ACTUARIAL RESEARCH CLEARING HOUSE
1993 VOL. 3

A NEW APPROACH TO DURATION

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The traditional calculation of duration is at the level of the individual financial instrument, e.g. a single bond. If one were going to calculate the duration of a portfolio of bonds, one would do so for each bond separately and then weight the results by the ratios of the individual bond market values to the total market value.

The new approach to calculate duration is simply to take a "macro" or portfolio perspective utilizing the idea of internal rate of return (IRR). It is presumed the user (a) has a model for projecting cash flows under different interest rate scenarios and (b) knows the current market value of the portfolio. While this article will use the term "portfolio" as if it applied only to assets, the method can be used on liabilities as well.

The macro approach views the portfolio as a unified entity. It has a complex set of cash flows which may vary with future interest rates. However, for any particular interest rate scenario, there is one IRR which equates the projected cash flows to the portfolio's current market value. Call this market value P.

To calculate the option-adjusted, or effective, duration of the portfolio using the macro approach and multiple interest rate scenarios, the steps are as follow:

1. Project the cash flows for each interest rate scenario which begins with current interest rates and find the IRR for each scenario. Save the interest rate scenarios and the IRR's for the next two steps.

2. Choose a change in interest rates, such as 50 basis points (0.5% or 0.005). Call it Δi. Project the cash flows assuming the interest rates in step 1 are all increased by Δi. For each scenario find the present value of the cash flows at the IRR from step 1 plus Δi. Calculate the average of such present values; call it P^{up}.

3. Repeat step 2 except use $-\Delta i$ in place of Δi and call the average P^{dn} (for down).

4. Then duration is $(P^{dn} - P^{up})/(2 \cdot \Delta i \cdot P)$, with Δi in decimal form.

What are the advantages of this method over the traditional one?

1. It utilizes the well understood concept of IRR. IRR is easily understood and has great intuitive appeal. Applied to a portfolio, just one number captures a very complex situation. One might claim that it oversimplifies, but the same could be said about the IRR of a single, coupon-paying bond when spot rates are available.

2. Consider calculating the duration of a portfolio containing an "at the money" interest rate swap, i.e. the market value is zero. The traditional method would falter, since the zero market value means that no weight is given to the swap. In contrast, the macro approach would capture the swap's effect on duration.

What are the disadvantages of the macro approach? One could argue that the financial markets give us good information about appropriate interest rates for the vast variety of securities traded there. Therefore, why not use this information? My response would be that the financial markets give us better information about prices, which the method does use, than interest rates, and interest rates are inferred from those prices. Also, the method assumes such interest rates are used in generating the scenarios. It simply doesn't use them for calculating present values when computing duration.

How do the results of this method differ from the traditional method? This method gives a little more weight to assets with higher IRRs. When such assets have longer durations than the average for the portfolio, the method will give a slightly higher duration number than the traditional method. In this case, using this method's duration to estimate a price change gives a better estimate than does the traditional method. The opposite holds when the assets with higher IRRs have shorter durations than the average for the portfolio. The differences from the traditional method are, of course, smaller if assets yields are reduced to reflect default risk, which is recommended for projecting cash flows.

The convexity of a portfolio can also be calculated with the present values calculated in the above steps. The formula is:

$$C = (P^{dn} + P^{up} - 2P)/(P(\Delta i)^2)$$

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September 17, 1993