ACTUARIAL RESEARCH CLEARING HOUSE
1993 VOL. 3

# SOLVENCY RISK

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## ABSTRACT

This paper presents a new perspective on three basic financial quantities: assets, liabilities and surplus. The new perspective is developed through a re-examination of the fundamental concepts inherent in these quantities. This re-examination clarifies the difference between the concept and the various rules commonly used to establish their numerical value. With the benefit of this clarification, an alternate approach to the development of surplus is presented.

The traditional actuarial/accounting paradigm creates surplus by first discounting and summing individual asset and liability cash flows to establish individual asset and liability values. These asset and liability values are then summed to establish a surplus value. The alternate paradigm first sums all anticipated annual itemized cash flows to establish a vector of projected net total cash flows. These total cash flows are then discounted and summed to develop a surplus value. The new paradigm does not create a new type of surplus. It creates a new way to calculate surplus. And the new way in which it calculates surplus allows and encourages the notion of variability.

The purpose of developing this new paradigm is to create a framework in which surplus variability can be recognized, defined and studied. By incorporating variability, the new framework allows a precise definition of solvency risk. In summary terms, solvency risk is defined as the degree of variability in surplus, where surplus is a function of its scenario (uncontrollable future events) and strategy (controllable future events)

determinants. The definition is shown to be consistent with the common understanding of solvency risk and provides a workable guide for comparing alternative management initiatives in terms of their solvency optimization potential.

# I. INTRODUCTION

In this paper a new actuarial and accounting paradigm is presented. This new paradigm was developed, and is presented here, because it provides a vantage point from which the issue of solvency can be examined and explored in a productive manner. In this examination and exploration, the advantage of this paradigm, compared to current accounting and actuarial methodology, lies in the fact that if allows very explicit recognition that surplus is variable in response to alternative futures. Variability is the essential ingredient of risk and the recognition that surplus is variable is key to understanding and defining solvency risk.

In Section II, some background is provided. The paradigm presented in this paper is the outcome of accumulated ideas generated in the course of confronting a number of practical financial reporting dilemmas. The resolution of these dilemmas has lead the author, over the last few years, to the ideas presented here. In this section as well some basic concepts are discussed. The meanings of commonly used words, like "asset", "liability" and "surplus", are examined and clarified.

Section three re-constructs the fundamental building blocks of assets and liabilities into a new paradigm based on net cash flow. Two demonstrations of the equivalence of surplus calculated in the traditional manner and the new manner are presented.

Section four defines solvency risk in the context of the new paradigm. Using these ideas, the relationship between net cash flow and current financial reporting methods is more fully explored. Also included are some general comments on exposure to risk and potential management actions in light of a better understanding of risk exposure.

The final section presents some thoughts on the relationship between a number of current actuarial initiatives and the paradigm presented here. Some speculation on the broader application of surplus defined as discounted net cash flow is also presented.

## II. BACKGROUND, PROBLEMS AND A NEW VIEW

This paper is written from a Canadian financial reporting background. The Canadian life insurance financial reporting scene is currently changing to a new statutory reserving and reporting method. The author has had the opportunity of managing a number of financial reporting related projects through this transition.

The policy premium method, called PPM, is the new Canadian financial reporting standard. [1] It was introduced officially with the proclamation of the new Insurance Companies Act on June 1, 1992. Unofficially, the PPM method has been "in-use",

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[1]Understanding Canadian financial reporting methods is not a prerequisite for this paper. However, the reader interested in the new Canadian reporting rules may wish to review section 4210 of the CICA Handbook [6] and the CIA's June 22, 1992 memorandum regarding the Appointed Actuary's Report [5] as an introduction.

discussed and dissected for the last four or five years. The heart of the PPM method is a gross premium actuarial reserve calculation, with future contingent events valued at the actuary's best estimate plus an explicit margin for adverse deviation.

Like many other Canadian actuaries, the author has spent some considerable time contemplating the PPM method, and the values it produces, and comparing these to values produced by earlier methods. A frequent question that occurs during this comparison is "which value, or answer, is better - and why?". A similar, but basically more fundamental question, usually asked by non-actuaries is, "which answer is right?".

The author has also, in the course of consulting work, enjoyed some exposure to other financial reporting methods, primarily U.S. GAAP and U.S. statutory and, to a lesser extent, U.K. methodologies, especially the development of embedded values. Much of this exposure has been in the context of appraisal and acquisition work. For those not familiar with appraisal techniques, the value of a life company is traditionally calculated as the sum of adjusted surplus, the present value of future profits on inforce business and the present value of future profits on future business. [2] In these calculations, future profits are calculated on a statutory basis. Different statutory reporting methods produce different answers. Should we be able to reconcile these different answers? Is one

[2]Those readers interested in more details on the development of appraisal values are referred to the ASB's Actuarial Standard of Practice No. 19, Actuarial Appraisals [1]. For an introduction to embedded values, the interested reader is referred to a February 1990 report by an Institute of Actuaries Working Party [8].

answer more correct than others? With, typically, very large amounts of real money involved, finding the correct answer becomes a very real and very important exercise.

One final noteworthy dilemma is the issue of insolvency. When is a life company insolvent and how does it become insolvent? Specifically, how does a company progress from a solvent position, sometimes a strong solvent position, at the end of one fiscal year to an insolvent position at the end of the next? Was the earlier solvent statement incorrect? If so, how and why? Did something devastating happen during the year - should it have been foreseen earlier - could it have been avoided?

To some extent, the questions raised above can be discussed in the context of our current financial reporting framework. Typically, however, the author has found that resolutions to these problems require a vantage point outside of the current financial framework. The most useful point of view comes from a re-consideration of the most basic financial building block - cash flow.

*Assets and liabilities*

Assets and liabilities are the essential elements that form the foundation of current financial reporting methods. Most accounting texts start with a definition of these two concepts and build from there. In *Accounting Fundamentals*, Moscove [10], defines assets ("economic resources of an organization that are of value to the organization") and liabilities ("debts owed to parties external to the organization") in the first 10 pages. The balance of this and most other texts then builds on these ideas to construct what this paper refers to as the current actuarial/accounting paradigm.

In this paper, we start with a more indepth look at the meanings of asset and liability. We begin by establishing a distinction between "concept" and "a numerical expression of value". For example, a liability is a promise to pay. This is the *concept* of liability. This meaning is resident in the root word "liable". Liable means obligated, or responsible, according to law or equity. The value of a liability is a numerical expression of the monetary value of a promise to pay. The value of a liability is not at all the same thing as the concept of liability. There can be many rules for establishing a numerical value, but there is only one concept. A good rule for determining a liability value should be consistent with the concept of liability. The liability exists because of the promise to pay and, conceptually, the liability is uniquely defined by the terms of that promise.

Accounting and actuarial rules and conventions are constructed to assist us in developing a numerical value for the promise to pay. Different rules arrive at different values for the same liability. In our common discussions, we use the word "liability" when we mean "the value of the liability, calculated according to a given set of rules". Sometimes, to explicitly identify the rules in use, we say "statutory liability" or "PPM liability".

Applying these terms to a life insurance contract, we can see that the insurer's obligation, or liability, with respect to that contract, is defined by the terms of the contract. The insurer's liability is to pay certain sums on the occurrence of certain contingent events. Different actuarial methods assign a different value to that policy's liability - but the liability is the same, regardless of the method of valuing it.

At the core of all sensible and meaningful methods of valuing a liability is an estimation of the future payments that the liability represents. This is merely a restatement of the definition of liability. If a liability is a promise to pay then a determination of its value means establishing the amount to be paid (this being the "pay" part of the definition) and when that payment will be made (this being the "promise" part of the definition). Determining the value of a liability means estimating the current economic value of the promise to pay. In other words, it means calculating the present value of one or more future cash payments.

Actuarial reserving stays close to the root concept of present value of future payments. Jordan [9], in his discussion of the nature of the reserve, refers to the present value of payments to be made by the insurer and the present value of premium payments still to be received. He says it is "... important for the insurer to have a measure of the amount which he should have on hand at any time to assure the payment of benefits, assuming of course that all future premium payments will be made by the insured during his lifetime as they fall due. The concept of a *reserve* arises from this necessity ..." .

We can probably assume that all current actuarial reserving methods in common use are sensible and meaningful. But this need not be so. It is instructive to explore the implications of valuation rules that are not sensible. For example, we can define a rule that states: "the reserve value for every whole life policy is $10 per $1,000 of face amount inforce". We can complete a valuation using this rule, and with that valuation define a company's surplus. Although we can express the $10 per 1,000 rule as the present value of one or more future cash payments, such an expression would be as meaningless as the rule itself. If the rule for determining a liability value cannot be expressed in terms of the core concept of liability, that being future payments, then the values developed by that rule, being the dollar amount of the liability value itself and the value of derivative items, like surplus, are meaningless.

Other insurance liability items, like claim reserves, also are defined in terms of future cash obligations. Sometimes, the actuarial/accounting rules require that future payments be discounted using an interest rate of 0% - a common convention in the development of incurred but not reported and outstanding claim reserves. Taking some general, non-insurance liabilities, we can see that the concept of discounted cash flow still, of course, applies. Short term liabilities, like expenses payable, are defined by reference to the amount that will be paid in the near future, usually not discounted. Long term liabilities, like bonds, mortgages or long term notes payable are valued as the discounted present value of the future interest and principal payments.

Can all liabilities be defined as the discounted present value of future cash payments, regardless of the type of liability or the accounting rule? The answer is yes - a formulation as discounted present value can be constructed. If the accounting rule is sensible and meaningful, that is to say, consistent with the meaning of the word "liability", like the actuarial reserves, claim reserves, pre-paid expenses and long term debts discussed above, then the projected cash flows and their discounted value will be meaningful. If the accounting rule is not meaningful and not consistent with the meaning of the underlying concept of liability, then the equivalent cash flow construction will be equally meaningless.

Turning now to assets, the fact that one party's liability is another's asset establishes a good starting point for the discussion of assets as anticipated future cash flows. That is, looking at liabilities amounts to looking at cash flows from the point of view of the payor. Looking at assets amounts to looking at the same cash flows, but now from the point of view of the recipient. Common examples are readily apparent: one party's accounts payable are another's receivable. One party's mortgage obligation is another's mortgage asset.

Conceptually, asset means a source of value, suitable for the payment of debts and other obligations. An asset is something that generates cash, either through future periodic payments of dividends or interest and principal, or through a single payment on sale, or through a combination of these forms of cash inflow. The value of an asset comes from its expected future cash flow. Asset valuation recognizes this, although this recognition is not always immediately apparent.

A common rule for determining asset value is to state that its worth is equal to the amount of cash paid to acquire it, ie. its value is its cost. Superficially, this asset valuation rule seems concerned only with the past, not with future cash flows. A closer examination reveals that this is not the case. The asset at cost rule assumes that individuals and businesses act in a rational manner. It assumes that the amount paid for an asset, its price, is a fair representation of its value to the company. And, of course, fundamentally, the price is calculated by reference to anticipated future cash flows. For example, the cost of a bond is the discounted present value of future principal and interest payments, likewise mortgages and short deposits. For debt securities anticipated future cash flows are determined according to contractual terms. For other types of assets, like real estate and shares, the purchase price is determined based on a best estimate of future payments. Consider the amount paid to acquire an apartment building. The buyer assesses the amount of expected future rental payments and future maintenance costs. Perhaps an assessment of potential future resale value is also developed. All of these considerations reflect future cash flow which are used to establish the cost of the asset, which in turn is used to establish its stated value.

Some recent developments in the Canadian financial reporting scene are noteworthy in their reaffirmation of future cash flows as the fundamental definition of asset value. In

November of 1992, the CICA Accounting Standards Board issued proposed recommendations with respect to impaired loans [5]. The recommendations propose that all impaired loans be explicitly valued as the discounted present value of expected future cash flows.

Reviewing other common assets provides further illustration of the fact that future cash flows are the fundamental building block in asset valuation. Plant and equipment held at cost are prepaid future operating costs. Depreciation reflects the shortening prepaid period. Deferred policy acquisition costs are valued as the discounted present value of a portion of future premiums - the balance of the anticipated future premiums are used to develop the reserve liability value. Goodwill is the anticipated present value of future earnings from the acquired entity in excess of the earnings recognized in assets and liabilities held at book value.

As with liabilities, we can define all asset values as the discounted present value of future cash receipts, regardless of the type of asset or accounting rule. If all assets can be defined in terms of their future cash flow and all liabilities can be defined in terms of their future cash flows, then what about assets less liabilities? Surplus is the discounted present value of net cash flow, net cash flow being the combined asset and liability cash flows. In the section following, two demonstrations of this expression of surplus are presented.

## III. DISCOUNTED PRESENT VALUE OF PROJECTED NET CASH FLOW

In this section the relationship between surplus defined in the context of the traditional actuarial/accounting paradigm and surplus defined with direct reference to the underlying future cash flows is explored. Two demonstrations are provided to show that identical values of surplus are developed if each paradigm uses the same assumptions. Demonstrating this equivalence allows cash flow to be used to provide an alternative perspective on surplus. This alternative perspective readily supports the recognition of variability and risk. As traditionally calculated, surplus appears static and does not provide an effective platform for studying the implications of variable future events. However, calculated as the discounted present value of projected net cash flow, surplus is seen to be variable in response to variations in projected events.

The demonstration begins by constructing a general formula defining liabilities and another defining assets. Simple algebra is used to re-arrange the terms in the formulas so as to present a definition of surplus in terms of projected cash flows. As an alternative demonstration, the section concludes with an explanation of the equivalence based on a representation of the various calculations as operations on a two dimensional matrix.

The actuarial reserve for policy number $x$ can be described by the following formula:

$$l(x) = \sum_{t=1}^{\omega} v(x,t)l(x,t) \tag{1}$$

where:

$$l(x,t) = c(x,t) + e(x,t) - p(x,t) \tag{2}$$

In equation (1) $v(x,t)$ is the discount function that creates, for policy number $x$, a present value at time zero of an occurrence at time $t$. In equation (2) $c(x,t)$ is the amount of claim and other benefit payments at time $t$, $e(x,t)$ is the amount of expense payments and $p(x,t)$ is the premium income. The term $l(x,t)$ can also be referred to as the policy cash flow at time $t$ and the time series of these terms will be referred to as the policy cash flow vector.

In defining the various terms in equation (2) the force of contingent events, like death and lapse, are not specifically identified but are assumed in the construction of the terms. For example, the amount of premium differs from one year to the next, being reduced by death and lapse. Note also that the formula is for illustrative purposes only. Time zero was chosen as the present and all events are assumed to occur together at annual intervals beginning one year hence. Some readers may prefer to consider the underlying events as continuous. In which case, the summation is replaced by an integral starting from time zero. The theory, and demonstration, is unaffected.

Equations (1) and (2) are a generic representation of the three common components that determine an insurance liability cash flow (benefits, expenses and premium) and the discount factor used to generate a present value representative of the cash flow. All different types of policy reserve methods follow the same general form. The values of the terms in equations (1) and (2) are those dictated by the reserving method in use. The values are those used to develop the numerical expression of the value of the liability. They are not necessarily what might be called realistic or "best estimate" cash flows. Indeed, some valuation methods assume cash flows that would generally be considered unrealistic. A net premium valuation, for example, assumes an expense component exactly equal to the difference between gross and net premium; or, what amounts to the same cash flow, it assumes expenses are zero and the only the net premium is received. Our objective in this section is to explore the relationship between alternative methods of developing surplus. In the next section, the impact of different values for cash flow and the discount function will be addressed.

Formula (1) is sufficiently general to describe claim and other non-policy reserves and, as well, non-actuarial liabilities. Claim reserves have a zero premium component. Only the first two terms in equation (2) are need to describe disabled life reserves, incurred but not reported reserves and outstanding claims. Liabilities other than reserves essentially amount to expense obligations. In the context of formula (2) such liabilities have neither a premium nor a claim component. Throughout the balance to this demonstration, we will, for ease of reference, use the terms "policy" and "policy liability" and assume they are understood in the broader context of all liability items (insurance policy, claim and non-insurance item) and their defined values.

The company's total liability value $L$ is created by summing all policy liability values, namely:

$$L = \sum_{x=1}^{m} l(x) \tag{3}$$

(Equation (3) assumes the company has $m$ policies).

*Assets*

The value of asset $y$ is described by the following formula:

$$a(y) = \sum_{t=1}^{\omega} v(y,t)a(y,t) \tag{4}$$

where:

$$a(y,t) = b(y,t) + i(y,t) - o(y,t) \tag{5}$$

In equation (4), $v(y,t)$ is the discount function that creates, for an asset identified as $y$, a present value at time zero of an occurrence at time $t$. In equation (5) $b(y,t)$ is the amount of the principal payments at time $t$, $i(y,t)$ is the amount of interest, coupon, net rent and other investment income and $o(x,t)$ are the associated expense payments. The term $a(y,t)$ is called the investment cash flow at time $t$ and the time series of these terms can be referred to as the investment cash flow vector.

All asset valuation methods can be described in the form of equation (4). As with liabilities, different methods amount to different rules regarding the selection of assumptions used to create the investment cash flow vector. For example, for debt securities a "cost" basis stipulates the use of a discount rate consistent with the rate in effect at the time of purchase. A "market" value uses a discount rate consistent with the market rate in effect on the valuation date.

For assets other than debt securities, like shares and undeveloped real estate, the bulk of their value lies in the ability to sell the asset for a higher dollar value than its cost of purchase. In the context of equation (4), the emphasis is on the principal payments component. Taking a common share with no coupons and no investment expenses as an illustration, holding the share value at cost assumes that at some future date $t$ the share (identified as share number $y$) can be sold for an amount equal to its cost divided by

$v(y,t)$. Holding the share at market assumes that at some future date it can be sold for its current market value divided by $v(y,t)$.

Equation (4) serves to explicitly express the underlying assumptions implicit in all asset valuations. Again taking a common share as an example, its value to the insurance company lies in the company's ability to sell that share for cash, cash which will in turn will be used to pay policy benefits or expenses or shareholder dividends. Establishing the share's value in the context of equation (4) forces a recognition of the implicit assumptions being made with respect to "principal repayment" and the variability associated with these assumptions.

As with liabilities, traditionally all asset values are summed to define the company's total asset value. Equation (6) states this in a simple formula.

$$A = \sum_{y=1}^{n} a(y) \tag{6}$$

where $n$ is the total number of assets.

An alternative formula can easily be constructed to create the company's total asset value. First, equation (7) defines the total company's investment cash flow at time $t$, called $A(t)$, as the sum of individual asset cash flows.

$$A(t) = \sum_{y=1}^{n} a(y,t) \tag{7}$$

Then the total asset value $A$ can be written as:

$$A = \sum_{t=1}^{\omega} v(t)A(t) \tag{8}$$

where $v(t)$ is the weighted average of the individual asset discount function defined by equation (9).

$$v(t) = \sum_{y=1}^{n} \frac{v(y,t)a(y,t)}{A(t)} \tag{9}$$

Equation (8) shows that the company's asset value can be calculated as the discounted present value of the total company's investment cash flow. With $v(t)$ defined as in equation (9) this value is identical to the value created by means of equation (6).

A similar approach can be used to develop an alternative calculation for the total company liability value previously defined in equation (3). Specifically, the total company liability value can be defined by equation (10).

$$L = \sum_{t=1}^{\omega} v(t)L(t) \tag{10}$$

The total company policy cash flow at time $t$ called $L(t)$, is defined by equation (11).

$$L(t) = \sum_{x=1}^{m} l(x,t) \tag{11}$$

With $v(t)$ calculated as in equation (12), the total company liability calculated by equation (10) is identical to the value calculated by equation (3).

$$v(t) = \sum_{x=1}^{m} \frac{v(x,t)l(x,t)}{L(t)} \tag{12}$$

*Surplus*

A company's surplus is traditionally calculated as the total value of its assets less the total value of its liabilities. Total asset value is traditionally calculated by equation (6), total liabilities by equation (3). Expanding equation (6) by substituting $a(y)$ from equation (4) and expanding equation (3) by substituting $l(x)$ from equation (1) creates equation (13).

$$\begin{aligned} S &= A - L \\ &= \sum_{y=1}^{n} a(y) - \sum_{x=1}^{m} l(x) \\ &= \sum_{t=1}^{\omega} \sum_{y=1}^{n} v(y,t)a(y,t) - \sum_{t=1}^{\omega} \sum_{x=1}^{m} v(x,t)l(x,t) \end{aligned} \tag{13}$$

Equation (13) presents the traditional view of surplus. Similar to the alternative calculation of total assets and the alternative calculation of total liabilities an alternative equation for surplus can be constructed.

First a single - asset or policy - identifier $z$ is created by means of equations (14) and (15).

Define $s(z,t)$ as follows:

If $1 \le z \le n$ then:

$$s(z,t) = a(z,t) \tag{14}$$

and if $n < z \le n+m$ then:

$$s(z,t) = -l(z-n,t) \tag{15}$$

Now equation (13) can be re-written in terms of $z$ rather than in terms of $x$ and $y$.

$$S = \sum_{t=1}^{\omega} \sum_{z=1}^{n+m} v(z,t) s(z,t) \tag{16}$$

Equation (16) is identical to equation (13). A generalized cash flow term $s(z,t)$ is used in place of $a(z,t)$, the investment cash flow and $-l(z,t)$, the negative of the policy cash flow. The negative of the policy cash flow is used to allow summation throughout rather than adding investment cash flows (these being cash in) and subtracting policy cash flows (these being cash out).

The total company net cash flow at time $t$, called $S(t)$, can be defined by equation (17).

$$S(t) = \sum_{z=1}^{n+m} s(z,t) \tag{17}$$

The total company net cash flow at time $t$ is equal to the sum of all cash flowing into the company less all cash flowing out of the company. $S(t)$ is equal to the total company investment cash flow, $A(t)$, less the total company policy cash flow, $L(t)$. The total company net cash flow can also be derived as the sum of all cash in (principal, interest and premium) less all cash out (investment expense, claims and other expenses).

Using the total company net cash flow, equation (16) can be re-written as equation (18), where $v(t)$ is the weighted average of the discount rates defined by equation (19).

$$S = \sum_{t=1}^{\omega} v(t)S(t) \tag{18}$$

$$v(t) = \sum_{z=1}^{n+m} \frac{v(z,t)s(z,t)}{S(t)} \tag{19}$$

The value produced by equation (18) is identical to the value produced by equation (13). Equation (18) is a formula describing the life company's discounted projected net cash flow. Equation (13) describes the company's surplus. Surplus and discounted projected net cash flow are equivalent if the same assumptions are used to construct each.

*A Different Order of Matrix Operations*

The determination of a life insurance company's surplus amounts to a simple sequence of calculations performed on an array of data elements. These data elements naturally form a two dimensional matrix. The calculation of surplus can be viewed from the perspective of this matrix and the operations that reduce it to a single scalar value. Such a view helps in understanding the equivalence of the value produced by equation (13) and that produced by equation (18). In summary, the difference between surplus calculated as assets less liabilities and surplus calculated as discounted net cash flow is only a difference in the order in which two simple operations are performed.

Consider a matrix of $n+m$ rows and $\omega$ columns defined by the values $s(z,t)$. An illustration of the matrix is provided below. The top part of this matrix, rows 1 to n, are the investment cash flows $a(z,t)$. The cash flow for each asset are shown in a single row. For each row, successive columns show the values of the cash in-flows from that asset for each successive year. The bottom part of this matrix, rows n+1 to n+m, are the negatives of the policy cash flows, namely $-l(z,t)$. Each row shows the cash in-flows for a policy for each year.

| ITEM (z) | TIME (t) | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | . . . | t | . . . | w |
| 1 | s(1,1) | s(1,2) | . . . | s(1,t) | . . . | s(1,w) |
| 2 | s(2,1) | s(2,2) | . . . | s(2,t) | . . . | s(2,w) |
| . | . | . | . | . | . | . |
| . | . | . | . . . | . | . . . | . |
| . | . | . | . | . | . | . |
| n | s(n,1) | s(n,2) | . . . | s(n,t) | . . . | s(n,w) |
| n+1 | s(n+1,1) | s(n+1,2) | . . . | s(n+1,t) | . . . | s(n+1,w) |
| . | . | . | . | . | . | . |
| . | . | . | . . . | . | . . . | . |
| . | . | . | . | . | . | . |
| n+m | s(n+m,1) | s(n+m,2) | . . . | s(n+m,t) | . . . | s(n+m,w) |

All traditional accounting and financial reporting methods operate on this matrix first row by row then column by column. The first traditional operation produces a single asset value for each asset held and a single liability value for each insurance policy. This operation reduces all elements within a row to a single value, or in the context of the matrix, the operation reduces all elements in all the rows into a single column. In order to conduct this operation the "non-additive" nature of the elements in a row needs to be accommodated. The successive elements in a row are projected cash flows in successive future years. Discounting each value to its present value at time zero allows the successive cash flows to be added. The first operation then is a combination of discounting (to create additivity) and summing. The illustration below highlights the order of this first operation.

| PV(t) | TIME (t) | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | . . . | t | . . . | w |
| a(1)=s(1) | s(1,1) | s(1,2) | . . . | s(1,t) | . . . | s(1,w) |
| a(2)=s(2) | s(2,1) | s(2,2) | . . . | s(2,t) | . . . | s(2,w) |
| . | . | . | . | . | . | . |
| . | . | . | . . . | . | . . . | . |
| . | . | . | . | . | . | . |
| a(n)=s(n) | s(n,1) | s(n,2) | . . . | s(n,t) | . . . | s(n,w) |
| -l(1)=s(n+1) | s(n+1,1) | s(n+1,2) | . . . | s(n+1,t) | . . . | s(n+1,w) |
| . | . | . | . | . | . | . |
| . | . | . | . . . | . | . . . | . |
| . | . | . | . | . | . | . |
| -l(m)=s(n+m) | s(n+m,1) | s(n+m,2) | . . . | s(n+m,t) | . . . | s(n+m,w) |

The second operation, in traditional accounting, sums the rows (now, after the first operation, each row has only a single element) to produce a single surplus value. The subtotal of the first n rows provides the company's asset value. The subtotal of the last m rows provides the company's liability value (more specifically, in the matrix here described, the negative of the liability value).

Alternatively, the projected cash flow method performs the same two operations, but in the opposite order. Namely, column by column then row by row. The cash flow method sums all of the elements in a column to produce a single row. The sum of the first n elements in each column is the company's investment cash flow. The sum of the last m elements is the company's policy cash flow (actually, in the matrix here described, the negative of the policy cash flow). Taken together, the sum of the elements in each column provides the projected net cash flow for the year represented by that column. This sum is the sum of all principal repayments, interest, coupon and other investment earnings and premium less all claims and benefits paid and all expenses paid, both investment expenses and insurance expenses. The illustration below highlights the order of the first operation in the cash flow method, in contrast to the sequence in the traditional method.

| ITEM (z) | TIME (t) | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | . . . | t | . . . | w |
| 1 | s(1,1) | s(1,2) | . . . | s(1,t) | . . . | s(1,w) |
| 2 | s(2,1) | s(2,2) | . . . | s(2,t) | . . . | s(2,w) |
| . . . | . . . | . . . | . . . | . . . | . . . | . . . |
| n | s(n,1) | s(n,2) | . . . | s(n,t) | . . . | s(n,w) |
| n+1 | s(n+1,1) | s(n+1,2) | . . . | s(n+1,t) | . . . | s(n+1,w) |
| . . . | . . . | . . . | . . . | . . . | . . . | . . . |
| n+m | s(n+m,1) | s(n+m,2) | . . . | s(n+m,t) | . . . | s(n+m,w) |
| | S(1) | S(2) | . . . | S(t) | . . . | S(w) |

In the cash flow method, the first operation produces the company's net cash flow vector. Because this operation deals with "like" and additive elements, no discounting is required. The second operation collapses the columns of what is now a vector into a single element. Each successive column, or vector element, represents the net cash flow in a successive future year. In order to allow the values to be added each is first discounted to its present value at time zero.

Starting with the same matrix of data elements, the cash flow method produces the same result as the traditional method. Only the order of operation of arriving at the result differs.

*Variability*

Both methods of calculating surplus, traditional and cash flow, use the same operations on the same elements. The difference in the methods is simply the order of the operations. The value produced is the same. Both procedures; surplus calculated as assets less liabilities and surplus calculated as discounted net cash flow, develop the same quantity. Surplus calculated as discounted net cash flow is a not a different type of surplus. It is the same in concept, and with the same cash flow and discount assumptions it produces the same value. Recognizing this equivalence is critical to the discussion of variability and risk that follows. Discounted net cash flow is not an alternative to surplus. It is surplus. It is surplus viewed from a different perspective - a perspective that allows for a discussion of solvency risk.

The projected net cash flow of a life insurance company is uncertain and variable. Surplus, the discounted projected net cash flow, inherits all of the uncertainty and variability of projected cash flow. The purpose of demonstrating the equivalence of surplus and cash flow is to reveal the variable nature of surplus. The susceptibility of surplus to random, and non-random, influences is the foundation on which a study of solvency risk can be built.

## IV. SOLVENCY RISK

Surplus is a prediction of the future not a record of the past. Future events impact the company's cash flow and hence its surplus. This section uses the fact that a company's surplus is equal to its discounted net cash flow to study the concept of solvency risk.

As in our previous discussion of assets and liabilities, we need to recognize the distinction between the concept and a set of rules. Conceptually, solvency is about a company's ability to meet its financial obligations. A company is said to be solvent if it can meet its obligations and insolvent if it cannot. In most jurisdictions, solvency is defined as a excess of assets, calculated according to statutory valuation rules, over statutory liabilities. Statutory solvency is one measure of solvency, based on statutory rules.

In the context of the traditional paradigm, a life company is solvent if the value of its assets exceeds the value of its liabilities. That is, with reference to equation (13), the company is said to be solvent if the value of the first term (assets) is greater than the value of the second (liabilities). In the new paradigm, with reference to equation (18), a company is solvent if the discounted present value of its net cash flow is greater than zero. Regardless of the paradigm, a company is or is not solvent, given the same set of assumptions about future debts and obligations.

Before a definition of solvency risk is proposed, two concepts need to be explored. These are the discount function and the nature of the variability of future events.

*The Discount Function*

In the matrix development described in the previous section, the discount function was described as an operator that overcomes the non-additive nature of the cash flow in successive years. This description reflects the common understanding that the discount rate adjusts for the "time value of money". Although not incorrect, this common understanding fails to provide sufficient insight into the meaning of the discount function in the context of future events. It leaves unanswered the question of what discount rate should be used and instead, seems to suggest that selecting the discount rate is a matter of preference; that, with-in an acceptable range, one rate is as good as another. The discount rate defined in this manner lacks the precision necessary to support a precise definition of solvency risk. A more meaningful understanding of the discount operation is needed.

In the context of surplus defined as discounted projected net cash flow, the discount function can be shown to be a quantification of the company's re-investment strategy and future interest rates. This demonstration is accomplished by following the development of a life company's cash balance as the accumulation of net cash plus investment earnings on that cash.

In equation (18), $v(t)$ is the value at time zero of one dollar at time $t$. Stated another way, one dollar at time zero is worth $1/v(t)$ at time $t$. Looking one year at a time, a dollar invested at time $t$ is worth $v(t)/v(t+1)$ at time $t+1$, one year later.

The net cash flows in equation (18) represent the difference between cash received and cash disbursed from all assets and policies held at time zero. As time passes, the cash accumulates with the addition of each successive year's cash flow. This cash is invested according to a re-investment strategy. The interest earnings from the invested cash also serve to augment the accumulating cash balance. Negative cash flows diminish the cash balance and a negative balance is further diminished by an interest charge, reflecting the cost of the money borrowed to make-up the cash shortfall.

Following the cash flows from the beginning serves to illustrate this process. Net cash flow in year 1, $S(1)$, if invested to yield returns consistent with discount function $v$ will amount to $S(1)x(v(1)/v(2))$ at time 2. Adding this cash to the net cash flow in year 2,

$S(2)$, brings the total cash balance at time 2 to $S(1)x(v(1)/v(2)) + S(2)$. One year later, at time 3, the opening cash balance amounts to $(S(1)x(v(1)/v(2)) + S(2))x(v(2)/v(3))$ or, simplifying the terms, $S(1)x(v(1)/v(3)) + S(2)x(v(2)/v(3))$. Adding the net cash flow at time 3, the net cash balance can be written as $S(1)x(v(1)/v(3)) + S(2)x(v(2)/v(3) + S(3)x(v(3)/v(3))$. Continuing in this manner, the cash balance at time $t$ is defined by equation (19).

$$K(t) = \sum_{i=1}^{t} \frac{v(i)}{v(t)} S(i) \tag{19}$$

At the last duration, time $\omega$, the value of the cash balance is described by formula (20).

$$\begin{aligned} K(\omega) &= \sum_{i=1}^{\omega} \frac{v(i)}{v(\omega)} S(i) \\ &= \frac{S}{v(\omega)} \end{aligned} \tag{20}$$

Equation (20) shows that the accumulated cash balance at the last duration is equal to the accumulations of the company's net cash flows invested following a re-investment strategy yielding returns described by the function $v$. The surplus value $S$ is simply that accumulated cash balance discounted to time zero, using the same function $v$.

These formulations of surplus and accumulated cash balance are consistent with the common notion of solvency. $K(\omega)$, and its corresponding value of surplus $S$, are measures of the funds available after all financial obligations have been meet. $K(\omega)$ is the value of those funds at the end, $S$ is the value at the beginning.

The discount function used to create surplus in equation (18) and the accumulated cash balance in equation (20) is a very real component in the expectation of future events which generates the company's total net cash flow and surplus. As a further illustration of this, the re-investment strategy supporting the discount function $v$ can be "internalized" in the net cash flow values $S(t)$. This is done by allowing for asset purchases, negative values of $b(y,t)$ in the terms defined for equation (5). This moves the discount operation into the cash flow calculation directly. Purchasing assets in each year $t$, in an amount exactly equal to the net cash flow otherwise created, would develop a total net cash flow vector (existing asset plus re-invested cash flow) equal to zero for all but the last term.

The discount function is thus one among many components in the expectation of future events that create the accumulated cash balance. "Reverse engineering" a discount function allows the development of $S$ from $K(\omega)$. Selecting a discount function to create a surplus value is part of the same exercise that requires the selection of a mortality assumption, or a lapse assumption or expense levels, etc. Like the selection of other assumptions, it represents one selection of the future among many possibilities.

*Strategies and Scenarios*

The words "strategy" and "scenario" have been used above in reference to future events. They have not been used interchangeably. In discussing solvency risk, it will be beneficial to distinguish between future events over which management has control, which we will call strategies, and those over which it does not have control, which we will call scenarios.

In the context of a re-investment strategy and the previous discussion of the discount rate, the decision to purchase one year government securities as a means of accumulating the net cash flow is an example of a strategy. The yields earned by those securities are the outcome of a scenario.

Distinguishing between scenarios and strategies allows a differentiation between random events that impact surplus and controllable events that impact surplus and its variability. Scenarios, of course, include many items other than new asset purchase yields. Defaults, calls and prepayments are scenario components. So are mortality rates, withdrawals and lapses, inflation and expenses. All of the random events that impact a company's cash flow impact its surplus. Unlike scenarios which are random and controllable, strategies are actions that management controls. Strategies include changes in premium rates (if these are adjustable), changes in the dividend scale, and changes in staff and other controllable expenses.

Both a strategy and a scenario need to be selected in order to establish a value for surplus. That the development of net cash flow and surplus requires the selection of mortality, morbidity, lapse and other assumptions (together called a scenario) appears self-evident. Selection of a strategy, even if the selected strategy is only to maintain the status quo, is equally essential. As an example, the strategy determines staffing levels and other controllable costs, which, in combination with the rate of inflation and other scenario components, determines expense assumptions. The strategy selected determines premium adjustments and dividend scales. Strategy determines investment policy - the sale and purchase of assets. In combination with the yield components in the scenario, the investment strategy determines new money yields and their derivative discount rates.

Since, as demonstrated previously, the discount function is derived from the selected strategy (through its investment policy component) and the selected scenario (most importantly through its new money yield rate assumptions), selecting a strategy and a scenario completely defines the surplus value. In other words, a company's surplus is a function of the strategy and scenario selected and the selected strategy and scenario completely define the value of the function. The term S[a,b] will be used to mean the surplus developed under strategy "a" and scenario "b".

Figure 1 shows a representation of the form of S[a,b]. The diagram shows a two dimensional surface in three dimensional space. The "z" axis defines the selected strategy, the "x" axis defines the scenario and the "y" axis records the dollar value of the surplus calculated based on the selected strategy and scenario.

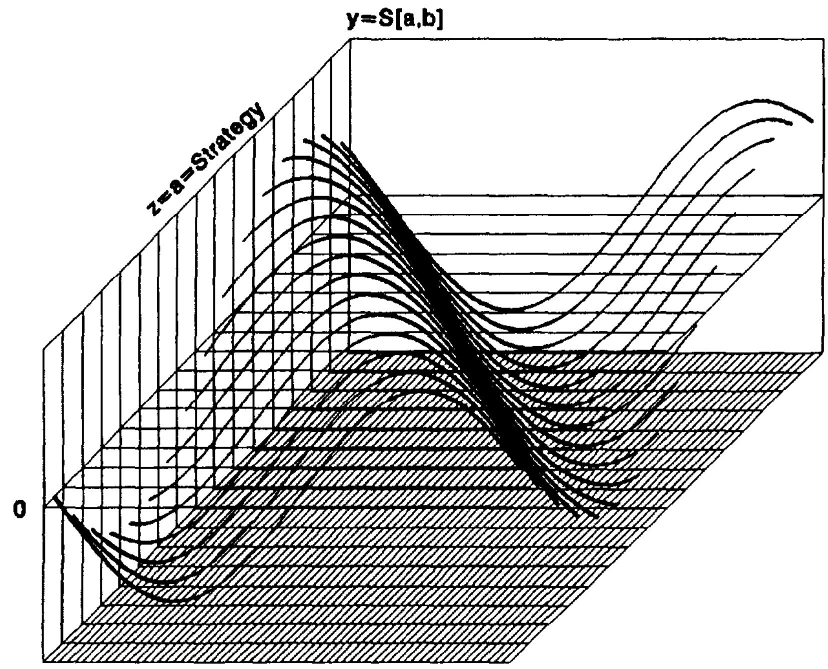


**Figure 1**

A company's solvency is contingent on the strategy and scenario selected. That is, whether or not S[a,b] is greater than zero depends upon the "a" and "b" used.

In the context of figure 1, solvency is measured by the vertical "distance" between each point on the surface and the "x-z" plane. Solvency risk is about changes in solvency due to changes in future events. Solvency risk is measured by measuring the variability of S[a,b] resulting from changes in "a" and "b". In figure 1, solvency risk is the variability or "undulation" in the two dimensional surface. A flat surface has zero solvency risk. The more variation in the surface, the greater the solvency risk.

Before proceeding to a further discussion of solvency risk, figure 1 provides an opportunity to clarify the relationship between the two paradigms. The surplus value created by each set of rules and assumptions in the traditional actuarial/accounting paradigm is represented by a point on the two dimensional surface shown in figure 1. Different valuation rules or assumptions create different points on the surface. In fact, theoretically, figure 1 can be created using the asset less liability equation of the traditional paradigm. Practically however, the construction of surplus in the traditional manner does not lend itself to a consideration of alternative futures; that is, strategies and scenarios. The traditional construction essentially closes the door to variability as each and every itemized asset and liability value is calculated. Because the traditional method suppresses the underlying cash flows very early in the process, consistency between and among asset and liability assumptions is lost. Without this consistency a sensible and useful analysis of alternative futures is exceedingly difficult.

*Optimal Strategy*

Figure 1 maps solvency in three dimensions. A small step in the direction of practical expediency can eliminate one dimension and bring the concept of solvency risk into sharper focus. For a single selected strategy, figure 1 collapses to a line in two dimensions, as shown in figure 2.

Since strategy, by definition, represents the actions controllable by management, the strategic dimension is not random in the same sense as the scenario dimension. Management can select an optimal strategy. Theoretically, the optimal strategy could be defined as that which yields the highest expected value of surplus over all scenarios with their probabilities of occurrence taken into account. Practically, the optimal strategy could be identified as the strategy selected and endorsed by management. Solvency risk can be defined and discussed in the context of surplus variability resulting from changes in scenarios given a single, optimal strategy. We can define solvency risk as a company's susceptibility to unexpected changes in its net cash flow. An unexpected change is a non-strategic change; that is, a change due to a scenario change.

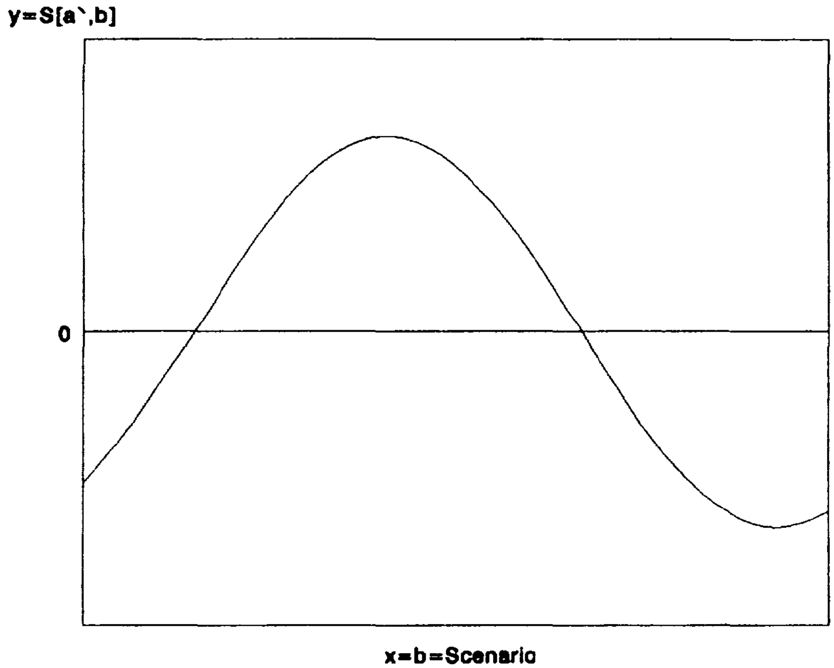


Figure 2

Referring to figure 2, solvency is measured by the distance from the horizontal axis; solvency risk is measured by the degree of variability in the line.

*Practical Application*

The calculation of surplus, as discounted projected net cash flow, is an exercise within the capability of every life company. The fundamental data elements, projected future cash flow for all assets and insurance policies, are available or can be constructed. If a company can calculate its surplus based on one scenario and one strategy, it can do so for others. If, for the moment, we suppress variations in strategy, a set of results, similar to that displayed in figure 3 can be created.

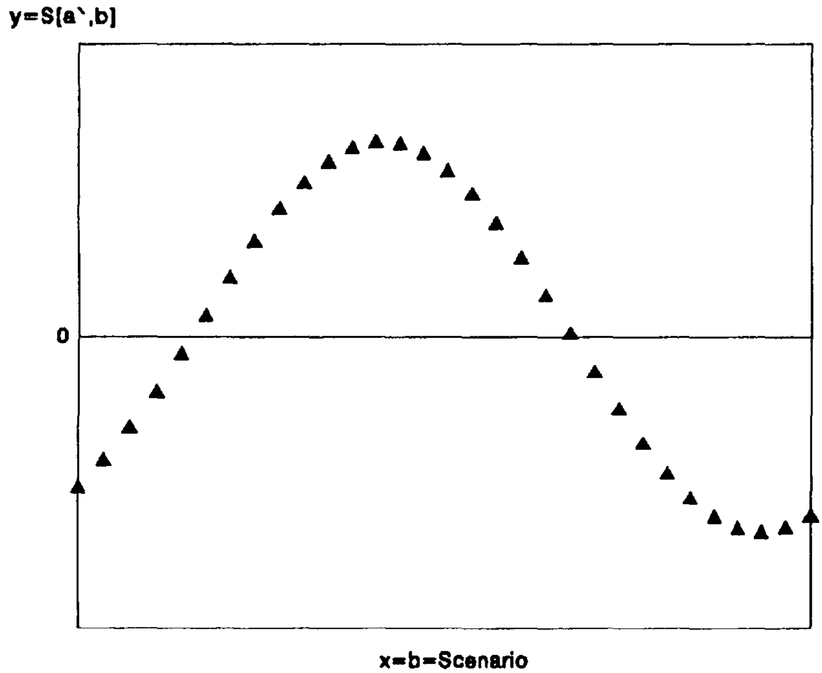


Figure 3

Unlike the theoretical values presented in figure 2, two practical constraints need to be recognized. First, the true universe of scenarios would seem to be unbounded and infinite. In practice, the analyst will need to limit the calculations and investigations to a finite subset. Second, scenarios, even a workable finite subset of scenarios, are not ordered. Each scenario is a combination of predictions on a variety of dimensions. Each of the dimensions addresses events for many years in the future. While each dimension, for each year, could be ordered, adding other years makes ordering difficult. Mortality in the first projection year, for example, could be ordered. A base case could be established then proportionately higher factors applied to the base case listed to its right, in a graph like figure 2, and lower factors listed to the left. But adding even a second year eliminates a natural single dimension order. Further, combining only two aspects of the future, say mortality and new money interest rates, makes the exercise of attempting an order practically insoluble.

In practice, figure 2 becomes figure 3. Scenario "x" is a set of assumptions about the future defined by the analyst. The order shown is arbitrary, simply based on the numerical identification assigned to each scenario. With information like that presented in figure 3 available, two solvency risk measurements can be performed. First, the relative solvency risk of one company compared to others can be developed. Second, for a given company, the riskiness of one strategy compared to another can be developed.

Solvency risk can be measured, in relative terms, by comparing the profile of one company with that of another, for the same subset of scenarios. If the subset of scenarios used for the comparison is representative of a reasonable universe of probable futures, then the comparison becomes a useful proxy for the relative solvency risk of the companies being compared. If a probability of occurrence could be assigned to each scenario then a single solvency risk value can be developed as the probability of the scenario's occurrence multiplied by the surplus developed in that scenario i.e., the expected value of surplus.

Implementing the means to calculate surplus under a variety of scenarios and constructing meaningful and internally consistent scenarios are challenging practical tasks. The difficult effort of additionally establishing a probability of occurrence for each scenario may not be warranted. A representative subset of scenarios can be developed. This subset could function like a "test suite". Calculating a company's surplus for every scenario in the suite is like a solvency test. Comparing the test results of companies provides a tool through which the comparative solvency risk can be measured - without needing to assign an exact probability to the various scenarios.

The second application of solvency risk measurement is the optimization of a company's strategy. Applying a test suite of scenarios over a range of strategies allows a comparison of the strategies. By comparing the results for different strategies, management can select the course of action that will minimize its solvency risk. Among the actions open to management are the selection of different new money investments, changes in controllable administrative costs and thus, renewal expense charges, and changes in adjustable premiums or policy dividends. Other actions are also open to management. Consideration of these other actions involves a re-examination of a basic premise inherent in traditional accounting.

*Advanced Strategic Options*

Traditional accounting methods calculate surplus as assets less liabilities. No new insurance policies are assumed, although lapses may be taken into account. Asset trades have no place in the calculation, although an attempt is made to recognize asset defaults. It is generally recognized that some consideration needs to be given to future income taxes payable and future policyholder dividends. But future shareholder dividends are nowhere recognized.

These same exclusions and limitations can be applied in the calculation of surplus via equation (18). In the discussions of strategy, up to this point, these exclusions and limitations have been adhered to. However, the new paradigm can advance our study of solvency, beyond these traditional limitations, by expanding the universe of available options. The purpose of this expansion is to provide a fuller understanding of solvency risk optimization.

Consider, as an example, asset trades. The traditional development of surplus deals only with the assets on hand at the valuation date. Since traditional surplus calculation precludes the notion of variability due to alternative future scenarios, consideration of asset trading is superfluous. If a 30-day treasury note is traded for a 30 year bond of equal value, surplus is unaffected. However, although its discounted present value may be the same, the net future cash flow is altered. The susceptibility of each net cash flow stream, the 30 day note stream compared to the 30 year bond, will differ, probably significantly. The solvency risk inherent in these two strategies differs.

Asset trades are a fertile source of strategic options. Rearranging the assets held changes the future net cash flow stream and hence changes the impact of different scenarios. That holding different assets can impact a company's solvency risk is generally well recognized. Other advanced strategic options are less well recognized but equally as interesting and perhaps as material in their impact on solvency risk.

The sale of new insurance policies and the elimination of entire portfolios of insurance policies, through their sale or reinsurance, represent significant strategic options. New sales are a part of almost every company's strategy and adding new insurance policies can significantly alter the company's projected net cash flow. While a study of projected net cash flow assuming no new sales, consistent with the traditional calculation of surplus, is interesting; including new policies brings the analysis closer to business reality. Further, adding new business allows consideration of the implications of product design.

Different product designs have different risk characteristics. Compare a typical North American whole life contract with guaranteed cash surrender values with a similar U.K.

equity linked product where the cash values are not guaranteed. In the context of the solvency analysis process described here, comparing these products would consist of testing a suite of scenarios for two different strategies and comparing the results. The first strategy assumes sales of fixed cash value business. The second strategy assumes sales of variable cash value business. Both strategies under one scenario, the scenario which would replicate traditional accounting surplus, would have the same surplus value. But other scenarios could be expected to produce significantly different net cash flow and surplus. Strategy 2, in which most of the investment risk is borne by the policyholder, would almost certainly exhibit less variability than strategy 1.

Different products react differently to the same scenario. An obvious example is the impact of increased mortality on life insurance (lower net cash flow) compared to its impact on vested annuities (higher cash flow). New business can be a vehicle used to balance the life company through immunization of non-investment risks. The elimination of a portfolio of policies could accomplish the same result, assuming appropriate assets are transferred.

In the context of surplus viewed as net cash flow, adding and removing insurance policy cash flow is akin to trading assets. The cash from both sources, policies and investments, has the same value and the same impact. As with asset trades, management can trade other cash flow elements and, in so doing, improve the company's solvency profile.

*Shareholder Dividends*

Traditional accounting assigns ownership of the difference between assets and liabilities to the shareholders (except if negative, in which case shareholders' limited liability precludes their having to fund the shortfall). Shareholders can, theoretically and practically, take the surplus. Such an action, of course, would have dramatic consequences on the future net cash flow and its variability. Including the cash paid to shareholders, in the form of dividends, among the company's disbursements brings our formula to a final net cash flow position. Whether or not dividends are paid and the timing and magnitude of their payment have a significant impact on a company's solvency. Postponing or reducing shareholder dividends represents an opportunity to reduce solvency risk. Studying a company's net cash flow and its variability under a strategy that includes the payment of shareholder dividends is a useful exercise for both the shareholders and the regulators.

*Using the new paradigm*

Returning to the three practical financial reporting problems mentioned at the beginning of the paper, some observation on how the cash flow paradigm can be applied may be useful.

In comparing two different financial reporting structures, for example, net premium Modified Canadian reserves (pre-PPM) compared to PPM gross premium reserves, how do we decide which is better? Can we decide one or the other is right? In the context of the new paradigm we see that different financial reporting methods amount to different assumptions about the future. The surplus values produced are those consistent with each specified set of assumptions.

Referring again to the two dimensional curve shown as figure 1, surplus values produced by different reporting structures amount to different points on the curve. Mathematically a number of options are available for defining the "best" method. Perhaps that which has the highest probability, or the one closest to the mean? Unfortunately both of these definitions require establishing probabilities for strategy and scenario combinations - not a trivial exercise.

Leaving mathematical precision aside, asking first the purpose of the surplus calculation exercise many help differentiate methods. If we are seeking the "most likely" value then using "most likely" assumptions would provide it. If we seek a "conservative" value, then, surely, conservative assumptions should be used. But here considerable caution is called for. Assumptions and methods that seem conservative are not always so. The author has encountered a number of circumstances where the surplus on a PPM basis (gross premium reserves with realistic assumptions plus a margin for adverse deviation) is lower than the supposedly conservative net-premium-with-conservative-assumptions value. The net premium reserve's lack of consideration for the impact of expenses is the principal culprit for this unexpected result. Completely ignoring a major source of cash flow is almost certain to misrepresent the future. Although the author has no direct experience with CRVM, logic would suggest that ignoring cash surrenders could have unintended results as well.

In comparing different methods, the reality is that none can be defined as "correct". Each is based on a view of the future and, until the future unfolds, actual cash flows and surplus are only speculation. Although it can be argued that different methods provide better or worse results, a more interesting view is that different methods provide different perspectives of the future. By constructing and comparing a number of these different perspectives, we can begin to construct a picture of surplus risk and the sorts of actions that will reduce that risk.

In the matter of appraised values, it should be readily apparent that they are also points on the two dimensional surface. The addition of new business (an advanced strategic option) is easily incorporated. As there is no correct surplus value, there is also no correct appraisal value. Each appraisal value reflects a view of the future and the best the appraiser can do is select a reasonable view and clearly describe how that view was constructed.

Finally, the issue of insolvency is where the new paradigm provides the most useful insight. If solvency means that surplus is greater than zero, then solvency depends on the view of the future selected to calculate surplus. We cannot say for certain that a company is solvent or not (except in the unlikely event that surplus if greater than zero, or less, for all possible futures). But by looking at surplus calculated based on a variety of futures, we can develop a picture of probable outcomes. If this picture is "negative", that is to say, there are many negative surplus values generated by reasonable future expectations, then we can say the company's ability to meet its obligations is impaired.

How does a company become insolvent? Recognizing that surplus is a prediction of the future helps to answer this question. As time progresses, the future becomes the past and the range of options, at least for the year just over, has narrowed. If the narrower range increases the likelihood of negative outcomes, we can say the company is becoming more impaired. In other words, it is not so much a matter of a company becoming insolvent as recognizing that it is more likely insolvent than not.

If we cannot say for certain if a company is solvent, does this mean an end to seizure or foreclosure by debt investors, solvency funds or regulators? Before answering this question, we should look at the purposes of establishing solvency. Two reasons come to mind.

First, if a company is likely to be insolvent, in other words, it will run-out of cash before all obligations have been meet, then an earlier recognition of this shortfall is generally seen as more equitable than a later recognition. To clarify this, let's look at the extreme of late recognition: allowing a company to continue to operate until all cash inflow has been exhausted. Up to the time of exhaustion, all cash outflow would be paid in full. After cash inflow has ceased, no further payments could be made on obligations. Debt investors might object to this if their payment falls due after assets are exhausted. Indeed, any legal priority order of payment would be eliminated in favour of a temporal priority - "first come first served". Most observers would see this as inconsistent with fair treatment between liability classes and inequitable within classes.

The existence of a solvency fund adds an interesting element. If the solvency fund guarantees all obligations, [3] a delay in foreclosure until all cash flow has stopped could be desirable. Why pay now rather than later, especially if strategies could be adopted that maximize the likelihood of scenarios leading to a positive surplus outcome?

The second reason for establishing insolvency is to reconsider strategic options. If a company is impaired (and assuming the rest of the industry is not) we can conclude that management's actions have not been effective at increasing or maintaining surplus. Perhaps products are improperly priced or inappropriate assets are being acquired. In

[3]Most don't, but the assumption simplifies the discussion. The argument is applicable regardless, only the mechanics are more complex.

any event, strategies should be reviewed and, likely, actions should be implemented to control the deterioration.

With these purposes in mind, seizure becomes a matter of assessing likelihood. If insolvency is likely, then regulators have an obligation to take control, in the best interest of the policyholders or depositors. Lenders, acting in their own best interest, have a similar obligation. Neither party can delay action until insolvency is a certainty because it will not be a certainty until all cash flows have ceased and then action is too late.

## V. SOME ADDITIONAL THOUGHTS

Although the author is not aware of any earlier discussion of surplus as discounted net cash flow, some of the sentiments expressed here have been voiced in recent actuarial work.

Immunization theory is concerned with strategies and scenarios in the interest rate and asset selection option subsets. The notion of discounted net cash flow is a central component. Indeed, Vanderhoof in The Interest Rate Assumption and the Maturity Structure of the Assets of a Life Insurance Company [11] begins a discussion of the basic theory of immunization with reference to liability cash flows (identified as $B_t$) and asset cash flows ($A_t$) essentially identical to the terms $l(x,t)$ and $a(y,t)$ used in this paper.

Two recent Canadian actuarial initiatives are also noteworthy. Mr. Stuart Wason's work on the Valuation of Single Premium Annuities [2] describes a cash flow valuation method that incorporates both asset and liability cash flows in the determination of a liability value. Essentially, the method proposes projecting net cash flows (asset inflows less liability outflows), adjusting the amount of assets until a zero net cash flow is achieved and setting the liability value equal to the statement value of the adjusted assets.

The second relevant Canadian initiative is the concept of dynamic solvency testing [3], [4]. Dynamic solvency testing, or DST, provides for the projection of surplus using a variety of alternative futures. Specifically, DST consists of projecting a company's balance sheet five year's into the future based on a variety of projection assumptions. All of the recommended assumptions are expected to generate negative results in the form of lower surplus. In this manner the surplus of the company is "tested" to determine how well it can withstand the impact of threatening future events. DST is a valiant effort, in the context of the current actuarial/accounting paradigm, to address the concept of solvency risk.

In the new paradigm, testing solvency is an integral component of the surplus development process. In the context of surplus as net cash flow, variability due to future events is inherently recognized in the calculation of "today's" surplus.

*Other applications*

The paradigm described in this paper is developed using a life insurance company as the subject of study. But none of the concepts are peculiar to a life company. The basic building blocks; namely, cash, assets and liabilities, are basic to all organizations and financial entities. The ideas presented in this paper are applicable to other insurance and risk endeavours (including pension plans), other financial institutions and, indeed, to every entity that reports a surplus position. The paradigm establishes a quantification of the risks faced by every entity and provides a comprehensive profile of the impact of future events on the solvency of the entity. It addresses solvency risk in the broadest sense.

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