ACTUARIAL RESEARCH CLEARING HOUSE
1991 VOL. 2

PROJECTIONS OF ACTIVE LIFE EXPECTANCIES

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ABSTRACT

Active life expectancy provides a measure of the expected number of years of independence in certain Activities of Daily Living (ADL). A recent research project produced tables of such expectancies for the noninstitutionalized elderly people of Massachusetts. Those expectancies did not allow for re-entry to the active life status of those who lost independence in ADL at an earlier age, but subsequently regained it. We will present the needed mathematical techniques and obtain values for active life expectancies in which dependent (in ADL) lives are followed after regaining such independence. Secondly, we will develop projections of active life expectancies to the year 2080, under three sets of assumptions, for 65 year old men and women. Finally, we provide estimates of the average numbers of years of dependency in ADL prior to death of 65 year old men and women, under alternate sets of assumptions.

KEY WORDS: Active life expectancy, Activities of Daily Living, projections of expectancies, years of dependency.

1. INTRODUCTION

The concept of life expectancy is very old, including some data from the Roman Empire. Sidney Katz and co-researchers (1983) presented some data for their newly developed idea of active life expectancy. They defined the end of active life to be loss of independence in certain activities of daily living (ADL), namely bathing, dressing, transfer, and eating. Katz et al in 1974 interviewed 1625 noninstitutionalized elderly people in Massachusetts, and constructed their ADL scores. In early 1976, data was gathered from 89% of the 1625 original respondents. From the 1225 people who in 1974 were independent in ADL, proportions were calculated of those who lost independence in ADL, or who died during the study period. Tables of active expectancy for males, females, by financial status, and for the aggregate population were developed.

Those expectancies did not allow for re-entry to the active lives column of those who lost independence in ADL at an earlier age, but subsequently regained it.

This paper has three purposes. We will obtain values for active life expectancies in which dependent (in ADL) lives are followed after re-entry to the independent (in ADL) lives column. Secondly, we will develop projections of active life expectancies to the year 2080, under three sets of assumptions, for 65 year old men and women. Finally, we provide estimates of the average numbers of years of dependency in ADL prior to death of 65 year old men and women under three sets of assumptions.

2. NOTATION

Assume that ℓ_x^a lives are independent in ADL, and ℓ_x^{na} are not independent in ADL, at exact age x.

Definition. The active life expectancy at age x (complete version) is denoted and defined by

$$(a^\circ e)_x = \int_0^{\omega-x} \frac{\ell_{x+t}^a dt}{\ell_x^a + \ell_x^{na}} ,$$

where ω is the terminal age for the life table.

The division by $\ell_x^a + \ell_x^{na}$ reflects the potential that each x year old person has to contribute active years to the aggregate total of active years.

If we assume that loss of independence in ADL occurs at midyear of age, then

$$(a^\circ e)_x \doteq \frac{\sum_{k=x}^{\omega-1} \ell_k^a}{\ell_x^a + \ell_x^{na}} + 0.5 .$$

3. REFINEMENT OF $(a°e)_{65}$ VALUES

The values of $(a°e)_x$ in Tables 1 and 2 of Katz (1983) were computed without allowance for re-entry to the independent status (in ADL) of those who lose their independence. Table 5 (loc.cit.) does provide statistics on those in the study who were initially dependent, but who regained their independence. We wish to now show how we computed $(a°e)_{65}$ values, with allowance for re-entry to the ℓ_x^a column.

Consider the following time diagrams, illustrative of possible chains of events.

	Active	Not Active	Active	Not Active	Death
65	$65+t_1$	$65+t_2$	$65+t_3$		$65+t_4$

	Not Active	Active	Not Active	Death
65	$65+t_1$	$65+t_2$		$65+t_3$

For each of the ℓ_{65}^a active people, and each of the ℓ_{65}^{na} non-active entrants, we want to measure their active years.

Let $(ri)_x$ = probability of regaining independence in ADL between ages x and $x+1$;

i_x^a = number who regain independence in ADL between x and $x+1$;

$q_x^{\ell.i.}$ = probability of losing independence in ADL between x and $x+1$;

$\overset{*}{q}{}_x^d$ = probability of dying between x and $x+1$ for a dependent (in ADL) person

q_x^a = probability of dying between x and $x+1$ for a person independent in ADL at age x ;

$\ell_x^a \cdot q_x^a = d_x^a$;

$\ell_x^a \cdot q_x^{\ell.i.} = d_x^{na}$;

$\ell_x^{na} \cdot (ri)_x = i_x^a$; and

$\ell_x^{na} \quad q_x^{*d} = d_x^{*na}$.

We have the recursion relations

$$\ell_{x+1}^{a} = \ell_x^{a} - d_x^{a} - d_x^{na} + i_x^{a} \tag{1}$$

and

$$\ell_{x+1}^{na} = \ell_x^{na} - d_x^{*na} + d_x^{na} - i_x^{a} \tag{2}$$

for $x = 65, 66, \ldots$

Drawing on the Katz (1983) study we will let

$\ell_{65}^{a} = 495$ and $\ell_{65}^{na} = 45$ for males, and

$\ell_{65}^{a} = 729$ and $\ell_{65}^{na} = 72$ for females.

We proceeded sequentially on age in this manner until obtaining $\ell_{100}^{a} = 0$.

For the age groups 65-69, 70-74, 75-79, 80-84, and $\geq$ 85, Table 2 of Katz (1983) gives values for the central rates ${}_n m_x$ for loss of independence in ADL or death, for men and women. Table 1 (loc. cit.) also provides values for ${}_n q_x$ = Prob [Losing independence in ADL, or dying in [x,x+n]], but for the Total Population Groups, rather than for the Male and Female Age Groups. For the male and female ${}_n q_x$ values, we use the Reed - Merrell equation:

${}_n q_x = 1 - \exp\{-n \cdot {}_n m_x - an^3 \cdot {}_n m_x{}^2\}$; see Spiegelman (1968), page 133.
For Katz (1983), $a = 0.008$, $n = 5$, and thus $an^3 = 1.0$.

For the age groups 85-89, and 90-94, we assumed that

${}_5 m_{90} = {}_{15} m_{85}$, and ${}_5 m_{85} = {}_{15} m_{85} - \frac{1}{2}({}_5 m_{90} - {}_5 m_{80})$ for men, and women. These assumptions seemed realistic, and were adequate. Thus, we obtained Table 1.

TABLE 1

Probabilities of Losing Independence in ADL or Dying

Age Group	Male $_5q_x$	Female $_5q_x$
65-69	0.3675	0.2231
70-74	0.3675	0.4300
75-79	0.4590	0.3675
80-84	0.5848	0.5620
85-89	0.7157	0.7309
90-94	0.8072	0.8463
90-100	1.0000	1.0000

We then graduated the data of Table 1 using an adaptation of a least-squares fit to a Gompertz curve. See London (1985) p.97. This gave values of q_x at each individual age. The values of q_x for $x > 85$ were then revised to lie on a cubic for which $q_{99} = 1$.

TABLE 2

Age x	Male q_x	Female q_x	Age x	Male q_x	Female q_x
65	0.0687	0.0475	83	0.1790	0.1660
66	0.0725	0.0511	84	0.1892	0.1774
67	0.0765	0.0548	85	0.1999	0.1895
68	0.0807	0.0588	86	0.2114	0.2023
69	0.0852	0.0632	87	0.2235	0.2159
70	0.0899	0.0678	88	0.2365	0.2317
71	0.0948	0.0727	89	0.2511	0.2514
72	0.1000	0.0780	90	0.2685	0.2764
73	0.1055	0.0837	91	0.2979	0.3084
74	0.1112	0.0898	92	0.3363	0.3488
75	0.1173	0.0962	93	0.3854	0.3993
76	0.1236	0.1031	94	0.4469	0.4614
77	0.1303	0.1105	95	0.5224	0.5366
78	0.1373	0.1184	96	0.6139	0.6264
79	0.1446	0.1267	97	0.7228	0.7325
80	0.1523	0.1356	98	0.8509	0.8564
81	0.1607	0.1452	99	1.0000	1.0000
82	0.1696	0.1553			

Next, we obtained $q_x^{\ell.i.} = q_x - q_x^a$, $x = 65, 66, \ldots, 100$ where we used Table 15 Wilkin (1981) for the q_x^a values. Table 15 (loc. cit.) gave Medicare probabilities of death for the same time period (1974) as Katz (1983). Its values of q_x were for exact ages 65.5, 66.5, ..., 99.5, but we used them as if they were for ages 65, 66, ..., 99.

Table 5 Katz (1983) gives values of ${}_{1.25}(ri)_x$. Thus, the observation period n = 15 months. The values were for 45 men (all ages), 72 women (all ages), and for the 117 total by age groups 65-74, 75- 84, and ≥ 85. After multiplying by 0.8 to reduce the probabilities to 12 month periods, and making other adjustments, we obtained Table 3.

TABLE 3

Age Group	Male $(ri)_x$	Female $(ri)_x$
65-74	0.22	0.28
75-84	0.18	0.24
≥ 85	0.02	0.06

Due to the scarcity of data a simple graphic graduation was applied to the data of Table 3 to obtain individual age values for $(ri)_x$.

TABLE 4

Probabilities of Regaining Independence in ADL

Age x	Male $(ri)_x$	Female $(ri)_x$	Age x	Male $(ri)_x$	Female $(ri)_x$
65	0.24	0.30	85	0.12	0.18
66	0.24	0.30	86	0.09	0.15
67	0.23	0.29	87	0.06	0.12
68	0.23	0.29	88	0.04	0.10
69	0.22	0.28	89	0.03	0.08
70	0.22	0.28	90	0.02	0.06
71	0.22	0.28	91	0.02	0.06
72	0.22	0.28	92	0.01	0.05
73	0.21	0.27	93	0.01	0.05
74	0.21	0.27	94	0.01	0.04
75	0.20	0.26	95	0.01	0.04
76	0.20	0.26	96	0.01	0.03
77	0.19	0.25	97	0.01	0.02
78	0.19	0.25	98	0.01	0.01
79	0.18	0.24	99	0	0
80	0.18	0.24			
81	0.17	0.23			
82	0.17	0.23			
83	0.16	0.22			
84	0.15	0.21			

Several sources were considered for individual age mortality rates q_x^{*d} for lives dependent in ADL. Tables III and IV Mortality Tables (1977) contain mortality rates for participants receiving disability payments from their pension plans, but not receiving Social Security disability benefits. The effective date is September 2, 1974. Mortality tables based on a million person study conducted by the American Cancer Society are presented by E.A. Lew and L. Garfinkel (1984). The tables are for the total population, for the ostensibly healthy, and for those not in good health. The data covered the period 1959-1972. We chose to use the graduated death rates from Table 4 (Mortality Rates - Persons Not in

Good Health) of that paper as the basis of our q_x^{*d} values. In order to reflect the difference in the population studied in Lew-Garfinkel (1984) versus that studied by Katz (1983), we adjusted the values in Table 4 of Lew-Garfinkel (1984) by multiplying by a graduated version of the Medicare ratios in Table 13 (loc. cit.) This should make the rates used more consistent with the other data used in this paper.

TABLE 5

Probabilities of Death for Lives Dependent in ADL

Age x	Male q_x^{*d}	Female q_x^{*d}	Age x	Male q_x^{*d}	Female q_x^{*d}
65	0.0463	0.0193	85	0.1725	0.1263
66	0.0487	0.0210	86	0.1812	0.1356
67	0.0517	0.0232	87	0.1903	0.1452
68	0.0553	0.0256	88	0.1996	0.1569
69	0.0594	0.0278	89	0.2132	0.1693
70	0.0635	0.0303	90	0.2296	0.1820
71	0.0678	0.0330	91	0.2469	0.1949
72	0.0728	0.0362	92	0.2647	0.2100
73	0.0779	0.0400	93	0.2829	0.2253
74	0.0823	0.0442	94	0.2984	0.2407
75	0.0869	0.0481	95	0.3135	0.2559
76	0.0924	0.0526	96	0.3280	0.2706
77	0.0993	0.0568	97	0.3417	0.2845
78	0.1066	0.0627	98	0.3545	0.2976
79	0.1137	0.0692	99	0.3665	0.3098
80	0.1224	0.0763			
81	0.1318	0.0849			
82	0.1419	0.0947			
83	0.1526	0.1054			
84	0.1638	0.1171			

We then were able to compute all the needed values for

$$(a^{\circ}e)_{65} \doteq \frac{\sum_{x=65}^{100} \ell_x^{a}}{\ell_{65}^{a} + \ell_{65}^{na}} + 0.5 \ , \quad \text{for males and females.}$$

Using values from Table 2 of Katz (1983), we obtained Table 6.

TABLE 6

Active Life Expectancy at Age 65

	Without Re-entry to ℓ_x^{a} Column	With Re-entry to ℓ_x^{a} Column
Male	9.3	11.9
Female	10.6	15.1

4. SOME PROJECTIONS OF $(a^{\circ}e)_{65}$.

The Office of the Actuary, Social Security Administration, develops projections for Social Security area populations, according to three sets of assumptions about fertility, mortality, and net immigration. Thus, Actuarial Study No. 105, by Alice Wade (1989), contains in Table 21 the July 1 populations in the Social Security area by Alternatives I, II, and III by age groups, sex, and marital status for the years 1990, 1995, 2000, 2020, 2040, 2060, and 2080. By using the alternative sets of assumptions, The Office of the Actuary develops sets of estimates of future income and expenditures of the Old-Age and Survivors Insurance and Disability Insurance (OASDI) program which are presented to the Congress for their financial guidance. The Alternative I set assumes low life expectancies, the Alternative II set assumes intermediate values for life expectancies, and the Alternative III set assumes high life expectancies. In order to develop the assumptions, possible future reductions in ten leading causes of death are considered.

By comparing values in our Table 6 and Table 3 Katz (1983), one obtains male and female values of 0.91 and 0.77 for $(a^{\circ}e)_{65}/\overset{\circ}{e}_{65}$ for the refined version (with re-entry to $\ell_x{}^a$ column) of $(a^{\circ}e)_{65}$, and the comparable Massachusetts values of $\overset{\circ}{e}_{65}$ for 1974. We will apply those values to the projected $\overset{\circ}{e}_{65}$ values to obtain projected values for $(a^{\circ}e)_{65}$. It is acknowledged that those ratios may change with time, but so far there is not enough data to suggest possible changes. The projected values of $\overset{\circ}{e}_{65}$ came from Table 10 Wade (1989). It is significant that the ratios $(a^{\circ}e)_{65}/\overset{\circ}{e}_{65}$ are 0.71 and 0.54 for men and women when the $(a^{\circ}e)_{65}$ values without re-entry to $\ell_x{}^a$ column are used. Those lower values lead to considerably smaller projections of $(a^{\circ}e)_{65}$, and similarly higher projected values of $\overset{\circ}{e}_{65}$ - $(a^{\circ}e)_{65}$, the average number of years of dependency in ADL prior to death of a 65 year old person.

TABLE 7

Projections of Life and Active Life Expectancies

	Alternative I				Alternative II			
	Male		Female		Male		Female	
Year	$\overset{\circ}{e}_{65}$	$(a^{\circ}e)_{65}$	$\overset{\circ}{e}_{65}$	$(a^{\circ}e)_{65}$	$\overset{\circ}{e}_{65}$	$(a^{\circ}e)_{65}$	$\overset{\circ}{e}_{65}$	$(a^{\circ}e)_{65}$
1990	15.0	13.65	18.8	14.48	15.1	13.74	19.0	14.63
1995	15.0	13.65	18.9	14.55	15.4	14.01	19.3	14.86
2000	15.0	13.65	18.9	14.55	15.6	14.20	19.6	15.09
2010	15.2	13.83	19.0	14.63	16.0	14.56	20.1	15.48
2020	15.3	13.92	19.2	14.78	16.4	14.92	20.5	15.79
2030	15.5	14.11	19.4	14.94	16.8	15.29	20.9	16.09
2040	15.7	14.29	19.6	15.09	17.1	15.56	21.4	16.48
2050	15.9	14.47	19.8	15.25	17.5	15.93	21.8	16.79
2060	16.0	14.56	20.0	15.40	17.8	16.20	22.2	17.09
2070	16.2	14.74	20.2	15.55	18.2	16.56	22.6	17.40
2080	16.3	14.83	20.3	15.63	18.5	16.84	23.0	17.71

TABLE 7 (continued)

Alternative III

Year	Male $\overset{\circ}{e}_{65}$	Male $(a^\circ e)_{65}$	Female $\overset{\circ}{e}_{65}$	Female $(a^\circ e)_{65}$
1990	15.2	13.83	19.1	14.71
1995	15.8	14.38	19.8	15.25
2000	16.2	14.74	20.4	15.71
2010	17.0	15.47	21.2	16.32
2020	17.8	16.20	22.0	16.94
2030	18.6	16.93	22.9	17.63
2040	19.3	17.56	23.7	18.25
2050	20.1	18.29	24.5	18.87
2060	20.9	19.02	25.4	19.56
2070	21.7	19.75	26.2	20.17
2080	22.5	20.48	27.0	20.79

5. FURTHER PROJECTIONS OF $(a^\circ e)_{65}$

We will now provide alternate projections of $(a^\circ e)_{65}$ based on some results of a Canadian study conducted by Wilkins and Adams (1983). They obtained values for "Disability-free life expectancy" for males and females for the years 1951 and 1978. Portions of their results are referenced in Manton (1988). Table 7.2 of Wilkins and Adams (1983) (or Table 2 of Manton (1988)) gives values of 8.2 and 9.9 for the male and female $(a^\circ e)_{65}$ for 1978. Table 6.1 of Wilkins and Adams (1983) (or Table 3 of Manton (1988)) reveals that $(a^\circ e)_0$ gained 1.3 (1.4) years for males (females) from 1951 to 1978. By contrast $\overset{\circ}{e}_0$ gained 4.5 (7.5) years for males (females) from 1951 to 1978. For our current purposes, we will assume that during each 25 year period a gain of 0.65 (0.70) years in $(a^\circ e)_{65}$ for males (females) will occur for the Alternate II projections. For the Alternate I and Alternate III projections, we will use gains of 0.33 (0.35) and 0.98 (1.05) years for each 25 year period for males (females). This leads to Table 8. We use the 1990 values from our Table 7 as beginning values.

TABLE 8

Alternate Projections of Life and Active Life Expectancies

	Alternate I				Alternate II			
	Male		Female		Male		Female	
Year	$\overset{\circ}{e}_{65}$	$(a^{\circ}e)_{65}$	$\overset{\circ}{e}_{65}$	$(a^{\circ}e)_{65}$	$\overset{\circ}{e}_{65}$	$(a^{\circ}e)_{65}$	$\overset{\circ}{e}_{65}$	$(a^{\circ}e)_{65}$
1990	15.00	13.65	18.8	14.48	15.1	13.74	19.0	14.63
2015	15.25	13.98	19.1	14.83	16.2	14.39	20.3	15.33
2040	15.70	14.31	19.6	15.18	17.1	15.04	21.4	16.03
2065	16.10	14.64	20.1	15.53	18.0	15.69	22.4	16.73

Alternate III

	Male		Female	
Year	$\overset{\circ}{e}_{65}$	$(a^{\circ}e)_{65}$	$\overset{\circ}{e}_{65}$	$(a^{\circ}e)_{65}$
1990	15.2	13.83	19.1	14.71
2015	17.4	14.81	21.6	15.76
2040	19.3	15.79	23.7	16.81
2065	21.3	16.77	25.8	17.86

It is reassuring to note that the values in Tables 7 and 8 are rather close together for the various years, and alternatives, for males and females. The biggest differences occur for Alternative III.

6. SIGNIFICANCE OF PROJECTIONS OF $(a^{\circ}e)_{65}$

An estimate of the average number of years of dependency in ADL prior to death of a 65 year old person is $\overset{\circ}{e}_{65} - (a^{\circ}e)_{65}$. Tables 7 & 8 permit us to calculate these quantities under the various sets of assumptions. We record the results as Tables 9 and 10.

TABLE 9

Expected Number of Years of Dependency in ADL = $\mathring{e}_{65} - (a^{\circ}e)_{65}$

	Alternative I		Alternative II		Alternative III	
Year	Male	Female	Male	Female	Male	Female
1990	1.35	4.32	1.36	4.37	1.37	4.39
1995	1.35	4.35	1.39	4.44	1.42	4.55
2000	1.35	4.35	1.40	4.51	1.46	4.69
2010	1.37	4.37	1.44	4.62	1.53	4.88
2020	1.38	4.42	1.48	4.71	1.60	5.06
2030	1.39	4.46	1.51	4.81	1.67	5.27
2040	1.41	4.51	1.54	4.92	1.74	5.45
2050	1.43	4.55	1.57	5.01	1.81	5.63
2060	1.44	4.60	1.60	5.11	1.88	5.84
2070	1.46	4.65	1.64	5.20	1.95	6.03
2080	1.47	4.67	1.66	5.29	2.02	6.21

TABLE 10

Alternative Version of Expected Number of Years of Dependency in ADL = $\mathring{e}_{65} - (a^{\circ}e)_{65}$

	Alternative I		Alternative II		Alternative III	
Year	Male	Female	Male	Female	Male	Female
1990	1.35	4.32	1.36	4.37	1.37	4.39
2015	1.27	4.27	1.81	4.97	2.59	5.84
2040	1.39	4.42	2.06	5.37	3.51	6.89
2065	1.46	4.57	2.31	5.67	4.53	7.94

The growth in the projected numbers of years of dependency has profound implications for those planning for nursing homes, and other care facilities, and those designing public and private insurance systems to meet the financial needs of the non-active population.

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