The Effects of Advanced Age Mortality Improvement on the Valuation of Variable Annuities with Guaranteed Death Benefits

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Presented at the Living to 100 and Beyond Symposium
Sponsored by the Society of Actuaries

Orlando, Fla.

January 12-14, 2005

Abstract

This paper studies the impact of improved advanced age mortality on the pricing and valuation issues of variable annuity (VA) contracts with guaranteed minimum death benefits (GMDBs). The contingent claim analysis is applied to evaluate the GMDBs, and the impact of the mortality improvement is then examined. The implications of the advanced age mortality improvement on GMDBs for various mortality models are discussed. A new hybrid mortality model is presented for the valuation of GMDBs.

1. Introduction

Modern technology and living have resulted in increasing numbers of people living into their 90s and beyond. Prospects of longer life have led to concern over their implications for social, financial, health-care and retirement systems. In the last decade, variable annuities with guaranteed benefits have been one of the largest developments in the life insurance industry. The variable annuity contracts provide minimum performance guarantee benefits, such as guaranteed minimum death benefits and guaranteed living benefits (VAGLBs). VAGLBs include guaranteed minimum accumulation benefits (GMABs), guaranteed minimum income benefits (GMIBs) and guaranteed payout annuity floor (GPAF). Variable annuity products (including GMDBs) have embedded options that affect their risk and returns. For example, popular GMDB provisions include "reset" and "ratchet" benefits. Death benefits on contracts with a "reset" provision will be reset to the fund value at various times during the life of the contract and can move up or down, but usually not below return of premium. Death benefits on "ratchet" contracts ratchet to the value of the fund at various times during the life of the contract, but only if the resulting benefit is higher than the one in force before the ratchet. Otherwise, the death benefit remains where it was before the ratchet date. A good overview of the state of the market is contained in Milevsky and Posner (2001).

The GMDB provision can be considered as an individual discrete lookback put, whose analytic solution was derived by Collin-Dufresne, Keirstad and Ross (1997). The analytic solutions are theoretically valuable but involve the cumulative multivariate normal distribution, which is difficult and time-consuming to evaluate in practice. Tiong (2000) uses the method of Esscher transforms pioneered by Gerber and Shiu (1994) to obtain analytic solutions for options embeded in equity-indexed annuities. Milevsky and Posner (2001) discussed at-the-money continuous lookback GMDB options on a variable fund only, when mortality follows some simple analytic forms.

Current studies on GMDB valuations have been focused on the market risk of the embedded options. Mortality calculation is, however, one of the key factors that affect the account value of the VA product. The assumed age distribution and the corresponding levels of mortality can be misestimated. This is very risky if the same asset-based charge is used at all ages. For example, even the slope of the mortality can have an adverse impact on the cost of the GMDB. Consider next the effect of the time to death. Since the put option embedded in a GMDB can only be exercised at time of death, a GMDB becomes more valuable as the time to death increases. Although it is uncertain when death will occur, the owner of a long-life GMDB has all the step-up opportunities open to the owner of a short-life GMDB. In 2002, when the financial markets were down, most VA account values were lower than the guaranteed level. Fortunately, the time to death for most VA accounts is long, due to the age factor—most policyholders purchased their VA contracts in their 50s during the 1990s, and therefore still have a low mortality risk.

This paper studies the impact of improved advanced age mortality on the pricing and valuation issues of GMDBs, where the value of the contract at death is the maximum value of the contract at any policy anniversary or the account value at death, if larger. It can also be extended to fit the general case, such as VAGLBs. The contingent claim analysis is applied to evaluate the GMDBs and the impact of the mortality improvement is then examined. Since the death benefit rollup or ratchet is guaranteed for life, both the expiration date and the strike of the embedded option depend on the mortality movement. The value of this GMDB depends on underlying movements such as the mortality distribution, the lapse distribution and the underlying market movement (such as Standard & Poor's 500 return). Moreover, the GMDBs depend on the interactions of all three underlying processes. The implications of the advanced age mortality improvement on the value of the GMDB under various mortality assumptions are discussed.

In Section 2, the basic features and valuation formulae are presented. Section 3 discusses the mortality factors in the valuation of GMDBs, especially the advanced age mortality. A case study and numerical examples are presented in Section 4. Section 5 summarizes the paper.

2. GMDB

VA policies are deferred life annuities with some guaranteed features for investment returns or benefit income. In addition to the tax benefits they share with traditional life annuities, VA products all have embedded options that affect their risk and returns.

Many VA products enhance their guaranteed return by periodically raising the minimum guarantee level ("step-up" or "ratchet"). For example, the GMDB level could be reset periodically in GMDB contracts. A GMDB would be based on a suitably defined highest anniversary account value. These anniversary step-up features vary widely from company to company; some policies offer an annual reset, while others require a 10-year wait, but the average is approximately five years. In the language of modern option pricing, the exercise price of the embedded put "floats" and increases to a new (higher) level every few years. The floating is based on the anniversary market value of the policy, $\max(S_{t_i}, S_T)$, where t_i is the i^{th} anniversary date. The allowable reset frequency is closely linked to the number of years the surrender charge is in effect.

VA products (including GMDBs) have embedded options that affect their risk and returns. Under a capital market model, a GMDB on a VA is essentially an increasing-strike put option on the underlying market movement with a stochastic maturity date. The embedded put option guarantees that the beneficiary of the VA can sell the investment contract back to the insurance company at a price better than the market price. Therefore, the usual factors that impact values of options, such as interest rates, underlying volatility, strike and market price and time to option maturity will affect the cost of the VA's guarantees. Other significant risk and cost factors include: mortality, persistency, active fund management, policyholder investment strategy, price risk and others.

For the mathematical models, consider a deferred variable annuity with a GMDB rider purchased at age *x*. The GMDB rider provides a death benefit that pays the greater of the guarantee, *G*, and the contract value at death. The value of the GMDB is then equal to the sum of the values of European puts at all durations multiplied by the probability that the individual has survived to that duration without lapsing his or her policy, and then dies at that exact instant. Mathematically, the value is:

$$V_{GMDB}(S,x)=\int_0^{\infty} {}_t p_x^{(\tau)} \mu_x^{(d)}(t) f(t,G \mid \sigma,r,q)dt \qquad (1)$$

where ${}_t p_x^{(\tau)}$ is based on the double-decrement model including both mortality and policy surrender, and $f(t,G \mid \sigma,r,q)$ is the value of a European put using Black-Scholes-Merton-type formula.

2.1 Rollup GMDB

The GMDB guarantees a return of premium with *g* percent compounded continuously for *n* years. Let *P* be the premium paid at begin of the contract. The guarantee at death, *T*, is $G=Pe^{gT}$.

Then Equation (1) is:

$$\begin{aligned} V_{RollUp}(S,x) &= \int_0^{n} {}_t p_x^{(\tau)} \mu_x^{(d)}(t) BSM(t,g \mid \sigma,r,q)dt \\ &= \int_0^{n} {}_t p_x^{(\tau)} \mu_x^{(d)}(t) \left[Pe^{gt}e^{-rt}N(-d_2)-Se^{-qt}N(-d_1)\right]dt \end{aligned} \qquad (2)$$

where

$$d_1(S,t)=\frac{\ln\left(\frac{S}{P}\right)+\left(r-q-g+\frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$d_2(S,t)=d_1(S,t)-\sigma\sqrt{t} \ .$$

2.2 Step-up GMDB

A step-up GMDB has the death benefit that pays the greater of the maximum value the contract attains on any policy anniversary and the contract value at death. For a step-up GMDB, the embedded option strike depends on the maximum fund value reached during the life of the annuity, and the GMDB is equivalent to a lookback put option with maturity date on the time of death. It is shown (see Hull, 2003, Chapter 19) that, $f(t, G \mid \sigma, r, q)$ of the integrand in Equation (1) is the value of a European lookback put, $BSMLB(t, S_{\max} \mid \sigma, r, q)$, given by:

$$V_{stepup}(S, x) = \int_0^n {}_t p_x^{(\tau)} \mu_x^{(d)}(t)\, BSMLB(t, S_{\max} \mid \sigma, r, q) dt \tag{3}$$

where

$$BSMLB(t, S_{\max} \mid \sigma, r, q) = S_{\max} e^{-rt} \left(N(b_3) - \frac{\sigma^2}{2(r-q)} e^Y N(-b_1) \right) + Se^{-qt} \frac{\sigma^2}{2(r-q)} N(-b_2) - Se^{-qt} N(b_2)$$

and

$$b_1 = \frac{\ln\left(\frac{S_{\max}}{S}\right) + \left(r - q - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$b_2 = b_1 - \sigma\sqrt{t}$$

$$b_3 = b_2 - \frac{2(r-q)}{\sigma}\sqrt{t}\,.$$

Most of the current studies on GMDB valuations have been focused on the market risk of the embedded options. Mortality calculation is, however, one of the key factors that affect the account value of the VA product. The assumed age distribution and the corresponding levels of mortality can be misestimated. This is very risky if the same asset-based charge is used at all ages. For example, even the slope of the mortality can have an adverse impact on the cost of the GMDB. The noticeable mortality improvement of the aged population might have significant impact on the valuation of GMDBs, as shown in the next section.

3. Effect of Advanced Age Mortality Improvement

Assume that death and surrender are independent, and the surrender follows a constant force of surrender, λ. Equation (1) is the same as

$$V_{GMDB}(S,x) = \int_0^n e^{-\lambda t} f(t, G \mid \sigma, r, q)\, {}_t p_x^{(d)} \mu_x^{(d)}(t) dt \,. \tag{4}$$

Since the purpose of this study is to study the mortality effect on the value of GMDB, we assume, without loss of generality, that surrender follows a constant force of decrement, λ . Equation (4) is then

$$V_{GMDB}(S,x) = \int_0^n e^{-\lambda t} f(t, G \mid \sigma, r, q)\; {}_t p_x^{(d)} \mu_x^{(d)}(t) dt \,. \tag{5}$$

Next, we discuss several specific mortality laws that are frequently used in actuarial practice. We then propose a new hybrid mortality model that incorporates the recent study for advanced age mortality improvement.

3.1 The Effect on GMDB under Constant Force of Mortality

Assume that the force of mortality is a constant, $\mu_x^{(d)}(t) = \mu$ for all (x) and t. The value of GMDB (Equation (5)) becomes

$$V_{GMDB}^{CF}(S,x) = \int_0^n e^{-\lambda t} f(t, G \mid \sigma, r, q)\,(\mu\, e^{-\mu t}) dt \,. \tag{6}$$

Although the mortality risk for x is independent of issue age x under the constant force of mortality, $V_{GMDB}^{CF}(S,x)$ is, however, still dependent on x because the endowment period n is age-dependent for advanced issue ages. For example, if 95 is the maximum age guaranteed, n might be specified in the VA contract as $n = \min(20, 95 - x)$.

Lemma 1. Under the constant force of mortality assumption, $\mu_x^{(d)}(t) = \mu$ for all (x) and t. Let n be the guarantee period. If

$$n \le \frac{1}{\mu} \;, \tag{7}$$

then $V_{GMDB}^{CF}(S,x)$ decreases in value with improved mortality as μ decreases.

Proof. Denote

$$h^{CF}(\mu, x) \equiv V_{GMDB}^{CF}(S,x) \,.$$

From Equation (6),

$$\frac{\partial h^{CF}(\mu, x)}{\partial \mu} = \int_0^n e^{-\lambda t} f(t, G \mid \sigma, r, q)(1-\mu t) e^{-\mu t} dt$$

and

$$\frac{\partial^2 h^{CF}(\mu, x)}{\partial^2 \mu} = \int_0^n e^{-\lambda t} f(t, G \mid \sigma, r, q)\, t\, (\mu t - 2) e^{-\mu t} dt .$$

One can see that if Condition (7) holds, then

$$\frac{\partial h(\mu, x)}{\partial \mu} > 0$$

and

$$\frac{\partial^2 h(\mu, x)}{\partial^2 \mu} < 0 \text{ if } n \le \frac{2}{\mu} .$$

For higher issue age *x*, it's more likely Condition (7) will be met.

Example. Based on the Annuity 2000 Mortality Table from age 7 to age 97 (most common maximum guarantee age), average value of μ is 0.022211 and the life expectation ($\frac{1}{\mu}$) of 45 years. Figure 1 shows the impact of the mortality improvement on a GMDB (*n*=20) under the constant force (CF) of mortality assumption. More details will be discussed in the case study presented in Section 4.

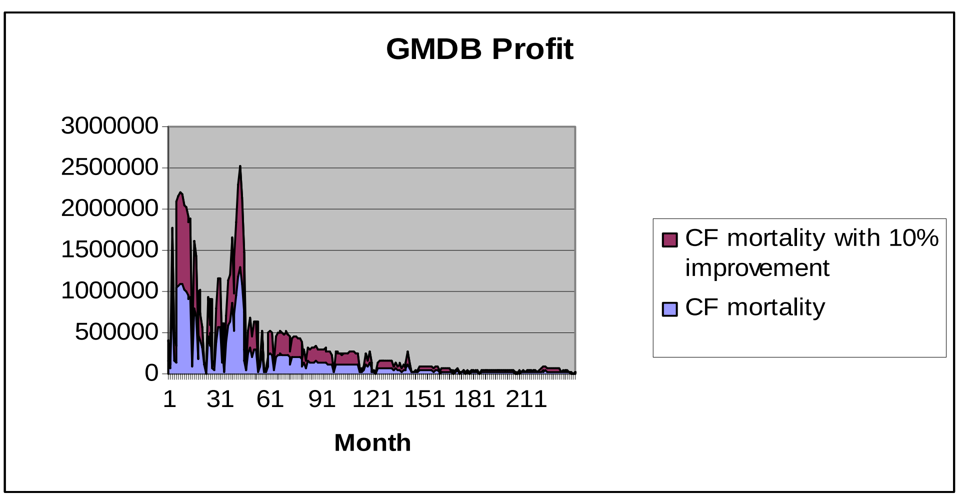


Figure 1. GMDB Profit under CF Mortality Assumption

3.2 The Effect on GMDB under De Moivre's Law

Under De Moivre's Law (DML), mortality assumption $\mu_x^{(d)}(t) = \dfrac{1}{\omega - x - t}$ where ω is the maximum survival age. Notice that $n \le \omega - x$.

The value of GMDB (Equation (5)) becomes

$$V_{GMDB}^{DM}(S,x) = \frac{1}{\omega - x}\int_0^n e^{-\lambda t} f(t, G \mid \sigma, r, q)\, dt \,. \tag{8}$$

Lemma 2. Assume mortality follows DML with the maximum survival age ω . If $n \le \omega - x$, then V_{GMDB}^{DM} is a decreasing function with respect to ω and an increasing function with respect to *x*.

Proof. Denote

$$h^{DM}(\omega, x) \equiv V_{GMDB}^{DM}(S, x) \,.$$

Note

$$\frac{\partial h^{DM}(\omega, x)}{\partial \omega} = -\frac{1}{(\omega - x)^2}\int_0^n e^{-\lambda t} f(t, G \mid \sigma, r, q) dt < 0$$

while

$$\frac{\partial h^{DM}(\omega, x)}{\partial x} = \frac{1}{(\omega - x)^2}\int_0^n e^{-\lambda t} f(t, G \mid \sigma, r, q) dt > 0$$

for all ω and *x*.

Therefore, V_{GMDB}^{DM} is a decreasing function with respect to ω and an increasing function with respect to *x*.

Consider a GMDB contract issued at age *x* with pricing assumption ω . As mortality experience improves for advanced ages, ω increases in value to $\hat{\omega}$ on the valuation date. The value of GMDB, V_{GMDB}^{DM} , decreases. On the other hand, as mortality improves for advanced ages, more seniors are likely to purchase the GMDB contract (*x* increases). In these cases, V_{GMDB}^{DM} will increase. Figure 2 shows the impact of the mortality improvement on a GMDB (*n*=20) under the DML of mortality assumption. More details will be discussed in the case study presented in Section 4.

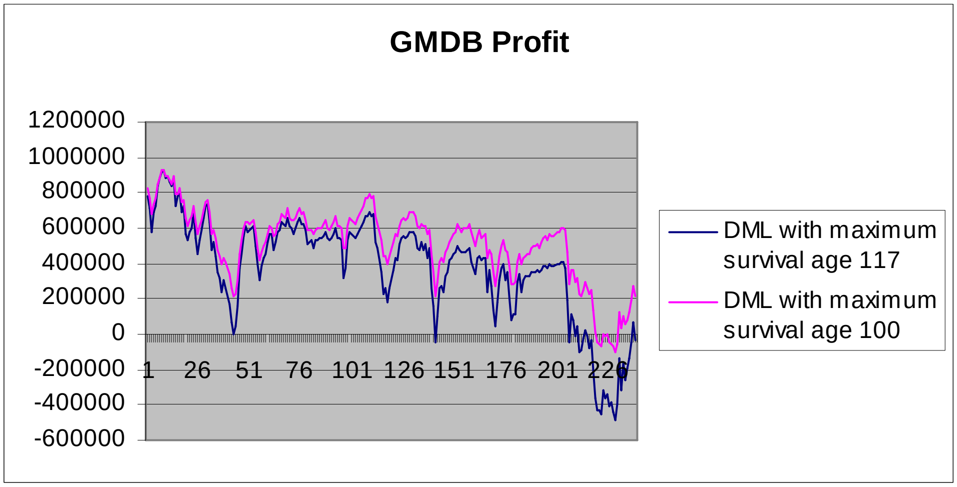


Figure 2. GMDB Profit under DML

3.3 The Effect on GMDB under Gompertz Law

In their study, Guo and Wang, 2001, show that Gompertz's Law (GL) is a more practical mortality assumption for advanced ages using Annuity 2000 data.

Under GL mortality assumption, $\mu_x^{(d)}(t) = BC^{x+t}$ where $B > 0, C > 1$.
The value of GMDB (Equation (5)) becomes

$$V_{GMDB}^{Gmpz}(S,x) = \int_0^n e^{-\lambda t} f(t, G \mid \sigma, r, q)(BC^x C^t e^{-mC^x(C^t - 1)})\, dt \qquad (9)$$

where

$$m = \frac{B}{\ln C}.$$

Lemma 3. Assume mortality follows Gompertz Law, $\mu_x^{(d)}(t) = BC^{x+t}$ where $B > 0, C > 1$. If

$$x > \frac{\ln(\ln C) - lnB}{\ln C} \qquad (10)$$

and

$$n \le \frac{\ln(B+\ln C)-\ln B}{\ln C} \tag{11}$$

hold, then V_{GMDB}^{Gmpz} is more valuable as mortality experience improves.

Proof. Denote

$$h^{Gmpz}(B,C;x) \equiv V_{GMDB}^{Gmpz}(S,x) .$$

Then

$$\frac{\partial h^{Gmpz}(B,C;x)}{\partial B} = \int_0^n e^{-\lambda t} f(t,G \mid \sigma,r,q) C^x C^t e^{-mC^x(C^t-1)}(1-mC^x(C^t-1))\, dt .$$

Note that

$$\frac{\partial h^{Gmpz}(B,C;x)}{\partial B} < 0$$

if Condition (10) holds and $h^{Gmpz}(B,C;x)$ is a decreasing function with respect to *B*.

Now consider the effect of *C* on $h^{Gmpz}(B,C;x)$:

$$\frac{\partial h^{Gmpz}(B,C;x)}{\partial C} = \frac{1}{C}\int_0^n e^{-\lambda t} f(t,G \mid \sigma,r,q) BC^{x+t} e^{-mC^x(C^t-1)}(x-mxC^x(C^t-1)-mtC^{x+t})\, dt$$

and

$$\frac{\partial h^{Gmpz}(B,C;x)}{\partial C} < 0$$

if Condition (10) holds. Therefore, V_{GMDB}^{Gmpz} is more valuable as mortality experience improves (as *B* and/or *C* decrease).

Finally,

$$\frac{\partial h^{Gmpz}(B,C;x)}{\partial x} = \ln C \int_0^n e^{-\lambda t} f(t,G \mid \sigma,r,q) BC^x C^t e^{-mC^x(C^t-1)}(1-m(C^t-1))\, dt .$$

For higher age *x*, the guarantee period *n* is likely to decrease and

$$\frac{\partial h^{Gmpz}(B,C;x)}{\partial x} > 0$$

If Condition (11) holds, it implies GMDB is more valuable for seniors if the endowment period (n) is limited by Condition (11).

In practice, C is usually between 1.05 and 1.20, and B is usually a number less than 1. Therefore, Condition (10) is always met for advanced ages, for example, $x > 60$. Figure 3 shows the effect of mortality improvement on GMDB profits.

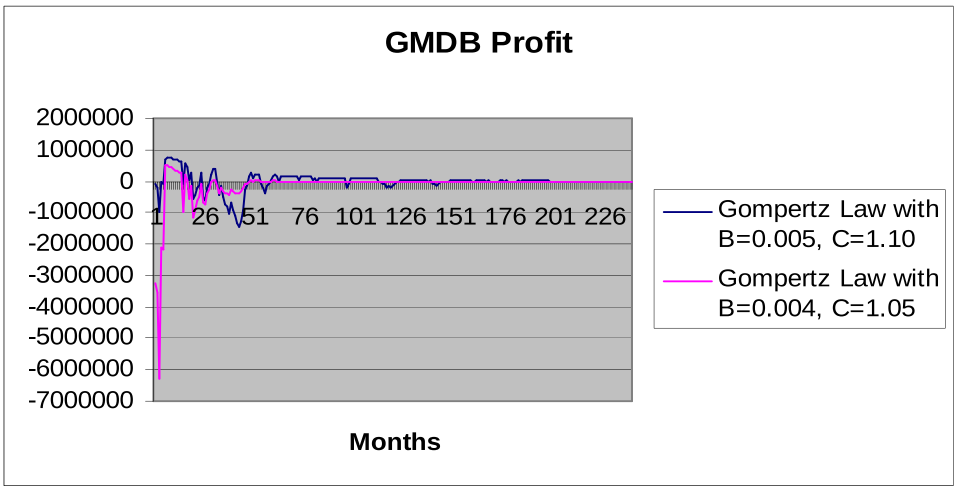
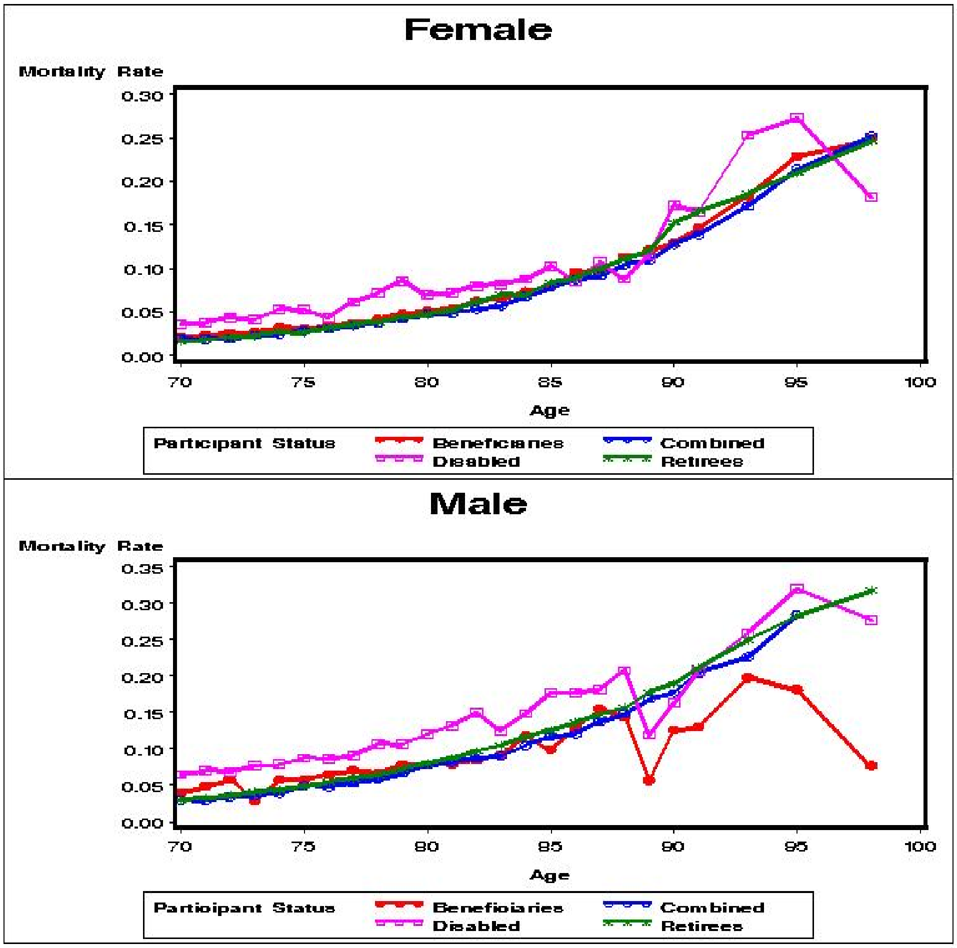


Figure 3. GMDB Profit under Gompertz's Law

3.4 New Mortality Model

In their study, Guo and Wang, 2002, show that "the mortality rate shows exponential growth from ages 50 to 69, linear growth between ages 70 and 85, and then exponential growth again for ages 86 and above." The following figure was shown in their paper.

Figure 15 Advanced Age Mortality Trend



Based on this observation, we proposed using a hybrid mortality model. Recall Equation (5),

$$V_{GMDB}(S,x)=\int_0^n e^{-\lambda t} f(t,G\,|\,\sigma,r,q)\ {}_t p_x^{(d)} \mu_x^{(d)}(t)dt\ .$$

Define

$$\mu_{new}^{(d)}(x)\equiv\begin{cases} B_Y C_Y^x & for \quad x<70 \\ cx & for \quad 70\le x\le 85 \\ B_O C_O^x & for \quad x>85 \end{cases}$$

Figure 4 shows an example of a hybrid mortality model.

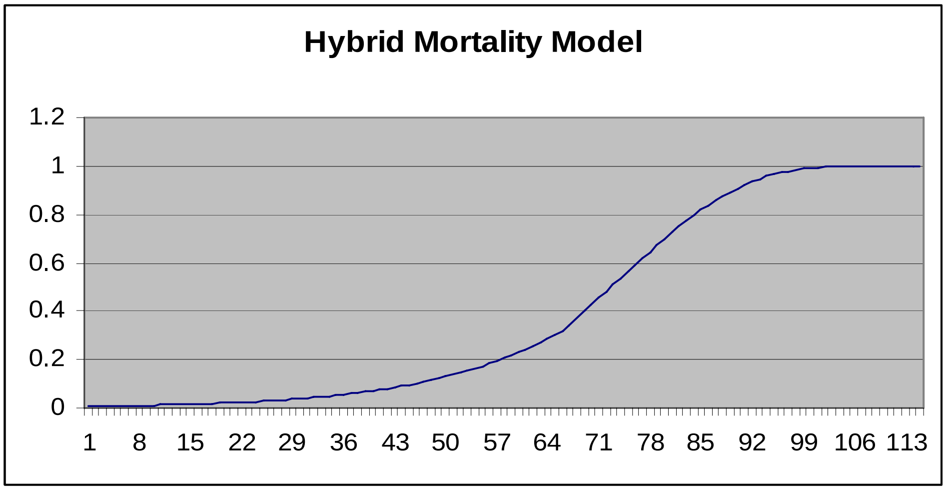


Figure 4. Hybrid Mortality Model

Then Equation (5) becomes

$$V_{GMDB}^{New}(S,x)=\int_0^n e^{-\lambda t} f(t,G\,|\,\sigma,r,q)\ {}_t p_x^{(d)}\mu_{new}^{(d)}(x+t)dt\,. \tag{12}$$

Figure 5 shows GMDB profit with the hybrid mortality model.

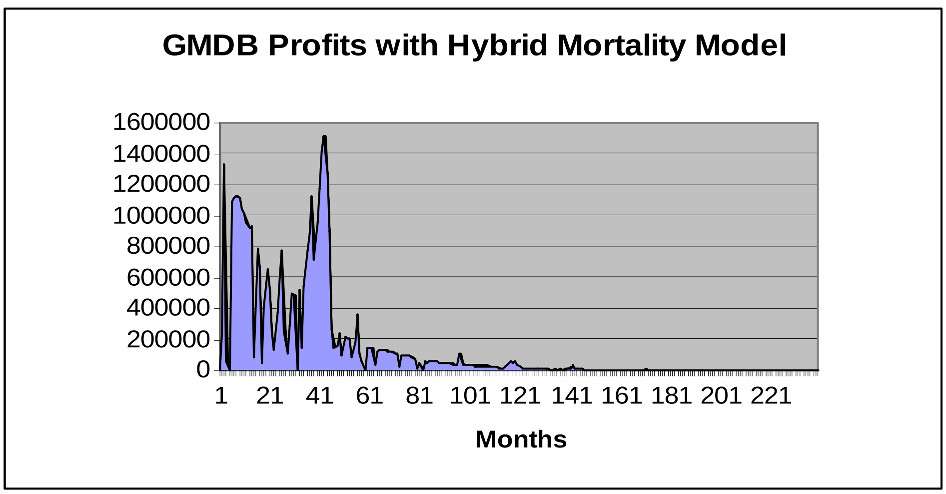


Figure 5. GMDB with Hybrid Mortality Model

The comparison of $V_{GMDB}(S,x)$ using the new mortality model and using the existing mortality law will be presented in the next section.

4. Case Study

Universal Century Financial Group (UCF) currently markets a VA named "UCF Better Life," a flexible-payment variable accumulation deferred annuity contract. This contract is available to individuals as well as to certain groups and individual retirement plans.

Policyholders can put their money into 30 investment choices: a fixed account and 29 subaccounts. Money put in the subaccounts is invested in a mutual fund portfolio. The investments in the portfolio are not guaranteed. Amounts in the fixed account earn interest annually at a fixed rate that is guaranteed by UCF to be at least 3 percent. UCF guarantees the interest, as well as principal, on money placed in the fixed account. Money can be transferred between any of the investment choices during both the accumulation period and the income phase, subject to certain limits on transfer from the fixed account.

For an additional charge, a compounding/monthly step-up death benefit may be selected. The guaranteed-minimum-income-benefit rider that guarantees a minimum amount of income payments if one annuitizes under one of the rider's payment options and the additional-earnings rider that may provide a supplemental death benefit may also be purchased.

Like all the deferred annuity contrasts, this contract has two phases: the accumulation period and the income phase. During the accumulation period, premiums are being paid and accumulated to cover the future benefits to be paid. The actuaries at UCF had made the following model assumptions.

Base Assumptions

- Single-premium deferred variable annuity
- No loads or surrender charges
- No expense charges
- No partial withdrawals or annuitizations
- 100 percent in equity funds
- Deaths at the end of the policy year using the SOA Annuity 2000 Mortality Table
- Lapses of 5 percent for the first year, increasing by 1 percent each year until year 10 and staying level thereafter
- Discount rate of 8 percent
- Base total return of 3.75 percent a year
- Annual lognormal volatility of 6 percent, net of all charges and fees (The ending policy value index is the beginning-of-year index times 1.0375, the result multiplied by a lognormal random variable centered on 1.00 with a standard deviation of 16 percent.)

For the mortality assumption, Annuity 2000 table is used. The male and female mortality based on Annuity 2000 experience is given in Figure 6.

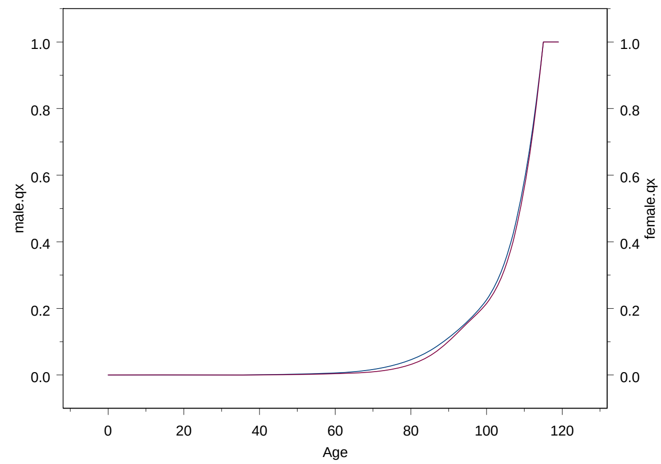


Figure 6. Annuity 2000 Mortality

The minimum death benefit is the greater of premiums paid or the accumulated policy value at the end of the policy year. Rollup and step-up are also modeled and shown in Figure 7.

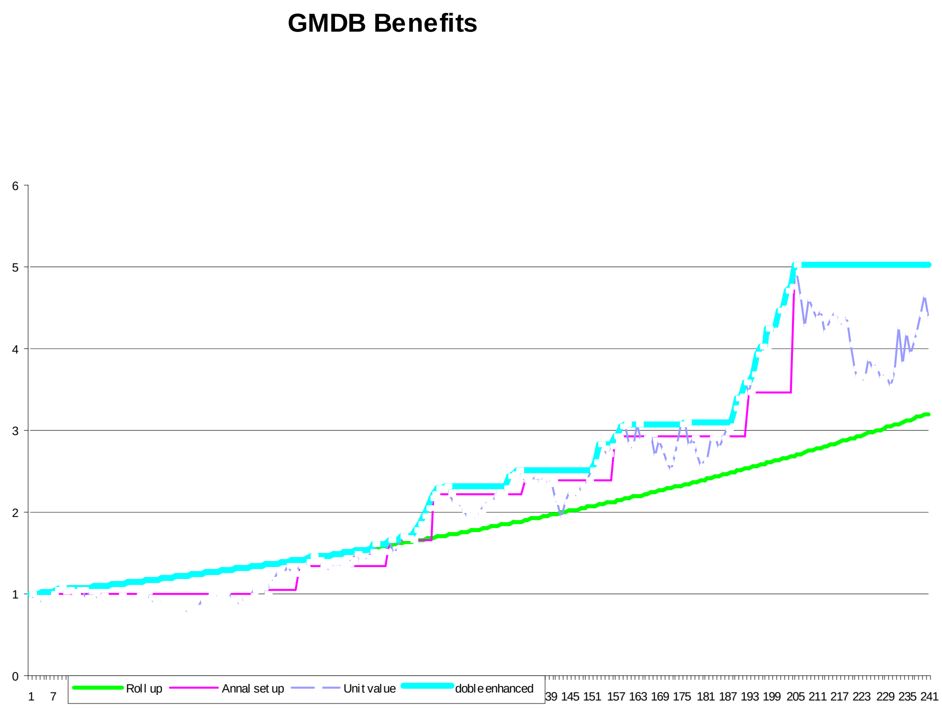


Figure 7. GMDB Death Benefits

The case study provided a numerical example of the mortality risk specifications that affect the account value of the variable annuity products with guaranteed features.

We start with the analysis of improved advanced age mortality on the valuation of GMDB with various features. Suppose the contract age is 70(x=70). Consider the 20-year endowment period. We study the impact of mortality improvement based on Annuity 2000 data on $V_{GMDB}(S,x)$.

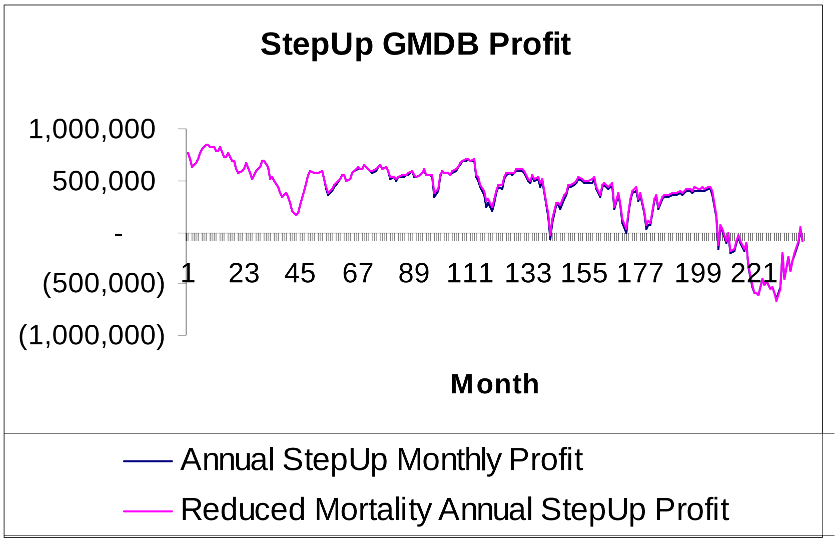


Figure 8. GMDB Profit with Step-Up Guarantees

Figure 8 shows the monthly profit for GMDB with step-up guarantees. With mortality improved by 0.005 for ages 70 to 89 in the Annuity 2000 table, the GMDB monthly profits increase. Figure 9 displays the GMDB profit trend of contribution for each value over time (months) with various mortality improvements.

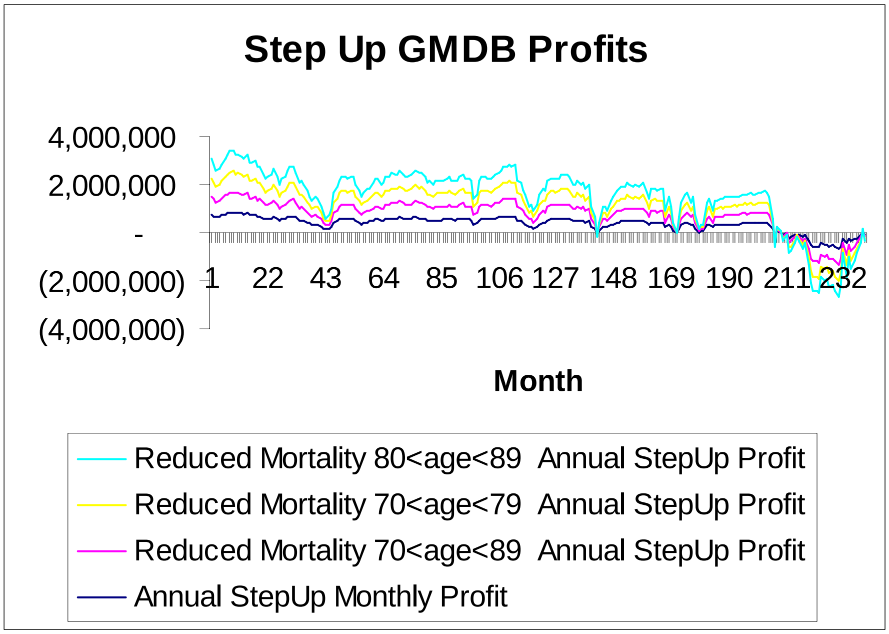


Figure 9. GMDB Profit Trends with Step-Up Guarantees

The study shows that mortality improvements in the seventies generate higher GMDB profits than mortality improvements in the eighties.

Figure 10 shows the monthly profit for GMDB with rollup guarantees. With mortality improved by 0.005 for ages 70 to 89 in the Annuity 2000 table, the GMDB monthly profits increase. Figure 11 displays the GMDB profit trend of contribution for each value over time (months) with various mortality improvements.

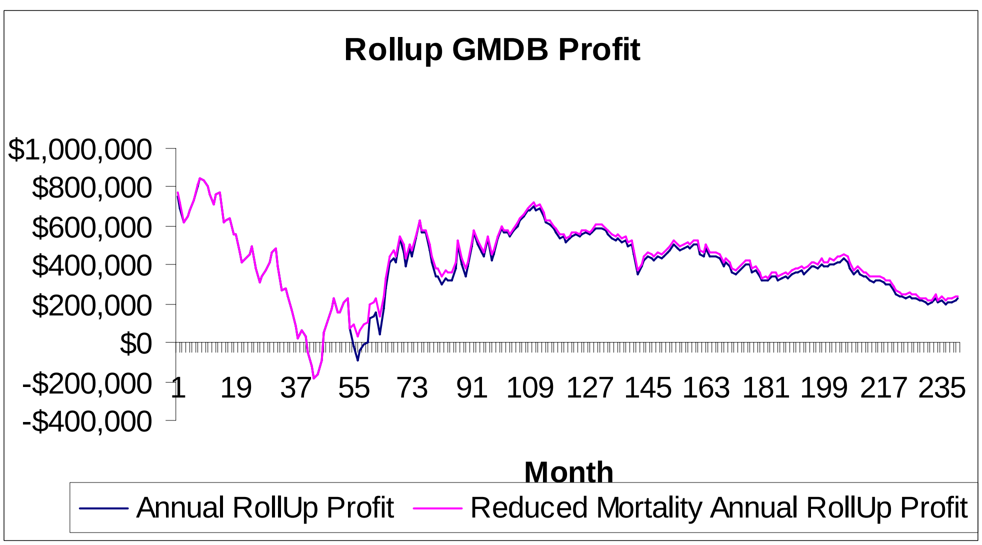


Figure 10. GMDB with Rollup Guarantees Profits

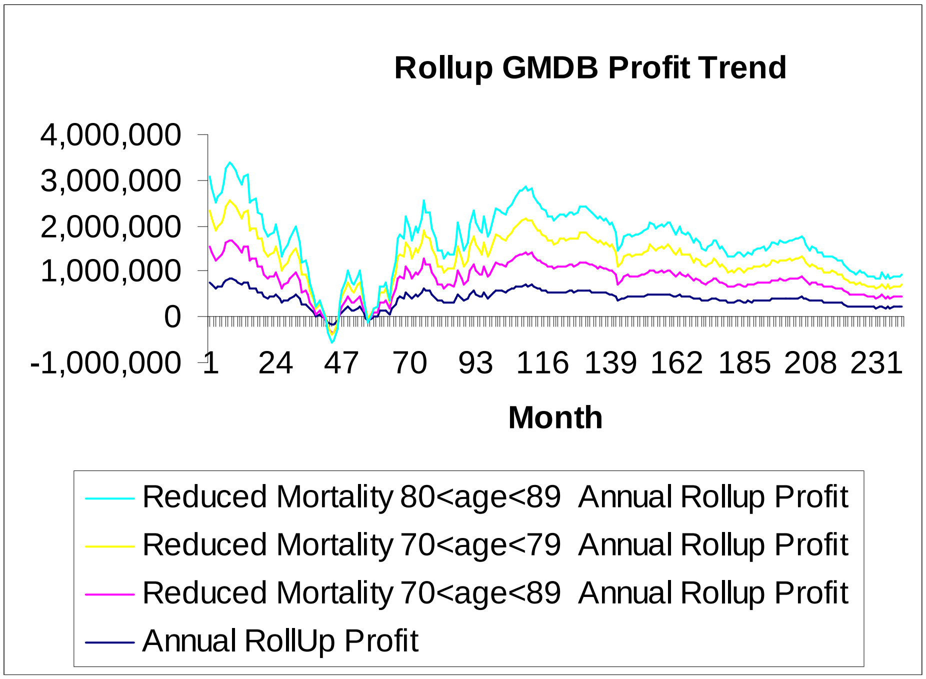


Figure 11. GMDB Profit Trends with Rollup Guarantees

The following table shows how the net present value (NPV) of the GMDB profits changes with respect to the mortality shocks. Columns 1 and 2 represent the change in mortality with all the other assumptions remaining the same. Columns 3 through 5 shows the NPV of the profit GMDB with various minimum guarantees. Column 6 shows the NPV for the GMDB with guaranteed death benefit that is the greater of the monthly step-up and rollup level.

Table 1. GMDB Profit with advanced age mortality improvements

69<x<80	79<x<90	Annual StepUp Present value of monthly Profit	Monthly StepUp Present value of monthly Profit	Annual RollUp Present value of monthly Profit	Double Enhanced Present value of monthly Profit
no change	no change	$45,908,971	$39,831,067	$38,021,038	$32,170,438
-0.001	-0.001	$46,062,164	$40,047,754	$38,306,795	$32,494,375
-0.002	-0.002	$46,216,924	$40,266,546	$38,593,806	$32,820,572
-0.003	-0.003	$46,373,273	$40,487,468	$38,882,079	$33,149,055
-0.004	no change	$46,310,972	$40,404,881	$39,044,104	$33,174,185
no change	-0.004	$46,124,386	$40,130,010	$38,145,754	$32,469,381
-0.004	-0.004	$46,531,231	$40,710,548	$39,171,625	$33,479,852
-0.005	no change	$46,412,681	$40,550,040	$39,301,733	$33,427,217
no change	-0.005	$46,179,486	$40,206,261	$38,177,352	$32,545,633
-0.005	-0.005	$46,690,822	$40,935,813	$39,462,454	$33,812,989

Finally, Figure 12 illustrates the effect of mortality improvement for GMDB profit of $V_{GMDB}^{Annuity\,2000}(S,x)$, $V_{GMDB}^{New}(S,x)$, $V_{GMDB}^{DML}(S,x)$, and $V_{GMDB}^{Gmpz}(S,x)$.

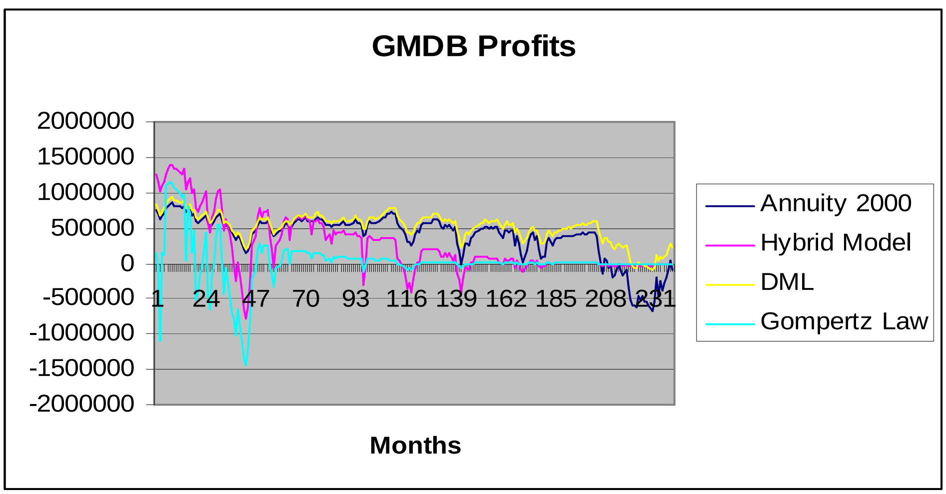


Figure 12. Effects of Various Mortality Assumptions on GMDB

Based on Annuity 2000 data, the proposed hybrid mortality model provides a better valuation tool than using Gompertz Law.

5. Conclusion

This paper studies how the advanced mortality improvement affects the valuation of a variable annuity with guarantees. The study shows the advanced mortality improvement does affect the valuation of variable annuities with GMDB features. The closed-form solutions for modeling GMDBs with mortality improvement for advanced ages are derived.

More importantly, the new method with hybrid mortality model presented in this paper is more useful and straightforward in practical applications than the traditional methods (such as Gompertz Law) of modeling GMDBs with mortality improvement for advanced ages. A case study provides a practical illustration of the effects of mortality changes to the GMDB profit with various minimum guarantees.

The effects of the mortality improvement for advanced age on VAGLB are even more significant. Although most the analysis is applied to GMDB, the study can be applied to VAGLB as well and will be included in a future study.

References

Collin-Dufresne, P., Keirstad, W. and Ross, M.P. 1997. "Martingale Pricing." *Equity Derivatives: Applications in Risk Management and Investment*, 233-243.

Gerber, H.U. and Shiu, E.S.W. 1994. "Option Pricing by Esscher Transforms." *Transactions of the Society of Actuaries*, 36:99-140.

Guo, L and Wang, M. 2002. "Data Mining Techniques for Mortality at Advanced Age." Society of Actuaries Living to 100 and Beyond Symposium.

Hull, J.C. 2003. *Options, Futures and Other Derivatives*. Prentice Hall.

Milevsky, M.A. and Posner, S.E. 2001. "The Titanic Option: Valuation of the Guaranteed Minimum Death Benefit in Variable Annuities and Mutual Funds." *Journal of Risk and Insurance*, 68(1):93-128.

Tiong, S. 2000. "Valuing Equity-Indexed Annuities." *North American Actuarial Journal*, 4(4):149-170.