Living to age 100 in Canada in 2000

Louis G. Doray, Ph.D., A.S.A.

Département de mathématiques et de statistique, Université de Montréal

C.P. 6128, Succursale Centre-Ville, Montréal, Québec, Canada H3C 3J7

(e-mail: doray@dms.umontreal.ca)

Key Words and Phrases: mortality rates, advanced ages, projected life table, force of mortality, Gompertz, Makeham, logistic, Beard, Kannisto models; maximum age.

ABSTRACT

With the annual number of deaths by age in Canada, published by Statistics Canada, we first construct cohort life tables at ages 80 and over for people born before 1900, using the method of extinct generations. We fit some statistical models to these data sets, using maximum likelihood theory to estimate the parameters of the models and obtain estimates of the standard error for the mortality rates. Goodness-of-fit tests are performed to check model adequacy. We then project the mortality rates for people who will attain age 80 in the future. Finally, the distribution of the maximum age at death that will be attained by a Canadian is investigated.

1 DATA SOURCES

Statistics Canada is a governmental organization responsible for making a detailed analysis of the Canadian population and publishing the data. From 1949 to 1997, we can find the annual number of deaths in Canada by sex and individual ages from 0 to 99 and grouped over age 100 (denoted 100+). This information is also available for each province and territory, but we will use only the total for Canada. Age is defined as the completed age in years.

Vincent (1951) developed the method of extinct generations to construct life tables from death statistics. Assuming that no migration takes place after age 80, we can calculate the number of males and females aged x ($x \geq 80$), living on January 1, Y, by summing the number of deaths in future years at successive ages,

$$l_x^Y = \sum_{k=0}^{\infty} d_{x+k}^{Y+k},$$

where l_x^Y is the number of people aged x living on January 1, Y and d_x^Y is the number of people dying between ages x and $x+1$ in calendar year Y. An assumption on the fraction of people dying at age x in year Y born in year $Y-x$ or $Y-x-1$ is necessary: it will be assumed that 50% are born in each year. The number l_x^Y was therefore approximated by

$$l_x^Y = \frac{1}{2}\sum_{k=0}^{\infty} d_{x+k}^{Y+k} + \frac{1}{2}\sum_{k=0}^{\infty} d_{x+k+1}^{Y+k},$$

and rounded to the nearest upper integer. The other assumption that no emigration or immigration to Canada occurs at those advanced ages is entirely reasonable.

Depoid (1973) notes that death statistics are much more reliable than census data in countries where a birth registration system has been in operation for a long time.

The empirical mortality rate at age x in calendar year Y can be calculated as

$$q_x^Y = 1 - l_{x+1}^{Y+1}/l_x^Y = d_x^Y/l_x^Y,$$

An empirical life table at ages 80-99 for calendar year Y is constructed by using the mortality rates $q_{80}^Y, q_{81}^Y, \ldots, q_{99}^Y$ and an arbitrary number of initial lives at age 80. On the other hand, a cohort life table at ages $80-99$ for somebody born in year Z would use the mortality rates $q_{80}^{Z+80}, q_{81}^{Z+81}, \ldots, q_{99}^{Z+99}$ and an arbitrary number l_{80} or the known number l_{80}^{Z+80}. Cohort tables will be constructed for certain birth periods before 1900.

2 MODELLING HUMAN MORTALITY

2.1 Statistical models for the force of mortality

To define the models which have been used by actuaries and demographers to describe the force of human mortality at adult or advanced ages, we will use the parametrization of Horiuchi and Coale (1990).

Gompertz (1825) assumed that the force of mortality μ_x at age x was an exponential function of age,

$$\mu_x = Be^{\mu x}.$$

Makeham (1860) added an extra parameter to this model to take into account the force of accidental death, assumed to be a constant independent of age, and obtained the model

$$\mu_x = A + Be^{\mu x}.$$

Marshall and Olkin (1997) mention that the Makeham model can be considered as a shock model (see Bowers et al. (1997)): if X, the lifetime of a person is Gompertz distributed, Y, the time to a fatal accident, has an exponential distribution, and the random variables X and Y are independent, then the minimum of X and Y has a Makeham distribution.

Gompertz and Makeham models were used for over a century. Depoid (1973) wrote that "For a long time, mortality tables stopped at age 85; to go beyond, insurance companies use Gompertz and Makeham formulas" (translation).

Another model developed by an actuary, Perks (1932), which has not received as much attention as the above two models is the logistic model, where the force of mortality at age x is given by the 4-parameter function

$$\mu_x = \frac{A + Be^{\mu x}}{1 + Ce^{\mu x}}.$$

By assuming that the parameter $A = 0$ in the logistic model, Beard (1963) obtained the 3-parameter model

$$\mu_x = \frac{Be^{\mu x}}{1 + Ce^{\mu x}}.$$

Kannisto (1992), a demographer, used the simple 2-parameter model

$$\mu_x = \frac{Be^{\mu x}}{1 + Be^{\mu x}}.$$

Note that the Gompertz ($A = 0,\ C = 0$), Makeham ($C = 0$), Beard ($A = 0$) and Kannisto ($A = 0,\ B = C$) models are all special cases of the logistic model; therefore, after successfully fitting the logistic and another one of the above models, a likelihood ratio test could be performed to check whether a model more parsimonious than the logistic one would be appropriate considering its smaller number of parameters.

Beard (1971) showed that the logistic model can arise in a heterogeneous population where each member has a Makeham force of mortality and where the parameter B varies among individuals according to a gamma distribution. This Makeham-gamma model is a frailty model. Thatcher et al. (1998) also mention two ways in which the logistic model could arise from stochactic processes. The logistic model can also be considered as a shock model: if the lifetime X follows a Beard distribution, the time to an accident Y is exponentially distributed and the random variables X and Y are independent, then $\min(X, Y)$ follows a Perks distribution

Using maximum likelihood, Thatcher et al. (1998) fit the Gompertz, logistic, Kannisto, and Weibull models as well as the Heligman & Pollard (1980) model

$$q_x = \frac{Be^{\mu x}}{1 + Be^{\mu x}}$$

and the quadratic model

$$\ln \mu_x = a + bx + cx^2$$

to mortality data of aged people in 13 industrialized countries for the periods 1960-70, 1970-80, 1980-90 and for the cohort born in 1871-80 using maximum likelihood theory. The data used were deaths at ages 85 and over for the quadratic model and ages 80 and over for all the other models. The 13 countries included in the study were Austria, Denmark, England and Wales, Finland, France, West Germany, Iceland, Italy, Japan, the Netherlands, Norway, Sweden and Switzerland. The best fit was consistently provided by the Kannisto and logistic models for all countries in each period and for the cohort data.

All models listed above produce very close values of μ_x at ages 80 to 95. After age 95, the Gompertz and Makeham forces of mortality continue to increase exponentially with age, while for the Kannisto, Beard and logistic models, μ_x tends asymptotically to a constant as x increases. This asymptote is equal to 1 for the Kannisto model and B/C for the Beard and logistic models.

Since, for Canada, we only had data at individual ages 80 to 99, and because Canadian mortality is comparable to that observed in the industrialized countries mentioned above, we also used the logistic and Kannisto models for mortality at advanced ages in Canada.

2.2 Estimation of the parameters

Let q_x represent the probability that a person aged x dies before attaining age $x+1$. In terms of μ_x, q_x equals (see Bowers et al. (1997))

$$q_x = 1 - exp\left(-\int_x^{x+1} \mu_y dy\right).$$

The complement of q_x, $p_x = 1 - q_x$ represents the probability that a person aged x survives at least to age $x+1$.

For the logistic model, we find that

$$\begin{aligned} q_x &= 1 - \exp\left(-\int_x^{x+1} \frac{A + Be^{\mu y}}{1 + Ce^{\mu y}} dy\right) \\ &= 1 - (e^{-A}) \left(\frac{1 + Ce^{\mu x}}{1 + Ce^{\mu(x+1)}}\right)^{(B-AC)/C\mu}; \end{aligned}$$

for Beard model,

$$q_x = 1 - \left(\frac{1 + Ce^{\mu x}}{1 + Ce^{\mu(x+1)}}\right)^{B/C\mu};$$

while for Kannisto model,

$$q_x = 1 - \left(\frac{1 + Be^{\mu x}}{1 + Be^{\mu(x+1)}}\right)^{1/\mu};$$

finally for Makeham model,

$$q_x = 1 - \exp\left[-A + (B/\mu)(1 - e^{\mu})e^{\mu x}\right].$$

In the Kannisto model, as x tends to ∞, q_x tends to $1 - e^{-1} = 0.632$, while it tends to $1 - e^{-B/C}$ in the Beard and logistic models; in the Gompertz and Makeham models, since μ_x is unbounded, q_x tends to 1.

Demographers often use the approximation

$$q_x \cong 1 - e^{-\mu_{x+1/2}},$$

obtained with the use of the midpoint rule, i.e. $\int_x^{x+1} \mu_y dy \cong \mu_{x+1/2}$. This approximation is very close to the true value. For Kannisto model, with the same values of the parameters as the ones obtained by Thatcher et al. (1998), the relative difference between the exact value of q_x and the approximate one obtained using the midpoint rule is only 0.03% for a male aged 80 and 0.0008% at age 100. The advantage of using the exact formula for q_x instead of the approximate one lies in the fact that an analytical expression can be found for the probability ${}_np_x$ that a person aged x survives to age $x + n$,

$${}_np_x = \exp\left[-\int_x^{x+n} \mu_y dy\right] = (e^{-nA})\left(\frac{1 + Ce^{\mu x}}{1 + Ce^{\mu(x+n)}}\right)^{(B-AC)/C\mu},$$

for the logistic model, and

$${}_np_x = \left(\frac{1 + Be^{\mu x}}{1 + Be^{\mu(x+n)}}\right)^{1/\mu},$$

for Kannisto model.

The use of the mid-point rule

$$ {}_np_x = \exp\left[-\int_x^{x+n} \mu_y dy\right] \cong \exp\left[-\mu_{x+n/2}\right] $$

would become less accurate as n gets larger, while using the mid-point rule over successive one-year intervals

$$ {}_np_x = \prod_{i=0}^{n-1} p_{x+i} \cong \prod_{i=0}^{n-1} \exp\left[-\mu_{x+i+1/2}\right] $$

would be more cumbersome.

Let θ be the vector of parameters of dimension p to be estimated, and $q_x(\theta)$ the value of q_x calculated with a parametric model. For the logistic model $p = 4$ and $\theta = (\mu, A, B, C)$, while for the Kannisto model, $p = 2$ and $\theta = (\mu, B)$. We will use the method of maximum likelihood to estimate the parameter vector θ. The likelihood function equals

$$ L(\theta) = \prod_{x=80}^{99} q_x(\theta)^{d_x} p_x(\theta)^{l_x - d_x}, $$

where d_x is the number of deaths between ages x and $x + 1$ and l_x is the number of people living at age x.

The loglikelihood function, denoted $l(\theta)$, defined as $l(\theta) = \ln L(\theta)$, equals

$$ l(\theta) = \sum_{x=80}^{99} d_x \ln q_x(\theta) + (l_x - d_x) \ln p_x(\theta). $$

The maximum likelihood estimator (MLE) of θ, $\hat{\theta}$, will maximize $L(\theta)$ or equivalently, $l(\theta)$. We can find the MLE $\hat{\theta}$ numerically, either by maximizing directly the loglikelihood function $l(\theta)$ or by solving the system of equations

$$\frac{\partial l(\theta)}{\partial \theta_j} = 0, \quad j = 1, \ldots, p,$$

where θ_j is the j^{th} component of θ.

The asymptotic variance-covariance matrix of $\hat{\theta}$, denoted Σ, is equal to the inverse of the observed information matrix

$$\Sigma = I_{\hat{\theta}}^{-1}, \text{ where } I_{\hat{\theta}} = \left(\left. \frac{-\partial^2 l(\theta)}{\partial \theta_i \partial \theta_j} \right|_{\theta = \hat{\theta}} \right).$$

The MLE $\hat{\theta}$ is a consistent estimator of θ and has an aymptotic multivariate normal distribution with mean vector θ and variance-covariance matrix estimated by $I_{\hat{\theta}}^{-1}$, enabling the construction of confidence intervals for the components of vector θ. The asymptotic variance of $\hat{q}_x(\theta)$ can then be calculated using the method of statistical differentials (see Lawless (1982)), and a confidence interval for $q_x(\theta)$ can be obtained for $x = 80, \ldots, 99$.

2.3 Goodness-of-fit tests

To test whether the fit of a model is appropriate to the data, the global χ^2 goodness-of-fit test statistic for males and for females can be used and compared with the critical value of the χ^2 distribution with the appropriate number of degrees of freedom. The χ^2 test statistic is defined as

$$\chi^2 = \sum_{x=80}^{100+} \frac{(d_x^{obs} - d_x^{exp})^2}{d_x^{exp}},$$

where d_x^{obs} is the observed number of deaths at age x, d_x^{exp} is the expected number of deaths at age x according to the parametric model used, and d_{100+} represents the number of deaths at age 100 and over, which is also equal to the number of people who survive to age 100.

2.4 Likelihood ratio tests

To select one model among two models which fit the data adequately according to the χ^2 goodness-of-fit test, a likelihood ratio test can be performed. Let $l(\hat{\theta}_{H_0})$ be the maximum value of the loglikelihood function under hypothesis H_0. The statistic

$$\lambda = 2[l(\hat{\theta}_{H_0}) - l(\hat{\theta}_{H_1})]$$

has an aymptotic χ^2 distribution with Δ degrees of freedom, where Δ is the difference between the number of parameters under H_0 and the number of parameters under H_1 and where the model under H_1 is a submodel of the model under H_0.

3 RESULTS

3.1 The data

We will illustrate the theory developed in the preceding section with Canadian mortality data. Table 1 contains the numbers of males living at ages

80 and over, for the 5 cohorts born in the periods 1869-1872, 1873-77, 1878-82, 1883-87 and 1888-92, and Table 2 the corresponding numbers for females. Those numbers were calculated with the method of extinct generations, as explained in section 1. It is assumed that the people of the first cohort born in the years 1869-1872 have all died, while all those who died before age 105 have been observed in the last cohort studied (people born in 1888-1892). The unknown number of people born in the years 1888-1892, still alive on 1/1/1998, who would be at least 105 year old, is not counted with the method of extinct generations. However, it is believed that this number is small and that it would not affect the parameter estimates of the mortality curves too much.

3.2 Estimated parameters

Table 3 contains the values of the parameters estimated with the maximum likelihood method for the five male cohorts with Kannisto model, as well as their standard error and the estimated covariance, and Table 4, the values for the female cohorts. The fit of the Kannisto model, as measured by the χ^2 goodness-of-fit test statistic, is not acceptable at the 5% level over the whole range of ages for all the cohorts. The same situation was observed when we used the Perks model. A possible reason is the extra variability introduced by the method of extinct generations, since, to obtain the l_x values, we need

Table 1: Male Cohorts

Age	1869-72	1873-77	1878-82	1883-87	1888-92
80	54812	81903	98229	111511	113437
81	49212	73748	88573	100641	102557
82	44053	65967	79355	90497	92132
83	38847	58179	70181	80253	81763
84	33657	50479	61182	70524	71852
85	28744	43369	52536	60934	62454
86	24463	36886	44870	52290	53809
87	20341	30756	37758	44160	45827
88	16631	25380	31209	36803	38591
89	13456	20524	25575	30261	32230
90	10739	16363	20661	24522	26699
91	8382	12860	16324	19559	21625
92	6473	9912	12734	15598	17294
93	4836	7488	9630	12114	13578
94	3561	5591	7253	9219	10428
95	2578	4010	5254	6927	7816
96	1813	2830	3806	5039	5702
97	1234	1968	2651	3620	4113
98	822	1292	1781	2543	2881
99	523	819	1209	1753	1937
100+	341	541	794	1145	1311

Table 2: Female Cohorts

Age	1869-72	1873-77	1878-82	1883-87	1888-92
80	59078	89468	109551	131130	150715
81	54002	82173	101322	121923	141024
82	49292	75426	93320	112879	131291
83	44397	68099	84800	103558	121063
84	39586	60891	76201	94008	110661
85	34953	53749	67914	84727	100310
86	30470	47044	59934	75577	90189
87	26171	40667	52292	66532	80325
88	22198	34686	45160	57917	71039
89	18526	29291	38455	49901	62231
90	15241	24313	32362	42432	53924
91	12400	19908	26892	35618	46027
92	10038	16192	21915	29535	38821
93	7847	12735	17375	24192	32136
94	6016	9835	13563	19492	26187
95	4505	7536	10396	15462	20894
96	3315	5645	7934	12043	16268
97	2384	4084	5943	9170	12411
98	1636	2964	4352	6826	9285
99	1158	2048	3135	4956	6751
100+	798	1349	2181	3465	4723

Table 3: Estimated Parameters for Males

Cohort	μ (variance)	B (variance)	Covariance
1869-72	3.186E-5 (1.284E-11)	0.10219 (1.732E-6)	-4.711E-9
1873-77	4.885E-5 (1.974E-11)	0.09716 (1.132E-6)	-4.725E-9
1878-82	4.362E-5 (1.260E-11)	0.09794 (9.037E-7)	-3.371E-9
1883-87	6.184E-5 (2.104E-11)	0.09335 (7.477E-7)	-3.961E-9
1888-92	8.482E-5 (3.710E-11)	0.08922 (6.987E-7)	-5.085E-9

Table 4: Estimated Parameters for Females

Cohort	μ (variance)	B (variance)	Covariance
1869-72	2.639E-5 (6.722E-12)	0.10178 (1.299E-6)	-2.951E-9
1873-77	2.643E-5 (4.298E-12)	0.10125 (8.249E-7)	-1.880E-9
1878-82	2.561E-5 (3.122E-12)	0.10078 (6.346E-7)	-1.406E-9
1883-87	2.758E-5 (2.821E-12)	0.09879 (4.903E-7)	-1.174E-9
1888-92	2.168E-5 (1.449E-12)	0.10053 (4.047E-7)	-7.647E-10

to add many random variables of death numbers, increasing the variance.

To illustrate the methods of section 2, we will use the parameter values for Kannisto model appearing in Tables 3 and 4. The maximum likelihood estimator of the mortality rates at ages 80 and over is equal to

$$\hat{q}_x = 1 - \left(\frac{1 + \hat{B} e^{\hat{\mu} x}}{1 + \hat{B} e^{\hat{\mu}(x+1)}} \right)^{1/\hat{\mu}},$$

by the invariance property of the MLE. The first five columns of Table 5 contain the estimated q_x values for males for the 5 male cohorts and Table 6, the values for females.

Table 5: Estimated and projected q_x for Males

Age	1869-72	1873-77	1878-82	1883-87	1888-92	1893-97	1898-1902
80	0.1009	0.1029	0.0986	0.0968	0.0955	0.0932	0.0909
81	0.1100	0.1116	0.1071	0.1047	0.1031	0.1004	0.0978
82	0.1197	0.1209	0.1161	0.1132	0.1111	0.1080	0.1050
83	0.1300	0.1308	0.1258	0.1222	0.1195	0.1160	0.1125
84	0.1410	0.1413	0.1361	0.1318	0.1285	0.1245	0.1206
85	0.1526	0.1523	0.1469	0.1419	0.1380	0.1336	0.1293
86	0.1649	0.1640	0.1584	0.1526	0.1480	0.1430	0.1382
87	0.1778	0.1762	0.1704	0.1638	0.1584	0.1529	0.1475
88	0.1913	0.1890	0.1831	0.1755	0.1694	0.1633	0.1575
89	0.2054	0.2023	0.1962	0.1877	0.1808	0.1742	0.1677
90	0.2201	0.2161	0.2099	0.2005	0.1927	0.1855	0.1785
91	0.2352	0.2303	0.2241	0.2136	0.2051	0.1973	0.1899
92	0.2507	0.2450	0.2387	0.2272	0.2178	0.2094	0.2014
93	0.2666	0.2599	0.2537	0.2412	0.2309	0.2222	0.2139
94	0.2828	0.2752	0.2689	0.2555	0.2444	0.2349	0.2258
95	0.2991	0.2907	0.2844	0.2701	0.2581	0.2481	0.2385
96	0.3155	0.3063	0.3001	0.2849	0.2721	0.2616	0.2515
97	0.3320	0.3219	0.3159	0.2998	0.2862	0.2752	0.2647
98	0.3484	0.3375	0.3316	0.3149	0.3005	0.2891	0.2782
99	0.3646	0.3531	0.3473	0.3299	0.3149	0.3031	0.2918

Table 6: Estimated and projected q_x for Females

Age	1869-72	1873-77	1878-82	1883-87	1888-92	1893-97	1898-1902
80	0.0834	0.0805	0.0757	0.0702	0.0641	0.0594	0.0551
81	0.0911	0.0879	0.0827	0.0766	0.0701	0.0650	0.0603
82	0.0994	0.0959	0.0903	0.0835	0.0767	0.0712	0.0661
83	0.1083	0.1045	0.0984	0.0910	0.0838	0.0779	0.0723
84	0.1178	0.1138	0.1072	0.0990	0.0914	0.0850	0.0790
85	0.1280	0.1236	0.1165	0.1076	0.0996	0.0927	0.0863
86	0.1388	0.1341	0.1265	0.1168	0.1084	0.1010	0.0941
87	0.1503	0.1452	0.1371	0.1266	0.1178	0.1099	0.1025
88	0.1624	0.1570	0.1484	0.1370	0.1279	0.1195	0.1116
89	0.1751	0.1694	0.1603	0.1480	0.1385	0.1292	0.1205
90	0.1885	0.1824	0.1728	0.1596	0.1498	0.1403	0.1314
91	0.2024	0.1960	0.1859	0.1719	0.1618	0.1518	0.1424
92	0.2169	0.2102	0.1996	0.1847	0.1743	0.1638	0.1539
93	0.2319	0.2248	0.2138	0.1980	0.1875	0.1765	0.1662
94	0.2472	0.2399	0.2285	0.2119	0.2012	0.1901	0.1796
95	0.2630	0.2554	0.2437	0.2263	0.2154	0.2035	0.1923
96	0.2790	0.2713	0.2592	0.2410	0.2301	0.2178	0.2058
97	0.2953	0.2873	0.2750	0.2562	0.2453	0.2327	0.2208
98	0.3116	0.3036	0.2910	0.2716	0.2608	0.2479	0.2357
99	0.3280	0.3199	0.3071	0.2873	0.2766	0.2635	0.2511

3.3 Projected mortality rates

We can estimate the improvement in the mortality rates of persons aged x born in different calendar periods and project this improvement in the future to obtain future mortality rates. Using the 4 cohorts 1873-77 to 1888-92, we calculated the average decrease in q_x over 5 years for each age x and projected this decrease to obtain q_x for the cohorts born in 1893-97 and 1898-1902 (last two columns of Tables 5 and 6).

3.4 Life expectancy at age x

The complete life expectancy at age x, defined as

$$\mathring{e}_x = \int_0^\infty {}_tp_x dt$$

can be calculated with the exact formula for ${}_tp_x$ in section 2.2 and the use of a symbolic programming language like MATHEMATICA to evaluate numerically the integral. Table 7 contains the values of $\mathring{e}_x$ for males and females aged 80 to 99 born in the years 1888 to 1892.

The actuarial present value (a.p.v.) at age x, of a continuous life annuity of 1 per annum, calculated at a force of interest δ, under the Kannisto model,

$$\bar{a}_x = \int_0^\infty e_t^{-\delta t} p_x dt = \int_0^\infty e^{-\delta t} \left(\frac{1 + Be^{\mu x}}{1 + Be^{\mu(x+t)}} \right)^{1/\mu} dt$$

is seen to be exactly equal to the complete life expectancy of a person aged x, under the Perks model with the four parameters $(\mu, \delta, B(1+\delta), B)$.

Table 7: Life expectancy at age x

Age	Males	Females
80	6.64	8.36
81	6.29	7.90
82	5.95	7.46
83	5.63	7.04
84	5.33	6.64
85	5.04	6.25
86	4.77	5.89
87	4.51	5.54
88	4.27	5.22
89	4.04	4.91
90	3.83	4.62
91	3.63	4.35
92	3.44	4.09
93	3.26	3.85
94	3.09	3.63
95	2.94	3.42
96	2.79	3.22
97	2.65	3.04
98	2.53	2.87
99	2.41	2.72

Similarly, it can be shown that the a.p.v. of a continuous life annuity of 1 per annum at age x, under the Perks model, is equal to the complete life expectancy of a person aged x, under the Perks model with the four parameters $(\mu, A+\delta, B+\delta C, C)$.

4 MAXIMUM ATTAINED AGE

From Tables 1 and 2, there were 113,437 males and 150,715 females living aged 80 born in the years 1888-1892. We can study the distribution of the maximum age-at-death attained by a male or a female. Using extreme value theory (see Reiss and Thomas (1997) or Thatcher et al. (1998)), the mode (most likely value) attained by a female would be the solution of the equation

$$150715 \ {}_{\omega}p_{80} = 1, \quad \text{for } \omega.$$

Howver, with Kannisto model, we find that ω is too high to accept that model for female mortality.

ACKNOWLEDGEMENTS

The author gratefully acknowledges the financial support of the Natural Sciences and Engineering Research Council of Canada and also the computational assistance of Étienne Chassé- St-Laurent.

References

Beard, R.E. (1963). A theory of mortality based on actuarial, biological, and medical considerations. In *Proceedings of International Population Conference, New York, 1961*, **1**, 611-625, Liège.

Bowers, N.L., Gerber, H.U., Hickman, J.C., Jones, D.A., Nesbitt, C.J. (1997). Actuarial Mathematics, Sociey of Actuaries, Schaumburg, IL.

Depoid, F. (1973). La mortalité des grands vieillards, *Population*, **28**, 755-792.

Gompertz, B. (1825). On the nature of the function expressive of the law of human mortality etc., *Phil. Trans. Roy. Soc.*, **115**, 513-585.

Heligman, L. and Pollard, J.H. (1980). The age pattern of mortality, *Journal of the Institute of Actuaries*, **107**, 49-80.

Horiuchi, S. and Coale, A.J. (1990). Age patterns of mortality for older women: an analysis using the age-specific rate of mortality change with age, *Mathematical Population Studies*, **2**, 245-267.

Kannisto, V. (1992). Presentation at a workshop on old-age mortality, Odense University, Odense, Denmark.

Lawless, J.F. (1982). Statistical Models and Methods for Lifetime Data, Wiley, NewYork.

Makeham, W.M. (1860). On the law of mortality and the construction of annuity tables. *Journal of the Institute of Actuaries*, **8**, 301-310.

Marshall, A.W. and Olkin, I. (1997). Probabilistic models for lifetime data, draft copy.

Perks, W. (1932). On some experiments on the graduation of mortality statistics, *Journal of the Institute of Actuaries*, **63**, 12-40.

Reiss, R.-D. and Thomas, M. (1997). Statistical Analysis of Extreme Values, Birhäuser Verlag, Basel.

Thatcher A.R., Kannisto, V. and Vaupel, J.W. (1998). The Force of Mortality at Ages 80-120, Monographs on Population Aging, 5, Odense University Press, Odense, Denmark.

Vincent, P. (1951). La mortalité des vieillards, *Population*, **6**, 181-204.