Property/Casualty Insurer Economic Capital Using a VaR Model

Thomas Conway, Principal

Mark McCluskey, Actuarial Analyst

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Abstract

The purpose of this paper is to build a bridge between the traditional methods of looking at financial risk and insurance risk. Currently, many regulatory and internal company models are attempting to combine insurance risk into a value-at-risk (VaR) modeling structure. VaR models view insurance-related risk differently than the traditional actuarial models since risk is defined as the one-year fluctuation in market value of insurance liabilities.

The paper starts by explaining the concepts behind VaR models of economic capital used in banking. Differences between actuarial and financial views of risk are reviewed and explained. The paper concludes with proposing, developing and parameterizing a true VaR model of insurance loss reserve risk which combines several lines of business.

1. Introduction

The appropriate measurement of required capital by modeling economic capital (EC) levels has become an increasingly important issue for property-casualty (P&C) insurers. Regulatory paradigms are emerging in the United Kingdom and continental Europe which require companies to build their own economic capital models to interface with regulators. In the United States, lacking a regulatory initiative, rating agencies are increasingly viewing internal economic capital models as a necessity. Company use of EC models is considered a key element of effective risk and capital management which is considered in the rating process.

While insurance companies desire to manage using EC tools, no universal methodology exists. The standard that has emerged in continental Europe, driven by Basel II and Solvency II, is a value-at-risk (VaR) methodology derived from banking risk management and capital analysis. Most P&C insurers in the United States have relied on factor-based methodologies borrowed from regulatory or rating agency formulas and dynamic financial analysis (DFA) models.

2. Background on VaR Methods

The traditional U.S. approaches take a much different view of insurance loss reserve risk than the VaR approach in Europe. VaR methodologies have their roots in financial risk management tools that were originally used on a daily basis to monitor the potential fluctuations of trading portfolios. VaR inherently views risk as the fluctuation in the market values of risky positions. Over time VaR methods have evolved into a broader set of applications, utilized over longer time horizons, which are now used in banking EC models to analyze potential fluctuations in the market value of a firm.

An excerpt describing the background, rationale and elements of the VaR view of EC used in a banking environment follows.

2.1 Market Value Definition of Risk[1]

Over the past decade, EC has steadily progressed toward market value models. Most commercial portfolio frameworks have by now discarded first-generation EC models based only on default risk, although these models persist in some cases for

[1] *Economic Capital, A Practitioner Guide*. London: Risk Publications, 2004.

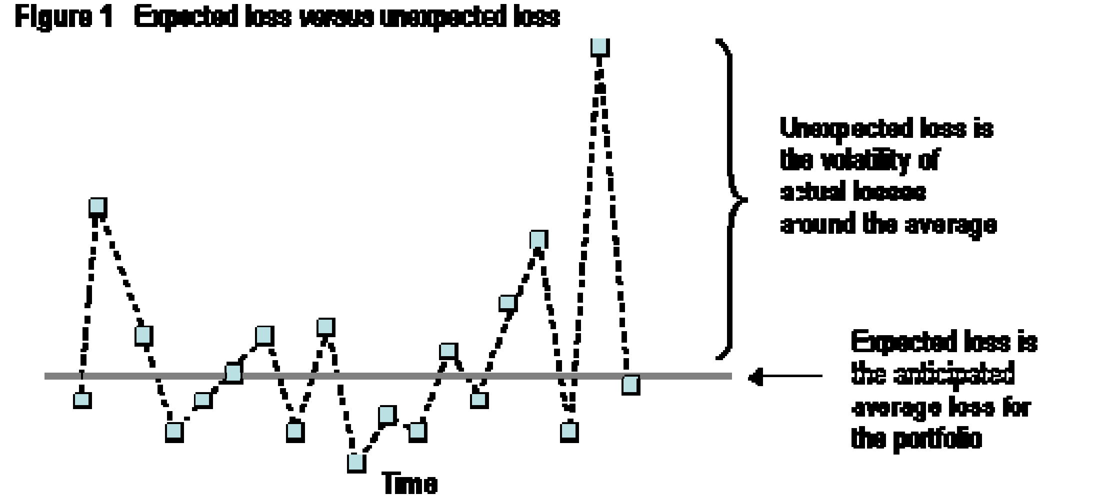
consumer portfolios. Given the goal of ensuring capital adequacy for a certain level of solvency, the volatility of market value is the best measure of a bank's risk and therefore its capital requirement.

Ultimately, shareholders are interested in the total return on their investment in the bank's stock and its risk in market value terms. They compare the return earned on their investment to a required return based on its risk. Bondholders also care about market values. The value of their fixed-income investment is a function of the credit spread of the bank, the level of interest rates and the expected cash flows of the debt. Since both stockholders and bondholders evaluate their investments based on market values, management should evaluate its opportunities with the same market value discipline. Defining risk in market value terms reinforces this discipline by aligning the interests of business managers with those of shareholders and bondholders.

2.2 Capitalization and Confidence Levels

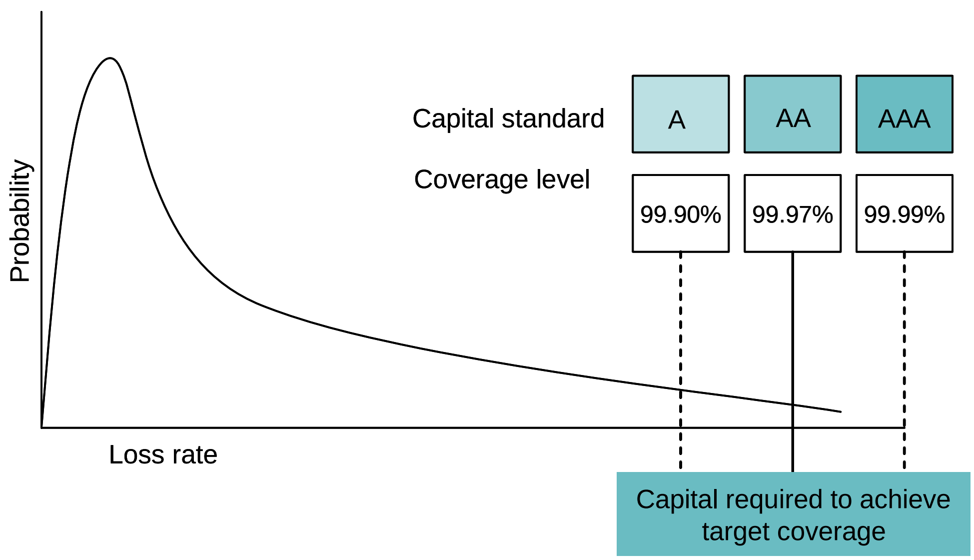
Two estimates describe a bank's risk profile: expected loss and unexpected loss. As illustrated in Figure 1, expected loss is the average rate of loss expected from a portfolio. If losses equaled their expected levels, there would be no need for capital. Unexpected loss is the volatility of losses around their expected levels. Unexpected loss determines the EC requirement.

Figure 1 Expected loss versus unexpected loss



To prevent insolvency, economic capital must cover unexpected losses to a high degree of confidence. Banks often link their choice of confidence level to a standard of solvency implied by a credit rating of A or AA for their senior debt. The historical one-year default rates for A firms and AA firms are approximately 10 and 3 basis points, respectively. These target ratings therefore require that the institution have sufficient equity to buffer losses over a one-year period with confidence levels of 99.90 percent and 99.97 percent (see Figure 2).

Figure 2 Achieving the desired coverage



For the purposes of this paper, the term VaR methods or models is being used in a generic context. While VaR models are sometimes thought of as risk models using a percentile or probability of ruin risk measure to determine EC, we are focused at a more basic level. A determination of the difference between EC risk models requires answering two key questions: "What triggers a need for additional capital in the risk model?" and "What is the time horizon over which the risk-producing elements are allowed to fluctuate?" A VaR model measures risk as an adverse change in market value over a one-year time horizon. Other EC risk models, such as DFA models, view risk as a change in accounting values over longer time horizons.

The choice of an appropriate risk metric, percentile measure or a tail value, is not relevant for the issues being discussed and is outside of the scope of this paper. Ample research has been done around appropriate risk measures with desirable properties. For example, see the discussion in the November 2002 *CAS Actuarial Review*. The discussion in this paper is not dependent on any specific risk measure.

3. A P&C Insurer's View of VaR

While much of the rationale and methodology discussed on using banking type VaR and EC could be applied to an insurance enterprise, construction of a similar VaR model would require three key items. First, an active market exists for the assets and liabilities held by a firm. The second assumption is, absent a market, a proxy for how the market value of the asset or liability would change under stress conditions could be developed. Finally, since most EC models using VaR methodology also focus on short time horizons (one year or less); the risk of the position must also manifest itself over the selected modeled horizon.

Unfortunately, the concept of market valuation of both assets and liabilities is less familiar to insurers. Historically, insurers in the United States have operated in the world of statutory accounting where the majority of investments, fixed income assets, were held at amortized value and loss reserves are held at an undiscounted nominal value. This accounting view is a significant deviation from the "mark to market" perspective that drove the development of VaR-based EC models.

Putting aside the U.S. statutory accounting model, most types of assets held by insurers can be analyzed in various historic market conditions due to the existence of long-term, active markets. A wealth of standardized and consistent financial market data also exists to create a needed proxy for market values of most other asset types.

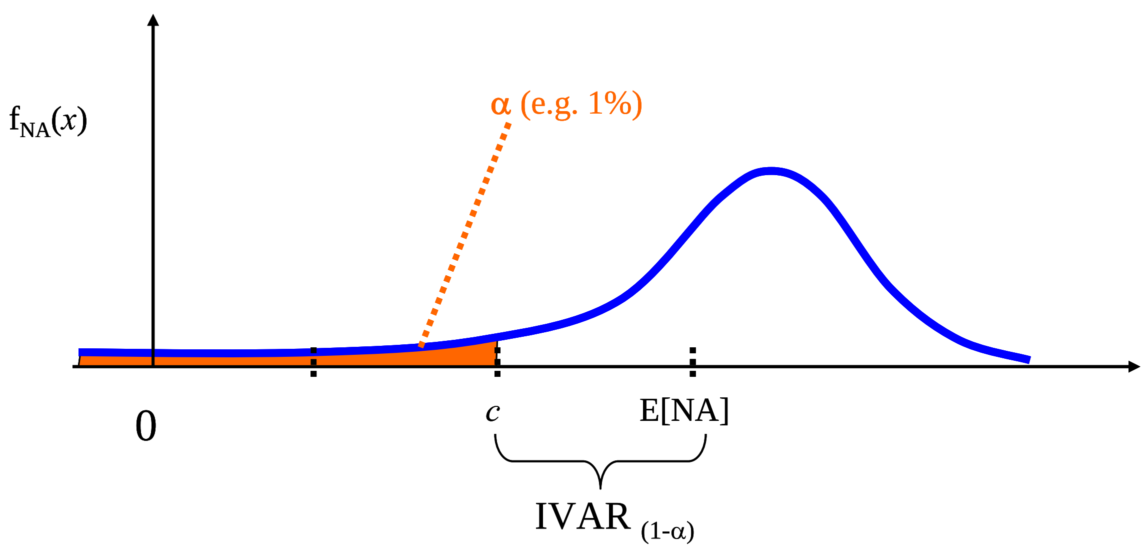
Insurance liabilities on the other end of the spectrum pose some unique challenges. No active market exists for insurance company liabilities. In a limited way, market prices can be observed through sales of companies, reinsurance transactions or securitizations. The numbers of transactions are small and information is not always public, so even this information is of limited value.

Given all of the issues mentioned above, attempts have been made to extend VaR and EC methodology into the P&C insurance world. The most notable example is the paper published by Nakada, Shah, Koyluoglu and Collignon in *The Journal of Risk and Finance* in 1999. A more recent example can be found in the *White Paper of the Swiss Solvency Test* published by the Swiss Federal Office of Private Insurance in November 2004.

Similar to prior attempts, we can define a VaR model for P&C insurance applications by relying on the banking concepts discussed above. Insurance VaR (IVaR), as in the banking applications, focuses on the unexpected loss in net assets (NA) at some percentile, c, where NA equals the market value of assets (MVA) less the market value of liabilities (MVL). The following equations and Figure 3 specify the model.

$$F_{NA}(x) = \Pr(NA \le x)$$
$$F_{NA}(c) = \Pr(NA \le c) = \alpha$$
$$IVAR_{(1-\alpha)} = \left|c - E[NA]\right|$$
$$IVAR_{(1-\alpha)} = EC_{(1-\alpha)}$$
Over a fixed time horizon, typically selected as one - year

Figure 3



4. Property-Casualty Reserve Liabilities in a VaR Context

In order to fully apply the IVaR model developed above, it is necessary to build and parameterize both a model of market value fluctuations for an insurer's assets and liabilities. The results of these two models could be derived simultaneously using common assumptions or combined through correlation algorithms. As was previously mentioned, the construction of VaR and EC models for assets is fairly well understood. Standard techniques, as well as commercial software, are available to model the potential fluctuations of investments to key risks such as interest rate, market and credit risk. The focus of this paper is creating an appropriate VaR model for P&C loss reserve liability risk so no further discussion of assets risk modeling is necessary. Two sources for additional information for an interested reader on asset-related topics are *Modern Risk Management, A History* and *VAR, Understanding and Applying Value-at-Risk*.

Focusing on loss reserve liability risk, the appropriate model to satisfy the IVaR structure will have several key characteristics. The characteristics are:

1. Produce a distribution of potential changes in loss reserve estimates.
2. Provide a proxy for market value estimates
3. Produce results over appropriate time horizons or time steps.
4. Parameterize with existing types of insurance data available.

The first characteristic simply requires that the model produce a distribution of results along with an expected value. Since a VaR model requires distribution percentiles, multiple model forms could satisfy this criterion. Those that have been used in practice include closed form distribution models, simulations and bootstrap sampling models.

The second criterion requires that a fair value, as a market value proxy, could be calculated for each of the outcomes that comprise the distribution of reserve estimates. Much research has been done around stating fair value of insurance liabilities for the purposes of implementing International Accounting Standards. The basic techniques involve discounting cashflows at an appropriate interest rate and then increasing the discounted value with a market value margin (MVM). The MVM is an adjustment that is meant to approximate a purchaser's risk margin or cost of committed capital required in a sale situation.

The concept behind a MVM creates many intriguing issues that are beyond the scope of this paper. One such issue is how the MVM would react in a stress situation where a large adjustment to reserves is made. Most methods currently contemplated assume the MVM is a fixed proportion of the expected value of the reserve liability. This is obviously a simplification that may be acceptable in a typical situation but is not in the extreme tail situations that drive the need for EC.

For purposes of this paper an adequate calculation of the fair value of reserve estimates will be the discounted value of the expected reserve payout cash flows at a risk-free rate. While this approach ignores some theoretical issues, the simplicity will aid in the discussion and development of an IVaR model for loss reserves.

The third criterion is probably the source of the most confusion and differences between the actuarial view of risk and the financial view of risk. Criterion three requires the model produce a distribution of the potential change in estimates over a selected time horizon. Unfortunately, standard actuarial models do not produce results over a discreet time horizon but rather results at "ultimate" or on a life of liability basis. The actuarial methods are different because they don't focus on an estimate moving to its final value but only the magnitude of the final value.

The reasons for the lack of time step in actuarial methods are primarily driven by the lack of relevance to the calculation for which they were intended. Current actuarial triangulation or chain ladder methods are used to produce best estimates of loss reserves. In addition, the methods, such as the Mack method, have been developed to produce estimates of the variability of reserve results using development triangle data.

P&C actuarial reserving methods are used to calculate values for financial statement purposes. The focus in this case is to set an adequate ultimate value and reasonable range of incurred losses. The point of the actuarial loss reserve estimation methods is to set a final value that will not change, while VaR focuses on how much the reserve estimate could change over the time horizon. Unfortunately, the best estimate and distributions produced by actuarial methods are related but not quite right for the IVaR calculation.

The last model criterion is a practical one. While the set of possible models is large, in the insurance world we are limited by the amounts and types of historical data available. Given likely data availability we will probably have to constrain ourselves to models that can be parameterized with loss development triangles and statutory annual statement financial data.

5. Specifying a VaR model for Loss Reserves

In order to satisfy the four criteria in the last section, a new type of loss reserve variability model will have to be specified. Prior to developing the model it is useful to discuss and clarify some basic concepts regarding the composition and estimation of loss reserves.

P&C loss reserves can be viewed as a portfolio of reserves which are composed of separate sub-portfolios from each accident year (AY). An AY is the underlying subgrouping of reserves used for statistical and financial statement purposes in insurance. AY contributions to a company's current reserve position can be viewed as different cohorts of reported open (RO) and incurred but not reported (IBNR) claims that when aggregated drive the company's reserve requirement as of an accounting date.

As of any accounting date, the total reserve contribution is derived from AYs with different levels of maturity or seasoning. Typically the current AY (corresponding with the accounting year) is seasoned by 12 months at a year-end accounting date. The first and second prior AYs are seasoned by 24 months and 36 months respectively. This pattern increments by an additional 12 months for each older AY and continues for as many years as an insurance company has been in business and claims are still open. In general the variability of an AY's final value should decrease as it matures since more claims are closed and more information is known about the RO claims the longer they have been reported to the insurer.

The AY cohort view of reserves described is analogous to a view of an asset portfolio compromised of groups of bonds. At any point in time a company will hold assets that are comprised of bonds from different issue years. For a set of bonds with the same term, the older the issue year, the more that is known about the underlying credit risk of the bonds and the closer they are to maturing.

The actuarial reserve estimation process involves analyzing AY development patterns from older, more mature AYs and imputing the same level of growth to less mature years. Development patterns are typically represented by the percentage growth observed in paid or case-incurred amounts by AY as they mature. Multiple actuarial methods utilizing different development patterns are typically used to produce loss reserve estimates.

Deciding on a reserve level to establish in company financial statements necessarily involves a set of judgments about the appropriate value for each AY in the face of uncertainty. Companies strive to reduce uncertainty by using multiple actuarial estimation methods, tracking price levels changes, understanding operational or data issues and using expert judgment. The financial statement reserve set at any point in time, therefore, is not the direct result of a mathematical calculation; rather it is the combination of judgments which weigh many factors. Along with the results of the actuarial calculations, factors such as future economic conditions, social attitudes and the state of the insurance market are considered. The reserving process has some similarities to how prices are set in an active market for financial instruments; the process is not always completely rational.

A model of the volatility of reserve estimates over a fixed time horizon, as required by a VaR method, cannot be reproduced by a simple mathematical method since it combines so many judgments. An appropriate solution requires observation of actual changes in estimates over time, similar to the way a study of the market price volatility of a financial instrument requires observing actual price changes over time.

As previously mentioned no open active market exists for loss reserve liabilities so other sources of information will have to be used. In the case of U.S. P&C insurers, the available data takes the form of the detailed information included in Schedule P of the annual statement. This schedule tracks AY reserve runoff for the prior 10 years and is available electronically from several sources. The data has been available electronically for many years so we were able to easily construct a database containing information for AYs going back to 1986.

The financial statement data has some shortcomings. As with any model-fitting process, the observations need to be collected over a long enough time to generate credible results. The best database would include information collected over several underwriting cycles, under a wide range of economic conditions for companies with similar reinsurance programs.

The data must also be relevant to the process we are modeling looking forward. As has been described in analyst publications such as *IBNR Weekly*, the industry is thought to go through reserving phases of "cheating," "recovery," and "restoration" as it moves through the underwriting cycle. To the extent that this phenomenon exits, current company attitudes may have changed due to increased insurance company regulation and scrutiny on reinsurance transactions, so the past may not be as predictive of the future. Using the available data will embed the assumption that all companies are equally diligent in setting reserves; any deviations over time are a result of process, parameter, model or operational risk and not deliberate bias introduced in the financial statement reserving process. With all of the shortcomings, the data is "the best we've got" if we want to build a time step model that encompasses the underlying complexities of the reserve setting process.

6. Building a VaR Model for Loss Reserves

Using a database that includes information from approximately 3,000 P&C insurers for each of 10 year-ends starting in 1995, we captured the key data in a form illustrated in the table below by AY for each statutory product line from Sch-P.

TABLE 1
Sample Loss Reserve Run-Off Database

	1 year	**2 years**	**3 years**	**4 years**	**5 years**	**6 years**	**7 years**	**8 years**	**9 years**	**10 years**
1986	$$$	$$$	$$$	$$$	$$$	$$$	$$$	$$$	$$$	$$$
1987	$$$	$$$	$$$	$$$	$$$	$$$	$$$	$$$	$$$	
1988	$$$	$$$	$$$	$$$	$$$	$$$	$$$	$$$		
:::::::	$$$	$$$	$$$	$$$	$$$	$$$	$$$			
:::::::	$$$	$$$	$$$	$$$	$$$	$$$				
:::::::	$$$	$$$	$$$	$$$	$$$					
2001	$$$	$$$	$$$	$$$						
2002	$$$	$$$	$$$							
2003	$$$	$$$								
2004	$$$									

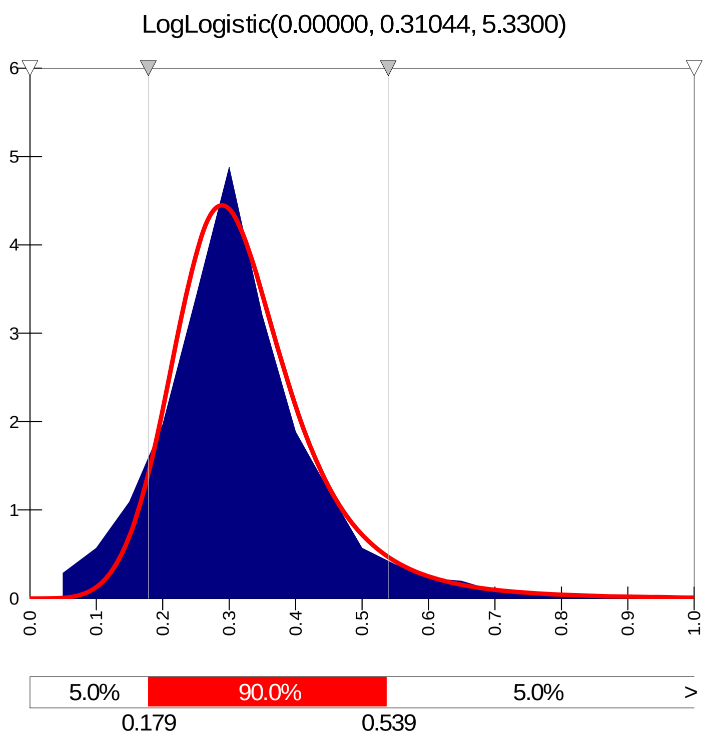
The database was populated with Sch-P Parts 2 and 3 which contains ultimate incurred loss and cumulative paid loss. Based on this the Sch-P data, another table of percentage movement in reserves was calculated for each AY level of maturity. In other words, we compiled separate observations for all AYs going from one year of age to two years, from two years to three years ending with nine years to 10 years. All percentages are expressed relative to the reserve balance held for each AY at the beginning of the interval period. An example of the full calculation is shown in Exhibit 1.

The purpose of collecting the data in such a way is two-fold. First, it allows for full use of all of the available data by breaking out the individual AY observations. This data format eliminates the problem of including an AY result in multiple observations. If we had taken the approach of tracking the run-off of aggregate blocks of loss reserves from subsequent year-ends, an individual AY result would have been counted more than once. By following this methodology, each AY result forms an independent observation for modeling purposes.

The second benefit of the data organization is that it allows separate models to be fitted for each AY maturity level which can be mixed back together. The ability to mix AYs will be necessary when applying the model to individual insurance companies since each has a unique volume of reserves and mix of business by AY.

The following graph shows a sample of the data produced in a histogram of the results. The data is combined across all years and is composed of about 8,000 observations for the Auto Liability line of business (LOB). This graph illustrates a shifted distribution of all AYs going from 24 months to 36 months of age and is typical of the shape for the movement at all ages of AY maturity.

Figure 4



Based on the shape of the distributions exhibited, we utilized a standard software package to fit the empirical distributions to a functional form. For ease of analysis we shifted all of the distributions by adding 0.3 to all of the observations. This eliminated the negative values and allowed the fitting algorithms to produce several options for curve fits.

It was interesting to note that all of the distributions had a mean close to zero before the scaling adjustment. This result was satisfying since any other result would have implied a bias in the reserve-setting process. We also noticed differentials between very small companies and the remainder of the company population. Since we are giving equal weight to each observation, the elimination of companies with less than $10 million of starting AY ultimate loss significantly reduced the volatility in the data. More work needs to be done to differentiate the population by size of company and also research some of the bimodality we observed at older maturity periods.

For the purposes of this paper, we are most interested in reasonable tail fits so behavior around the mean of the distribution can be further researched at a later time. Several options were evaluated for curve fits; the lognormal and loglogistic seemed to fit the data best especially in the tail. Figure 4 above, illustrates one of the loglogistic curves fit to the empirical data.

Using the results of the distribution fits, we produced the following table which shows the total undiversified capital charge by AY. The table below utilizes the Industry Aggregate Sch-P as of 12/31/04 and the 95th percentile EC charges for both the lognormal and loglogistic distributions. The table shows an interesting pattern of the volatility of the remaining reserve increasing with AY maturity.

TABLE 2
Industry Aggregate
Private Passenger Auto

				LogLogistic		Lognormal	
AY	Ultimate	Paid	Reserves	Charge **	Capital	Charge **	Capital
Prior	28,596,430	25,950,651	2,645,779	0.779	2,061,062	0.735	1,944,648
1995	45,723,271	45,539,873	183,398	0.779	142,867	0.735	134,798
1996	46,982,083	46,753,230	228,853	0.779	178,276	0.735	168,207
1997	47,285,673	46,920,942	364,731	0.777	283,396	0.747	272,454
1998	48,432,036	47,808,990	623,046	0.678	422,425	0.630	392,519
1999	51,810,005	50,715,549	1,094,456	0.726	794,575	0.656	717,963
2000	55,423,365	53,241,753	2,181,612	0.465	1,014,450	0.406	885,734
2001	57,110,245	52,661,358	4,448,887	0.390	1,735,066	0.343	1,525,968
2002	59,723,830	50,355,718	9,368,112	0.298	2,791,697	0.266	2,491,918
2003	59,848,713	41,640,458	18,208,255	0.239	4,351,773	0.218	3,969,400
2004	62,391,010	24,468,224	37,922,786	0.217	8,229,245	0.195	7,394,943
Total	563,326,661	486,056,746	77,269,915	0.285 *	22,004,832	0.258 *	19,898,552

* denotes (Total Capital) / (Total Reserves)
** Charge determined as (95% Percentile - Mean)

Revisiting the AY portfolio view of reserves discussed above, total reserve volatility can be viewed as a conflict between the decay in an aging AY's reserve level and the increase in volatility as an AY ages. The increasing pattern of volatility in Table 2 is plausible since typically the less severe, easier to settle claims are paid out first leaving the difficult more volatile claims in the remaining population. For auto liability the oldest claims would tend to be long-term personal injury protection (PIP) claims requiring lifetime medical payments.

Obviously, the desired result of this analysis is the diversified EC for the total portfolio of reserves recorded at any accounting date. In order to perform this last step of the analysis, we performed a rank correlation analysis on the Auto Liability AY data. The correlation analysis was used to determine how closely movement in one AY was related to another.

Intuition would say that movement in an AY cannot be completely independent of other AYs. Judgments that impact the decision to set reserve levels for one AY must also impact reserve decisions in other AYs. For example, reasonability of AY to AY trends such as claim severity, loss ratios and IBNR to case reserve ratios are all examined for groups of clustered AYs. Any changes in judgment for one AY can have a ripple effect and change the reserves established for several AYs.

TABLE 3
Indicated Rank Correlations by AY Maturity

AY Maturity	12	24	36	48	60	72	84	96	108
12	1	0.279	0.226	0.168	0.091	0.094	0.076	0.037	0.014
24	0.279	1	0.338	0.232	0.196	0.06	0.101	0.035	0.028
36	0.226	0.338	1	0.344	0.231	0.175	0.114	0.122	0.012
48	0.168	0.232	0.344	1	0.313	0.202	0.149	0.139	0.034
60	0.091	0.196	0.231	0.313	1	0.178	0.124	0.195	0.021
72	0.094	0.06	0.175	0.202	0.178	1	0.114	0.095	0.04
84	0.076	0.101	0.114	0.149	0.124	0.114	1	0.092	0.052
96	0.037	0.035	0.122	0.139	0.195	0.095	0.092	1	0.045
108	0.014	0.028	0.012	0.034	0.021	0.04	0.052	0.045	1

AY Maturity

The pattern of the correlations shown in the table above tends to match intuition. The darker the area in Table 3, the higher the correlation indicated. For an individual AY the movement in a calendar year is closely related to the movements in the adjacent AYs. The strength of this pattern is fairly strong until 60 months of age. Past 60 months the relationship is weak, possibly indicating that changes in the reserves for older AYs are more related to the actual claims open in that year rather than changes in common assumptions across AYs.

The next step in our analysis was to utilize the correlation matrix developed to create aggregate results across AYs that reflect the appropriate amount of diversification. In order to aggregate the results, we developed a simulation routine utilizing the Iman-Conover procedure. A discussion of the procedure was published by the CAS Working Party on Correlation in the *CAS Forum*, Winter 2005. The results of the aggregation procedure, including the simulation results at combined portfolio percentiles, are shown in Table 4 below.

TABLE 4

Industry Aggregate
Private Passenger Auto
Data From Schedule P, Parts 2 and 3

				Simulated Change in Ultimate at Confidence Level											
				LogLogistic						Lognormal					
AY	**Ultimates**	**Paid**	**Reserves**	**0.9**	**0.95**	**0.975**	**0.99**	**0.995**	**0.999**	**0.9**	**0.95**	**0.975**	**0.99**	**0.995**	**0.999**
2004	62,391	24,468	37,923	4,217	3,248	14,957	15,222	21,603	13,722	2,371	6,134	7,395	9,814	12,342	15,750
2003	59,849	41,640	18,208	2,688	3,050	640	2,926	4,345	22,129	5,051	3,001	3,685	1,245	5,585	8,252
2002	59,724	50,356	9,368	-242	7,570	-36	2,161	608	3,838	836	1,692	1,474	3,019	1,719	-524
2001	57,110	52,661	4,449	-125	16	1,221	1,079	-226	1,817	117	634	1,236	3,062	1,001	812
2000	55,423	53,242	2,182	2,673	232	318	345	485	495	619	883	156	79	-24	-449
1999	51,810	50,716	1,094	229	124	3	152	-162	-7	45	-139	472	731	72	-33
1998	48,432	47,809	623	66	209	107	1,151	-50	44	284	-101	479	349	-100	-14
1997	47,286	46,921	365	-15	-31	60	0	67	57	43	-3	17	561	-33	297
1996	46,982	46,753	229	-39	287	21	-17	47	62	6	20	187	-29	-28	-13
1995	45,723	45,540	183	57	-26	-20	6	79	19	14	113	-4	-13	22	-37
Prior	28,596	25,951	2,646	973	-391	366	-136	496	-582	191	180	-118	-508	-126	730
Total	563,327	486,057	77,270	10,482	14,288	17,638	22,889	27,292	41,596	9,577	12,415	14,979	18,310	20,430	24,772
Ratio of Chg in Ultimates to Carried Reserves				0.136	0.185	0.228	0.296	0.353	0.538	0.124	0.161	0.194	0.237	0.264	0.321

It is worth noting the differences between the results at the 95th percentile on Table 4 and the results on Table 2. The differences in the risk charges, 0.185 and 0.285 which are expressed as a percentage of carried reserves, represent the impact of the AY portfolio diversification effect.

In order for the model to be useful for a multi-line company, we need to extend the process to include additional lines of business. In order to test the procedure we selected a longer tailed line, other liability occurrence (OL), to determine if the same types of distributions and patterns would hold. The distributions by AY maturity followed the same general shape and were fit to both loglogistic and lognormal distributions. A sample distribution is shown below for OL AYs going from 24 months to 36 months of age as was displayed for auto liability. Table 5 below shows the final results for OL in the same format as Table 4. As we would expect, the reserve charges are significantly higher for OL.

Figure 5

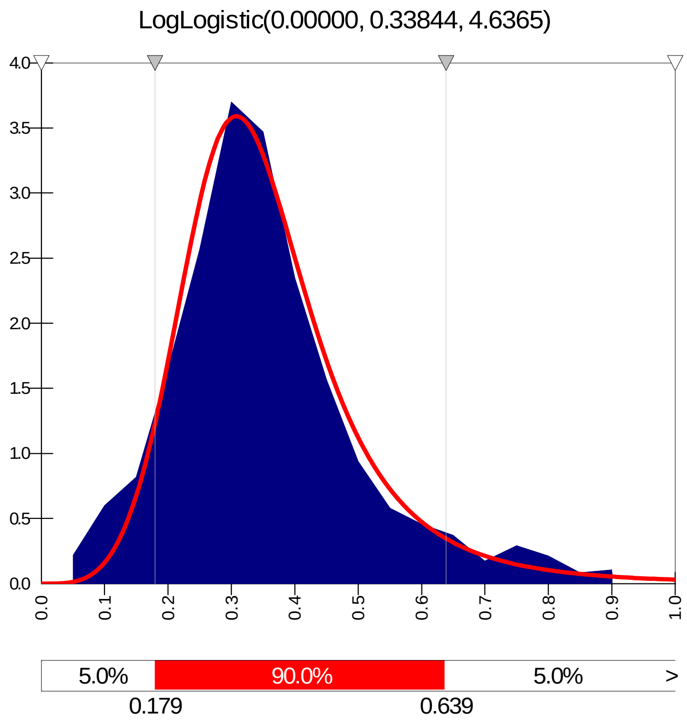


TABLE 5

Industry Aggregate
Other Liability - Occurrence
Data From Schedule P, Parts 2 and 3

				Simulated Change in Ultimate at Confidence Level											
				LogLogistic						Lognormal					
AY	**Ultimates**	**Paid**	**Reserves**	**0.9**	**0.95**	**0.975**	**0.99**	**0.995**	**0.999**	**0.9**	**0.95**	**0.975**	**0.99**	**0.995**	**0.999**
2004	17,280	1,984	15,295	4,933	6,090	-2,684	5,438	5,344	9,438	1,205	1,736	994	4,341	-266	2,140
2003	14,420	3,747	10,673	-122	1,201	335	1,411	3,490	2,525	-1,051	1,900	888	1,036	2,091	552
2002	12,861	5,869	6,992	561	1,210	374	8,846	7,120	-481	96	-637	1,517	1,116	557	1,351
2001	12,504	7,771	4,733	-50	-45	-448	693	1,214	1,762	311	212	475	1,710	935	-268
2000	11,918	8,755	3,164	-634	816	472	318	2,030	-410	-21	726	582	612	149	1,283
1999	11,664	9,503	2,161	102	0	-4	5,396	-113	83	-41	-411	487	1,242	1,486	21
1998	12,895	10,364	2,531	-629	514	116	-351	31	-4	244	2,031	1,466	3,227	-84	-143
1997	10,499	9,250	1,249	122	-179	-149	1,513	-78	-101	-69	-9	1,351	357	621	832
1996	9,139	8,217	922	30	40	423	-135	1,000	338	-131	262	221	317	234	296
1995	8,612	7,943	669	119	169	38	-87	26	308	270	-7	188	-45	-52	407
Prior	56,390	37,300	19,091	10,610	10,258	27,166	12,767	25,713	66,011	12,984	11,809	13,285	12,301	25,535	33,806
Total	178,182	110,702	67,480	15,043	20,074	25,640	35,809	45,777	79,468	13,797	17,613	21,455	26,214	31,205	40,276
Ratio of Chg in Ultimates to Carried Reserves				0.223	0.297	0.380	0.531	0.678	1.178	0.204	0.261	0.318	0.388	0.462	0.597

The last step in the IVaR model development was to combine the results for Auto and OL to generate an overall distribution of combined reserves. The results were accomplished in two steps. In the first step, we reviewed the simulation output that produced the results in Tables 4 and 5. The simulation output from the separate aggregated LOB distributions was refit to a combined distribution of reserve volatility across all AYs for Auto and OL respectively. In the second step, we performed a simulation and again used the Iman-Conover procedure to generate correlated results. The correlation process relied on a two-by-two matrix developed between the lines based on aggregate calendar year movements in reserves that showed weak correlation. The fitted distributions, results of the aggregation procedure, and fair value proxy calculation are shown below in Figures 6 and 7 and in Table 6.

Figure 6
Sample Fitted Distribution for Aggregate Auto Liability Reserves

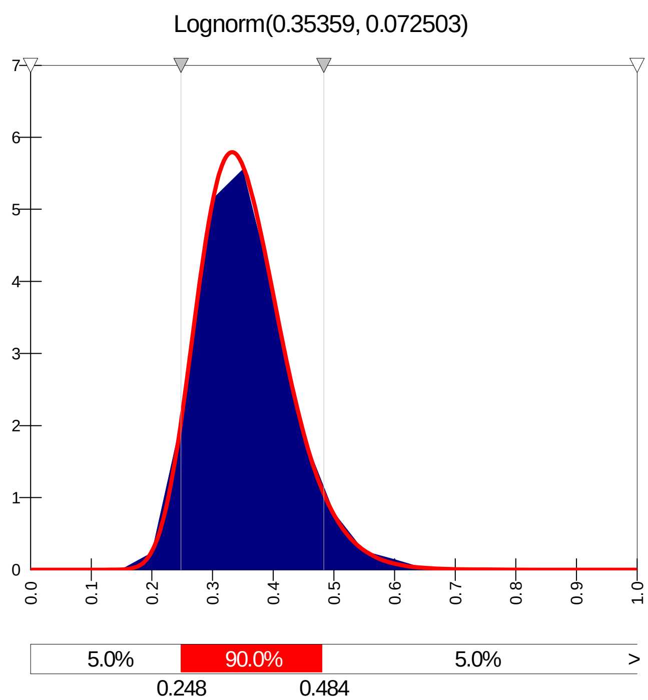
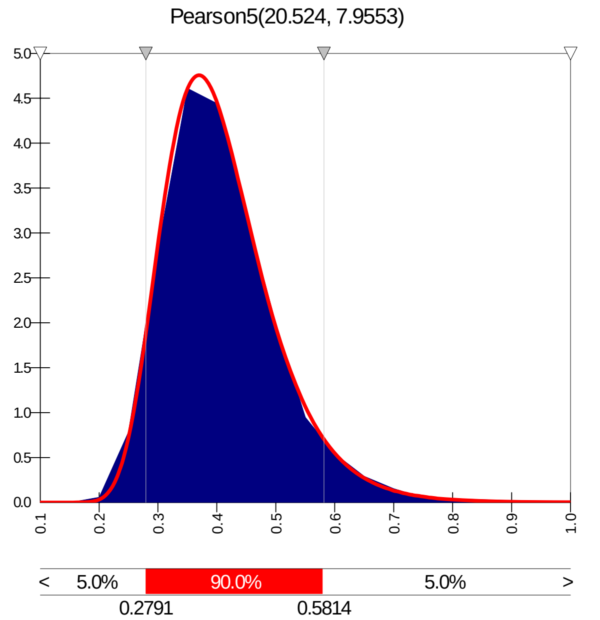


Figure 7
Sample Fitted Distribution for Aggregate Other Liability Reserves



It is interesting to note that the two distributions showing the best fits to the aggregate LOB are the lognormal and the inverse gamma distribution for Auto and OL respectively. These were both utilized in the simulation results shown in Table 6 since loglogistic distribution did not fit as well to the aggregate LOB results.

TABLE 6

Simulation Output - Iteration with Aggregate at Percentile

	Change in Ultimates over Prior Year			
Percentile	B - Auto	H - OL	Total	Discounted
0.900	17,606	5,552	23,157	21,680
0.950	24,602	3,004	27,606	25,845
0.975	15,204	16,237	31,441	29,435
0.990	11,516	24,737	36,253	33,941
0.995	4,219	35,791	40,011	37,459
0.999	19,080	32,082	51,162	47,898

	Ratio to Carried Reserve			
Percentile	B - Auto	H - OL	Total	Discounted
0.900	22.8%	8.2%	16.0%	15.0%
0.950	31.8%	4.5%	19.1%	17.9%
0.975	19.7%	24.1%	21.7%	20.3%
0.990	14.9%	36.7%	25.0%	23.4%
0.995	5.5%	53.0%	27.6%	25.9%
0.999	24.7%	47.5%	35.3%	33.1%

7. Conclusion

We have developed a model for P&C insurer loss reserve risk that conforms to the structure of a VaR model. The IVaR model completely incorporates the correct time horizon as well as LOB diversification.

This important step is necessary for insurers to be able to integrate risk models into existing and emerging EC modeling paradigms. Full integration requires that all risk sources, whether from assets or liabilities, are expressed in common time horizons. The common horizon, typically selected as one year, allows the risk distribution to be aggregated.

In addition, VaR models are usually calibrated at very high percentiles, which has proved difficult to reasonably match in insurance applications. With less available data the tails of distributions are hard to parameterize with confidence at the extremes. The IVaR method utilizes the data available in a way to maximize the sample size and also seems to produce plausible results at the extreme percentiles.

Exhibit 1
Sample Data for Private Passenger Auto Industry Aggregate

Carried Reserves (in $1,000,000s)

		Evaluation Age in Months									
AS Yr	**AY**	**12**	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**
1995	Prior	15,921	9,098	5,247	3,241	2,326	1,824	1,514	1,402	1,398	1,212
1995	1986	17,857	9,566	5,310	2,872	1,566	877	522	336	226	177
1996	1987	20,723	10,682	5,839	3,130	1,681	936	506	315	193	173
1997	1988	23,293	12,075	6,624	3,630	1,931	1,076	620	378	255	172
1998	1989	26,291	13,633	7,570	3,977	2,134	1,162	634	385	236	187
1999	1990	26,591	14,311	7,599	4,082	2,158	1,185	645	374	255	165
2000	1991	28,260	14,693	7,834	4,092	2,176	1,124	575	358	211	155
2001	1992	30,278	15,338	8,211	4,361	2,254	1,144	599	355	234	126
2002	1993	31,563	15,862	8,594	4,440	2,175	1,148	638	344	230	157
2003	1994	32,257	16,135	8,539	4,366	2,181	1,094	545	293	189	154
2004	1995	32,227	16,113	8,398	4,370	2,124	1,063	570	320	214	183
2004	1996	32,300	15,881	8,462	4,203	2,053	1,047	571	355	229	
2004	1997	31,811	15,602	8,141	4,093	1,927	950	610	365		
2004	1998	30,934	15,128	8,047	4,021	1,994	1,069	623			
2004	1999	31,297	15,336	8,077	4,220	2,058	1,094				
2004	2000	32,486	15,860	8,493	4,360	2,182					
2004	2001	34,285	16,646	8,794	4,449						
2004	2002	36,425	17,854	9,368							
2004	2003	37,680	18,208								
2004	2004	37,923									

Change in Ultimates (in $1,000,000s)

AS Yr	AY	Development Period 12:24	24:36	36:48	48:60	60:72	72:84	84:96	96:108	108:120
1995	Prior	310	50	16	142	181	82	130	154	-91
1995	1986	251	0	-52	-27	-41	-47	-22	-15	-35
1996	1987	-331	-98	-46	-33	-61	-38	-28	-46	1
1997	1988	-368	-221	-149	-215	-108	-111	-89	-21	-39
1998	1989	-244	-139	-417	-159	-178	-170	-59	-54	-129
1999	1990	-618	-737	-346	-352	-271	-187	-66	-145	-97
2000	1991	-1,303	-905	-748	-444	-359	-181	-67	-115	-4
2001	1992	-1,826	-1,232	-854	-606	-316	-173	-126	-29	-30
2002	1993	-1,835	-1,132	-1,028	-532	-340	-182	-90	-20	-11
2003	1994	-1,670	-1,358	-692	-612	-297	-135	-57	-9	1
2004	1995	-1,726	-1,158	-613	-451	-168	-54	-37	3	26
2004	1996	-1,943	-940	-596	-134	-39	-1	-8	27	
2004	1997	-1,872	-770	-270	-150	-48	-16	-5		
2004	1998	-955	-173	-155	3	-51	13			
2004	1999	-95	-50	102	-93	-1				
2004	2000	359	168	84	-1					
2004	2001	-253	-96	32						
2004	2002	-703	-299							
2004	2003	-2,041								
2004	2004									

Exhibit 1, Page 2
Ratio of Change in Ultimates to Reserves Carried at the Beginning of the Period

AS Yr	AY	Development Period 12:24	24:36	36:48	48:60	60:72	72:84	84:96	96:108	108:120
1995	Prior	0.019	0.005	0.003	0.044	0.078	0.045	0.086	0.110	-0.065
1995	1986	0.014	0.000	-0.010	-0.009	-0.026	-0.054	-0.043	-0.046	-0.156
1996	1987	-0.016	-0.009	-0.008	-0.011	-0.036	-0.040	-0.054	-0.147	0.003
1997	1988	-0.016	-0.018	-0.022	-0.059	-0.056	-0.104	-0.144	-0.055	-0.154
1998	1989	-0.009	-0.010	-0.055	-0.040	-0.083	-0.146	-0.093	-0.141	-0.546
1999	1990	-0.023	-0.051	-0.046	-0.086	-0.126	-0.158	-0.103	-0.389	-0.380
2000	1991	-0.046	-0.062	-0.095	-0.109	-0.165	-0.161	-0.116	-0.321	-0.018
2001	1992	-0.060	-0.080	-0.104	-0.139	-0.140	-0.151	-0.210	-0.081	-0.126
2002	1993	-0.058	-0.071	-0.120	-0.120	-0.156	-0.159	-0.141	-0.058	-0.048
2003	1994	-0.052	-0.084	-0.081	-0.140	-0.136	-0.123	-0.105	-0.032	0.007
2004	1995	-0.054	-0.072	-0.073	-0.103	-0.079	-0.051	-0.066	0.011	0.119
2004	1996	-0.060	-0.059	-0.070	-0.032	-0.019	0.000	-0.015	0.077	
2004	1997	-0.059	-0.049	-0.033	-0.037	-0.025	-0.017	-0.009		
2004	1998	-0.031	-0.011	-0.019	0.001	-0.025	0.012			
2004	1999	-0.003	-0.003	0.013	-0.022	0.000				
2004	2000	0.011	0.011	0.010	0.000					
2004	2001	-0.007	-0.006	0.004						
2004	2002	-0.019	-0.017							
2004	2003	-0.054								
2004	2004									

Note: AS Yr denotes Annual Statement Year

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