SOCIETY OF ACTUARIES

Article from:

The Actuary

June 1989 – Volume 23, No. 6

The Newsletter of the Society of Actuaries
VOL. 23, NO. 6
JUNE 1989

THE Actuary

Was Fibonacci an actuary?

by Donald R. Sondergeld

"I read with interest the article by Sheri Lynn Levine in the November 1988 issue of The Fibonacci Quarterly *on 'Suppose More Rabbits Are Born.' Levine, like Fibonacci, assumes that rabbits never die and made additional assumptions regarding rates of reproduction. As an actuary, I found these assumptions unreasonable as the purpose of reproduction seems related to death. In the case of rats, I'm told that rates of reproduction are related to the food supply. Perhaps I should have titled this article 'On the Immortality of Rabbits.' Let me be somewhat serious and comment on the Multinacci series of the above article and provide some further generalizations."*

The above was the first paragraph of a discussion I wrote on Levine's Multinacci series

$F_n = F_{n-1} + q\,F_{n-2}$,

where q represents the number of pairs of rabbits in a Fibonacci litter.

Fibonacci Background

For those of you not familiar with the Fibonacci series, a few words may be helpful. Leonardo Fibonacci (alias Leonard of Pisa) was born in 1175. His reputation began when at age 27 he wrote Liber Abaci (the book of Abacus), in which he demonstrated the great advantage of the Arabic system of notation over the Roman. However, a puzzle proposed by Fibonacci in Liber Abaci was the famous Rabbit Problem: "Suppose there is one pair of rabbits in the

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Pioneered in Canada

Advance payment of death proceeds

by Thomas W. Reese

Some life insurers in Canada are pioneering a creative new noncontractual approach to meeting the financial needs of terminally ill insureds who may need funds for medical and living expenses.

Funds are made available by lending a portion of the death benefit to the terminally ill insured. This is a loan against future death proceeds; it is independent of loans against any cash value, which would be used first.

The loaned amount, increased by interest at the rate that the company would have earned on those funds, is deducted from the proceeds payable at the time of death. This creates a "win-win" situation: The insured obtains access to needed funds, and the insurer's financial position is the same as if the claim were paid only at death.

This idea originated at the Canadian branch of the Prudential Insurance Company of America. At least two other Canadian companies have adopted Prudential's practice, and others are expected to follow. No U.S. company has begun this practice, but it is being studied.

This new concept comes in the wake of much recent publicity about special life insurance policies and riders that can provide living benefit payments. Examples are "long-term care" and "dread disease" riders that pay part of the death benefit to a living insured under certain conditions.

The Canadian programs have received good publicity and favorable public reaction. The approach has been praised as a humanitarian gesture that is much welcomed from the insurance industry.

Special considerations

Several questions need careful consideration in developing a noncontrac-

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Advance payments cont'd

income the excess of the amount ceived from Living Benefits over the policyholder's cost basis of the policy.

Living Benefits requires all persons having an ownership or beneficiary interest in the policy to sign release documents. The insured must allow Living Benefits access to all medical records. There is a 15-day "grace period" during which the insured may cancel the arrangement made with Living Benefits.

However, the outright sale of an insurance policy to a third party has a number of potentially adverse implications for the insured, especially considering the personal stress accompanying his or her illness. With this approach, the terminally ill insured must decide whether to trade a deferred death benefit for the beneficiary for a much lower amount paid to the insured immediately. The insurance company approach described above preserves the full amount of the death proceeds and likely avoids other disadvantages as well.

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Fibonacci cont'd

Month of January which breed a second pair in the Month of February and that thereafter these produce another pair monthly, that each pair of rabbits produce another pair in the second month following birth and thereafter one pair per month." The problem was to find the number of pairs the following December.

The general equation for the number of pairs of rabbits at the end of the month (F_n) is

$$F_n = F_{n-1} + F_{n-2} .$$

The Fibonacci series is 1,1,2,3,5,8,13,21, 34,55,89,144,233, etc. (The first 1 represents the number of pairs of rabbits at the end of the first December, and 233 represents the number of pairs at the end of the second December, or F_{13}.)

(The limit of $F_n \div F_{n-1}$ as n increases is $(1+\sqrt{5})/2$, the "Golden Ratio," which is approximately equal to 1.6.)

The "Golden Ratio" called Φ (Phi) is also found by dividing a line segment, such that the lesser is to the greater, as the greater is to the sum: $(1-x) \div x = x \div 1$. This produces the equation $x^2+x-1 = 0$. Alternatively, x is the number that is exactly one less than its reciprocal (i.e., $x = (1 \div x) - 1$).

The rectangle supposedly most pleasing to the eye has a ratio of length to width of Φ. (A 3×5-inch postcard is close, as is 8.5×14-inch legal size paper.) Phidias, a Greek sculptor, made use of it and the Φ symbol came into use as the first letter of that Greek's name.

The logo of the Fibonacci Association at the University of Santa Clara, California, is the pentagram (Φ is the ratio of a diagonal to the side of the internal pentagon). Jakob Bernoulli (1654-1705) was so fascinated by the mathematical beauty of the equiangular spiral that he asked that it be engraved on his tombstone – and gave it the name logarithmic spiral.

There are many books on Φ. I will mention three: *The Divine Proportion – A Study in Mathematical Beauty* by H.E. Huntley, *The Curves of Life* by Theodore Andrea Cook, and *The Geometry of Art and Life* by Matila Ghyka. They are all available from Dover Publications, 180 Varick Street, NY, NY, 10014.

Multinacci and Sondergeldnacci

Levine defined the Multinacci series

$$F_n = F_{n-1} + q\, F_{n-2}$$

where q represents the number of pairs of rabbits in a litter. When $q = 1$, we have the famous Fibonacci series.

In my discussion of the Levine article, I developed the Sondergeldnacci:

$$S_n = S_{n-g} + q\, S_{n-g-m}$$

which is more general than the Multinacci. Here g is the number of months in the gestation period, m is the number of months it takes rabbits to mature to be able to reproduce, and q is the number of pairs of rabbits in Levine's Multinacci litter.

If B_n equals the number of pairs of baby rabbits born in month n, we can derive another formula for S_n, where $B_n = S_n - S_{n-1}$.

$$S_n = S_{n-g} + q\, S_{n-g-m}$$
$$S_{n-1} = S_{n-q-1} + g\, S_{n-g-m-1}$$
$$S_n - S_{n-1} = (S_{n-g} - S_{n-g-1}) + q\,(S_{n-g-m} - S_{n-g-m-1})$$
$$B_n = B_{n-g} + q\, B_{n-g-m}$$

The new formula is:

$$S_n = S_{n-1} + B_n$$

where B_n is calculated from the recursion formula

$$B_n = B_{n-g} + q\, B_{n-g-m} .$$

This brings us to the "actuarial-nacci."

Actuarialnacci

Had Fibonacci been an actuary studying the demography of rabbits, he might have further modified my formula for S_n to take into account rates of fertility, death, immigration, emigration, etc.

We can now modify the above formula to recognize deaths. If rabbits live d months, then

$$A_n = A_{n-1} + B_n - D_n$$
$$= A_{n-1} + B_n - B_{n-d}$$

as $D_n = B_{n-d}$.

However, we need a new formula for B_n as I assume rabbits don't reproduce after they are dead. Thus, B_n are births that occur in month n from rabbits born in the last d months.

A pair of rabbits will produce a finite number of litters in the $d-1$ months after birth. The first litter occurs $m+g$ months after rabbits are born, the second occurs $m+2g$ months after rabbits are born, the third occurs $m+3g$ months, etc.

$$B_n = q \sum_{i=1}^{d-1} K_{n-i}\, B_{n-i}$$

If rabbits only produce litters during the first fertile (f) months of their life, replace d with f in the above.

If $m+k\,g = i$, where k is any integer, a litter will be born in month n from rabbits born in month $n-i$. If k is an integer greater than zero, $K_{n-i} = q$, otherwise $K_{n-i} = 0$.

Summary

Fibonacci may have been an actuary.

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Problem workshops for 110 and EA-2

Intensive problem-solving workshops for Course 110 (2 days) and for the EA-2 exam (3 days) will be given by Actuarial Study Materials in October in New York City. For details, write to A.S.M., P.O. Box 522, Merrick, NY 11566.