



SOCIETY OF ACTUARIES

Article from:

# The Actuary

January 1990 – Volume 24, No. 1

# Proof that $1 = -1$ using Makeham's Law

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It is well known in actuarial circles that  $1 = -1$ , but rigorous proof of this deceptively simple equality has seldom been attempted. A proof using Makeham's Law of mortality is presented in hopes of stimulating discussion.

Makeham's Law can be written  $\mu_x = A + Bc^x$ . Note that, mathematically, negative ages are as valid as positive ages. This leads one to write:

$$\textcircled{1} \quad 1 = -1 - \frac{2(C^{40} - C^{39})}{C^{79}(C^{-40} - C^{-39})} = -1 - \frac{2(\mu_{40} - \mu_{39})}{B}$$

A useful property of Makeham's Law is that a joint life status consisting of any two ages is equivalent to a joint life status with equal ages. The equal age is somewhere between the original ages. Assume a Makeham table has been constructed so that  $\bar{A}_{41:41} = \bar{A}_{40:42}$ . This implies that:

$$\textcircled{2} \quad \mu_{41+t} + \mu_{41+t} = \mu_{40+t} + \mu_{42+t}$$

For the proof, proceed as follows:

$$\frac{C^{40}}{C^{39}} = \frac{C^{41}}{C^{40}}$$

$$\frac{\mu_{40} - A}{\mu_{39} - A} = \frac{\mu_{41} - A}{\mu_{40} - A}$$

$$(\mu_{40} - A)^2 = (\mu_{39} - A)(\mu_{41} - A)$$

$$\mu_{40}^2 - 2A\mu_{40} + A^2 = \mu_{39}\mu_{41} - A\mu_{39} - A\mu_{41} + A^2$$

$$\mu_{40}^2 = A(\mu_{40} + \mu_{40}) + \mu_{39}\mu_{41} - A(\mu_{39} + \mu_{41})$$

$$\mu_{40}^2 = \mu_{39}\mu_{41} \text{ (using } \textcircled{2} \text{ with } t = -1)$$

$$\mu_{40}^2 = \mu_{39}(2\mu_{40} - \mu_{39}) \text{ (again using } \textcircled{2} \text{ with } t = -1)$$

$$\mu_{39}^2 - 2\mu_{39}\mu_{40} + \mu_{40}^2 = 0$$

$$(\mu_{39} - \mu_{40})^2 = 0$$

$$\mu_{39} = \mu_{40}$$

$$1 = -1 \text{ (by substitution into } \textcircled{1})$$

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## Coffee cont'd

Instead of allowing the Vietnamese to get the best of them, Coffee said the POWs continued to follow the American Fighting Man's Code of Conduct to the best of their ability, even though they were ordered not to follow those rules. They tapped messages to each other on their cell walls and learned foreign languages and even poetry through that method.

Poetry brought new meaning to their circumstance, Coffee said, because of the beauty and strength it provided. "We locked onto the verse in the poem, 'If,' that says, 'If you can force your heart and nerve and sinew to serve their term long after they are gone and yet hold on when there is nothing left within you except the will that says to them hold on, hold on.'"

Coffee cited volunteerism and leadership as an example of "faith in ourselves," and said the Society is a perfect example of both.

The second aspect of faith, faith in one another, also is of great importance. "Our motto there in the prison system was very simple," Coffee said. "Unity over self. (That's) not a bad corporate motto or association motto." He said strength lies in unity and togetherness, and that is why the Viet-

namese kept the POWs separated in isolation. "We weren't allowed to communicate with each other and if you're caught communicating with another American, you're punished severely. But we communicated all the time anyway."

Coffee asked why people now make communicating so much more difficult than it needs to be. "We don't have to tap on the walls from the bedroom to the kitchen, or from office to office in our workplaces," he said. "(We need) faith in one another professionally, really believing that every other man or woman is doing his or her best to maintain the same standards of professionalism, quality, dedication that have seen your profession through a hundred years now."

The Vietnamese made the third aspect of faith, faith in your country, a bit difficult to follow, Coffee said. Every day while he was kept prisoner, he was forced to listen to everything that was bad about the United States through loudspeakers located in each cell. "And I'll tell you, after four, five, six years of that overwhelming wave of negative propaganda about our country and our cause, you'd have to say to yourself, 'Wait a minute, don't believe that junk. You're an American,

you've lived there, that's your home. This isn't the place to change your mind, dummy. Keep faith."

The final aspect of faith, faith in God, is possibly the foundation for it all, Coffee said. When first imprisoned, he found two English words scratched on the wall of his cell: God = strength. "I was never, ever totally alone," he said. "I could always find just a little bit more strength when I needed it."

Upon his return to the United States in 1973, Coffee said he was overwhelmed by the greeting he received from friends as well as strangers. They told him how proud they were of his patriotism. He told his audience they should feel proud because they had a part somehow in the outcome of Coffee's return and the spirit that helped guide him through his experience. That's because, he said, "We are so much alike. I'm from you, you know, we are all the same clay and spirit, we derive our strengths from the very same sources. All those years the Vietnamese tried to break our spirit, our confidence, our faith and they couldn't do it. And the reason that they couldn't (is because) in great part right here before me today, truly every one of you."