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An Analysis of Long-Term Care Data from Hamilton-Wentworth, Ontario

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Abstract

The aging populations of both Canada and the United States have created an increasing interest in long-term care for the elderly. However, little published data is available to those involved in assessing long-term care risks. This paper will present some preliminary results of an analysis of data from Hamilton-Wentworth, a regional municipality in Ontario. The data includes information on the optimal level of care at one or more points in time for each individual under study. This optimal level of care was modelled using a Markov process. The transition intensities were estimated by maximum likelihood. The paper will discuss the analysis and present the resulting parameter estimates.

1 Introduction

In order to estimate future long-term care costs it is necessary to assess future demand for the services available. Since the demand is greatly influenced by the care requirements of the population, one must be able to determine probabilities that individuals will need the various levels of care. Unfortunately, data which enables one to estimate these probabilities is scarce. Not only is it difficult to find longitudinal data providing information on long-term care requirements, but the determination of such requirements is rather subjective.

This paper describes a data set which has been used to estimate the parameters necessary to obtain care requirement probabilities. The data provides information on individuals assessed by the Hamilton-Wentworth Placement Coordination Service during a six-year period. This data was also used in a survival study by Haight et al (1991).

Section 2 provides some background information on long-term care in the province of Ontario. The source and characteristics of the data set are discussed in section 3. The stochastic model used in the analysis is presented in section 4. It is suggested there that a Markov process is appropriate for modelling long-term care requirements. Section 5 describes the maximum likelihood parameter estimation along with the results of the analysis. Some brief remarks on the use of these results are made in section 6.

2 Long-Term Care in Ontario

The province of Ontario has classified the levels of institutional long-term care as follows:

1. Residential Care
   This type of care is appropriate for those who require primarily supervision and/or assistance with the activities of daily living.

2. Extended Health Care
   This type of care is appropriate for individuals requiring personal care on a continuing 24-hour basis with medical and professional nursing supervision.

3. Chronic Care
   This type of care is for those needing a range of therapeutic services, medical management and skilled nursing care.

In addition to institutional care, non-institutional long-term care services are currently available in many regions. These include home care, vacation care, homemaker services, meals on wheels and friendly visiting. For the purposes of this analysis, non-institutional services have been included with residential care since the difference between these care requirements is largely due to personal circumstances. Therefore, the model described in section 4 allows for three levels of long-term care.

3 The Data

The analysis was based on data from the Hamilton-Wentworth Placement Coordination Service (PCS). The purpose of the Service was described as follows in the 1982-83 Annual Report:

- to promote better assessment of the needs of persons with long-term disabilities utilizing the personal physician and other health personnel closely associated with the patient
- to find appropriate programs that could meet these needs and to identify whatever modifications or new approaches which may be required
- to provide a resource for the education of health personnel in the complex needs of the chronically ill and handicapped

The Hamilton-Wentworth PCS was established in 1971 and was the first such agency in Ontario. There are now twenty-two regions with placement coordination services. They are funded by the Ontario Ministry of Health.

Individuals requiring long-term care services for the first time, or those needing changes to their care programs, are referred to the PCS for an assessment. They may also approach the PCS on their own. The assessment process begins with the completion of two standardized PCS Referral Forms. The first form contains demographic and functional information and the second contains medical information. These forms are reviewed by a counsellor at the PCS who prepares a recommendation. The recommendation along with the final placement decision are recorded on a PCS Action Form. Also recorded on this form is the optimal placement which may differ from the recommended placement depending on availability. The optimal placements coded were very important to the analysis because they, although subjective, indicate care requirements.

The information recorded on the Referral and Action Forms is kept on the PCS database. In addition, the PCS reviews death records regularly to identify files which may be closed. The data set used in the analysis consisted of a record for each assessment completed during the period from April 1, 1980 to March 31, 1986 with follow-up to March 31, 1987. Only individuals aged 65 and over were included.

The distribution of individuals included in the study by sex and number of assessments is shown in table 1. After eliminating individuals for whom important information was not recorded, the total number in the study was 9348. Most of these were females due to their longer life expectancy and smaller likelihood of having a caregiver spouse. Most individuals were assessed only once during the study period. A significant number, however, were assessed more than once. These individuals were important because of the information they revealed about transitions amongst the levels of care requirements.

Table 1: **Distribution by Sex and Number of Assessments**

Number of Assessments	Males	Females	Total
1	2509	4572	7081
2	571	1089	1660
3	171	265	436
4	48	71	119
5	19	17	36
6+	8	8	16
Total	3326	6022	9348

Individuals who were assessed multiple times provided snapshots of their care requirements at several points in time. This information was somewhat less than ideal since the actual times of transition were not given. Fortunately, the model parameters could still be estimated.

A much greater problem was the fact that the observation (assessment) times were not independent of the care requirements of the individuals. Often an assessment is performed as a result of a change in an individual's care requirement. This creates a bias in the observations. Assessments can be initiated for a number of other reasons including changes in individuals' personal situations and changes in the availability of services. However, the fact that assessments tend to result from changes in care requirements reduces the probability of more than one transition (change in care requirement) between two consecutive assessment times. To reflect this and therefore reduce the bias, the following assumptions were made for each individual in the study:

- no more than one transition occurred between two consecutive assessment times,
- no transitions occurred between the final assessment time and death or the end of the study period.

4 The Stochastic Model

In order to estimate the desired probabilities, a stochastic model is needed. Since we are interested in modelling long-term care requirements, we begin by letting $\{X(t); t \geq 65\}$ be the stochastic process representing an individual's care requirement at age t. The state space for the process is $\{1,2,3,4\}$. Individuals in state 1 require residential care (or non-institutional services). Individuals in state 2 require extended health care. Individuals in state 3 require chronic care. Individuals in state 4 are dead (an absorbing state). We will assume that $\{X(t)\}$ is a Markov process. Therefore, for all $t \geq s$, and $i, j, x(u) \in \{1,2,3,4\}$,

$$P\{X(t) = j | X(s) = i, X(u) = x(u), 65 \leq u < s\} = P\{X(t) = j | X(s) = i\}.$$

The Markov assumption is intuitively appealing. It seems reasonable to believe that an individual's future health depends only on his/her current health and not on the individual's health history. It is difficult, however, to assess a person's current health adequately. As a result, health history can be valuable in predicting future changes in health. Thus, in some situations a semi-Markov process may be more appropriate, but also more complicated.

Let the transition probability function be given by

$$P_{ij}(s,t) = P\{X(t) = j | X(s) = i\},\ 65 \leq s \leq t,\ i,j \in \{1,2,3,4\}.$$

Also, assume the existence of transition intensities or forces of transition defined by

$$\lambda_{ij}(t) = \lim_{h \to 0^+} \frac{P_{ij}(t, t+h)}{h},\ i \neq j.$$

We will begin by assuming that the forces of transition are constant, i.e. $\lambda_{ij}(t) = \lambda_{ij}$ for all t. Thus, the process is time-homogeneous. This assumption might be suitable since we are considering a restricted age range (65 and over) and forces of transition are conditional on current care requirements. Ross (1983) discusses the properties of time-homogeneous Markov processes (continuous-time Markov chains).

Figure 1: The Stochastic Model

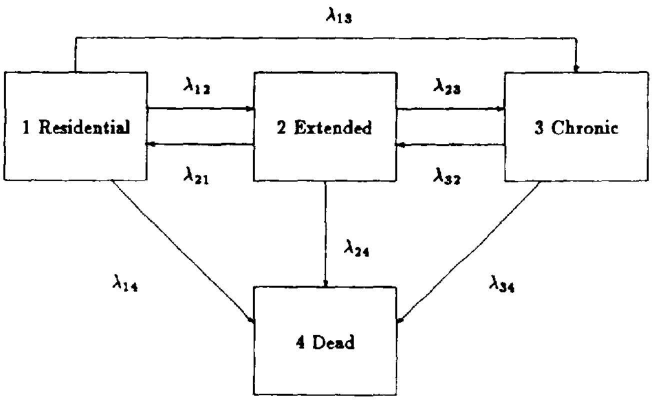


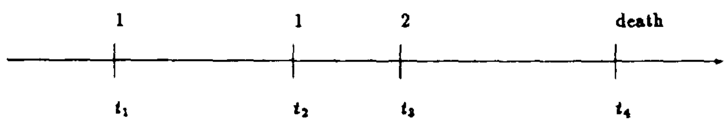
Figure 1 illustrates the possible transitions amongst the care requirement states. It is interesting to note that while transitions from state 1 to state 3 are possible, direct transitions from state 3 to state 1 are not allowed. The reason is that 1-3 transitions can result from traumatic events, but recoveries must be gradual. The model does not include the state in which an individual requires no care at all even though this state is very important in forecasting future long-term care requirements. Unfortunately, the associated transitions cannot be analyzed using the Hamilton-Wentworth data because no information is provided about individuals who require no care. Thus, for the purposes of this analysis, the model involves only four states and eight possible transitions.

5 Parameter Estimation

The forces of transition were estimated by the method of maximum likelihood. A description of the method is given by Silvey (1975). The likelihood contributions are easy to find since the distribution of the time spent in each state is exponential. To illustrate this, consider an individual who is observed as indicated in figure 2. At times t_1 and t_2 the individual is in state 1. At time t_3 the individual is in state 2. The individual dies at time t_4. Letting $\lambda_1 = \lambda_{12} + \lambda_{13} + \lambda_{14}$ and $\lambda_2 = \lambda_{21} + \lambda_{23} + \lambda_{24}$, the likelihood contribution for this individual is given by

$$
\begin{aligned}
L &= \int_{t_2}^{t_3} e^{-\lambda_1(s-t_1)}\lambda_{12}e^{-\lambda_2(t_4-s)}\lambda_{24}\,ds \\
&= \frac{\lambda_{12}\lambda_{24}}{\lambda_2-\lambda_1}\left[e^{-\lambda_1(t_3-t_1)}e^{-\lambda_2(t_4-t_3)} - e^{-\lambda_1(t_2-t_1)}e^{-\lambda_2(t_4-t_2)}\right].
\end{aligned}
$$

Figure 2: Example



The log of this expression can be differentiated (twice) to obtain the contributions to the score and information. Newton-Raphson iteration was used to maximize the likelihood. Starting values were obtained by assuming that all transitions occur halfway between the assessment times.

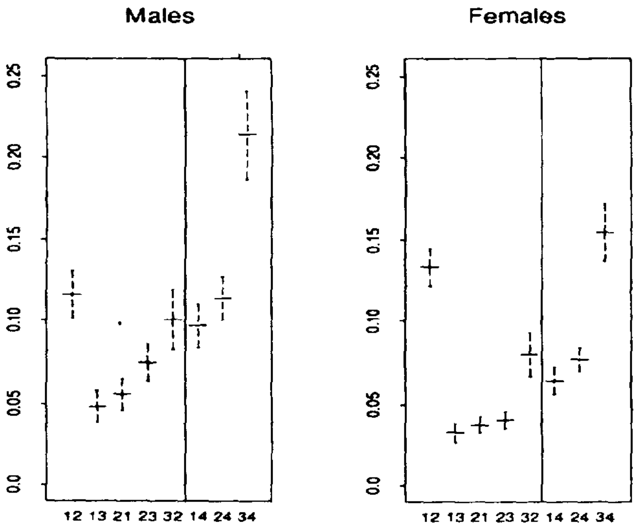
Table 2 gives the results of the estimation. The estimated forces of transition are shown along with their standard deviations based on the estimated asymptotic covariance matrix. These results are shown graphically in figure 3. Each of the vertical intervals in figure 3 extends from the estimate minus two standard deviations, to the estimate plus two standard deviations. The horizontal line in the middle of each interval gives the estimate. The right-hand portion of each box shows the results for the forces of mortality, and the left-hand portion shows the results for the other forces of transition.

Table 2: Estimated Forces of Transition/Mortality

	Males		Females	
Transition	Estimate	Std. Dev.	Estimate	Std. Dev.
1-2	.1162	.0074	.1335	.0059
1-3	.0485	.0048	.0330	.0029
1-4	.0966	.0067	.0642	.0041
2-1	.0557	.0047	.0377	.0025
2-3	.0746	.0055	.0408	.0026
2-4	.1139	.0068	.0768	.0035
3-2	.1003	.0092	.0796	.0064
3-4	.2139	.0134	.1546	.0089

The same general pattern of results occurs for males and females. It is easiest to verify the reasonableness of the results by looking at the mortality transitions. As expected, the estimated force of mortality increases as the level of care requirement increases. The relationship between male and female mortality is also what one would expect. The actual magnitudes of the forces of mortality appear to be reasonable. When compared to life tables for Canadian lives during 1980-1982 (see Statistics Canada, 1984), the forces of mortality for the three levels of care requirements correspond to mortality at ages 80, 82 and 90 for males and 81, 83 and 90 for females. It is much more

Figure 3: Estimated Forces of Transition/Mortality



difficult to determine whether the other forces of transition are reasonable since we have very little prior knowledge as to how these should behave.

In order to test the constant force assumption, the data was broken into three age groups: 65 to 74 (young-old), 75 to 84 (middle-old) and over 84 (old-old). The estimation was repeated for each of these age groups. The resulting estimates are given in table 3. The results are also shown graphically in figures 4 through 7. The dotted line in each box indicates the level of the estimate for all ages combined. The plots are very helpful in assessing the appropriateness of the constant force assumption. If the dotted line intersects all three intervals and there is no clear trend, then one might be justified in assuming a constant force of transition. Otherwise, piecewise constant forces of transition using three age groups would be preferable.

Table 3: **Estimated Forces of Transition by Age Group**

	Males			Females		
Transition	Young	Middle	Old	Young	Middle	Old
1-2	.1035	.1073	.1541	.0978	.1204	.1842
1-3	.0560	.0512	.0308	.0378	.0337	.0270
1-4	.0877	.0953	.1123	.0555	.0643	.0698
2-1	.0745	.0543	.0424	.0421	.0441	.0289
2-3	.0660	.0744	.0820	.0402	.0385	.0436
2-4	.1033	.1036	.1399	.0559	.0789	.0831
3-2	.0730	.1184	.1102	.0810	.0878	.0733
3-4	.1679	.2368	.2445	.1322	.1717	.1528

Most of the plots in figures 4 through 6 show obvious trends with age. The 1-2 force of transition increases with age, as one would expect. Here, it would seem difficult to justify using a constant force, especially for females. The 1-3 force of transition appears to decrease with age. This is somewhat surprising. However, if we consider the fact that these transitions typically result from traumatic events which are more difficult to survive at older ages, the decreasing trend becomes plausible. For the other non-death transitions, the assumption of a contant force of transition may be reasonable.

The estimated forces of mortality are illustrated in figure 7. It is quite clear that conditioning the force of mortality on care requirement removes

Figure 4: Estimated Forces of Transition

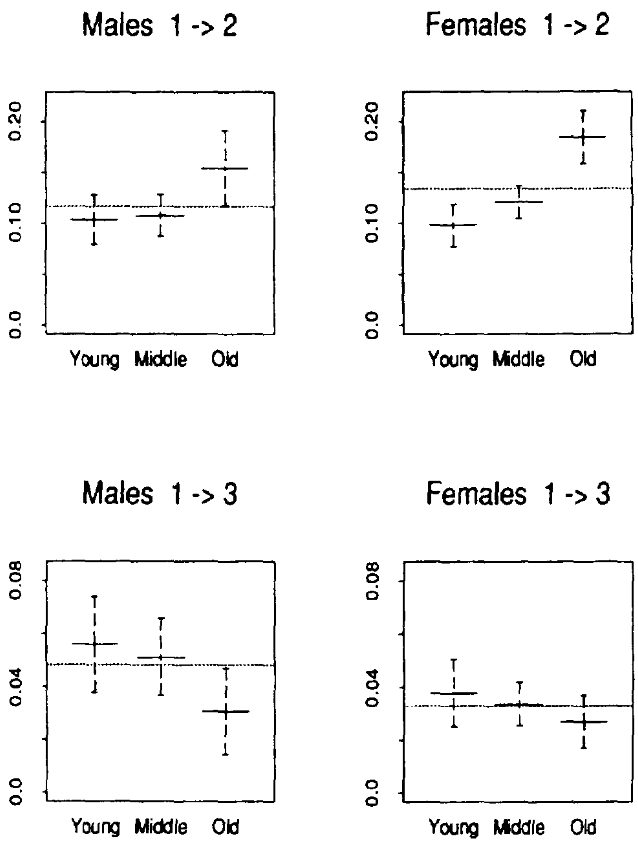


Figure 5: Estimated Forces of Transition

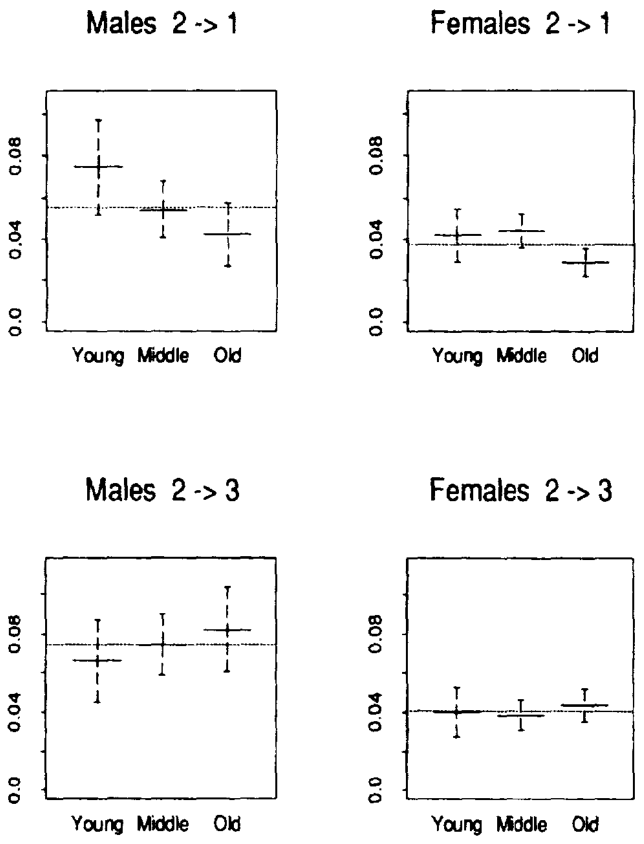


Figure 6: Estimated Forces of Transition

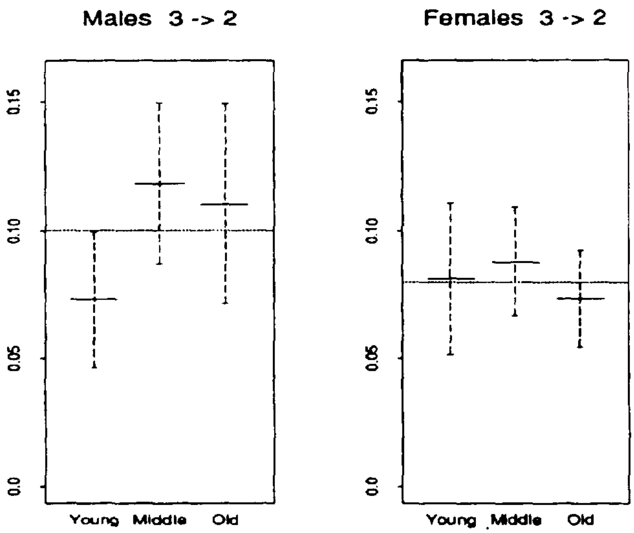
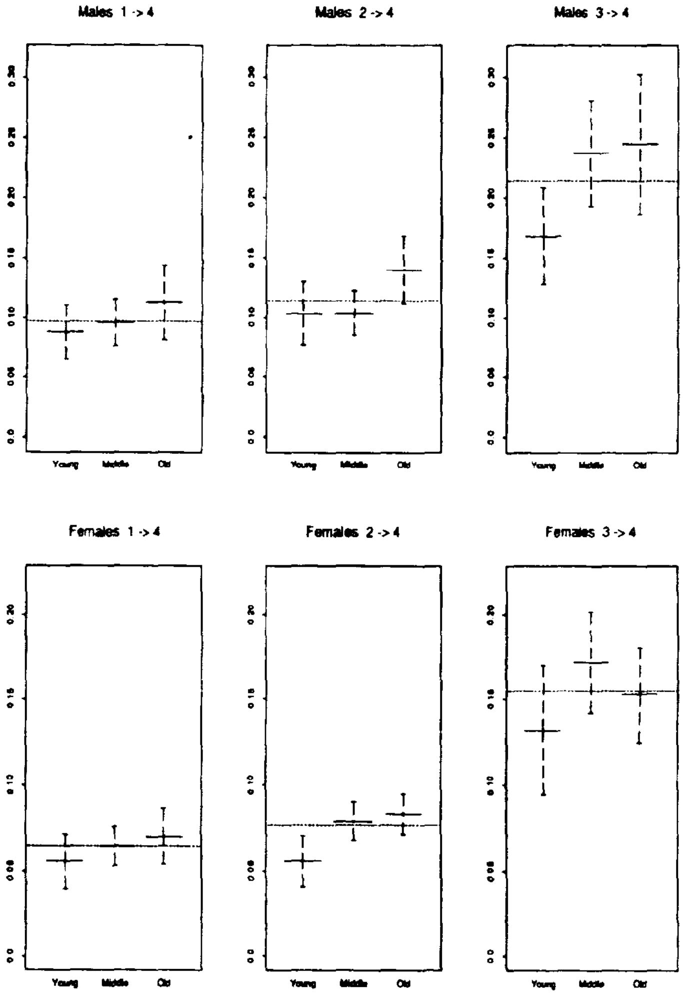


Figure 7: Estimated Forces of Mortality



most of the effect of age. This is consistent with the results of Haight et al (1991). While increasing trends are apparent, one might still investigate the impact of assuming constant forces of mortality.

6 Conclusion

This paper has presented an analysis of long-term care data using a Markov process model. Results are presented under both the assumption of constant forces of transition and the assumption of piecewise constant forces. In either case, the forces of transition can be used to obtain transition probability functions. The task is somewhat more cumbersome with piecewise constant forces, but no more difficult mathematically. The decision of whether or not to use a model with constant forces of transition should be based on the use of the model. It remains to be seen what impact this assumption has on financial values.

Explicit formulas for the transition probability functions in terms of the forces of transition are very difficult to obtain using a Markov process with four states. However, the spectral representation of the transition intensity matrix discussed by Cox and Miller (1965) is very useful in determining these functions, given the estimates of the forces of transition.

Once the transition probability functions have been obtained, one may begin to analyze financial values. Expressions for actuarial (expected) values are very easy to find. Also, Ramlau-Hansen (1988) used the theory of counting processes to show how expressions for variances of financial random variables can be obtained in a Markov model setting.

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