Leveraging up Return on Equity by Issuing Subordinated Indebtedness

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Under Section 84 of the proposed new *Insurance Companies Act* of Canada (Bill C-28), Canadian insurance companies will be able to issue subordinated indebtedness (debentures and notes) subordinate to insurance liabilities. Subordinated indebtedness will be considered as part of the insurance company's Tier 2 capital. This note presents a formula for the expected *rate of return on shareholders' equity (R.O.E.)* when an insurance company raises capital by issuing subordinated indebtedness. The R.O.E. formula is also applicable to the case of raising capital by issuing nonconvertible preferred shares.

First, we derive a R.O.E. formula for the case with no subordinated indebtedness. To fix ideas, let us introduce two yields – a net rate for assets and a gross rate for liabilities:

y = expected **net** yield on assets
= expected investment yield *minus* investment expenses (expressed as yield)

g = **total** yield required to support the liability
= interest rate for crediting the customer's account (for a product such as G.I.C. this interest rate is normally determined by the marketplace) *plus* acquisition and maintenance expenses (expressed as yield)

Suppose that, for each $1 that the insurance company receives from its customer, the shareholders contribute $E for backing up the business. (For example, E is around 0.04 for the typical G.I.C. business.) The question of interest is: What is the expected rate of return on shareholders' contribution?

For simplicity, consider that the liability is a one-year G.I.C. For each $1 received from the customer, the company needs $(1 + g) at the end of the year to pay the customer, sales commissions and expenses. On the other hand, at the end of one year, the total value of assets is expected to be $(1 + E)(1 + y). The difference

$$\$[(1 + E)(1 + y) - (1 + g)]$$

belongs to the shareholders. Since the shareholders contribute $E at the beginning of the year, the expected rate of return on shareholders' equity (before tax) is

$$\frac{[(1 + E)(1 + y) - (1 + g)] - E}{E},$$

which, after simplification, is

$$y + \frac{y - g}{E}.$$

Notice that this pre-tax R.O.E. formula is the sum of two terms. The first term, y, is the expected investment net yield. The second term, $(y - g)/E$, is the *leverage* term. It is the difference between the investment net yield and liability gross yield divided by shareholders' equity expressed in terms of per unit of customer's fund. It represents the *excess* yield that the shareholders expect to get by investing in the insurance company instead of just investing in the assets directly.

Suppose that we let τ denote the tax rate. Then the after-tax expected rate of return on shareholders' equity is

$$(1 - \tau)\left(y + \frac{y - g}{E}\right). \qquad (1)$$

Formula (1) is not a new formula. An approximate variant of (1) can be found on page 584 in Volume 41 (1989) of the *Transactions of the Society of Actuaries*. Formula (1) also appears on pages 928 and 1650 in Volume 16 (1990) of the *Record of the Society of Actuaries;* however, the symbol E on these two pages is said to denote equity as a percentage of assets, not as a percentage of liabilities.

It is obvious that, for a fixed positive yield differential $(y - g)$, the lower is the equity/liability ratio E, the higher is the expected rate of the return on equity. However, the amount of capital required to back up a line of business should be determined by the overall riskiness of the business, including the attendant investment operations. That is, the required amount of capital should be determined by the combined effect of the business's C-1, C-2 and C-3 risks; see [SoA], [CLHIA MCCSR Formula], [Ha], [Co] and the Appendix in [Ki]. To raise capital by issuing preferred shares or subordinated debentures is a way to reduce shareholders' equity and, thereby, increase the expected return on equity.

Let us now suppose that, for each $1 of liability received by the insurance company, $D of subordinated debenture is issued and the shareholders contribute $E for backing up the business.

Thus for each \$1 of liability, there is \$(D + E) of capital. (We do not use the same symbol as before to denote shareholders' equity because we want to emphasize that, with the issuing of subordinated indebtedness, we expect E to be less than E. In general, $E \approx D + E$.)

Let g_d denote the **total** yield required to service the subordinated debenture. Again, we simply consider the liability as a one-year G.I.C. For each \$1 received from the customer, the company needs \$(1 + g) at the end of the year to pay the customer and expenses, and $\$D(1 + g_d)$ to repay the subordinated debenture and cover its issuing cost. On the other hand, at the end of the year, the total asset value is expected to $\$(1 + D + E)(1 + y)$. The difference

$$(1 + D + E)(1 + y) - [(1 + g) + D(1 + g_d)]$$

belongs to the shareholders. Since the shareholders contribute \$E at the beginning of the year, the expected rate of return on shareholders' equity (before tax) is

$$\frac{[(1 + D + E)(1 + y) - (1 + g) - D(1 + g_d)] - E}{E},$$

which, after simplification, is

$$y + \frac{y - g}{E} + \frac{D(y - g_d)}{E}. \qquad (2)$$

This pre-tax R.O.E. formula is a sum of three terms. The first term, y, is the expected investment net yield. The last two terms are the *leverage* terms. To show the leverage more clearly, let us rewrite the two terms as

$$\frac{1}{E}(y - g) + \frac{D}{E}(y - g_d).$$

Note that the "1" in the numerator stands for the \$1 received from the customer. Rewriting (2) as

$$\frac{E(y - 0) + 1(y - g) + D(y - g_d)}{E}$$

shows that the R.O.E. formula can easily be generalized to the case of multiple asset and liability classes, each with a different yield rate.

With τ denoting the tax rate, the after-tax expected rate of return on shareholders' equity is

$$(1-\tau)\left[y + \frac{1}{E}(y - g) + \frac{D}{E}(y - g_d) \right]. \qquad (3)$$

We note that (3) is also valid for the case of raising secondary capital by issuing nonconvertible preferred shares, if g_d denotes the pre-tax equivalent yield rate needed to pay the dividends on and issuing costs of the preferred shares.

Remarks

If we define

$$\gamma = \frac{1}{1+D} g + \frac{D}{1+D} g_d ,$$

then formula (2) can be written as

$$y + \frac{1+D}{E}(y - \gamma), \qquad (4)$$

which is essentially the same as formula (1). One can interpret γ as an "average" liability gross yield rate.

As E is less than E, it follows from formulas (1) and (4) that the shareholders could expect a higher rate of return with the issue of subordinated debentures. However, the higher return comes with higher risk. Let Y denote the random variable of asset return rate, net of investment expenses. The expected value of Y is y. If we assume that the other parameters are constant, the variance of the return on equity without issuing debentures is

$$\left(1 + \frac{1}{E}\right)^2 \mathrm{Var}(Y),$$

and the variance of the return on equity in the presence of debentures is

$$\left(1 + \frac{1+D}{E}\right)^2 \mathrm{Var}(Y).$$

As E is less than E, the second expression gives a larger value than the first one. On the other hand, since the debentures are subordinate to insurance liabilities, the policyholders' security position remains the same, whether the capital comes only from the shareholders or from the shareholders and subordinated debentures.

For further discussion on leverage and capital structure, we refer the interested reader to [CW, Chapter 13] and [KR]. The Nobel Memorial Prize Lecture [Mi] is illuminating.

Acknowledgment

I gratefully acknowledge the support from the Great-West Life Assurance Company and thank A.D. Alleyne, B.G. Challes, A.K.C. Lam, D.E. Steven and P.D. Tilley for their comments.

References

[CLHIA MCCSR Formula] Canadian Life and Health Insurance Association Sub-Committee on Solvency Testing. *CLHIA Formula for Minimum Continuing Captial and Surplus Requirements*. Toronto: CLHIA, 1991.

[Co] Cody, D.D. "Probabilistic Concepts in Measurement of Asset Adequacy," *Transactions of the Society of Actuaries*, 40 (1988): 149-160; Discussion 161-172.

[CW] Copeland, T.E., and Weston, J.F. *Financial Theory and Corporate Policy*, 3rd ed. Reading, Massachusetts: Addison-Wesley, 1988.

[Ha] Letter from R.M. Hammond, Deputy Superintendent, Insurance and Pensions Sector, Office of the Superintendent of Financial Institutions Canada, to F. Speed of the Canadian Life and Health Insurance Association (July 6, 1990).

[Ki] Kischuk, R.K. "Strategic Management of Life Insurance Company Surplus," *Transactions of the Society of Actuaries*, 38 (1986): 105-121; Discussion 123-144.

[KR] Kopcke, R.W., and Rosengren, R.S. (ed.) *Are the Distinctions between Debt and Equity Disappearing?* Proceedings of a Conference Held at Melvin Village, New Hampshire, in October 1989. This book is available free of charge from the Federal Reserve Bank of Boston, which sponsored the conference to examine the changes in business financing.

[Mi] Miller, M.H. "Leverage," *Journal of Finance*, 46 (1991): 479-488.

[SoA] Society of Actuaries Committee on Life Insurance Company Valuation Principles. *The Valuation Actuary Handbook*. Itasca, Illinois: Society of Actuaries, 1987.