Ending the Mortality Table

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Abstract

One question that always arises in constructing a mortality table is where and how to end the table. The last age in the table is called the "ultimate age." Until recently, the mortality rate at this ultimate age was always set equal to 1.000. While that is still the common practice for life insurance and annuity tables, many recent national tables have been constructed without an ending value of 1.000. This paper explores how mortality tables have been ended in the past and addresses the question as to whether an ending rate of 1.000 is most appropriate. It discusses how to adjust the rates approaching the ultimate age to blend into the 1.000 rate if that method is used. Finally, if 1.000 is not the best rate at the ultimate age, options for ending a table and their relative effects are considered.

The paper shows that the choice of how to end the mortality table has no significant financial impact on large pension plans, and that is probably the case for almost all life and post-retirement health insurance plans. However, actuaries and demographers should be encouraged to give thought to how to end a mortality table and use methods that are best supported by current knowledge about mortality patterns at the oldest ages.

1. Methods Used to End Mortality Tables

Mortality tables are based on a body of experience of exposures and deaths. There are usually not enough data at the oldest ages observed in the population to determine valid rates at those ages. For instance, there may be 10 people over age 100 in a population with the oldest age being 105. There is not enough valid data over age 100 and probably not over age 95. The oldest age should be at least 105, but even with the extension of graduated rates there is not enough information in the population to derive valid mortality rates over age 100.

Graduation methods permit projection of some ages beyond the age with the last valid data. However, the actuary must speculate on the level and pattern of rates beyond that age. Actuaries must pick the ultimate age at which to end the table and a method for setting rates between the last valid rate and the ultimate rate.

In the past, the ultimate age in a mortality table has often been lower than the oldest age observed in the population used to build the tables. For instance, the actuary developing a table from the above-mentioned experience might have chosen to end the table at age 100. Until the advent of modern computers, one consideration leading to using an earlier ultimate age than the oldest observed age was that each additional age in the table resulted in a substantial increase in the number of calculations, since insurance and annuity tables were calculated backward from the ultimate age. Adding 10 or 20 ages to a mortality table today only requires the additional time it takes for the actuary to enter the 10 or 20 values in a spreadsheet.

When the ultimate age was lower than the oldest age in the population, insurers often gained substantial publicity by handing a check for the full amount of life insurance to insureds who had "outlived the mortality table." Since the insurer accumulated the full face amount at the ultimate age, these payments did not create a financial burden. With so many people reaching the ultimate age of the older tables and with changes in the tax law, it would no longer be practical to hand out checks to insureds who "outlive the mortality table."

Actuaries have often used the Gompertz or Makeham theory to fit mortality tables for most of the span of life. These theories have been shown to be valid over a large span of life, including the ages of most importance for annuities or life insurance. Before modern computers, the theories also permitted joint life annuities to be calculated directly from a single life table, which was an important simplification. The alternative was to prepare a separate joint life table for each combination of husband and wife ages.

The Gompertz and Makeham theories do not produce a mortality rate of 1.000 at some older age, but they do create rates that are high enough for the 1.000 to be a reasonable continuation at the ultimate age. Four methods have been used to end mortality tables:

The Forced Method: Select an ultimate age and set the mortality rate at that age equal to 1.000 without any changes to other mortality rates. This creates a discontinuity at the ultimate age compared to the penultimate and prior ages.

The Blended Method: Select an ultimate age and blend the rates from some earlier age to dovetail smoothly into 1.000 at the ultimate age.

The Pattern Method: Let the pattern of mortality continue until the rate approaches or hits 1.000 and set that as the ultimate age.

The Less-Than-One Method: Select an ultimate age but end the table at whatever rate is produced at that age so that the ultimate rate is less than 1.000.

I reviewed a number of tables available through the Society of Actuaries for this study. These tables are available in a Table Manager data file at www.soa.org.

2. History of Tables through 1960

Table 1 shows the method used to end the mortality table for many prominent tables in use before 1960. This includes the first age at which the blending started for the Blended Method. For instance, the rates in the Northampton table increased by 13 percent at 91 and 92, but the increase was 31 percent from 92 to 93 and about the same for 94 through 96.

Six of the 14 tables used the Forced Method and six used the Blended Method. The remaining two tables used the Pattern Method. None of the tables used the Less-Than-One Method. Even if the developers of an older table believed that the ultimate rate was less than 1.000, limitations on the computational methods available would have lead to use of an ultimate rate of 1.000.

Some of the tables do not fit neatly under one of the four types. For instance, the 1893 Om[5] table increased 7 percent at 99 and 100. The table then increased by 17 percent at 100 and ended with a 50 percent increase to get to 1.000 at 101. This is categorized as the Blended Method, but it is also very close to the Forced Method. Most of the tables before 1958 were either identified as male or the gender was not stated. The male table was used for the tables that had both male and female rates.

TABLE 1
Method Used to End Commonly-Used Tables Before 1960

Year and Table	SOA Table Number*	Ultimate Age	Type
1793 Northampton UK	250	96	Blended from 92
1843 Actuaries Combined UK	252	99	Blended from 97
1868 American Experience US	300	95	Pattern
1869 Hm UK	253	97	Blended from 92
1893 Om[5]	255	102	Blended from 100
1899 McClintock's Annuitant US	800	105	Forced
1906 Standard Industrial US	302	98	Forced
1918 AM(5) US	301	103	Forced
1928 Combined Annuity US	803	106	Forced
1934 A1924-29 UK	256	121	Pattern
1937 Standard Annuity US	806	109	Forced
1941 Commissioners Standard Ordinary (CSO) US	3	99	Blended from 95
1949 a-1949 US	808	109	Forced
1958 CSO US	5	99	Blended from 96

* Number of table in Table Manager data file available at www.soa.org.

3. Method Used in Recent Tables

A total of 22 recent tables that were both graduated and had an ultimate age of 100 or higher were reviewed. The tables were the most recent tables in the Society of Actuaries data file from 15 different countries.

The most frequent type of ending, used by nine of the tables, was the Less-Than-One Method. However, eight of these nine tables were national life tables (such as the 1990 Canadian population table) and were not intended for life insurance or annuity purposes. The breakdown of the 22 tables by type of method used to end the table is:

Forced Method	6
Blended Method	5
Pattern Method	2
Less-Than-One Method	9

The designers of the United States UP 1994 Table were convinced that continuation of an increasing mortality rate for ages over 100 would not be an accurate reflection of the actual pattern of mortality. They capped the rate at 0.500. However, they decided that the table should end at some point so that computer programs would not spin on forever, so they forced the rate to 1.000 at 120. This concept was also used in the RP-2000 Table that succeeded the UP 1994 Table as the current predictor of pensioner mortality in the United States.

The most recent insurance mortality table in the United States is the CSO 2001 Table. The designers of that table agreed with the ultimate age of 120 used in the RP-2000 Table, but were uncomfortable with a level mortality rate before that age, since that would produce discontinuities in the reserve factors as policyholders lived beyond age 100. Their solution was to set a rate that would blend smoothly from an earlier age to 120. The male composite table, for instance, set an increase rate of 5.8 percent from age 95 through age 120.

4. Shape of Mortality Curve at Oldest Ages

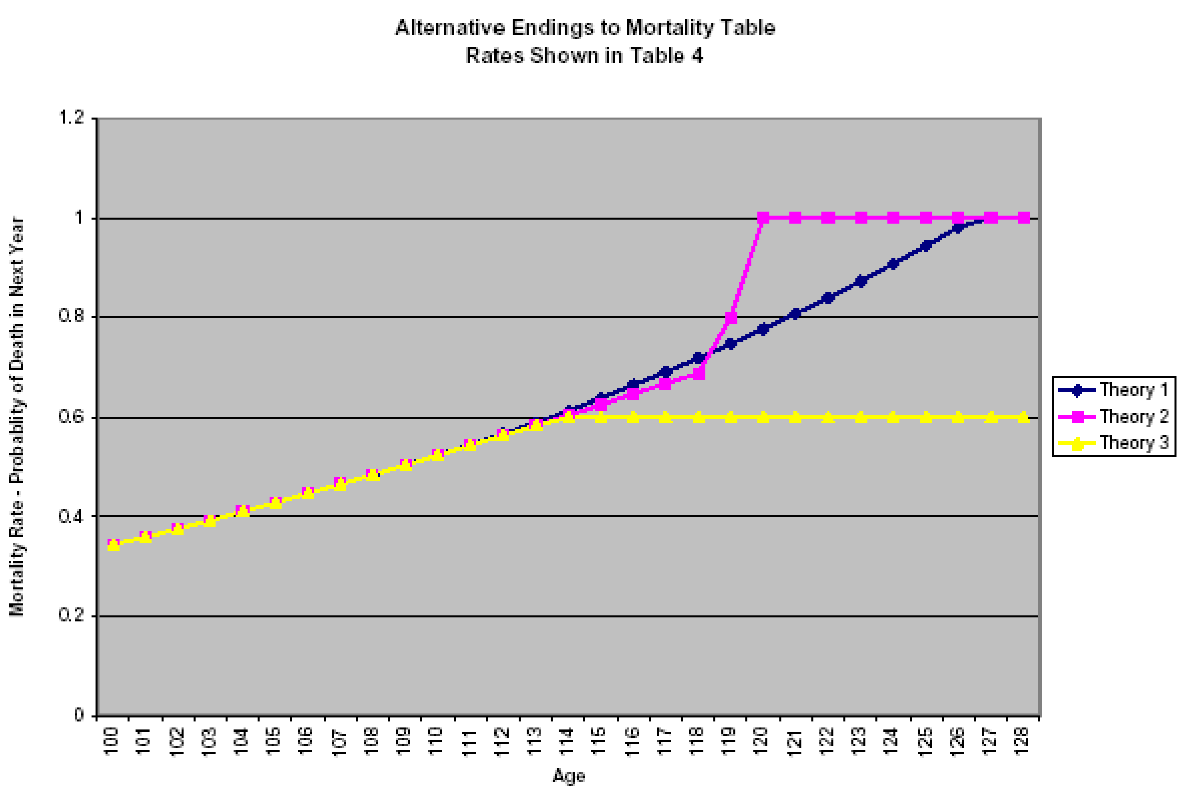
Before suggesting how and where to end the mortality table, it is useful to discuss what is known about the shape of mortality tables at the oldest ages. Many of the papers to be presented at this seminar deal with the shape of the mortality table at the oldest ages. Hence, the following analysis may be superseded, or at least modified, by those papers and discussions.

Three theories have been put forward concerning the shape of the mortality table at the oldest ages. One is that mortality rates continue to increase with age, but probably at a lower rate than for the younger ages, until the rate of mortality approaches 1.000. Proponents of this theory would probably accept the fact that the mortality rate is never actually 1.000, but would argue that it eventually becomes close enough to 1.000 for practical purposes in designing a mortality table. This is the Pattern Method for ending mortality tables.

A second theory is that there is a natural limiting wall to the life span and that the rate of mortality jumps to a rate very close to 1.000 at the ultimate age, irrespective of the shape of the curve before the ultimate age. This is the Forced Method for ending mortality tables.

A third theory is that the rates are asymptotic to an ultimate rate that is well below 1.000. For instance, if 0.500 is set as the ultimate rate, the rates probably slowly decelerate until they hit but do not exceed 0.500. This is the Less-Than-One Method for ending mortality tables.

The graph illustrates the three theories using the rates from Table 4 below.



5. Ultimate Age

For any of the theories, an important question is the age at which the mortality rate approaches 1.000 or, in the case of the third theory, levels off at a rate below 1.000. That discussion should begin with consideration of the oldest ages that have been verified.

The pattern at the oldest ages is obscured by both the paucity of data and the uncertainty of the accuracy of the ages for the very old. Demographers have long noticed a tendency of the old to overstate their ages. For example, the U.S. Census reported almost 1,400 people as 110 or older in the 2000 census, but this is much higher than could reasonably result from those reported as age 100 or older in the 1990 census.

Claims of the oldest ages have been carefully studied to determine if they are valid. An excellent collection of such studies was compiled in *Validation of Exceptional Longevity*, edited by Jeune and Vaupel. The conclusion of the researchers was that there is solid evidence supporting the longevity of Katherine Plunkett who lived to age 110 in 1930; Jeanne Calment who lived to 122; and Marie Louise Meilleur who was 117 in 1997. The oldest verifiable age for a man was Chris Mortensen, who was 115 in 1997. The supercentenarian Web site[1] adds Sarah Kraus, who died at age 119 in 1999, and reports a total of 10 verified life spans of 115 to 122.

Ages of 115 or more have rarely been reached in the past because even the third method would predict few people from the current population who reach an age of 110 or more. For example, the analysis of 1980-1990 mortality by Kannisto began with 70,000 people from 14 countries who reached age 100. The mortality rates presented for Theory 3 in Table 4 would predict that only one person would survive to 117, and there would only be a 5 percent chance that that person would survive to age 120.

Given the above considerations, I suggest that 120 is a reasonable ultimate age for tables today. Theory 1 and Theory 3 would have the ultimate age increase past 120 as the number at the oldest ages increases in the future. Theory 2 would leave the ultimate age at 120.

6. Mortality Rates at the Oldest Ages

The rate of mortality at ages 100 and over is difficult to determine even if the population includes a large number of people over age 100. Bodies of data maintained on insured lives for even the largest insurance companies have few lives over age 100.

[1] Supercentenarian.com

Large, healthy populations, as in the United States or Japan, report many residents over age 100 in recent censuses. However, the tendency for the very elderly to overstate their ages and the fact that censuses are not taken annually means that it is difficult to construct a valid mortality table for the very old from raw census data.

Kestenbaum and Ferguson carefully investigated the information on Social Security recipients with ages over 85 in the U.S. Social Security files to estimate the actual deaths per 1,000 by age. The most reliable set of data was for Social Security/Medicare beneficiaries who were receiving regular monthly benefits and the Medicare Part B premium was being paid by the beneficiary and not by a third party. The resulting mortality rates are shown in Table 2.

TABLE 2
Probability of Dying at Ages 100 and Over, 1990-1999
From Table 1 of Kestenbaum (Column 5)
Subset of the Social Security Master Beneficiary Record File

Age	Males	Females
100	.375	.332
101	.389	.358
102	.420	.381
103	.447	.395
104	.458	.413
105	.472	.438
106	.451	.450
107	.479	.490
108	.535	.507
109	.545	.567
110	.409	.448

Kannisto (1994) carefully analyzed data from 28 countries and selected 14 as providing data of good quality. The probability of dying at ages 100 and over for those 14 countries is shown in Table 3.

TABLE 3
Probability of Dying at Ages 100 and Over, 1980-1990, Pooled Data for 14 Countries
Annex Table 9 of Kannisto

Age	Males	Females
100	.421	.368
101	.425	.383
102	.430	.397
103	.464	.420
104	.442	.433
105	.451	.459
106	.453	.455
107	.486	.477
108	Not sufficient data	.562
109	Not sufficient data	.603

Both tables support increasing mortality rates after age 100. However, the increase in the male rates between 103 and 107 appears to be slowing. The female rates seem to be increasing faster than the male rates. This indicates that the male and female rates of mortality may approach parity at the oldest ages.

Robine, Saito and Jagger examined the mortality rates by five-year age groups for Japanese citizens. They estimated that the mortality rate was around 0.35 for males and 0.30 for females in the 100-104 age group and around 0.45 for both males and females between 105 and 109. Thatcher, Kannisto and Vaupel concluded that the mortality rate at 120 "is between about 0.5 and 0.65 for both males and females."

My conclusion from a review of the above-mentioned studies is that the rate of mortality increases at the oldest ages but the rate of increase appears to decelerate. This could support any of the three theories on the shape of the curve. Under Theories 1 and 3, the ultimate age will increase and there will probably always be a paucity of data in the 10 years before the ultimate age. Therefore, we may never know enough to choose between those two theories. On the other hand, if 120 or some other age is a true wall, then future data will permit us to test and accept or reject Theory 2.

The increase in the rate of mortality averaged around 10 percent a year from age 50 through age 89 in the 24 recent Society of Actuaries (SOA) tables. The rate of increase dropped to around 7.5 percent in the 90s. The five-year moving average of rates from the Kestenbaum study decreased from 6.1 percent at age 100 to 3.7 percent at age 109 for males and from 7.7 percent at age 100 to 5.7 percent at age 109 for females. The average increase for the Kannisto study mortality rates shown in Table 3 is 2.2 percent for males and 5.7 percent for females.

The developers of the United States Life Tables for 2000 also observed the deceleration of the rate of increase in the mortality rate after age 85. They developed a set of "k" factors related to the natural logarithms of the mortality rates. The rate of increase in the mortality rate (k) at age x was set equal to a function of the mortality rates at age x and age x-1 for the 2000 life tables[2]. The increase in the mortality rates declined from 9.3 percent at age 85 to 5.9 percent at age 99 for males and from 10.3 percent to 6.1 percent for the same ages for females.

My conclusions are that the increase in the rate for males is between 5 percent and 7 percent at age 100 and that the deceleration in the rates observed in the 90s continues after age 100. The rate of increase for females also decelerates but from a higher level at age 100. With a higher rate of increase in the mortality rates, the female mortality rates will eventually catch up to the male mortality rates.

Research done and reported by Ronald Lee [3] suggests that the mortality in all countries is either merging toward the mortality in Japan, the country with the lowest mortality, or running parallel to the rates for Japan. If this is true, then the above patterns would eventually be appropriate for most countries but with the ultimate mortality rates being somewhat higher at each age for some countries.

7. **Construction of Alternative Mortality Tables**

The RP-2000 Table from the www.soa.org database was used as the basis for building alternative tables that fit the three theories. The three sets of rates are shown in Table 4.

The rate of increase in the RP-2000 mortality rates averages 11 percent a year from age 60 through age 90 with little variation. The rate of increase declines from 10.2 percent at age 90 to 0.5 percent at age 106 where the mortality rate is 0.400. The rate of increase first drops below 5 percent at age 99. The RP-2000 mortality rates continue at

[2] $q_x = \ln(q_x) - \ln(q_{x-1})$

[3] "Mortality Forecasts and Linear Life Expectancy Trends"

0.400 through age 119 with a forced rate of 1.000 at the ultimate age of 120. The RP-2000 Table is not a Theory 2 table but a Theory 3 table with an arbitrary ultimate age for computational purposes.

I began the construction of the three tables by changing the rate of increase from 4.7 percent to 5.0 percent at age 99 since I propose 5.0 percent as the lowest increase at age 100. For Theory 1, I assumed the rate of increase would decline for 10 years to 4.0 percent and then stay at that rate until the mortality rate hits 1.000 at age 127.

For Theory 2, I used the Theory 1 rates to age 110, but then assumed that the rate of increase would continue to decline after 110. I assumed that the wall would be at age 120, so I set the mortality rate to 1.000 at this age. I used a rate of 0.800 at age 119 for some smoothing into the ultimate rate.

I used the Theory 2 rates for Theory 3 until age 113. I set the Theory 3 rates at 0.600 at age 114 and older, when the Theory 2 rates first exceed 0.600. The annuity rates and present values in Tables 5 and 6 ended the Theory 3 rates with a value of 0.600 at age 127.

TABLE 4
Mortality Rates after 100 for the Three Theories based on RP-2000

Age	Theory 1	Theory 2	Theory 3
100	0.345	0.345	0.345
101	0.359	0.359	0.359
102	0.375	0.375	0.375
103	0.393	0.393	0.393
104	0.410	0.410	0.410
105	0.428	0.428	0.428
106	0.447	0.447	0.447
107	0.466	0.466	0.466
108	0.485	0.485	0.485
109	0.504	0.504	0.504
110	0.524	0.524	0.524
111	0.545	0.544	0.544
112	0.567	0.564	0.564
113	0.590	0.584	0.584
114	0.613	0.605	0.600
115	0.638	0.625	0.600
116	0.663	0.646	0.600

TABLE 4
Mortality Rates after 100 for the Three Theories based on RP-2000

Age	Theory 1	Theory 2	Theory 3
117	0.690	0.666	0.600
118	0.718	0.687	0.600
119	0.746	0.800	0.600
120	0.776	1.000	0.600
121	0.807	Not applicable after 120	0.600
122	0.839		0.600
123	0.873		0.600
124	0.908		0.600
125	0.944		0.600
126	0.982		0.600
127+	1.000		0.600

8. Financial Effect of Different Methods for Ending the Mortality Table

Table 5 shows the present value of an annuity of $1 payable monthly for the rest of life at 8 percent interest for the three mortality tables shown in Table 4. There is no difference in the annuity rate to four decimal places through age 100. The first difference to show up in the fourth decimal is at age 105. The difference is still small at 110 and only becomes significant at 115 and 120.

TABLE 5
Present Value of $1 per Year at 8% Interest for Life

Age	Theory 1	Theory 2	Theory 3
50	$11.0479	$11.0479	$11.0479
55	10.5219	10.5219	10.5219
60	9.7849	9.7849	9.7849
65	8.8627	8.8627	8.8627
70	7.7807	7.7807	7.7807
75	6.5621	6.5621	6.5621
80	5.2909	5.2909	5.2909
85	4.0713	4.0713	4.0713
90	3.0460	3.0460	3.0460
95	2.3405	2.3405	2.3405
100	1.8823	1.8823	1.8823
105	1.5077	1.5079	1.5079

TABLE 5
Present Value of $1 per Year at 8% Interest for Life

110	1.2017	1.2057	1.2067
115	0.9382	0.9665	1.0466
120	0.7088	0.4583	1.0464

The small differences in Table 5 result in a miniscule difference when used to determine reserves for a typical pension plan. Table 6 shows the difference of the Table 5 factors when applied to a group of annuitants who retired at or after their full retirement age under a large state pension plan. For a total liability of over $6.2 billion, the difference between the three tables at a typical 8 percent discount rate is less than $300, or .000003 percent. A typical post-retirement medical valuation might incorporate a set of assumptions that are approximately equal to a 2 percent interest rate. As shown in Table 6, there is a greater, but still miniscule, difference for post-retirement medical insurance calculations at an effective discount rate of 2 percent.

TABLE 6
Present Value of Benefits for Retirees in a Typical Pension Plan using the Table 5 Annuity Factors

	Theory 1	Theory 2	Theory 3
At 8% interest	$6,233,853,891	$6,233,854,063	$6,233,854,108
Percent increase over Theory 1		0.000003%	0.000003%
At 2% interest	$9,725,121,988	$9,725,122,304	$9,725,122,407
Percent increase over Theory 1		0.000003%	0.000004%

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