A New Risk Metric for Defined Benefit Pension Plans

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Presented at
Enterprise Risk Management Symposium
Society of Actuaries

Chicago, IL

April 23–26, 2006

Abstract

In order for actuaries to play a valuable role in the realm of enterprise risk management (ERM), they must provide value-added advice and quantitative analyses. In this paper, the authors use stochastic simulation technology to present a risk metric for defined benefit pension plans that provides improved measures of the plan's solvency and provides a tool for pension plan managers to determine the value of risk mitigation activities.

1. Introduction

The ERM framework presented in this paper was recently developed for the pension plan of a large tax-exempt religious organization in the United States, so many accounting and ERISA issues were not important considerations. These constraints add substantial complexity to the analysis of pension plans, and consequently, it has been helpful in developing an ERM framework for defined benefit (DB) pension plans to begin in a simplified environment. From this framework, one can then move on to consideration of such issues as ERISA compliance, Generally Accepted Accounting Principles (GAAP) and the pressure of quarterly corporate reporting.

An ERM process should include two components:

1. identification and management of specific risks; and
2. risk quantification.

There are a number of institutions that have produced lists of specific risks, such as the Committee of Sponsoring Organizations of the Treadway Commission (COSO). For our framework, we used the American Academy of Actuaries' adaptation for Life Insurance Risk Mapping of the U.S. Federal Reserve Risk Categorization.

This paper will focus on a risk quantification methodology that helps pension plan stakeholders in understanding the solvency of a pension plan and provides risk managers with a tool for evaluating risk mitigation activities.

As life insurance consulting actuaries, our initial response when this pension plan asked for assistance in measuring risk was to explain the concept of capital. The more risk there is, the more capital there should be. But capital is an alien concept for a pension plan, and any risk measures submitted on this basis might seem inappropriate. For a pension plan, there is really no difference between capital and funding—it is all fungible within the pension trust. [We will leave aside the legal issue of who owns the surplus of a DB plan.] Moreover, for a church plan, the more funds put into the pension plan, the less is available to do the good works of the sponsoring organization.

The primary risk for a DB pension plan is the question of its ability to meet its payment obligations. To measure this risk, one would have to take into account the future viability of the pension sponsor, but that would make the whole quantitative process too subjective, complicated and unreliable. A more realistic measure of risk would be to calculate the probability of meeting the pension obligations taking into

account the plan's existing trusteed assets, and potentially, future contributions from the plan sponsor.

It is interesting to note that calculations of liabilities do not figure prominently in this analysis. That is because the liability value may be useful for accounting purposes, but it is not helpful for risk management purposes. Our view is that the obligation of a pension plan is the stream of contractual payments. Too much noise is added when this obligation is translated into a single number.

This can be demonstrated by use of a simplified example of a lump sum obligation of $1 million payable in 30 years. Funding will be a single contribution at time zero. There is not a clear consensus as to how to report this obligation as a liability on financial statements. A property and casualty insurance company would record the obligation as $1 million because that industry doesn't favor discounting to reflect the time value of money. By contrast, a U.S. life insurance company would record the liability as $200,644, using a discount rate of 5.5 percent as required by the NAIC for 2005. And a corporate pension plan might report a liability as $114,221 based on a discount rate of 7.5 percent.

Taking a different tack, we could ask the plan sponsor's chief investment officer (CIO) how much cash she would want to invest in order to meet this lump sum obligation. Let's assume her answer is $75,371, because she is calculating that the funds could be invested in a diversified stock portfolio which would earn an average annual return of 9 percent over the next 30 years. But what if another criterion is added—being 100 percent certain of having enough funds to pay the obligation in 30 years? Then, she would want $286,892 to invest—the cost of a 30-year zero coupon government bond.

From a risk management viewpoint, the objective isn't getting the liability right—five different numbers could be presented—but rather determining the probability of satisfying the obligations given a specified funding amount and a specified investment strategy.

Initial Contribution	Probability of Success
$75,371	75% (if stocks earn 9.0%)
$286,892	99.99%

The 75 percent probability is determined through a stochastic simulation that assumes a 9 percent average annual return over the complete set of simulation results and also assumes that the variability of the stock market, as measured by the Standard & Poor's index, continues at historical levels. The 99.99 percent is a guess to

demonstrate that there is only a very small chance that the zero coupon government bond would default and that there is no interest rate risk, reinvestment risk or C3 risk.

To assist pension trustees, CFOs and risk managers make risk adjusted decisions that make sense from both an economic and actuarial standpoint, we propose a risk metric that measures the probability of success in satisfying the pension obligations.

While a single probability value has as little value as a single liability value, the proposed risk metric is the cumulative distribution function of the calculated surplus or deficit of the quantum of assets backing the pension liability. The measure of both assets and liabilities will be determined by their respective cash flows, and the surplus or deficit would be calculated as follows:

$$\sum_{t=s}^{\infty}(a_t - b_t)v^t$$

where a_t = expected asset cash flow at time t

b_t = expected benefit cash flow at time t

v^t = discounted interest rate at the risk-free yield curve

s = start time for the calculation.

Further discussion of "expected asset cash flows" is required. For ease of classification, we will define all investment vehicles with contractual payments as the fixed-income investment class and those that don't have contractual payments as the equity investment class. For the equity class, the expected cash flow will be defined as the market value of the investment vehicle at time of sale.

For the fixed income investment asset class projected asset cash flows are well understood. Most asset management systems will produce such cash flows (even taking into account defaults) for even the most esoteric debt instruments, such as collateralized mortgage obligations or interest rate swaps.

For the equity investment class, obviously common stocks easily fit into this category, but so do real estate, private equity and hedge funds. The difficulty for the actuary is deriving realistic future simulations of market values for these investment vehicles.

For the equity investment class, the algorithm will assume a sale at time t when $a_t^{FI} < b_t$, for the number of units of the equity class such that the market of those units equals $b_t - a_t^{FI}$ where $a_t^{FI} =$ fixed income expected cash flow at time t.

To illustrate the surplus/deficit risk metric, we will continue to use the 30-year lump sum example:

TABLE 1
Surplus/Deficit Risk Metric

Percentile	Year 0	Year 10	Year 20	Year 30
10.0%	858,636	1,318,608	2,128,347	3,777,817
25.0%	448,038	648,514	1,087,550	1,971,275
50.0%	171,292	228,667	396,096	753,650
75.0%	7,585	9,116	17,109	33,374
90.0%	(83,270)	(114,591)	(194,198)	(366,370)
95.0%	(120,979)	(181,862)	(289,916)	(532,284)
99.0%	(167,232)	(285,817)	(442,732)	(735,784)
99.5%	(176,563)	(320,368)	(478,432)	(776,841)
99.9%	(201,720)	(398,896)	(550,977)	(887,524)

Cumulative Distribution Function (CDF)

Looking at the formula for the surplus/deficit risk metric, one sees that the risk metric is a function of the investment strategy. The CDF is created using stochastic simulation. This CDF was created using the American Academy of Actuaries' published scenarios and running a simulation of all 10,000 scenarios of the S&P 500 investment class with a 1.5 percent parallel shift to get an average annual return of 9.0 percent per annum. The risk-free yield curves are also simulated using the AAA scenarios.

The CDF is presented at different times in the future in order to provide an understanding of how both assets and liabilities could change in value, yet the pension obligations are still satisfied. A positive value in the chart represents excess funds after all pension obligations are met, discounted to the specific point in time. A negative number represents the additional funds that would have to be contributed at that time in order to satisfy the pension obligations.

In this simulation, since we started with equities worth $75,371, there is a 75 percent chance that in year 30, when the lump sum payment is required to be paid, we will have sufficient assets to make that payment. According to the simulation, there is a 25 percent chance that the pension sponsor will have earned at least another $1,971,275, but there is also a 5 percent chance the sponsor will have to make an additional

payment of at least $532,284 to satisfy the obligation and a 1 percent chance that the additional payment will be at least $735,784.

We believe there need to be subsidiary metrics to better understand this primary risk metric. These subsidiary risk metrics are:

1. market value of assets used in the surplus/deficit risk metric;
2. a measure of the liabilities used in the surplus/deficit risk metric; and
3. CDF of ratio of asset/liabilities.

2. Assets

For the presentation of assets, we use the market value of assets at the specified time.

TABLE 2
Assets

Percentile	Year 0	Year 10	Year 20	Year 30
10.0%	75,371	466,652	1,787,668	4,777,817
25.0%	75,371	319,015	1,311,918	2,971,275
50.0%	75,371	199,418	772,540	1,753,650
75.0%	75,371	169,615	509,175	1,033,374
90.0%	75,371	146,410	238,328	633,630
95.0%	75,371	148,931	204,294	467,716
99.0%	75,371	87,976	97,537	264,216
99.5%	75,371	76,076	110,967	223,159
99.9%	75,371	57,007	88,672	112,476

These values represent the market value of assets underlying the surplus/deficit risk metric from Table 1. In this example, the plan was funded with a contribution of $75,371. This table shows there is 10 percent chance that after 30 years, these assets could grow to at least $4,777,817, leaving a surplus of $3,777,817. There is also a 5 percent chance that the starting assets will grow to no more than $467,716, leaving a deficit of at least $532,284.

Remember that the asset values listed in the table are not a CDF of possible asset values, but rather the market value of assets underlying the surplus/deficit CDF. That is why in year 10, for example, the asset value referenced at the 90th percentile ($146,410) is less than the asset value referenced at the 95th percentile ($148,931). What will have caused this seeming inconsistency is the changing value of the liabilities caused by changing yield curves.

3. Liabilities

For the presentation of liabilities, we use the present value (PV) of all future pension payments calculated at the then current risk-free yield curve according to the specific scenario (i.e., the fair value).

TABLE 3
Liabilities

Percentile	Year 0	Year 10	Year 20	Year 30
10.0%	227,284	323,710	635,749	1,000,000
25.0%	227,284	322,053	718,088	1,000,000
50.0%	227,284	300,156	676,047	1,000,000
75.0%	227,284	373,865	705,548	1,000,000
90.0%	227,284	453,610	499,714	1,000,000
95.0%	227,284	573,715	546,829	1,000,000
99.0%	227,284	498,512	439,015	1,000,000
99.5%	227,284	490,978	590,106	1,000,000
99.9%	227,284	480,672	615,564	1,000,000

These reported liabilities relate to the specified percentiles of the surplus/deficit CDF. This explains why the liability values are not always increasing with the increasing percentiles.

This table is more important to prepare pension plan stakeholders for the range of possible reporting values in the future, as opposed to eliciting meaningful management initiatives. because we know for sure $1 million will have to be paid in year 30, and that figure does not change over the course of the 30 years.

4. Ratio of Asset/Liabilities

A final subsidiary metric that should assist pension fiduciaries is a CDF of the asset/liability ratio. This is the ratio of the market value of assets at time *t* to the PV of liabilities discounted by the risk-free yield curve prevailing at time *t*. This metric is valuable in providing information as to the variability of two forces: asset growth and change in the fair value of the pension obligation.

TABLE 4
Asset/Liability Ratio

Percentile	Year 0	Year 10	Year 20	Year 30
10.0%	33.2%	144.2%	281.2%	477.8%
25.0%	33.2%	99.1%	182.7%	297.1%
50.0%	33.2%	66.4%	114.3%	175.4%
75.0%	33.2%	45.4%	72.2%	103.3%
90.0%	33.2%	32.3%	47.7%	63.4%
95.0%	33.2%	26.0%	37.4%	46.8%
99.0%	33.2%	17.6%	22.2%	26.4%
99.5%	33.2%	15.5%	18.8%	22.3%
99.9%	33.2%	11.9%	14.4%	11.2%

At time zero, there is no variance in the ratio because the assets are equal to the contribution of $75,371, and the liability value of $227,284 is calculated using the initial risk-free yield curve.

The percentages presented are the asset/liability ratios at specified times in the future. For example in year 10, there is a 25 percent chance that the ratio will be about 100 percent. This means that there is a 25 percent chance that by year 10, the equity assets will grow to a high enough level to permit converting them into a risk-free bond and thereby perfectly hedging the obligation. Note that in year 30, the ratio must be in excess of 100 percent to satisfy the pension obligation.

While the asset and liability values on those subsidiary metrics were reported on the basis of the surplus/deficit percentiles, this asset/liabilities ratio is a CDF based on its own values.

It is interesting to note that an asset/liability ratio of 33 percent is sufficient to meet the obligation, with 75 percent probability. This is the case because the assets are assumed to grow at an average annual rate of 9.0 percent, and the liabilities are assumed to grow at an average rate of 5.06 percent at time zero. (This can be viewed as a form of equity arbitrage in which the downside of relying on equity investments to pursue higher returns is the increased probability of not being able to meet the obligation because of variability of equity returns.)

5. Case Study

We have created a test portfolio of 10,000 payout annuities over various ages and various guarantee periods. The inforce block has starting monthly income of $5.3 million. On a PV basis at a level 6.5 percent, the PV of expected benefit payments is $717.9 million. (The PV of expected benefits based on the risk-free yield curve is $843.0 million). For demonstration purposes, the $717.9 million of assets will be invested in two funds:

35% Domestic Bond Fund
65% S&P 500 Index Fund

Both funds will be considered equity class investments. Sales of assets to meet benefit cash flows will be done in the same 35%/65% ratio.

The results of surplus/deficit risk metric are as follows:

TABLE 5
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Surplus/Deficit Risk Metric

Percentile	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Year 10	Year 20	Year 30
10.0%	2,411,749	5,418,316	6,297,945	6,564,935	6,666,571	6,697,916	5,796,191	5,418,341	7,925,778
25.0%	817,243	1,631,384	1,905,628	2,029,073	1,909,499	1,934,021	1,763,490	1,631,408	2,789,151
50.0%	65,354	131,966	148,078	150,223	142,059	157,037	114,919	131,993	303,114
55.0%	2,283	7,457	5,268	5,650	6,867	5,971	5,066	7,491	9,264
56.0%	(4,456)	(5,524)	(7,106)	(6,193)	(5,805)	(5,557)	(8,231)	(9,509)	(18,059)
75.0%	(105,668)	(119,910)	(126,854)	(131,463)	(132,705)	(134,985)	(174,895)	(185,103)	(47,101)
90.0%	(216,168)	(228,481)	(237,950)	(246,951)	(253,414)	(270,618)	(337,467)	(215,738)	(51,143)
95.0%	(262,369)	(288,995)	(302,058)	(312,121)	(325,343)	(329,613)	(419,588)	(225,594)	(52,915)
99.0%	(327,053)	(364,485)	(390,535)	(416,890)	(444,303)	(452,866)	(542,440)	(243,877)	(56,489)
99.5%	(344,704)	(392,912)	(425,004)	(436,711)	(465,945)	(493,848)	(570,264)	(248,801)	(57,032)
99.9%	(371,958)	(452,022)	(477,091)	(497,533)	(501,583)	(554,374)	(601,773)	(256,347)	(58,509)

The first thing to note is that we have expanded the number of time periods presented relative to our previous example. This has been done because it is important to show the early years in order to manage the expectations of pension plan fiduciaries as to the possible range of near-term fluctuations. These early year results are also important in managing liquidity risk.

The results show there is only a 55.4 percent chance of meeting the benefits obligations. This percentage does not change by time period because all calculations are on a prospective basis. In year 30, the range of possible surpluses or deficits extends from a surplus of $7.9 billion to a deficit of $58 million. This broad range shows the difficulty of the risk management function because lowering the mismatch risk means giving up the potential for very big gains. This new risk metric provides pension plan management with a better analysis of the true costs of risk mitigation.

The deficits presented in this table for future years can be misleading in that they are based only on future cash flows. This model assumes that the plan would borrow money to meet its obligations when all assets are depleted, and we decided to show the full extent of the downside risk in the assets metric.

The results of the asset valuations backing the surplus/deficit risk metric are as follows:

TABLE 6
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Assets

Percentile	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Year 10	Year 20	Year 30
10.0%	717,890	788,671	857,254	937,578	958,514	1,282,828	1,319,140	1,536,431	2,789,010
25.0%	717,890	732,059	702,197	543,097	678,520	998,715	890,111	773,203	1,503,148
50.0%	717,890	658,844	796,133	595,829	622,903	865,154	457,505	308,506	111,127
55.0%	717,890	760,329	623,852	553,562	791,630	694,445	473,095	225,801	46,237
56.0%	717,890	741,688	612,645	585,754	751,526	416,464	500,918	220,620	24,196
75.0%	717,890	769,229	653,290	479,295	709,449	395,621	246,905	(307,026)	(1,516,194)
90.0%	717,890	635,195	718,958	433,874	389,256	482,100	237,568	(460,802)	(133,254)
95.0%	717,890	648,082	736,955	435,411	400,491	431,297	104,881	(487,693)	(2,116,422)
99.0%	717,890	551,977	405,675	466,963	297,195	318,206	62,791	(102,059)	(703,279)
99.5%	717,890	510,461	488,380	397,076	199,289	274,807	(8,829)	(337,558)	(370,139)
99.9%	717,890	397,121	449,851	449,708	402,145	341,065	(24,319)	(98,295)	(1,717,585)

This table presents the market value of assets underlying the surplus/deficit risk metric. It shows the initial assets of $717.9 million have a 10 percent chance of growing to at least $2.8 billion over the 30 years after deducting the required pension payments.

As noted above, if the assets are depleted, the system will begin a borrowing program (with assumed borrowing costs of 10 percent per annum). The reason for this design is to ensure proper understanding of the full deficit position in the surplus/deficit risk metric. Since the risk metric is calculated on a prospective basis, it doesn't take into account the missed pension payments that would actually have occurred once the assets were depleted.

It is important to note that these asset values are calculated on a market value basis, whereas in the surplus/deficit calculation the value of the assets would be determined as $\sum_{t=s}^{\infty} a_t v^t$ where s is the presentation time for future cash flows and v is the discounted risk-free yield curve. This methodology will usually produce a higher value than the market value. This difference highlights the anomaly that actuaries value equities more than the markets do because actuaries take into account future assumed values of stocks.

In the results above, for example, at the 10th percentile for year 30, the surplus is projected to be $7.9 billion. But the market value of assets at that time for that percentile is only $2.8 billion. The difference is that the surplus calculation takes into account future increases in market value that will be realized when the assets are sold. This phenomenon gets magnified in the later years when there are minimal pension payments still to be paid. We think this calculation approach is appropriate because the stocks can be held for a number of years before they need to be sold, making it possible to take advantage of time diversification. It is understood this is only true if stocks continue to provide average annual returns in excess of fixed income rates.

The results of the liability valuations—on a fair value basis—backing the surplus/deficit risk metric are as follows:

TABLE 7
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Liabilities

Percentile	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Year 10	Year 20	Year 30
10.0%	843,026	829,901	814,513	769,198	780,844	638,400	511,882	194,120	43,420
25.0%	843,027	817,194	730,455	572,166	678,653	642,280	510,874	193,456	43,530
50.0%	843,027	820,912	735,836	888,273	606,103	673,784	470,078	174,851	46,552
55.0%	843,027	852,987	766,787	589,191	794,200	617,174	314,472	185,988	41,569
56.0%	843,025	781,196	794,293	601,425	698,799	581,669	507,577	195,528	43,335
75.0%	843,029	824,425	792,700	697,920	688,959	577,646	435,712	185,103	47,101
90.0%	843,028	911,250	782,078	698,916	674,951	666,994	531,501	215,738	51,144
95.0%	843,024	822,047	806,732	703,945	735,592	752,649	532,345	225,594	52,915
99.0%	843,027	810,494	786,157	777,380	728,806	777,457	599,069	243,878	56,490
99.5%	843,027	893,641	930,190	833,919	733,594	751,041	570,264	248,801	57,032
99.9%	843,025	911,538	888,985	961,720	908,019	906,738	601,773	256,348	58,509

The liability cash flows were projected assuming RP-2000 Healthy Annuitant mortality. The underlying deaths of the annuitants were not simulated—the payment obligations were deterministically calculated. The differences in results are solely related to the assumed range of risk-free yield curves developed in the stochastic simulation.

The results of the asset/liability ratio metric are as follows:

TABLE 8
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Asset/Liability Ratio

Percentile	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Year 10	Year 20	Year 30
10.0%	85.2%	101.0%	110.0%	119.7%	130.4%	143.8%	229.4%	978.0%	8838.6%
25.0%	85.2%	93.0%	98.8%	103.9%	109.9%	116.3%	159.8%	478.3%	3400.6%
50.0%	85.2%	86.3%	86.7%	89.4%	90.9%	91.2%	101.7%	140.5%	461.9%
55.0%	85.2%	84.9%	84.8%	86.6%	87.0%	86.8%	94.0%	98.3%	104.6%
56.0%	85.2%	84.7%	84.4%	85.9%	86.1%	86.1%	91.5%	89.9%	55.8%
75.0%	85.2%	79.0%	77.0%	75.1%	73.8%	72.1%	59.8%	0.0%	0.0%
90.0%	85.2%	72.9%	67.4%	64.1%	60.5%	56.1%	31.4%	0.0%	0.0%
95.0%	85.2%	68.3%	62.8%	57.6%	53.8%	48.5%	16.7%	0.0%	0.0%
99.0%	85.2%	59.3%	52.4%	46.9%	43.8%	37.3%	0.0%	0.0%	0.0%
99.5%	85.2%	57.4%	50.5%	44.7%	38.0%	33.1%	0.0%	0.0%	0.0%
99.9%	85.2%	43.6%	45.1%	37.8%	27.2%	25.6%	0.0%	0.0%	0.0%

A ratio greater than 100 percent means the pension plan could achieve perfect cash flow matching because there are enough assets to buy the risk-free yield curve. A ratio of zero means the assets of the pension plan have been exhausted.

6. Scenario Testing

The power of this methodology lies in its ability to test different assumptions in order to arrive at an improved risk profile.

Three scenarios were tested:

1. Liquidate the bond fund and use the proceeds to cash flow match the early durations.
2. Liquidate the bond fund and part of the equity fund to cash flow match the first seven years.
3. Increase the funding by 15 percent but maintain the same asset allocation.

In the first scenario, by liquidating the bond fund and using the proceeds to cash flow match the early durations, the probability of meeting all pension obligations is increased to 59.6 percent from 55.4 percent. (See Table 1.1 in the appendix. Note that the full results of testing all of these scenarios, which are summarized in the text, are

presented in full in the appendix.) The assets have a 10 percent chance of being at least $5.93 billion (see Table 1.2) in year 30 ($2.79 in base case) with a 5 percent chance of being short by at least $0.94 billion ($2.12 in base case). The reason for this improvement is that there is increased time diversification on the equity portfolio since no equity assets have to be sold for 4.25 years [the period of benefit cash flows that can be matched with the proceeds of the bond fund].

In the second scenario, the cash flow match is extended from 4.25 years (where it was in the previous scenario) to seven years. The additional funds come from the sale of a portion of the equity fund. This investment strategy actually reduces the probability of meeting all pension obligations to 56.2 percent (see Table 2.1) from the previous 59.6 percent, and it also decreases the upside to a 10 percent chance of being at least $3.43 billion in year 30 (see Table 2.2) from $5.93 billion in previous scenario. But the downside risk has been reduced. There is a 5 percent chance of being out $0.64 billion (from a shortfall of $0.94 billion in previous scenario). The improved cash flow match of Scenario 2 has actually reduced the likelihood of meeting the pension obligations by 3.4 percent, reduced the upside potential by $2.49 billion with a reduced downside of $0.30 billion. By lowering the mismatch risk, the plan has actually given up some of the opportunity to earn higher investment returns because it has sold some of the equity assets.

The results of this sensitivity testing demonstrate both the power of the tool but also a potential weakness. The results show that the investment strategy of the base case would be improved by cash flow matching in the early years instead of using a bond fund. This can be seen from the fact that Scenario 1 has improved the probability of success while offering a higher upside and lower downside. The sensitivity testing also shows that cash flow matching for too many years is actually counter-productive because the pension plan loses the potentially higher reward that can come from holding equity investments.

It is clear that one criticism of the tool is that the analysis hinges on the assumption that future investment returns on various asset classes will be similar to historical norms. In all the underlying presentation of probabilities involving a diversified stock portfolio, for example, we have used the AAA stochastic simulations which have a "bias" of 7.5 percent/ average annual returns. This methodology does not measure the risk of being wrong in that assumption. Obviously there would be great distress if equities don't live up to expectations. It should be the responsibility of the actuary to show to the pension fiduciaries in advance what would happen to the solvency of the plan if equities produce lower returns.

Finally, in the third scenario, we show the value of additional plan sponsor contributions in increasing the likelihood of meeting all pension obligations. We increased the assets to $780.8 billion from $717.9 billion. This new asset level was determined by discounting all future pension cash flows at 5.5 percent rather than 6.5 percent. The original asset allocation was retained. With these additional assets, the probability of meeting all pension obligations is 69.4 percent (compared to 55.3 percent in the base case). This probability could be further increased by cash flow matching in the early years.

These scenarios demonstrate the power of this tool in providing pension plan managers with analytics that can be custom-tailored to their own situation in order to show the upside and downside of specific risk mitigation tactics. These additional analytics don't necessarily make the decision easier, but they do make it more disciplined.

7. Further Research Required

There are several areas in which additional research would be beneficial:

1. Active life funding
   Include active lives and the new contributions they bring. This provides the plan sponsor with a new risk mitigation opportunity and generates new cash for the plan, but it also adds a new pricing risk by raising the issue of whether the new obligations can be satisfied by the new funding.

2. Capital market solutions
   As can be seen in Table 5, there is a non-symmetrical result curve. In year 30 there is a 10 percent chance of being $7.9 billion overfunded with a 5 percent chance of having a $2.1 billion deficit (as measured by the asset metric in Table 6). Is there a capital market solution to trade that upside for downside protection?

   This is where the adherents of financial economics may get their revenge because it is our guess that if you are able to execute such a transaction it will likely cost the pension plan so much that the returns will be lowered to effectively a risk-free rate.

3. Modifications for corporations

The plan sponsor presented in this report was a tax-exempt religious organization. No assumption was made as to how any asset insufficiency in its pension plan would be satisfied. In the case of a corporate pension plan, the resources of the corporation would be available to the retirees. In this setting, modeling should take into account two elements:

- Whether the funding percentage is sufficient.
- Whether the asset allocation and investment strategy is appropriate given the risk variables that could affect the health of the corporate plan sponsor.

8. Criticism of the Methodology

The most glaring weakness of the system is the reliability of the assumption of future growth in the asset classes. In all the underlying presentation of probabilities involving a diversified stock portfolio, we have used the AAA stochastic simulations which have a "bias" of 7.5 percent per annum average growth in the simulations. This methodology does not measure the risk of being wrong in that assumption. At this time what can be—and should be—presented to pension fiduciaries is scenario testing with lower assumed equity growth rates to at least put borders around the risk.

9. Summary

We believe the risk metrics described in this paper are a start at improving the quantitative analysis necessary for managing the risk of a DB pension plan.

Appendix

Results of Scenario Testing

Scenario 1

Liquidate bond fund and use proceeds to cash flow match early (4.25) years.

TABLE 1.1
('000)
Surplus/Deficit Risk Metric

Percentile	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Year 10	Year 20	Year 30
10.0%	3,161,858	7,083,337	7,640,882	8,156,794	8,510,722	8,466,621	6,978,543	7,065,187	9,999,711
25.0%	1,101,703	2,401,601	2,619,375	2,711,851	2,746,658	2,628,803	2,437,291	2,166,175	3,752,248
50.0%	140,112	306,958	342,739	339,793	327,495	340,338	280,539	293,428	755,822
59.0%	10,175	23,022	23,752	27,682	20,276	23,171	15,672	22,451	45,092
60.0%	(1,940)	(4,103)	(4,567)	(2,451)	(1,926)	(2,178)	(3,006)	(6,331)	(11,609)
75.0%	(99,110)	(113,898)	(119,060)	(122,112)	(128,337)	(126,369)	(162,991)	(184,289)	(46,308)
90.0%	(218,996)	(241,995)	(251,409)	(260,969)	(268,372)	(279,955)	(355,830)	(215,384)	(50,775)
95.0%	(278,564)	(306,831)	(317,283)	(324,879)	(333,129)	(346,686)	(442,228)	(225,407)	(52,763)
99.0%	(341,234)	(385,053)	(404,197)	(439,682)	(453,334)	(482,954)	(550,086)	(243,091)	(56,490)
99.5%	(352,587)	(414,414)	(441,620)	(454,713)	(500,612)	(532,203)	(575,579)	(244,901)	(57,032)
99.9%	(399,073)	(467,275)	(476,458)	(497,299)	(551,168)	(582,470)	(601,773)	(249,054)	(58,509)

TABLE 1.2
('000)
Assets

Percentile	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Year 10	Year 20	Year 30
10.0%	717,890	682,712	951,037	664,586	871,543	634,533	807,330	1,678,864	5,927,095
25.0%	717,890	720,355	609,361	577,056	525,480	1,215,326	1,311,280	892,529	1,878,132
50.0%	717,890	818,342	569,272	850,909	603,019	411,752	442,418	432,703	355,260
59.0%	717,890	685,834	695,204	737,860	641,521	569,410	515,625	244,347	67,127
60.0%	717,890	512,119	465,912	491,797	517,305	500,206	407,072	269,163	36,385
75.0%	717,890	657,221	793,038	485,131	877,317	460,915	287,767	(783,631)	(810,281)
90.0%	717,890	665,830	576,020	616,924	316,632	523,462	87,012	(563,048)	(402,742)
95.0%	717,890	515,669	565,847	462,163	407,979	381,538	35,489	(289,862)	(941,901)
99.0%	717,890	644,907	473,163	456,938	294,317	313,079	(23,085)	(128,241)	(884,463)
99.5%	717,890	637,874	572,197	599,209	447,280	490,201	(1,642)	(215,439)	(256,168)
99.9%	717,890	781,911	696,223	427,916	293,741	306,640	(63,861)	(169,207)	(1,260,051)

TABLE 1.3
('000)
Liabilities

Percentile	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Year 10	Year 20	Year 30
10.0%	843,027	783,589	848,220	794,687	784,719	698,808	600,491	193,928	46,463
25.0%	843,026	787,995	748,958	784,716	818,283	687,618	361,962	221,003	29,152
50.0%	843,028	786,046	806,102	669,165	842,168	767,895	524,767	221,215	39,049
59.0%	843,027	829,834	750,462	750,816	731,498	686,557	459,534	209,352	49,130
60.0%	843,028	878,549	814,179	697,920	661,758	632,822	519,259	218,563	52,231
75.0%	843,026	855,899	763,511	719,689	658,342	634,032	532,463	184,289	46,308
90.0%	843,029	720,069	806,769	749,320	656,842	718,586	431,408	215,384	50,775
95.0%	843,027	699,500	772,105	713,710	665,137	778,485	481,978	225,407	52,763
99.0%	843,026	856,556	888,985	812,081	772,977	751,041	550,086	243,091	56,490
99.5%	843,025	910,872	839,967	797,508	767,709	824,590	575,579	244,901	57,032
99.9%	843,027	879,416	865,871	961,720	896,028	864,217	601,773	249,054	58,509

TABLE 1.4
('000)
Assets/Liability Ratio

Percentile	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Year 10	Year 20	Year 30
10.0%	85.2%	99.8%	109.2%	119.3%	133.3%	151.4%	260.6%	1160.4%	10709.6%
25.0%	85.2%	92.0%	97.6%	104.1%	111.8%	119.7%	176.4%	591.4%	4346.2%
50.0%	85.2%	85.5%	86.2%	88.9%	91.0%	92.9%	109.8%	194.5%	961.3%
59.0%	85.2%	82.8%	82.8%	84.1%	84.5%	84.4%	90.2%	102.9%	140.7%
60.0%	85.2%	82.6%	82.3%	83.7%	84.0%	83.7%	88.3%	89.8%	69.7%
75.0%	85.2%	78.6%	76.0%	75.4%	73.6%	72.5%	60.4%	0.0%	0.0%
90.0%	85.2%	72.4%	66.4%	62.6%	59.8%	56.7%	28.8%	0.0%	0.0%
95.0%	85.2%	68.4%	61.9%	57.6%	51.9%	47.1%	12.8%	0.0%	0.0%
99.0%	85.2%	58.9%	52.5%	45.1%	42.3%	35.0%	0.0%	0.0%	0.0%
99.5%	85.2%	57.6%	50.2%	43.7%	40.1%	33.3%	0.0%	0.0%	0.0%
99.9%	85.2%	41.8%	45.4%	39.1%	26.5%	24.7%	0.0%	0.0%	0.0%

Scenario 2

Cash flow match first seven years.

TABLE 2.1
('000)
Surplus/Deficit Risk Metric

Percentile	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Year 10	Year 20	Year 30
10.0%	2,324,069	5,213,365	5,846,578	6,247,427	6,378,420	6,161,986	5,169,316	5,153,163	7,670,332
25.0%	794,363	1,724,913	1,928,081	1,976,160	1,906,610	1,851,062	1,783,209	1,549,601	2,625,585
50.0%	80,144	158,530	167,311	176,521	178,018	170,900	153,989	141,560	358,367
56.0%	1,523	2,453	3,065	2,722	3,110	3,018	2,913	2,569	8,608
57.0%	(5,264)	(8,093)	(7,687)	(7,279)	(7,709)	(8,008)	(6,926)	(13,399)	(27,176)
75.0%	(100,059)	(112,058)	(114,805)	(116,700)	(120,326)	(124,734)	(154,373)	(185,225)	(46,659)
90.0%	(191,829)	(214,245)	(222,519)	(229,635)	(233,525)	(243,050)	(309,670)	(216,462)	(51,020)
95.0%	(233,756)	(257,843)	(267,258)	(277,353)	(284,890)	(299,881)	(375,907)	(227,068)	(52,837)
99.0%	(292,513)	(324,917)	(345,426)	(388,069)	(387,766)	(411,241)	(490,475)	(243,091)	(56,330)
99.5%	(312,352)	(343,200)	(363,709)	(401,213)	(447,218)	(459,723)	(544,708)	(244,901)	(57,032)
99.9%	(358,969)	(422,264)	(437,748)	(433,639)	(521,264)	(552,404)	(601,773)	(249,054)	(58,509)

TABLE 2.2
('000)
Assets

Percentile	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Year 10	Year 20	Year 30
10.0%	717,890	743,026	766,804	765,233	639,159	648,440	687,433	2,644,843	3,433,459
25.0%	717,890	680,351	629,741	808,749	761,089	912,516	646,196	410,919	1,220,645
50.0%	717,890	668,671	649,168	663,107	539,954	608,636	733,591	1,012,894	328,326
56.0%	717,890	682,560	512,684	596,388	611,896	613,769	494,423	245,701	46,731
57.0%	717,890	633,975	589,512	685,356	686,812	567,020	725,056	126,186	20,823
75.0%	717,890	638,887	731,471	570,680	486,873	588,721	309,645	(594,286)	(115,377)
90.0%	717,890	654,914	682,402	549,889	473,800	392,259	159,326	(599,551)	(519,976)
95.0%	717,890	701,215	674,374	553,380	473,675	347,351	214,601	(279,581)	(639,855)
99.0%	717,890	704,349	630,893	581,844	450,660	524,423	48,488	(116,543)	(34,776)
99.5%	717,890	691,201	571,485	420,008	377,084	325,233	57,577	(219,333)	(406,343)
99.9%	717,890	741,772	516,257	435,088	362,523	363,210	(8,212)	(83,351)	(616,102)

TABLE 2.3
('000)
Liabilities

Percentile	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Year 10	Year 20	Year 30
10.0%	843,025	893,976	817,543	767,235	646,903	688,591	640,313	229,972	49,182
25.0%	843,025	849,920	863,914	635,824	753,355	622,747	515,637	210,459	41,295
50.0%	843,027	829,882	735,329	623,044	688,426	754,572	479,827	168,682	41,968
56.0%	843,028	805,117	804,226	761,450	755,902	642,021	490,362	205,350	48,169
57.0%	843,027	806,189	757,008	732,156	738,078	709,821	518,508	152,602	49,621
75.0%	843,025	804,821	806,720	646,606	724,249	628,051	443,489	185,225	46,659
90.0%	843,028	864,669	745,552	834,859	764,715	709,859	509,644	216,462	51,021
95.0%	843,027	874,677	789,621	775,463	661,680	733,219	594,214	227,068	52,837
99.0%	843,026	826,953	736,761	890,469	841,590	851,160	546,895	243,091	56,330
99.5%	843,027	843,645	839,967	809,383	733,062	751,457	583,651	244,901	57,032
99.9%	843,027	879,416	899,735	777,380	896,028	864,217	601,773	249,054	58,509

TABLE 2.4
('000)
Assets/Liability Ratio

Percentile	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Year 10	Year 20	Year 30
10.0%	85.2%	94.2%	99.2%	107.4%	117.6%	129.5%	213.6%	892.1%	8130.7%
25.0%	85.2%	88.6%	90.9%	96.3%	102.2%	107.4%	153.0%	467.2%	3200.1%
50.0%	85.2%	83.6%	82.4%	84.6%	86.7%	87.3%	99.8%	144.9%	554.0%
56.0%	85.2%	82.3%	80.3%	82.1%	83.6%	82.9%	88.8%	98.5%	97.0%
57.0%	85.2%	82.0%	80.0%	81.6%	83.3%	82.2%	87.2%	87.7%	42.0%
75.0%	85.2%	78.4%	74.3%	74.0%	73.1%	73.0%	62.1%	0.0%	0.0%
90.0%	85.2%	73.7%	67.7%	65.6%	62.9%	60.9%	36.3%	0.0%	0.0%
95.0%	85.2%	70.8%	64.2%	61.0%	57.6%	54.6%	26.2%	0.0%	0.0%
99.0%	85.2%	62.9%	56.4%	52.0%	49.1%	44.1%	8.9%	0.0%	0.0%
99.5%	85.2%	61.6%	55.9%	50.3%	47.0%	42.0%	2.5%	0.0%	0.0%
99.9%	85.2%	50.2%	51.1%	48.3%	39.8%	39.1%	0.0%	0.0%	0.0%

Scenario 3

Increased funding.

TABLE 3.1
('000)
Surplus/Deficit Risk Metric

Percentile	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Year 10	Year 20	Year 30
10.0%	3,270,124	7,449,750	8,316,354	8,699,108	8,965,853	8,604,417	7,645,688	7,449,693	10,077,279
25.0%	1,236,304	2,431,156	2,906,674	3,077,754	2,947,600	2,930,989	2,839,036	2,431,151	4,049,149
50.0%	255,039	511,056	618,657	555,151	587,998	595,732	494,773	511,075	1,146,961
69.0%	6,571	8,626	17,973	15,055	12,113	13,096	11,676	8,648	41,514
70.0%	(3,991)	(6,925)	(5,969)	(4,258)	(4,137)	(4,652)	(5,452)	(8,266)	(13,300)
75.0%	(37,142)	(40,037)	(46,205)	(48,379)	(48,558)	(51,858)	(57,744)	(97,719)	(42,461)
90.0%	(161,656)	(174,486)	(179,279)	(186,164)	(190,379)	(198,764)	(250,284)	(208,326)	(49,737)
95.0%	(210,794)	(237,710)	(244,647)	(256,663)	(266,339)	(267,712)	(346,146)	(222,005)	(52,027)
99.0%	(289,563)	(329,273)	(335,938)	(363,386)	(401,221)	(398,930)	(495,129)	(241,440)	(55,989)
99.5%	(308,425)	(347,219)	(375,886)	(392,473)	(418,734)	(446,300)	(527,739)	(244,901)	(56,572)
99.9%	(330,897)	(396,700)	(424,695)	(434,451)	(448,973)	(486,348)	(600,395)	(249,054)	(58,509)

TABLE 3.2
Assets

Percentile	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Year 10	Year 20	Year 30
10.0%	780,843	846,168	726,889	827,199	949,494	816,900	883,410	2,334,290	3,536,066
25.0%	780,843	818,923	990,666	529,677	689,229	734,069	426,439	1,557,267	2,260,296
50.0%	780,843	945,268	674,194	757,798	687,545	839,111	1,189,971	1,051,543	597,683
69.0%	780,843	709,632	642,126	1,100,609	729,133	666,513	593,919	189,891	87,667
70.0%	780,843	851,562	716,107	499,038	802,710	687,164	603,704	182,478	24,718
75.0%	780,843	716,985	742,839	673,558	633,080	507,948	340,756	113,861	(1,037,689)
90.0%	780,843	662,940	672,193	443,465	633,585	393,925	239,659	(429,510)	(974,878)
95.0%	780,843	786,813	557,032	503,937	468,100	403,300	144,874	(83,760)	(1,360,580)
99.0%	780,843	720,320	553,255	460,603	449,615	372,727	38,627	(774,570)	(713,837)
99.5%	780,843	560,828	567,019	370,602	330,139	327,168	17,169	(199,852)	(637,230)
99.9%	780,843	437,549	500,338	506,671	388,688	400,189	(15,315)	(33,838)	(1,157,647)

TABLE 3.3
('000)
Liabilities

Percentile	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Year 10	Year 20	Year 30
10.0%	843,026	853,255	802,621	686,264	697,949	575,567	466,821	234,550	43,420
25.0%	843,026	852,173	769,658	703,273	717,149	728,206	485,694	193,787	50,465
50.0%	843,027	787,574	777,885	674,362	784,616	562,388	525,264	212,296	51,044
69.0%	843,028	809,722	859,191	779,458	796,687	733,303	512,535	184,164	45,024
70.0%	843,024	813,686	686,894	756,382	708,399	691,114	439,787	193,853	39,326
75.0%	843,026	799,652	722,696	751,403	566,839	632,822	566,600	208,110	42,461
90.0%	843,026	761,383	804,761	692,155	665,235	625,119	512,644	208,326	49,738
95.0%	843,028	821,637	821,757	778,036	750,502	669,643	483,036	222,005	52,028
99.0%	843,026	910,872	785,739	912,757	841,590	777,457	538,841	241,441	55,990
99.5%	843,026	893,641	899,735	783,933	772,977	751,041	544,690	244,901	56,573
99.9%	843,025	911,538	888,985	961,720	896,028	906,738	600,395	249,054	58,509

TABLE 3.4
('000)
Assets/Liability Ratio

Percentile	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Year 10	Year 20	Year 30
10.0%	92.6%	110.5%	121.1%	132.8%	145.6%	162.3%	268.0%	1210.7%	11535.2%
25.0%	92.6%	101.8%	109.0%	115.7%	123.2%	131.2%	191.9%	657.8%	4954.6%
50.0%	92.6%	94.6%	95.8%	99.8%	102.4%	104.0%	125.6%	268.6%	1505.7%
69.0%	92.6%	88.7%	88.5%	87.9%	88.0%	88.2%	90.6%	99.3%	149.8%
70.0%	92.6%	88.5%	87.4%	86.8%	87.4%	87.2%	89.0%	94.3%	62.9%
75.0%	92.6%	86.6%	85.1%	83.9%	83.7%	83.0%	80.3%	48.3%	0.0%
90.0%	92.6%	80.0%	74.7%	71.6%	68.9%	65.1%	48.7%	0.0%	0.0%
95.0%	92.6%	74.9%	69.7%	64.8%	61.3%	57.6%	31.4%	0.0%	0.0%
99.0%	92.6%	65.2%	58.3%	53.0%	50.8%	44.1%	9.6%	0.0%	0.0%
99.5%	92.6%	63.1%	56.1%	50.5%	44.3%	40.7%	3.6%	0.0%	0.0%
99.9%	92.6%	48.0%	50.5%	43.0%	32.1%	32.0%	0.0%	0.0%	0.0%