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ESTIMATING ACCIDENT YEAR INCURRED LOSSES FROM CALENDAR YEAR PAID LOSSES

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An actuary is often asked to provide an estimate of incurred losses for some future accident year. When provided with historical accident year losses, development patterns, and trend factors, such an exercise can be fairly routine. Recently, however, my staff and I were called upon to estimate incurred losses for a future accident year, and the only information available was aggregate paid losses by calendar year. Following is a description of a method, based on our experience, that one can use to develop an estimate of accident year incurred losses if the only available data is aggregate paid losses by calendar year for a number of calendar years. An exhibit demonstrating the use of this method follows this description.

One can summarize the theoretical relationship that exists between calendar year paid losses and accident year incurred losses as follows:

Let A_m = Accident year m incurred losses.

$L_{i,m}$ = Accident year m incurred losses paid in year i+m.

e.g., $L_{2,6}$ is accident year 6 incurred losses paid in year 8.

$p_{i,m}$ = the portion of accident year m incurred losses paid in year i+m.

e.g., $p_{2,6}$ is the portion of accident year 6 incurred losses paid in year 8. Furthermore, one can make the simplifying assumption that $p_{i,m} = p_{i,m+n} = p_i$ for all n. That is, the loss payout pattern will not vary by accident year. This is the same assumption often made when one uses the paid loss development method to estimate ultimate losses.

t = pure premium trend, such that $L_{i,m+n} = (1+t)^n L_{i,m}$

C_m = Calendar year m paid losses.

Then (1) $p_i = (L_{i,x} / A_x)$ by definition.

(2) $A_m = L_{0,m} + L_{1,m} + \ldots = \sum_{i=0}^{\infty} L_{i,m}$

(3) $L_{i,m+n} = (1+t)^n L_{i,m}$ by definition.

(4) $A_{m+n} = L_{0,m+n} + L_{1,m+n} + \ldots = (1+t)^n A_m$

(5) $C_m = L_{0,m} + L_{1,m-1} + L_{2,m-2} + \ldots$

$= L_{0,m} + (1+t)^{-1} L_{1,m} + (1+t)^{-2} L_{2,m} + \ldots$

$= A_m[(L_{0,m}/A_m) + (1+t)^{-1}(L_{1,m}/A_m) + (1+t)^{-2}(L_{2,m}/A_m) + \ldots]$

$= A_m[p_0 + (1+t)^{-1} p_1 + (1+t)^{-2} p_2 + \ldots]$

(6) $= A_m \sum_{i=0}^{\infty}[(1+t)^{-i} p_i]$

Similarly,

(7) $C_{m+n} = A_{m+n} \sum_{i=0}^{\infty}[(1+t)^{-i} p_i]$

(8) $= (1+t)^n A_m \sum_{i=0}^{\infty}[(1+t)^{-i} p_i]$

(9) $= (1+t)^n C_m$

Now, dividing both sides of (9) by A_m yields

(10) $C_{m+n}/A_m = (1+t)^n C_m/A_m$

$= (1+t)^n A_m \sum_{i=0}^{\infty}[(1+t)^{-i} p_i]/A_m$

(11) $= (1+t)^n \sum_{i=0}^{\infty}[(1+t)^{-i} p_i]$

Now, rearranging (11) implies that

(12) $A_m = C_{m+n}/\{(1+t)^n \sum_{i=0}^{\infty}[(1+t)^{-i} p_i]\}$

It is the relationship summarized in formulas (11) and (12) that one can use to estimate accident year incurred losses, given aggregate paid losses by calendar year. Formula (12) is particularly important. It means that, given the aggregate paid losses for <u>any</u> calendar year (C_{m+n}), a pure premium trend factor (t), and a loss payout pattern (p_i), one can develop an estimate of the accident year incurred losses for a specific accident year (A_m). In Section IV of the exhibit, the quantity $(1+t)^n$ is referred to as the Pure Premium Trend Adjustment Factor and the quantity $(1+t)^n/\sum_{i=0}^{\infty}[(1+t)^{-i}p_i]$, which is the expected ratio of C_m to A_{m+n}, is shown on row 1.

The method described above requires three items in order to estimate A_m:

(i) aggregate paid losses by calendar year for a number of years,

(ii) pure premium trend (t), and

(iii) the loss payout pattern (p_i).

Following is a description of how one can obtain each of these items.

As mentioned earlier, the one piece of data available is aggregate paid losses by calendar year for a number of calendar years (C_{m+n} for a number of different n's). In Section I of the exhibit, it is assumed that six years of aggregate paid losses by calendar year are available. The method can be used regardless of the number of calendar years for which aggregate paid losses are available. However, the more calendar years for which aggregate paid losses are available, the greater the confidence one might feel in the result obtained from this method. After gathering the calendar year paid loss data, one next needs to obtain estimates for and p_i and t.

Given that the only available loss data is aggregate paid losses by calendar year, it is not possible to derive a meaningful estimate of p_i from the data. One can begin by considering the number of years needed for all of an accident year's claims to be closed. The number of years needed for all of an accident year's claims to be closed will vary and may depend on the line of business, company payment philosophy, legal environment, etc. One can derive estimates for each p_i ($i = 0, 1, 2, 3, . . .$) from industry wide payout patterns and insurance company payout patterns for risks with similar characteristics. Finally, one should verify the reasonableness of the selected loss payout pattern with individuals such as the client's risk manager and insurance company claims personnel. In Section II of the exhibit, a payout pattern is selected such that all claims from an accident year will be closed five years after the inception of the accident year. Now, given a payout pattern, one next needs to select a pure premium trend factor, t.

One can use formula (9) to derive an estimate for t. Note that formula (9) implies that $C_{m+n}/C_m = (1+t)^n$. Thus, one can use the calendar year paid losses to develop an estimate for t. Given several years of calendar year paid losses, one can estimate a value of t by performing exponential regression on the paid losses for the years for which calendar year paid losses are available. Given the volatility possible in calendar year paid losses, one should also compare this estimate of t to some industry value. Then, based on these two estimates, one can select a value for t. This procedure is demonstrated in Section III of the exhibit.

Now, with actual aggregate paid losses for a number of calendar years, estimates for p_j and t, and the relationship between calendar year paid and accident year incurred losses derived in equation (11), one can develop several estimates of the accident year incurred losses for a specific accident year (A_m). This is done in Section IV of the exhibit for accident year 0. Note that one can obtain as many estimates of the incurred losses for the specific accident year as one has years of calendar year paid losses. Also, observe that in Section IV of the exhibit, one uses formula (11), with m=0.

Now, based on the estimates of the incurred losses for the specific accident year, one needs to select a single estimate. One could use a variety of techniques for this process. In Section IV, row 4 of the exhibit, a value slightly less than the average was judgmentally selected in order to reflect the lower level of the aggregate paid losses by calendar year for the more recent years.

Finally in Section V of the exhibit, given the selected estimate of A_m, one can use the relationship set forth in equation (4) to estimate the accident year incurred losses for other accident years.

While this method could no doubt be embellished and enhanced, it should provide a basic foundation from which one can estimate incurred losses for any accident year if the only available information is aggregate paid losses by calendar year.

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I. Calendar Year Paid Losses

Calendar Year(m + n)	C_{m+n}
0	1,200,817
1	1,732,582
2	2,029,849
3	1,789,535
4	1,682,467
5	1,461,680

II. Accident Year Payout Pattern

Year (i)	p_i
0	0.350
1	0.250
2	0.200
3	0.150
4	0.050

III. Pure Premium Trend

** Estimate based on regression on logs of calendar year paid losses	2.2%
** Industry	6.0%
Selected Pure Premium Trend (t)	5.0%

IV. Estimation of A_m (m = 0)

Pure Premium Trend Adusiment Factor		Accident Year	Calendar-Year (m + n) 0	1	2	3	4	5	6	7	8	9	10
$(1+t)^{-4}$	0.8227	−4	0.0411										
$(1+t)^{-3}$	0.8638	−3	0.1296	0.0432									
$(1+t)^{-2}$	0.9070	−2	0.1814	0.1361	0.0454								
$(1+t)^{-1}$	0.9524	−1	0.2381	0.1905	0.1429	0.0476							
$(1+t)^{0}$	1.0000	0	0.3500	0.2500	0.2000	0.1500	0.0500						
$(1+t)^{1}$	1.0500	1		0.3675	0.2625	0.2100	0.1575	0.0525					
$(1+t)^{2}$	1.1025	2			0.3859	0.2756	0.2205	0.1654	0.0551				
$(1+t)^{3}$	1.1576	3				0.4052	0.2894	0.2315	0.1736	0.0579			
$(1+t)^{4}$	1.2155	4					0.4254	0.3039	0.2431	0.1823	0.0608		
$(1+t)^{5}$	1.2763	5						0.4467	0.3191	0.2553	0.1914	0.0638	
$(1+t)^{6}$	1.3401	6							0.4690	0.3350	0.2680	0.2010	0.0670

	0	1	2	3	4	5	Average
1. Expected ratio of C_{m+n} to A_m	0.9402	0.9872	1.0366	1.0884	1.1428	1.2000	
2. C_{m+n}	1,200,817	1,732,582	2,029,849	1,789,535	1,682,467	1,461,680	
3. Estimate of A_m (m = 0) (1 + 2)	1,277,177	1,755,007	1,958,211	1,644,169	1,472,189	1,218,092	1,554,141
4. Selected estimate of A_m (m = 0)	1,500,000						

V. Estimation of A_{m+n}

Using formula (4): $A_{m+n} = (1+t)^n A_m$

(m = 0 from Section IV above)

n	$(1+t)^n$	A_m	A_{m+n}
−2	0.9070	1,500,000	1,360,544
−1	0.9524	1,500,000	1,428,571
0	1.0000	1,500,000	1,500,000
1	1.0500	1,500,000	1,575,000
2	1.1025	1,500,000	1,653,750