ACTUARIAL RESEARCH CLEARING HOUSE
1993 VOL. 3

A BOND MANAGER'S METHOD FOR ALM

August 26, 1993

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Abstract

Over the past few years, many worthwhile efforts have been made to extend the concepts of asset liability matching analysis beyond simple duration and convexity. Although these have moved us in the right direction, the methods developed to date generally revolve around spot rates and require fairly laborious calculations. This paper describes a very simple method of analyzing the impact of a change in interest rate levels on the present value of a stream of cash flows. The method described will be referred to throughout the paper as the "bond manager's method". It has the following features :

1. A stream of cash flows can be represented by a "notional" portfolio of "benchmark" bonds with approximately the same sensitivity to changes in interest rate levels as the cash flow stream itself.

2. The amount of each benchmark bond in the notional portfolio can be easily determined using a simple algorithm. These amounts are referred to as "benchmark weights".

3. The impact of a change in interest rate levels on the present value of a stream of cash flows can easily be approximated by multiplying the price changes on the benchmark bonds by the benchmark weights and summing the results.

4. The calculation of spot rates required by most traditional methods of analysis is not required.

5. In practical situations, the method produces results which are not materially different from those produced by more complex methods using spot rates.

It is hoped that the simplicity of this method will allow all parties involved in the asset liability risk management process to develop a better feel for the the risks involved ultimately leading to more informed decision-making.

The paper deliberately avoids long complex mathematical derivations. Instead, concepts are illustrated using simple examples. It is hoped that this approach will be easier for the average practitioner to follow.

The Traditional Approach

Most traditional approaches to asset liability matching analysis require that a set of spot rates be available for all terms at which there are cash flows. Although the market for strip bonds has become more active over the last several years, it is not possible to determine a reliable set of spot rates by observing market quotations for strip bonds. Instead, most practitioners observe the quotes for normal coupon bonds and use a well-known recursive formula to derive spot rates from the prices of these bonds. The details of how this is done may vary from practitioner to practitioner. However, the following description is fairly typical :

1. Prices are determined for a set of benchmark bonds based on current market quotations. These benchmark bonds might include coupon bonds with maturities of 1, 3, 6 and 12 months and 3, 5, 7, 10, 20 and 30 years for a total of 10 benchmark bonds.

2. In order to determine a full set of spot rates using a recursive formula, coupon bond rates and prices are required at six-month intervals. These are normally determined by interpolation from the benchmark bond coupon rates and prices already known. Although, linear interpolation is often used, a more realistic result can probably be obtained using higher-order interpolations which follow the general shape of the curve in the area being interpolated.

3. Once the coupon rates and prices are known at six-month intervals, the calculation of spot rates at six-month intervals is a relatively simple process.

4. If cash flows are more frequent than semi-annual, additional spot rates must be determined from the set of spot rates already obtained. This might be done by linear interpolation.

For further information on this process, please review Appendix C.

Once a set of spot rates representing the current market has been determined, these can easily be used to discount cash flows and obtain a present value for the set of cash flows. Usually several alternate interest scenarios are also assumed and the present value calculated and compared to the value on the base scenario. Alternate scenarios can be defined as a modification of the base scenario either using the yields to maturity on the benchmark bonds or using the spot rates.

If yields to maturity are used, the spot rates need to be re-determined and the cash flows rediscounted. When this process is used, it is often difficult to understand why certain scenarios produce the results they do. If spot rates are used, the new spot rates can be used directly to determine a new present value. However, it may be necessary to convert the assumed spot rates to coupon rates in order to have a basis of comparison that can be presented to management.

The Bond Manager's Method

The method introduced in this paper allows the impact of a change in interest rate levels on the present value of a stream of cash flows to be directly determined from the coupon rates and prices on benchmark bonds without the intermediate step of determining spot rates. The procedure is as follows :

1. The coupon rates and prices of a set of benchmark bonds are determined by observing current market quotations. This is the same as for the traditional approach.

2. Coupon rates and prices for all intermediate terms (monthly if cash flows are monthly) are determined by interpolation. Linear interpolation can be used but a higher-order method is suggested. Note that an interpolation method should be adopted that allows each interpolated value to be readily stated as a linear combination of benchmark values. Interpolation using Lagrange polynomials is one such method. The reason for this will become clear later. (Note that the traditional approach described earlier requires essentially the same interpolation but implements it in two steps.)

3. Calculate the par values of a notional portfolio of the bonds in step 2) which would have cash flows exactly equal to the cash flows being analyzed. Fortunately this is a relatively simple process which will be illustrated later. Let's call the set of par values determined in this way **W(t)** where **t** is the term to maturity of each bond. We will also refer to **W(t)** as the par value weights.

4. The present value of the cash flows can now easily be determined by multiplying each **W(t)** by the price of each bond (say **P(t)**) and summing the results or **SUM(W(t) * P(t))**.

5. Only a few of the prices **P(t)** are actually known from market data. The remainder were obtained by interpolation in step 2). We can substitute the equations for these interpolated values into the equation for present value in step 4) and obtain an equation that only contains the benchmark prices, the par value weights **W(t)** and the interpolation factors from step 2). By re-grouping the terms of the equation, we can easily derive an equation that represents a linear combination of benchmark prices. Let's represent the equation as **SUM(B(t) * P(t))** where **t** represents the term to maturity for each benchmark bond and **B(t)** will be known as the benchmark weights. This process will be illustrated by a simple example in the following section.

Having obtained the benchmark weights **B(t)**, we can very easily determine the effect of a change in a yield rate for a benchmark bond on the present value of the cash flows. The benchmark bond with **t** years to maturity has a price **P(t)**. On an alternate interest scenario, this same bond has a price **P'(t)**. (This price can easily be determined given the yield to maturity of the bond.) The effect of this change in yield on the total present value of the stream of cash flows can be calculated as **B(t) * (P'(t) - P(t))**. Furthermore, this same approach can be used when more than one benchmark rate is changed. The individual impacts can simply be added to get the total impact. Note also that the benchmark weights **B(t)** remain constant and each scenario can be evaluated simply by multiplying these weights by the changes in price of the benchmark bonds and adding up the results. In a typical situation where there are only 10 benchmark bonds involved, this leads to much simpler computations than would be required with the traditional approach.

A Simple Example

The method can probably best be understood by considering a simple example rather than a long series of symbolic manipulations.

Suppose we have the following cash flows, and benchmark bonds :

Term	Coupon Rate	Price	Cash Flow
1	5.0 %	1	1,140
2	???	???	3,615
3	6.0	1	2,450
4	???	???	4,330
5	7.0	1	1,070

The first step required is to determine the unknown coupon rates and prices for terms of 2 and 4 years. To keep the example simple, we will do this by linear interpolation. The results are as follows :

Term	Coupon Rate	Price	Cash Flow
2	5.5	1	3,615
4	6.5	1	4,330

The next step is to determine the amount of par value of 5 year bonds we need to buy to produce a cash flow of 1070 in year 5. Clearly $1000 of 5-year bonds will produce a maturity value of $1000 plus a coupon of $70 to give us the desired total. (Note we are assuming annual coupons to keep the example simple.) This bond will also produce cash flows of $70 in years 1 to 4. The following table shows our original cash flows and a set of modified cash flows which are the original ones reduced by the cash flows on the $1000 of 5-year bonds.

Term	Cash Flow	5-Year Bond Cash Flow	Modified Cash Flow
1	1,140	70	1,070
2	3,615	70	3,545
3	2,450	70	2,380
4	4,330	70	4,260
5	1,070	1,070	0

The previous step can now be repeated for the last non-zero cash flow which is the $4260 in year 4. A 4-year bond with a par value of $4000 will produce a maturity value of $4000 and a coupon of $260 giving the desired total. In addition, this bond will produce coupons in years 1 to 3 of $260. Deducting these cash flows from the modified cash flows in the previous table, we get :

Term	Previous Table	4-Year Bond Cash Flow	Modified Cash Flow
1	1,070	260	810
2	3,545	260	3,285
3	2,380	260	2,120
4	4,260	4,260	0
5	0	0	0

This process can be continued until we determine that the set of cash flows being analyzed can be represented exactly by the following set of bonds :

$500 of 1-year
$3,000 of 2-year
$2,000 of 3-year
$4,000 of 4-year
$1,000 of 5-year

In this simple example, we have chosen to have all of the bonds at par. The present value of the cash flows can therefore easily be determined as the sum of the above values or $10,500. To determine the impact of an alternate scenario on this value, we can easily re-calculate the price for each bond and multiply the difference in price on each bond by the par value weights **W(t)** obtained above.

This process can be generalized to situations with monthly or even daily cash flows. However, if we had 30 years worth of monthly cash flows, we would need to calculate 360 bond prices, do 360 multiplications and 359 additions to get a present value. This would not appear to be a big improvement over the traditional approach. However, we can introduce one more step to the process to greatly simplify the calculations. In this step we substitute the equations for the interpolated price of each bond into the equation for the present value of the cash flows. In this example, the equation for the present value of the cash flows is :

$$W(1) * P(1) + W(2) * P(2) + W(3) * P(3) + W(4) * P(4) + W(5) * P(5)$$

Where **P(t)** are the prices of the bonds and **W(t)** are the par value weights determined previously.

P(2) is a linear interpolation between **P(1)** and **P(3)** so :

$$P(2) = 1/2 * P(1) + 1/2 * P(3)$$

Similarly :

$$P(4) = 1/2 * P(3) + 1/2 * P(5)$$

We can now re-write the equation for the present value (after re-arranging some terms) as :

$$(W(1)+1/2*W(2)) * P(1) + (1/2*W(2)+W(3)+1/2*W(4)) * P(3) + (1/2*W(4)+W(5)) * P(5)$$

In the particular example shown above, we get a present value of :

$$2000*P(1) + 5500*P(3) + 3000*P(5)$$

This allows us to calculate the present value as a linear combination of the prices of only the benchmark bonds. In this example, we have only eliminated two of the terms of the formula. However, in a typical example where we might have 360 months of cash flows and 10 benchmark bonds, we can reduce the formula from 360 terms to just 10 terms.

A Real Life Example

Appendix A contains a set of asset and liability cash flows for each of 360 months. Appendix B contains a base scenario of bond rates stated in terms of coupons and yield rates for a set of benchmark bonds and 6 alternate scenarios stated in terms of yield rates. The fact that most of these bonds are at par on the base scenario is not a requirement of the method. This has been done simply to make it easier for the reader of this report to reproduce the results presented. The present values of these cash flows have been calculated using a traditional approach based on spot rates. The details of how the spot rates were calculated from the benchmark rates in the scenario are described in Appendix C. The results of the discounting of these cash flows under each of the scenarios is as follows (in thousands) :

Scenario	Assets	Liabilities	Surplus
Base Scenario	$8,778,553	$8,706,491	$72,062
Flat Increase	$8,490,304	$8,414,548	$75,756
Flat Decrease	$9,094,212	$9,026,193	$68,019
Increased Slope	$8,756,704	$8,686,261	$70,443
Decreased Slope	$8,802,943	$8,729,153	$73,790
Bump	$8,616,216	$8,545,612	$70,604
Trough	$8,948,947	$8,875,401	$73,546

The following table shows the changes with respect to the base scenario (in thousands) :

Scenario	Assets	Liabilities	Surplus
Flat Increase	$(288,249)	$(291,943)	$3,694
Flat Decrease	$315,659	$319,702	$(4,043)
Increased Slope	$(21,849)	$(20,230)	$(1,619)
Decreased Slope	$24,390	$22,662	$1,728
Bump	$(162,337)	$(160,879)	$(1,458)
Trough	$170,394	$168,910	$1,484

Using the bond manager's method, we execute the following steps :

1. *Interpolate in the table of benchmark bond rates to determine coupon rates and prices for all months from 1 to 360.* This was done using 3 cubic polynomials over the ranges 1 to 12 months, 1 year to 7 years, and 7 years to 30 years. The cubic polynomials selected are unique because there are 4 benchmark points in each range and they exactly reproduce the benchmark values.

2. *Determine the par value weights* **W(t)** *as illustrated in the simple example above.* Note that these weights represent the amount of par value of bonds of each term which must be purchased to exactly reproduce the cash flows being analyzed.

3. The present value of the cash flows can be calculated as **SUM(W(t)*P(t))** where **P(t)** are the prices of the bonds at each term.

4. We can simplify the formula in step 3) by substituting for each **P(t)** obtained by interpolation, the formula for its value in terms of benchmark prices. After some re-arranging, we get a formula for present value in the form **SUM(B(t)*P(t))** where only benchmark values are involved in the summation. Appendix D documents the development of the benchmark weights **B(t)** for terms of 1, 3, 6 and 12 months and 3 and 5 years. For the cash flows shown in Appendix A, the full set of benchmark weights is as follows (in millions) :

Term	Assets	Liabilities	Surplus
1 month	1,014.5	448.6	565.9
3 months	(516.6)	(672.8)	156.2
6 months	1,322.1	1,848.0	(525.9)
12 months	1,161.7	1,328.1	(166.4)
3 years	2,738.4	2,885.4	(147.0)
5 years	870.3	507.4	362.9
7 years	129.7	199.6	(69.9)
10 years	733.9	803.2	(69.3)
20 years	964.7	1,051.4	(86.7)
30 years	237.0	188.9	48.1

5. Given the above benchmark weights **B(t)**, we can easily calculate the effect of an alternate interest scenario using the formula **SUM(B(t)*(P'(t)-P(t)))** where only benchmark bonds are involved in the summation, **P(t)** is the price of each benchmark bond on the base scenario and **P'(t)** is the price on the alternate scenario.

The following table illustrates how the impact of the flat increase scenario (see Appendix B) on the present value of assets would be calculated (figures in $ millions) :

Term	Weight (2)	Base Scenario Price (3)	Alternate Scenario Price (4)	Impact on Value (2) x ((4)-(3))
1 month	1,014.5	1.000036	0.999223	(0.824)
3 months	(516.6)	1.000068	0.997635	1.257
6 months	1,322.1	1.000000	0.995145	(6.418)
12 months	1,161.7	1.000000	0.990460	(11.083)
3 years	2,738.4	1.000000	0.973444	(72.720)
5 years	870.3	1.000000	0.959696	(35.075)
7 years	129.7	1.000000	0.948045	(6.738)
10 years	733.9	1.000000	0.934677	(47.940)
20 years	964.7	1.000000	0.911494	(85.384)
30 years	237.0	1.000000	0.902085	(23.202)
Total Impact				(288.127)

Note that the value obtained using the bond manager's method, $288.127 million is only marginally different from the value obtained using the traditional approach, $288.249. The discrepancy is less than 0.1 %.

By now, the power of the method should be evident. For example, you can easily tell from the benchmark weights **B(t)** calculated above that surplus will be reduced if rates from 6 months to 3 years fall or rates from 7 years to 20 years fall or if any of the other rates rise. This conclusion would be much more difficult to draw using a traditional approach based on spot rates.

The complete set of results using the bond manager's method is presented on the following page (in thousands) :

Scenario	Assets	Liabilities	Surplus
Flat Increase	$(288,127)	$(291,832)	$3,705
Flat Decrease	$315,536	$319,588	$(4,052)
Increased Slope	$(21,897)	$(20,255)	$(1,642)
Decreased Slope	$24,442	$22,690	$1,752
Bump	$(162,354)	$(160,841)	$(1,513)
Trough	$170,423	$168,876	$1,547

Note the similarity to the results obtained using the traditional approach. The largest error is in the trough scenario where the impact on surplus is estimated to be $1,547 versus $1,484 (in thousands) using the traditional approach.

Closing Comments

The bond manager's method provides a means of analyzing the effect of a change in interest rate levels on the present value of a stream of cash flows which gives results that are not materially different from those obtained by more complex methods. In addition, it provides other advantages :

1. Once the benchmark weights, **B(t)** have been determined, the evaluation of the impact of a change in interest rate levels is a very simple calculation. In fact, because the calculation is so simple, it is fairly easy to draw conclusions about the general nature of scenarios that will cause losses or produce gains. This should help management develop a better understanding of mismatching risks and lead to better strategic decisions.

2. The determination of the benchmark weights is a relatively simple calculation and can easily be implemented on a spreadsheet. In addition these weights only need to be calculated once for a given set of cash flows and can then be used to evaluate various interest rate changes.

3. The method completely eliminates the need to calculate spot rates.

4. Since the method is based directly on price changes of benchmark bonds, it is likely to be much more easily understood by bond managers.

5. For any investment instrument, the benchmark weights can easily be calculated. This would allow a bond manager to calculate the impact on the benchmark weights of any proposed trading strategy.

6. A side benefit of this method is that we can easily define a trading strategy which will reduce the sensitivity of surplus to changes in interest rate levels to almost zero. The trades to be made are simply to sell bonds in the amount of **B(t)** at all terms where **B(t)** is positive and buy bonds in the amount of -**B(t)** at all terms where **B(t)** is negative. Because benchmark bonds are typically far more liquid than other bonds, this strategy can be implemented at less cost than most other trading strategies. Once these trades have been completed, the benchmark weights will all be zero and the portfolio will act as if it is completely cash flow matched even though it is not. However, note that this conclusion is based on the belief that prices for non-benchmark bonds can be determined using a cubic interpolation of the prices for benchmark bonds. To the extent that this is not true, there will still be some small sensitivity to changes in interest rate levels.

7. The method can very easily be applied to annual, semi-annual, quarterly, monthly or even daily cash flows.

8. The general approach used to develop the bond manager's method can be applied to other actuarial problems. Two examples are mentioned in Appendix E.

Appendix A. Asset and Liability Cash Flows

Asset Cash Flows (in $ millions, rounded)

	Month											
Year	1	2	3	4	5	6	7	8	9	10	11	12
1	737.3	184.6	130.6	149.1	142.3	194.0	139.4	233.4	298.4	162.4	143.1	141.0
2	121.5	163.7	173.3	126.2	167.2	135.4	141.5	216.3	130.3	129.3	113.8	165.0
3	158.9	101.1	83.2	115.3	106.3	125.3	115.3	167.8	111.6	120.0	0.0	53.0
4	131.8	116.1	95.3	122.7	143.8	92.5	119.0	94.0	92.4	144.4	119.9	108.1
5	123.2	54.0	69.6	104.6	133.2	89.8	55.8	127.3	110.1	81.7	79.3	39.0
6	102.4	49.8	82.2	47.1	21.5	31.8	21.1	20.1	42.3	22.6	20.3	6.5
7	19.7	20.7	49.5	24.9	23.1	34.8	13.4	20.8	33.9	49.8	14.2	23.7
8	19.5	21.2	30.4	24.4	24.3	24.6	11.1	21.9	19.2	41.9	16.0	16.6
9	26.8	12.1	15.0	12.9	20.7	24.4	12.5	54.0	21.3	19.4	18.4	28.8
10	17.7	12.6	32.8	15.1	12.4	22.1	10.5	11.6	14.1	26.6	10.4	18.4
11	16.8	18.4	24.4	13.1	20.6	34.7	13.9	5.2	11.4	21.1	9.1	45.9
12	13.3	4.8	22.5	15.7	26.3	23.9	14.0	4.9	17.5	28.1	14.2	13.5
13	10.9	4.3	22.6	14.9	12.2	13.4	9.9	7.1	25.1	26.9	16.2	25.7
14	11.0	7.6	10.2	12.6	8.6	47.0	15.8	19.4	17.1	19.4	27.0	26.3
15	12.6	9.4	20.9	11.1	8.2	17.5	20.0	23.3	48.1	17.0	4.8	10.8
16	12.4	8.6	14.9	38.2	13.6	13.1	10.2	1.7	37.0	19.3	11.8	26.7
17	11.8	8.1	60.9	24.9	5.7	22.2	5.7	14.9	62.6	46.6	7.0	11.0
18	15.3	4.9	11.9	17.1	3.1	13.8	11.4	19.5	30.9	5.0	26.9	12.4
19	25.9	8.8	10.8	19.1	8.7	13.5	29.7	6.7	12.6	45.6	7.3	8.4
20	7.2	6.5	11.6	8.3	6.9	8.8	0.0	14.9	14.9	8.7	0.0	8.3
21	7.0	6.5	7.6	6.4	1.8	10.4	1.1	8.3	111.7	11.6	0.2	13.8
22	0.2	17.7	0.0	3.1	0.3	12.6	6.3	0.0	1.0	9.1	0.9	62.6
23	0.0	3.9	7.3	16.8	12.5	5.2	0.0	2.0	6.0	8.7	0.9	5.4
24	0.0	7.7	2.0	1.1	2.5	10.4	4.2	1.7	0.9	1.6	3.4	4.2
25	8.2	3.1	2.7	0.1	5.9	1.6	5.1	4.9	11.5	3.9	0.1	8.9
26	4.0	2.1	6.1	0.0	0.0	4.3	2.4	7.4	5.4	0.1	0.0	9.1
27	0.0	0.0	4.1	3.2	2.0	10.3	0.0	1.3	0.0	39.5	5.0	5.3
28	5.9	14.8	5.3	3.8	16.9	11.6	0.0	0.0	19.8	0.0	0.0	33.5
29	4.5	46.5	2.5	0.0	0.0	7.0	2.7	0.0	0.0	0.5	2.0	94.8
30	35.2	25.5	0.0	2.8	2.4	0.0	3.4	0.0	0.0	2.4	4.6	0.0

Liability Cash Flows (in $ millions, rounded)

Year	Month 1	2	3	4	5	6	7	8	9	10	11	12
1	79.0	160.4	205.5	208.1	243.4	202.9	229.3	306.4	392.4	190.9	175.4	167.1
2	126.7	143.6	204.3	160.0	139.8	118.7	142.4	151.5	235.6	131.4	143.8	147.9
3	124.8	133.2	202.3	154.8	125.6	117.1	135.8	147.2	156.1	78.3	72.2	85.1
4	74.8	91.9	127.2	115.9	124.6	77.6	83.6	96.8	113.7	59.6	67.9	75.9
5	74.4	80.5	91.5	95.1	86.9	80.7	95.5	101.7	123.0	80.5	59.4	60.2
6	29.6	27.6	29.0	19.8	25.9	25.9	35.2	29.5	28.0	24.0	25.1	21.1
7	26.0	22.0	28.3	26.8	23.3	25.4	36.6	22.9	18.4	18.5	22.5	24.7
8	28.6	23.0	20.2	27.2	20.6	28.3	38.4	25.0	21.2	27.4	26.2	25.0
9	27.6	21.2	24.8	29.7	25.5	28.7	31.2	23.7	24.0	23.9	26.7	23.3
10	21.0	24.1	27.9	25.3	24.3	23.5	31.1	21.8	19.4	26.9	16.6	16.4
11	24.0	22.5	22.2	18.5	19.5	15.7	22.7	23.1	18.4	19.2	18.4	20.2
12	17.1	15.3	19.8	17.6	14.7	21.6	19.7	20.2	16.9	17.6	18.3	15.4
13	20.0	18.6	14.6	14.1	22.5	24.5	26.1	18.8	21.0	20.9	19.8	19.7
14	18.8	18.0	16.4	20.2	20.9	21.6	26.0	23.0	20.0	18.1	21.2	18.3
15	18.3	15.6	21.6	16.7	13.6	21.3	16.8	15.5	13.9	21.9	15.3	18.2
16	21.1	17.2	10.7	13.3	16.2	20.1	21.9	13.1	16.1	12.8	11.4	16.9
17	20.1	16.0	14.0	18.4	10.7	22.7	19.2	18.1	16.9	14.2	13.5	13.8
18	14.7	18.3	10.2	15.2	19.2	16.4	17.1	21.4	15.0	13.7	14.4	13.9
19	18.1	15.0	17.4	13.4	12.5	13.5	16.0	9.3	8.1	12.6	17.3	14.8
20	17.3	8.8	17.2	15.3	14.4	16.5	12.1	10.6	17.5	14.4	8.0	8.0
21	10.7	10.4	10.8	16.1	11.8	10.8	8.3	10.8	10.9	6.6	15.4	8.4
22	5.4	13.8	10.3	11.2	10.6	12.2	7.3	12.4	6.8	9.0	5.7	5.7
23	10.9	8.7	12.0	4.9	7.2	8.2	8.8	7.4	13.7	11.0	8.8	5.6
24	13.4	3.9	12.0	12.2	14.8	9.3	5.7	8.2	5.6	11.9	11.2	4.9
25	13.2	10.9	7.3	11.1	10.6	8.7	5.7	7.7	6.4	2.7	11.9	11.2
26	4.0	4.6	2.5	8.8	10.9	3.7	4.2	3.9	0.0	0.0	0.0	5.0
27	34.8	2.2	5.0	3.7	0.0	2.2	29.8	0.0	1.0	0.0	0.0	1.1
28	36.9	3.6	0.0	0.0	0.0	0.7	34.4	4.6	0.0	0.0	0.0	0.0
29	31.9	0.0	1.0	4.2	0.0	0.0	27.8	3.1	0.0	0.0	1.0	0.0
30	29.3	1.0	3.7	3.8	0.0	0.0	27.3	0.0	3.0	2.3	1.0	0.0

Appendix B. Interest Rate Scenarios

Months	Coupon	Base Yield	Flat Incr	Flat Decr	Slope Incr	Slope Decr	Bump	Trough
1	4.60%	4.60%	5.60%	3.60%	4.10%	5.10%	4.62%	4.58%
3	4.75	4.75	5.75	3.75	4.27	5.23	4.80	4.70
6	5.00	5.00	6.00	4.00	4.54	5.46	5.10	4.90
12	5.40	5.40	6.40	4.40	4.99	5.81	5.60	5.20
36	6.20	6.20	7.20	5.20	6.00	6.40	6.80	5.60
60	7.25	7.25	8.25	6.25	7.25	7.25	8.25	6.25
84	7.50	7.50	8.50	6.50	7.54	7.46	8.42	6.58
120	7.90	7.90	8.90	6.90	8.00	7.80	8.70	7.10
240	8.55	8.55	9.55	7.55	8.85	8.25	8.95	8.15
360	8.60	8.60	9.60	7.60	9.10	8.10	8.60	8.60

Note that coupons are semi-annual except for 1 and 3 months. The 1 month bond pays 1/12th of the stated coupon. The 3 month bond pays 1/4 of the stated coupon. All yields are semi-annual. This means that the 1 and 3 month bonds will trade slightly above par on the base yield scenario.

Appendix C. Development of Spot Rates

The spot rates for the analysis presented in this paper were developed as follows :

1. Lagrange polynomials of degree 3 were determined separately for the ranges 1 to 12 months, 1 to 7 years, and 7 to 30 years. Each range contains 4 benchmark points which allows a unique cubic to be determined for the range. These cubics were used to determine interpolated coupons and prices at 6-month intervals.

2. The 1, 3 and 6 month yield rates were taken as the spot rates for those terms converted to an annualized basis.

3. The 12-month spot rate was determined by discounting the coupon payable at six months (using the six-month spot rate) and subtracting this from the price for the 12-month bond. *The coupon plus the maturity value of the 12-month bond is then divided by the remaining* price to determine the spot rate. The calculation looks like this :

   $$.05478 = ((1+.054/2)/(1-.054/2/(1+.05/2)))^{(12/12)}-1$$

4. A similar calculation can be done for the 18-month spot rate subtracting from the price the value of the coupons payable at 6 and 12 months. The calculation looks like this :

   $$.05603 = ((1+.0552/2)/(1-.0552/2/(1+.05/2)-.0552/2/(1+.05478)))^{(12/18)}-1$$

   Note that the coupon rate of .0552 was obtained by interpolation.

5. This process can be continued until all of the annualized spot rates have been determined at six month intervals.

6. *Spot rates not determined by the above process were obtained by linear interpolation* between adjacent spot rates.

Appendix D. Development of Benchmark Weights

The yield curve is broken into 3 sections to perform the interpolations. This appendix shows two of those sections : 1 to 12 months and 13 to 84 months. The following table shows the interpolation factors applied to each benchmark bond to obtain interpolated values. The par value weights are also shown along with the interpolation factors multiplied by these weights. The sum of these items gives the benchmark weight for 1, 3 and 6 month bonds and part of the benchmark weight for the 12 month bond.

	----- Interpolation Factors ----				Par Value	Weighted Interpolation Factors			
Months	P(1)	P(3)	P(6)	P(12)	Weights	P(1)	P(3)	P(6)	P(12)
1	1.000	0.000	0.000	0.000	682.0	682.0	0.0	0.0	0.0
2	0.364	0.741	(0.111)	0.007	138.5	50.3	102.6	(15.4)	0.9
3	0.000	1.000	0.000	0.000	81.1	0.0	81.1	0.0	0.0
4	(0.145)	0.889	0.267	(0.010)	103.6	(15.1)	92.1	27.6	(1.0)
5	(0.127)	0.519	0.622	(0.013)	104.4	(13.3)	54.1	64.9	(1.4)
6	0.000	0.000	1.000	0.000	151.1	0.0	0.0	151.1	0.0
7	0.182	(0.556)	1.333	0.040	99.8	18.1	(55.4)	133.0	4.0
8	0.364	(1.037)	1.556	0.118	190.5	69.3	(197.5)	296.3	22.4
9	0.491	(1.333)	1.600	0.242	250.9	123.2	(334.5)	401.4	60.8
10	0.509	(1.333)	1.400	0.424	119.4	60.8	(159.2)	167.2	50.7
11	0.364	(0.926)	0.889	0.673	107.8	39.2	(99.8)	95.8	72.6
12	0.000	0.000	0.000	1.000	101.9	0.0	0.0	0.0	101.9
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Total (benchmark weights)						1014.5	(516.6)	1322.1	310.9*

* add to total from following table

The following table shows the same process for terms 13 to 84 months. The remainder of the benchmark weight for the 12 month bond is derived giving a total of 1,161.7 (310.9 + 850.8). In addition the benchmark weights for the 36 and 60 month bonds are calculated. The benchmark weight for the 84 month bond cannot be calculated because this requires the next section of the interpolation which is not shown here.

	----- Interpolation Factors ----				Par Value	Weighted Interpolation Factors			
Months	P(12)	P(36)	P(60)	P(84)	Weights	P(12)	P(36)	P(60)	P(84)
13	0.925	0.121	(0.059)	0.013	84.5	78.1	10.2	(5.0)	1.1
14	0.854	0.233	(0.111)	0.024	125.6	107.3	29.3	(14.0)	3.1
15	0.786	0.337	(0.157)	0.034	132.3	104.0	44.6	(20.8)	4.5
16	0.721	0.433	(0.197)	0.042	86.4	62.3	37.4	(17.0)	3.7
17	0.660	0.521	(0.230)	0.049	134.8	88.9	70.2	(31.0)	6.6
18	0.602	0.602	(0.258)	0.055	99.1	59.6	59.6	(25.5)	5.4
19	0.546	0.675	(0.280)	0.059	106.7	58.3	72.0	(29.9)	6.3
20	0.494	0.741	(0.296)	0.062	181.7	89.7	134.6	(53.8)	11.2
21	0.444	0.800	(0.308)	0.063	92.9	41.3	74.3	(28.6)	5.9
22	0.398	0.852	(0.314)	0.064	91.8	36.5	78.2	(28.8)	5.9
23	0.354	0.898	(0.316)	0.064	85.1	30.1	76.4	(26.8)	5.4
24	0.313	0.938	(0.313)	0.063	131.4	41.1	123.2	(41.1)	8.2
25	0.274	0.971	(0.305)	0.060	127.1	34.8	123.4	(38.8)	7.7
26	0.238	0.999	(0.294)	0.057	71.6	17.0	71.5	(21.0)	4.1
27	0.204	1.021	(0.278)	0.054	48.4	9.9	49.4	(13.5)	2.6
28	0.173	1.037	(0.259)	0.049	80.4	13.9	83.3	(20.8)	4.0
29	0.144	1.048	(0.237)	0.044	80.0	11.5	83.8	(18.9)	3.6
30	0.117	1.055	(0.211)	0.039	95.4	11.2	100.6	(20.1)	3.7
31	0.093	1.056	(0.182)	0.033	87.1	8.1	92.0	(15.9)	2.9
32	0.070	1.053	(0.150)	0.027	140.3	9.8	147.7	(21.1)	3.8
33	0.050	1.046	(0.116)	0.021	78.2	3.9	81.8	(9.1)	1.6
34	0.031	1.034	(0.080)	0.014	87.4	2.7	90.5	(7.0)	1.2
35	0.015	1.019	(0.041)	0.007	(23.9)	(0.4)	(24.4)	1.0	(0.2)
36	0.000	1.000	0.000	0.000	26.0	0.0	26.0	0.0	0.0

	Interpolation Factors				Par Value	Weighted Interpolation Factors			
Months	P(12)	P(36)	P(60)	P(84)	Weights	P(12)	P(36)	P(60)	P(84)
37	(0.013)	0.977	0.042	(0.007)	106.2	(1.4)	103.8	4.5	(0.7)
38	(0.024)	0.952	0.087	(0.014)	92.8	(2.3)	88.3	8.0	(1.3)
39	(0.034)	0.923	0.132	(0.021)	64.3	(2.2)	59.3	8.5	(1.3)
40	(0.042)	0.891	0.178	(0.027)	92.8	(3.9)	82.7	16.5	(2.5)
41	(0.049)	0.857	0.226	(0.033)	119.1	(5.9)	102.1	26.9	(4.0)
42	(0.055)	0.820	0.273	(0.039)	66.3	(3.6)	54.4	18.1	(2.6)
43	(0.059)	0.782	0.322	(0.044)	96.7	(5.7)	75.6	31.1	(4.3)
44	(0.062)	0.741	0.370	(0.049)	73.6	(4.5)	54.5	27.3	(3.6)
45	(0.063)	0.698	0.419	(0.054)	63.5	(4.0)	44.3	26.6	(3.4)
46	(0.064)	0.654	0.467	(0.057)	117.5	(7.5)	76.8	54.9	(6.7)
47	(0.064)	0.609	0.515	(0.060)	99.1	(6.3)	60.3	51.0	(6.0)
48	(0.063)	0.563	0.563	(0.063)	84.0	(5.3)	47.3	47.3	(5.3)
49	(0.060)	0.515	0.609	(0.064)	104.1	(6.3)	53.6	63.4	(6.6)
50	(0.057)	0.467	0.654	(0.064)	36.1	(2.1)	16.9	23.6	(2.3)
51	(0.054)	0.419	0.698	(0.063)	42.7	(2.3)	17.9	29.8	(2.7)
52	(0.049)	0.370	0.741	(0.062)	81.6	(4.0)	30.2	60.4	(5.0)
53	(0.044)	0.322	0.782	(0.059)	115.7	(5.1)	37.2	90.4	(6.8)
54	(0.039)	0.273	0.820	(0.055)	68.6	(2.7)	18.8	56.3	(3.8)
55	(0.033)	0.226	0.857	(0.049)	40.2	(1.3)	9.1	34.5	(2.0)
56	(0.027)	0.178	0.891	(0.042)	110.6	(3.0)	19.7	98.6	(4.7)
57	(0.021)	0.132	0.923	(0.034)	84.7	(1.7)	11.2	78.2	(2.9)
58	(0.014)	0.087	0.952	(0.024)	61.5	(0.8)	5.3	58.5	(1.5)
59	(0.007)	0.042	0.977	(0.013)	65.8	(0.5)	2.8	64.3	(0.9)
60	0.000	0.000	1.000	0.000	20.1	0.0	0.0	20.1	0.0
61	0.007	(0.041)	1.019	0.015	88.3	0.6	(3.6)	90.0	1.3
62	0.014	(0.080)	1.034	0.031	37.0	0.5	(2.9)	38.3	1.2
63	0.021	(0.116)	1.046	0.050	59.8	1.2	(7.0)	62.6	3.0
64	0.027	(0.150)	1.053	0.070	29.2	0.8	(4.4)	30.7	2.0
65	0.033	(0.182)	1.056	0.093	10.4	0.3	(1.9)	11.0	1.0
66	0.039	(0.211)	1.055	0.117	13.7	0.5	(2.9)	14.4	1.6
67	0.044	(0.237)	1.048	0.144	10.2	0.5	(2.4)	10.7	1.5
68	0.049	(0.259)	1.037	0.173	8.7	0.4	(2.3)	9.0	1.5
69	0.054	(0.278)	1.021	0.204	22.1	1.2	(6.2)	22.6	4.5
70	0.057	(0.294)	0.999	0.238	5.8	0.3	(1.7)	5.8	1.4
71	0.060	(0.305)	0.971	0.274	9.6	0.6	(2.9)	9.3	2.6
72	0.063	(0.313)	0.938	0.313	(11.1)	(0.7)	3.5	(10.4)	(3.5)
73	0.064	(0.316)	0.898	0.354	9.1	0.6	(2.9)	8.2	3.2
74	0.064	(0.314)	0.852	0.398	9.7	0.6	(3.0)	8.2	3.8
75	0.063	(0.308)	0.800	0.444	30.2	1.9	(9.3)	24.1	13.4
76	0.062	(0.296)	0.741	0.494	8.2	0.5	(2.4)	6.1	4.1
77	0.059	(0.280)	0.675	0.546	12.8	0.8	(3.6)	8.6	7.0
78	0.055	(0.258)	0.602	0.602	16.8	0.9	(4.3)	10.1	10.1
79	0.049	(0.230)	0.521	0.660	3.1	0.2	(0.7)	1.6	2.1
80	0.042	(0.197)	0.433	0.721	10.1	0.4	(2.0)	4.4	7.3
81	0.034	(0.157)	0.337	0.786	15.7	0.5	(2.5)	5.3	12.3
82	0.024	(0.111)	0.233	0.854	33.5	0.8	(3.7)	7.8	28.6
83	0.013	(0.059)	0.121	0.925	4.3	0.1	(0.3)	0.5	4.0
84	0.000	0.000	0.000	1.000	6.3	0.0	0.0	0.0	6.3
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Total (benchmark weights)						850.8*	2738.4	870.3	

* add to total from previous table (310.9 + 850.8 = 1161.7)

Appendix E. Other Applications

The first step of the derivation of the bond manager's method is the determination of a notional portfolio of coupon bonds which has exactly the same cash flows as the cash flows being analyzed. Clearly the present value of the cash flows being analyzed is equal to the price of the portfolio. Although this approach to calculating present value isn't new, it is not in common use in the actuarial profession where discounting using spot rates is the norm.

Using this approach to calculating present value, it is possible to state many actuarial problems in a very different (but equivalent) form. In this different form, it will often be possible to arrive at a solution to a problem more quickly and easily than might be possible using discounting with spot rates. This paper is just one example of a problem that can be solved this way.

The second step of the bond manager's method introduces an interpolation for non-benchmark prices. This step will often allow a solution derived using step one to be restated in terms of benchmark bonds alone. This may yield a much simpler solution without a material loss in accuracy. Again, this paper presents only one example of where this can be done. This appendix describes two additional applications of this approach which are being developed for implementation in the near future.

Spread Measurement in Asset Pricing

It is common in pricing assets such as mortgages and corporate loans to assess the potential profitability of the asset in terms of the difference (or spread) between the rate charged and the rate available for Government of Canada bonds of the same term trading at par. Of course, a 10-year fully amortizing loan has a very different set of cash flows than a 10-year Government of Canada bond. It is debatable whether the spread in this case has any real meaning. Furthermore, the slope of the yield curve has a very significant influence on the amount of this spread.

The approach used to develop the bond manager's method can be used here to state the amortizing loan in terms of benchmark loans which are not amortizing and which have the company's desired spread over government of Canada bonds. (Note that the non-amortizing loans are directly comparable with the bonds.) If the sum of the opening principal on the benchmark loans is greater than the opening principal on the loan in question, then the spread on the loan is adequate.

This is one simple conclusion that can be drawn. Further research should lead to others.

Group Life and Health Deposit Accounts

It is common in the Group Life and Health business to hold client funds on deposit with the intention of eventually using those funds to make claims payments. However, unless the terms of the deposit accounts are carefully chosen, a change in interest rate levels can cause the funds held on deposit to become inadequate (or overly adequate) for making the claims payments. If the business is experience-rated, this will result in a gain or loss to the client. However, in practice it is often difficult to explain this type of loss to a client. So, there can be a lot of pressure on the company to absorb losses and pass on gains.

The first step of the method outlined in this paper can be applied directly to determine a portfolio of deposit accounts of different terms with cash flows exactly equal to the anticipated claims. However, there would be a lot of deposit accounts resulting in increased administration costs and difficulty explaining the approach to the client. The second step of the method allows these deposit accounts to be re-formulated in terms of benchmark accounts with a similar sensitivity to interest rate changes. This reduces the number of accounts and simplifies communication with the client while at the same time significantly reducing the potential for gains and losses resulting from changes in interest rate levels.