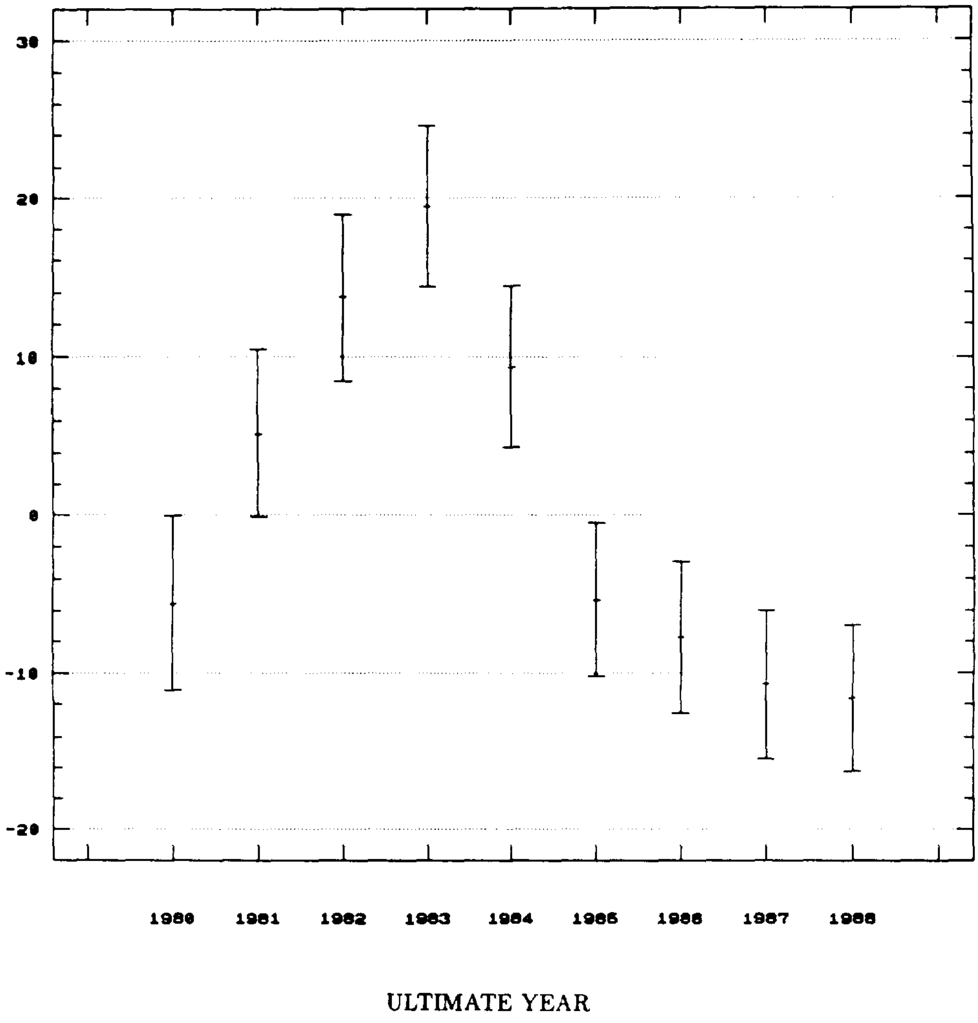
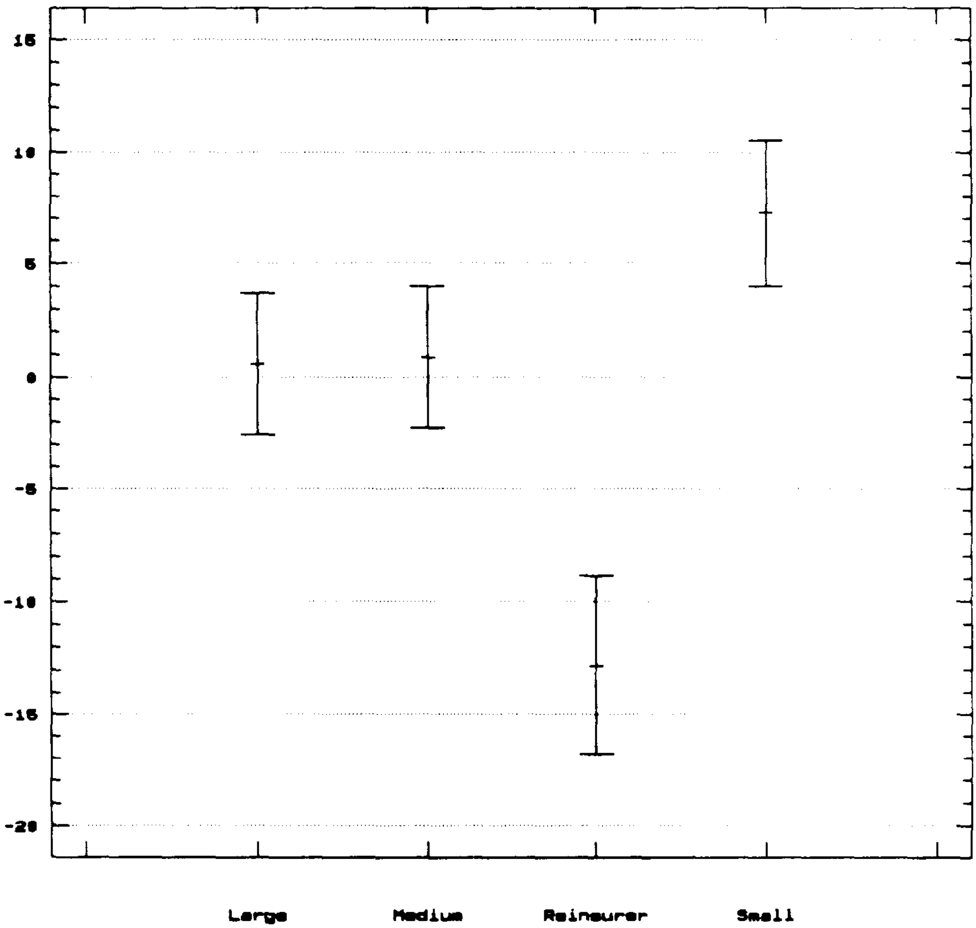


FIGURE 4

95% CI FOR MEANS BY SIZE

ALL INSURERS

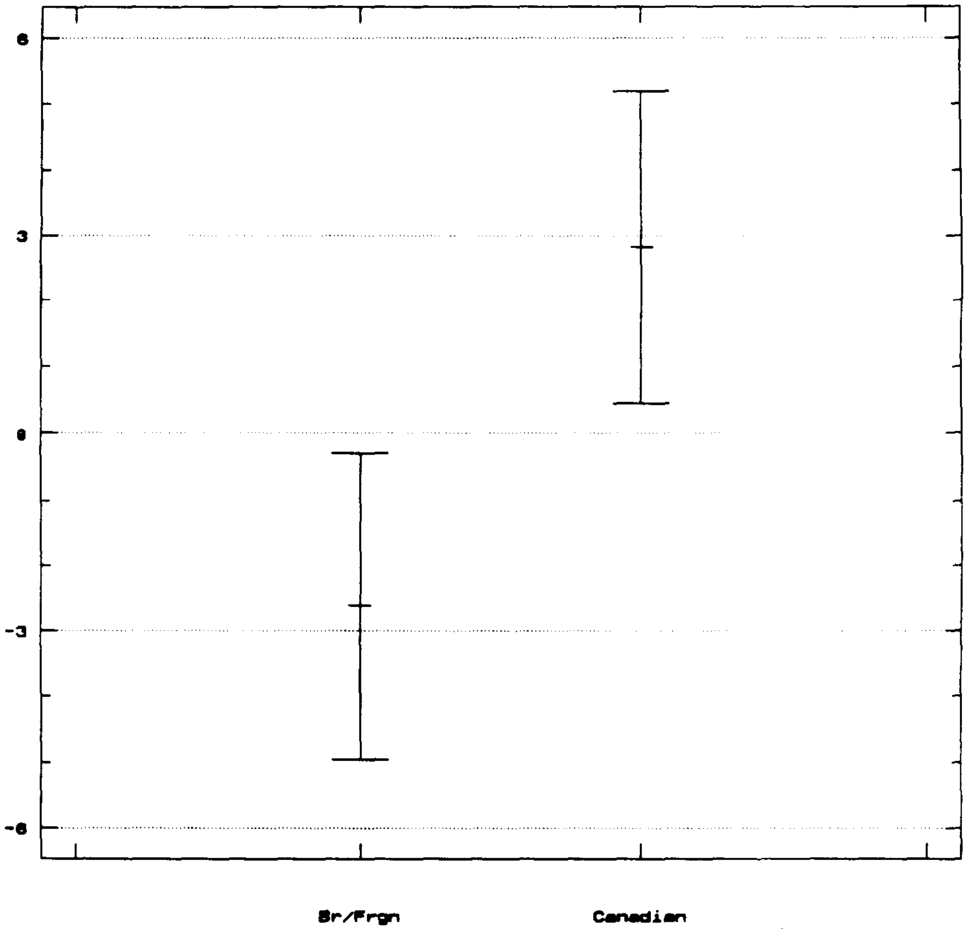



COMPANY SIZE

FIGURE 5

95% CI FOR MEANS BY TYPE

ALL INSURERS




TYPE

Summary statistics and percentiles for the residuals $\epsilon_{ijkl}$ of the analysis of variance given in Table 2 are given in Table 3. From the percentiles, it can be seen that the distribution is more tightly packed around zero than a normal distribution with the same standard deviation since the 75th, 80th, 85th, 90th and 95th percentiles of the normal distribution are 20.9, 26.0, 32.1, 39.7 and 50.9 each of which is larger than the corresponding percentile of the empirical distribution of the residuals. This phenomenon can also be illustrated from the hanging histogram in Figure 2 which also shows that the distribution of residuals is reasonably symmetric (as given in Figure 1). The symmetry of the residuals is desirable for the least-squares method of the analysis of variance to be valid. Figures 3, 4 and 5 show 95% confidence intervals for the level of excess/deficiency of reserve by year, size and type. Figure 3 shows general under-reserving in 1975 with rapid reserve strengthening through 1978 (during a period of price controls) and weakening of reserves in subsequent years. Figure 4 shows that reinsurers were under-reserved relative to direct insurers while Figure 5 shows that foreign insurers were slightly under-reserved while Canadian insurers were slightly over-reserved.

The results in Table 2 show that most (85.7%) of the variation remains unexplained. In order to explain more of the variation, we include the "company" variable. The resulting model is

$$X_{ijkl} = \mu + y_i + s_j + t_k + c_l + \epsilon_{ijkl} \tag{3}$$

The results of the analysis of variance are given in Table 4. It is seen that 66.3% of the total variation can be explained by adding in the "company" variable with 52.7% coming from the "company" variable alone. This means that about half of the total variation is a result of companies consistently under-reserving or consistently over-reserving.

Table 5 gives the summary statistics and percentiles of the distribution of the residuals. Figure 6 gives the histogram while Figure 7 gives the "hanging histogram" again indicating a distribution that is reasonably symmetric but more peaked than a Normal distribution with the same mean and variance.

TABLE 4

Analysis of Variance for Excess/Deficiency

| Source of variation | Sum of Squares | | d.f. | Mean Square | F-ratio | Sig. level |
|---|---|---|---|---|---|---|
| MAIN EFFECTS | 993,162.67 | 66.3% | 185 | 5,258.10 | 12.05 | .000 |
| YEAR | 152,700.32 | | 8 | 19,087.54 | 43.74 | .000 |
| SIZE | 47,024.15 | | 3 | 15,674.72 | 35.92 | .000 |
| TYPE | 3,815.23 | | 1 | 3,815.23 | 8.74 | .001 |
| COMPANY | 789,623.05 | | 173 | 4,564.30 | 10.46 | .000 |
| RESIDUAL | 504,884.02 | 33.7% | 1157 | 436.37 | | |
| TOTAL | 1,498,046.70 | 100% | 1342 | | | |

TABLE 5

Summary Statistics of Residuals of ANOVA in Table 4

| | |
|---|---|
| Sample size | 1343 |
| Average | 0.00 |
| Median | 0.08 |
| Standard deviation | 21.22 |
| Minimum | -124.69 |
| Maximum | 161.65 |
| Lower quartile | -10.47 |
| Upper quartile | 10.58 |
| Skewness | 0.24 |
| Kurtosis | 8.28 |

| Percentages | Percentiles |
|---|---|
| 50 | 0.08 |
| 55 | 2.01 |
| 60 | 3.95 |
| 65 | 6.04 |
| 70 | 8.19 |
| 75 | 10.58 |
| 80 | 12.93 |
| 85 | 15.98 |
| 90 | 20.57 |
| 95 | 29.31 |

## 6. Discussion

A provision for adverse durations (PAD) is intended to provide for the inherent instability in both the claim process as well as the reserving process. We have shown that there are some significant effects that have influenced the direction of the excess/deficiency of the loss reserves in the past. We have extracted these effects and are left with residual variation of about 35% of the original variation. This residual variation represents the degree of inherent instability of reserves of an individual company under the assumption that reserves are unbiased estimates of outstanding claims. The analysis showed that about half of the total variation was due to the tendancy of individual companies to consistency under-reserves or consistency over-reserves. We feel that it is the responsibility of the actuary to estimate the reserve in an unbiased way and that a provision for adverse deviation should be for the deviation in results for each company; independent of the performance of loss reserve estimates of other companies. Hence, we removed the company "bias".

The distribution of residuals given in Table 5 and Figures 6 and 7 gives a description of the inherent variability of individual company's results, given that the company's results are unbiased (i.e., corrected for the biases above). Figures 6 and 7 show that the residuals are symmetrically distributed and that they are more tightly distributed than the normal distribution. As a result, it would be inappropriate to base the PAD on the standard deviation. A percentile approach gives a direct interpretation of the degree of security associated with a particular PAD.

The results of Table 5 suggest that companies should hold a provision for adverse deviation of about 10% (13%) to ensure that reserves will be adequate 75% (80%) of the time.

Clearly we have not addressed the issue of variability by line of business. This would require a more detailed breakdown of data than we had available to us. Furthermore, if this was done it would be necessary to combine each company's lines of business since PAD's are not additive.

We hope that the results of this study may be useful in the discussion concerning the general size of a provision for adverse deviations.

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FIGURE 6
HISTOGRAM OF RESIDUALS
ALL INSURERS

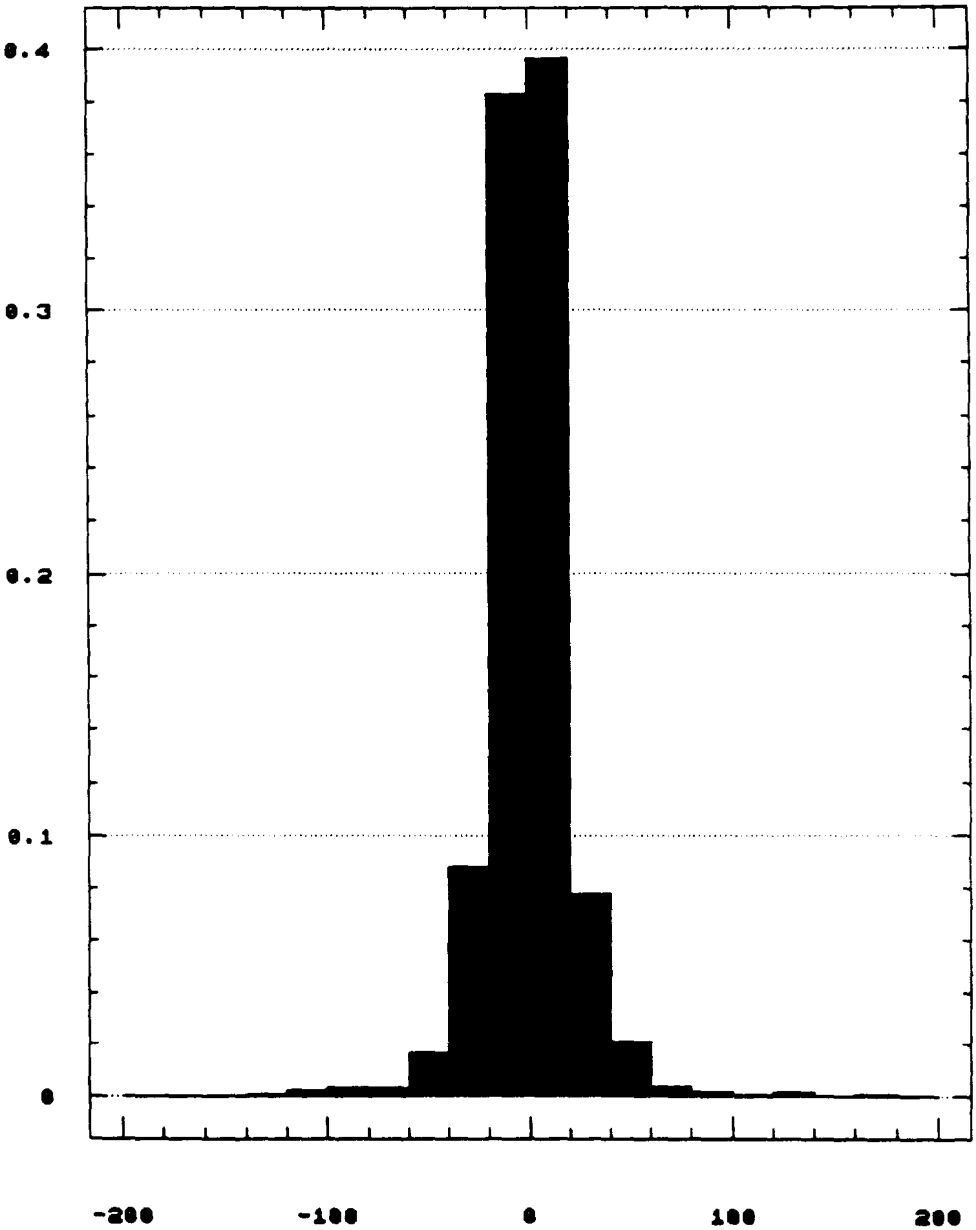
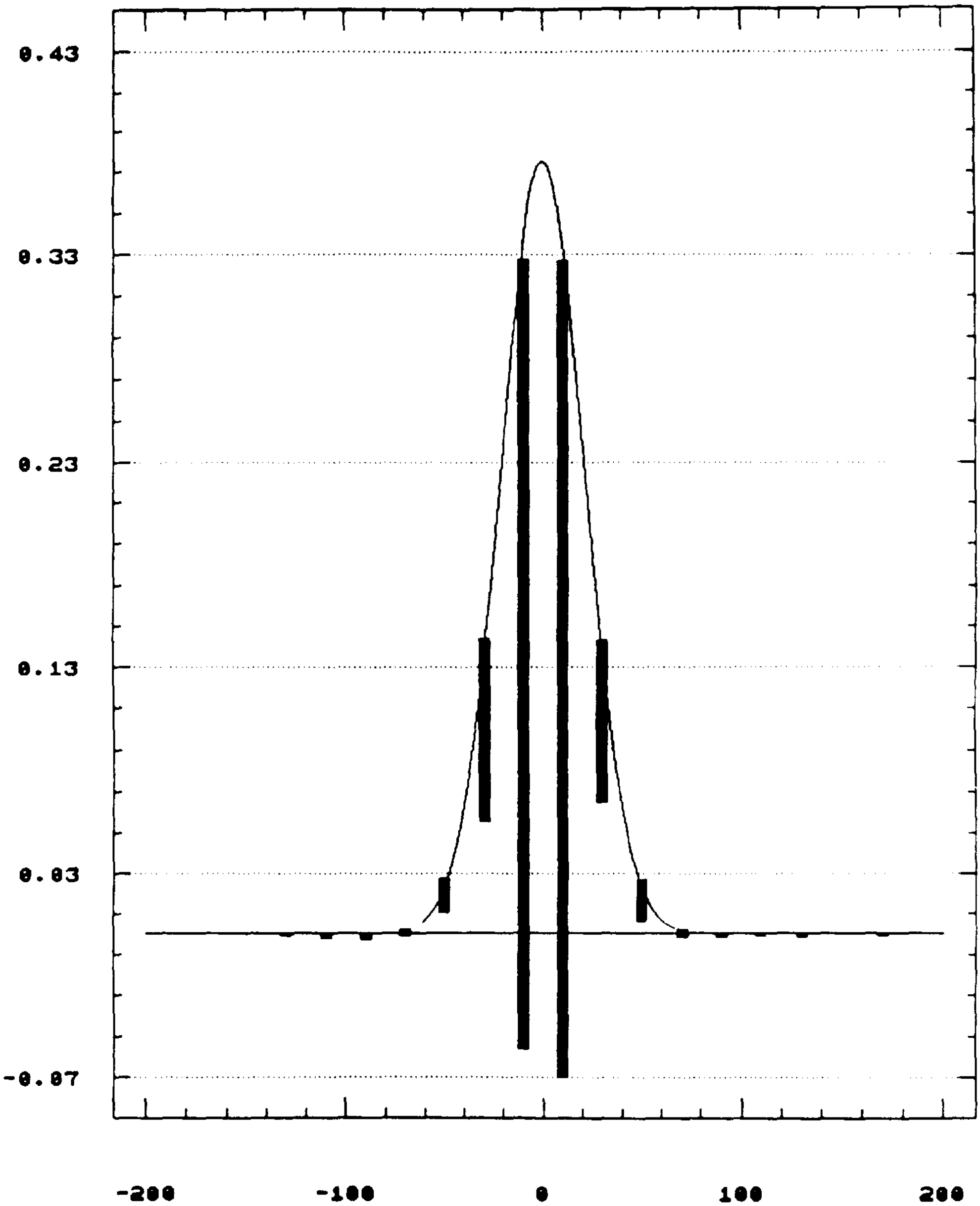


FIGURE 7
HANGING HISTOGRAM FROM A NORMAL DISTRIBUTION
ALL INSURERS




RESIDUALS