ACTUARIAL RESEARCH CLEARING HOUSE
1995 VOL. 1

Can the Insurance Market Support both Agency and Direct Writers?

An Economic Analysis of the Structure of the Property-Casualty Insurance Industry

SOA SUBJECTS: Casualty Insurance, Economics

Mary V. Kelly
Faculty of Commerce and Business Adminstration
University of British Columbia, Vancouver, British Columbia

Abstract

This paper is a theoretical examination of the structure of the property-casualty insurance industry in North America. The industry is composed of two types of insurance companies: agency writers who have low start up costs and an inefficient distribution network for their product, and direct writers, who have high initial sunk costs and a more efficient distribution system. Without further assumptions, standard economic theory asserts that the agency writers would not exist in equilibrium.

The first aim of this paper is to construct an equilibrium in which both writers exist by recognizing that direct writers compete spatially. The second goal of this paper is to examine the standard Rothschild-Stiglitz (1976) results within this context given that direct writers can differentiate between consumers and agency writers cannot. It is shown that informational rents due to the agency writers' inability to distinguish between consumers accrues to the direct writers. The paper concludes with a discussion of future work including the examination of the policy implications arising from the two models and possible empirical extensions.

1 Introduction

In North America, property-casualty insurance is marketed in two basic manners. Most insurance companies distribute their products through an agency system. In this structure, independent brokers represent large numbers of companies and sell the policies on commission to the public on behalf of these companies. Companies which sell insurance through mail order, through their own sales force or through exclusive agents are called direct writers. Direct writers have the advantage of being able to sell insurance at a much lower cost than agency insurers once the (costly) retail network is in place. Commission scales for direct writers may also distinguish between new business and renewals further reducing the costs of the direct writers.

Normative economic theory concludes that the agency system, with its inefficient technology, should not survive in the long run. sunk costs are discounted over an infinite time period, all that matters is marginal cost

Models of perfect competition predict that in equilibrium only direct writers should offer insurance and that they would sell the product at its marginal cost of production.

The lack of justification of the market structure of the property-casualty insurance market by existing economic models has been previously noted.[1] The purpose of this paper is to justify the existence of both the direct and agency distribution systems in equilibrium. To do this, two types of models are presented: the first embeds a full information insurance market within a spatial framework and the second model expands the first to include private information.

Equilibria are constructed based on the assumptions that agency writers are perfectly competitive, direct writers compete in prices and consumers incur transportational costs when buying insurance from direct writers. In order to characterise equilibria in both models, a symmetric sequential entry rule for the direct writers is assumed.

The first model characterises the conditions under which both agency writers and direct

[1] Van Cayseele (1992) is a statement as chair in insurance economics at Katholieke Universiteit (Louvain, Belgium) includes this problem in a list of important unanswered questions in insurance economics.

writers exist in equilibrium. Direct writers will operate as local monopolists with the agency writers entering between the captive markets of the direct writers to sell insurance to those consumers whom direct writers find too expensive to serve.

It is a simplification to assume that all consumers are the same, since each insurance consumer faces a different level of risk. Therefore the second half of the paper extends this above model to include two types of consumers with differing claim frequencies. Using agency theory arguments, it is assumed that direct writers can differentiate between the two types of consumers, but that agency writers cannot. Insurance companies are not constrained by common carrier requirements[2] but they are not allowed to price discriminate. This extension produces a Rothschild-Stiglitz (1976) model with two types of insurance companies with differing fixed and variable costs and consumers that incur transportation costs.

In the equilibrium with two consumer types, direct writers, acting as local monopolists, sell full insurance to the good risks only. Agency writers serve all the bad risks and those good risks whom direct writers find to costly to serve. The conditions under which the agency writer will offer a single contract and a menu of contracts to the two risk types are derived.

A comparison of the full information and the asymmetric information models yields the following results. Because agency writers cannot differentiate between the two types of consumers, direct writers earn informational rents. Good consumers are better off due to the existence of the direct writers and their ability to distinguish between consumers, while high risk consumers are never better off.

The setup of the paper is as follows. First, in Section 2, some empirical evidence illustrating the cost differences of the two types of insurance distribution systems is given. Section 3 first discusses some of the assumptions before presenting the basic model under

[2] A common carrier requirement is a statutory provision requiring a firms to sell its product to all who wish to purchase it. Airlines, railroads and utilities, for example, are constrained by common carrier requirements.

the conjecture of symmetric information. The symmetric equilibrium in which direct writers and agency writers co-exist is characterised. The fourth section of the paper extends the basic model to include asymmetric information. Section 5 specifies areas of future research and Section 6 concludes.

2 Empirical Evidence

An abundance of data has already been published illustrating the difference in the expenses between direct writers and agency writers. Joskow, in his examination of the United States property-casualty insurance industry for the years 1970 to 1971, states that "*expense ratios of direct writers average 10.82 percentage points less than the agency companies ceteris paribus*". Similar results over the time period 1968 through 1976 were reported by Cummins and VanDerhei (1979), and by Barrese and Nelson (1992) for 1978 to 1990. A Canadian study by Quirin *et al* (1974) notes that as the prominence of direct writers increased in the early 1960's the average commission paid to brokers fell from 25% to a range of 8% to 15% in an attempt to stay competitive.

The data in Table 1, collected for Canadian companies in 1988 on both direct and agency writers,[3] corroborate the difference in expense ratios. Other statistics listed in the table provide other information about the structure of the industry.

The data suggest that direct writers occupy a oligopoly position in the industry. Only 11.6% of insurance companies are direct writers but they wrote 19% of all premiums in 1988. These twenty-three companies include both small local firms and large federally registered companies which operate in several provinces. The seventeen national direct writers account for 17.8% of all net written premiums. The average expense ratio for a direct writer is

[3]No available guide provides a concise listing of the distribution system for each insurer. Most information was provided by the Stone and Cox's General Insurance Register. Where ambiguity remained (usually with respect to the smaller companies) the author's best judgement was used.

	Direct Writers	Agency Writers
Number of Companies	23	176
Average* Expense Ratio	27.16%	38.09%
Total Net Written Premiums ($ 000)	$2 241 862	$9 542 564
Average* Return on Net Earned Premiums	24.1%	21.8%
Average* Asset Level ($ 000)	$201 651	$106 630

* **Average is taken over direct and agency writers separately for 1988**

Source: *The Blue Chart Report 1988*. Stone and Cox Limited, Toronto.

Table 1: Selected Data for Direct and Agency Writers in Canada

significantly below that of an agency writer and the average return on net earned premiums is higher for direct writers than for agency writers, but this difference is not statistically significant at the 5% level.

3 The Model

In this section, the symmetric information spatial insurance model is presented. Two critical assumptions which differentiate this paper from previous work are discussed. Before the equilibrium conditions are derived the behaviour of the utility maximising consumers and profit maximising insurers are described. The full information equilibrium then can be characterised in terms of the exogenous variables. The section concludes with a discussion on the possibility of partial insurance contracts.

3.1 Assumptions

Two key assumptions used to derive the equilibrium conditions are that insurance companies compete in prices and that consumers incur transportation costs when purchasing insurance from direct writers. This paper differs from earlier research on the structure of the property-casualty insurance industry in that it is assumed that direct writers compete on price.

Joskow (1973) hypothesises that perhaps the true reason that direct writers have not taken over the market is that they choose only to underwrite the better risks (cream-skimming) and therefore do not wish to write insurance for much of the market. Agency writers exist because of supply side rationing; direct writers do not have the capacity to serve all of the market. There are two difficulties with this hypothesis. First, there is zero probability that the number of low risk consumers equals exactly the size of the market that the direct writers would like to optimally serve and so it is impossible to characterise an equilibrium without first introducing a set of rationing rules: this cream-skimming story alone is not sufficient to explain the structure of the insurance market.

And secondly, the supply side rationing of the direct writers is consistent with profit-maximising oligopolistic behaviour of a small group of firms with superior technology insulated from entry and constrained by short run capacity limitations. The unique equilibrium resulting from such oligopolistic behaviour has been shown to be the Cournot-Nash equilibrium, where firms compete in quantities and not prices (see Kreps and Scheinkman (1983)). There are many reasons why the hypothesis that direct writers compete in quantity is unlikely. Many industries do operate via Cournot competition, but the characteristics of the insurance product makes it doubtful that insurers compete in this fashion. Slade (1993) hypothesises that firms that engage in Cournot competition are in homogeneous-product intermediate-goods industries, while firms in differentiated-product retail-goods industries, like insurance, compete in a Bertrand fashion.

Furthermore various economic papers in insurance, such as Rothschild and Stiglitz (1976), Wilson (1977), Spence (1978), Sobti (1988), Hoy (1989) and Eisenhauer (1993), treat insurance companies as firms in a competitive industry, competing in prices. Schlesinger and Venezian (1986) introduce a market with one insurer with a monopoly in loss prevention goods and a competitive fringe without this additional good where firms compete on price.

Therefore Joskow's cream-skimming hypothesis will not be pursued. Like most economic papers on insurance, it will be assumed that direct writers compete in prices, and as such the equilibrium characterised in this section is Bertrand-Nash.

The second fundamental conjecture is the spatial framework. The use of spatial models (such as Salop's (1979) circular city) in insurance has been previously proposed by Clapp (1985) and Schlesinger and von der Schulenburg (1991). Both papers follow Archibald, Eaton and Lipsey's (1982) definition of location on the circumference of the circle as location in characteristic or attribute space. Clapp's use of a spatial model yields a Nash pooling equilibrium among homogeneous insurance companies within the Rothschild-Stiglitz framework. Schlesinger and von der Schulenburg develop a model of entry into the insurance market under the hypothesis that all consumers have search costs.

This model defines the location of a direct writer on the circle as its physical location. The distance between a direct writer and a consumer is the distance between the insurer's office and the consumer's home. The transportation cost can be regarded as a true transportation expense. The insurance offered by the brokers is, in Salop's terminology, the outside good and as such brokers are assumed to be located continuously around the circle so the transportation cost for a consumer to purchase insurance from an agency writer is zero.

This interpretation is not unreasonable. To purchase insurance for the first time, an individual must travel to the office of either the broker or direct writer. A major difference between the two types of insurance distribution systems is the accessibility; brokerage offices can be found almost everywhere, whereas direct writers have few offices in big cities and perhaps no offices in smaller towns. For example, the *1992 Vancouver Yellow Pages* lists 458 brokerage offices, while the largest direct writer in Canada, the Co-operators, has four offices in the metropolitan Vancouver area, and All State insurance, another large direct writer, has two.

3.2 Demand Side

The consumers' behaviour in this model will now be characterised. There L identical risk-averse consumers located uniformly about the circle where each consumer is endowed with wealth W and is assumed to have constant absolute risk aversion.[4] Therefore, a consumer's utility function is given by $-e^{-\alpha W}$. These consumers live in a world where there is only one time period and 2 states. With probability ρ, the consumer suffers a loss of d and with complementary probability, $1-\rho$, no loss occurs.[5] Both ρ and d are known to the consumers and the potential insurers. It is assumed that there is no adverse selection or moral hazard in this framework.

An individual can insure against loss either by purchasing insurance from a perfectly competitive agency writer at the price p_a or by purchasing insurance from a direct writer for p_d. There are n identical direct located symmetrically about a circle of unit circumference, so if the consumer purchases insurance from the direct writer she will incur a transportation cost of t times the distance travelled. There are no transportation costs associated with purchasing insurance from agency firms since brokers are located continuously about the circle.

It will be assumed that the insurance companies offer only full insurance contracts, restricting each consumer's choice to full coverage or no insurance. It will be shown later, in Section 3.5, that full insurance contracts are sustainable in equilibrium.

The consumer then is faced with three choices: she may purchase no insurance, she may purchase full insurance from the agency writer or she may purchase full insurance from a direct writer. Assuming that the consumer is a utility maximiser, her preference will be the

[4]This utility function is used so that wealth effects can be ignored. Similar results for Section 3 can be obtained if other concave von Neumann-Morgenstern utility functions are used.

[5]Assume for simplicity that this probability is uncorrelated across consumers. Since the insurance market exists because of the law of large numbers, the existence of the insurance industry is a function of the number of customers and the correlation between consumers.

option that gives her the highest expected utility.

If she purchases no insurance, her expected utility is $V_0 = -(1-\rho)e^{-\alpha W} - \rho e^{-\alpha(W-d)}$.

A consumer who purchases insurance from the agency writer has expected utility of $V_a(p_a) = -e^{-\alpha(W-p_a)}$. However given that the agency writers act competitively, each insurer charges the expected cost per policy of $\rho d + e_a$, where e_a is the cost to the insurer of writing one policy. Therefore the consumer's utility can be rewritten as

$$V_a = -e^{-\alpha(W-\rho d-e_a)}. \tag{1}$$

Let ℓ be the distance between a consumer and the closest direct writer. Under the assumption of linear transportation costs, the effective cost of purchasing insurance from a direct writer is $p_d + t\ell$. Thus the consumer's utility is given by

$$V_d(p_d) = -e^{-\alpha(W-p_d-t\ell)}. \tag{2}$$

The decision to purchase insurance from an agency writer or to self-insure does not depend on the location of the consumer. The consumer will purchase insurance as long as

$$V_a - V_o \propto (1-\rho) - e^{\alpha d}(e^{-\alpha((1-\rho)d-e_a)} - \rho) \geq 0.$$

Figure 1 gives, for differing levels of d and α,[6] values of e_a and ρ for which insurance will be purchased.[7] Each curve represents values of e_a and ρ for a given d and α, at which the consumer is indifferent between purchasing insurance and going without. The area under each of the curves illustrate combinations of e_a and ρ for which the consumer would strictly prefer to purchase insurance. Insurance would not be purchased for combinations of e_a and ρ located above each curve.

[6]The levels of α examined were suggested by Haubrich (1994) in a study of risk aversion of company executives.

[7]Implicit in this figure is the assumption that the size of the insurance premium is less than the initial wealth W.

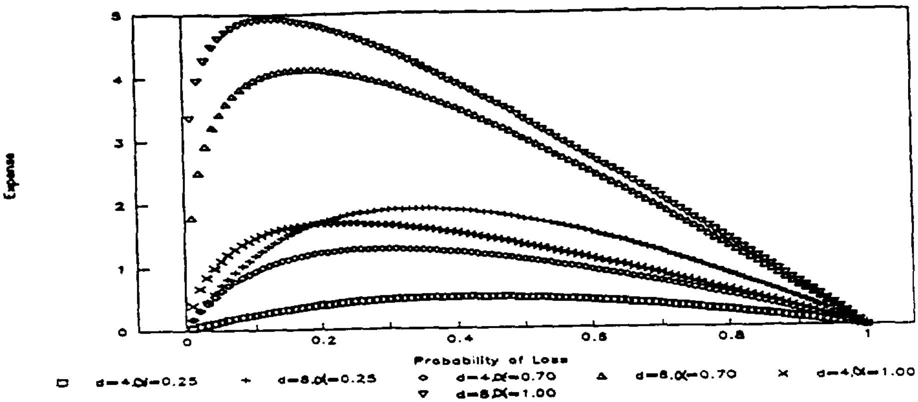

Figure 1: Risk Characteristics of Insurance Purchasers

At very high probabilities of loss, consumers will purchase insurance only if the expense goes to zero, regardless of their risk aversion coefficient because the actuarial fair value of insurance is approaching the size of the loss. As the probability of loss approaches zero, the expense level at which consumers would purchase insurance also falls. As also expected, as the risk aversion coefficient increases, the amount of expense that an individual is willing to pay at any probability of loss also increases. Consumers are also willing to pay higher expenses for higher sizes of loss.

It will be assumed that buying insurance from the agency writer would always be preferable to no insurance.[8]

The consumer will prefer to purchase insurance from the direct writer if $V_d(p_d) \geq V_a(p_a)$, a decision which depends on the location of the consumer. From (1) and (2), a consumer will purchase insurance from a direct writer if

[8]This assumption assures the existence of the agency writers. If no agency writers exist, the size of the market faced by a direct writers and the number of direct writers that enter the market will be different, but the flavour of the analysis will be maintained.

$$\ell \leq \frac{\rho d + e_a - p_d}{t}. \quad (3)$$

Therefore if the transportation cost is very high or if there is not much difference between the prices charged by the direct and agency writers, then a consumer is more likely to buy insurance from a broker.

3.3 Supply Side

As assumed earlier, there are n identical direct writers who enter sequentially and locate symmetrically about a circle of unit circumference. Unlike Salop's original model it is assumed here that relocation costs are so prohibitive that once a direct writer has chosen its location, it cannot move. Each direct writer first incurs a capital cost of F_d and then chooses a price p_d at which to sell insurance. The direct writer expends $e_d < e_a$ per policy.[9] Agency writers are located continuously about the circle and possess ample capacity to absorb the entire market's demand at the price $p_a = \rho d + e_a$.

Sequential entry and high relocation costs are necessary to derive the characterisation of the model and the symmetry produces a tractable equilibrium. The entry configuration of direct writers follows closely to the pattern discussed in Eaton and Wooders (1985). Since it is the purpose of this paper to explain the co-existence of direct and agency writers, the following analyses will concentrate on equilibrium conditions which lead to the existence of both types of insurers.

Before the equilibrium is characterised, the profit maximising behaviour of the direct writer both acting as a local monopolist and competing with other direct writers will be examined.

[9] A further restriction on e_d and e_a is $e_a - e_d \leq t$. This restriction ensures that one direct writer cannot enter and capture the entire market.

Lemma 1 *If a direct writer can act as a local monopolist, it will sell its product at a price of* $p_d^m = \rho d + \frac{1}{2}(e_a + e_d)$ *and earn monopoly profits of* $\Pi(p_d^m) = \frac{L}{2t}(e_a - e_d)^2 - F_d$. *The length of the monopoly market is given by* $2\hat{\ell}^m = \frac{e_a - e_d}{t}$, *where* $2\hat{\ell}^m$ *denotes market length evaluated at the monopolist's profit maximising price.*

Proof: If a direct writer acts as a local monopolist, it competes with the agency writer for the marginal consumer. As such, the demand for the direct writer's product is given by

$$q_d^m = 2L\ell^m = \frac{2L}{t}[\rho d + e_a - p_d], \tag{4}$$

where, from (3),

$$\ell^m = \frac{1}{t}[\rho d + e_a - p_d] \tag{5}$$

is the location of the consumer indifferent between purchasing insurance from the direct writer or the agency writer. All consumers closer to the direct writer would prefer to purchase insurance from the direct writer.

The profit maximising direct writer will

$$\max_{p_d} \Pi(p_d) = (p_d - \rho d - e_d) \cdot \frac{2L}{t}(\rho d + e_a - p_d) - F_d,$$

which yields a monopoly price of $p_d^m = \rho d + \frac{1}{2}(e_a + e_d)$ and a profit of $\Pi(p_d^m) = \frac{L}{2t}(e_a - e_d)^2 - F_d$. Substituting for p_d^m in equation (5) shows that the equilibrium length of a local monopoly market is

$$2\hat{\ell}^m = 2\ell^m|_{p_d = p_d^m} = \frac{e_a - e_d}{t}. \quad \blacksquare \tag{6}$$

Lemma 2 *If a direct writer competes with other direct writers in a Bertrand manner, it charges a profit maximising price of* $p_d^c = \rho d + e_d + \frac{t}{n}$ *and earns profits of* $\Pi(p_d^c) = \frac{tL}{n^2} - F_d$.

Under free entry, the number of direct writers that would exist in equilibrium is $n^c = \sqrt{tL/F_d}$, *and the length of each direct writer's market is* $2\hat{\ell}^c = \frac{1}{n^c}$.

Proof: In this situation, agency writers do not exist and direct writers compete with neighbouring direct writers. The consumer who is indifferent between two adjacent direct writers, j and j^+ is located ℓ^c from writer j, where, using equation (2)

$$\ell^c = \frac{1}{2t}(p_{dj^+} - p_{dj}) + \frac{1}{2n}, \tag{7}$$

and p_{dj} is the price charged by the j^{th} direct writer. From (7), the demand for the product of the direct writer located at j is given by $q^c_{dj} = \frac{L}{2t}(p_{dj^-} - p_{dj}) + \frac{L}{2t}(p_{dj^+} - p_{dj}) + \frac{L}{n}$, where j^- and j^+ are j's closest neighbours.

Given this demand function, the direct writer located at j will

$$\max_{p_{dj}} \Pi_j(p_{dj}) = (p_{dj} - \rho d - e_d)q^c_{dj} - F_d$$

taking p_a and p_{dk} for $k \neq j$ as given. Solving this maximisation problem and appealing to the symmetry of the direct writers, yields an equilibrium price of $p^c_d = \rho d + e_d + \frac{t}{n}$. The profit earned by each direct writer in the market is $\Pi(p^c_d) = \frac{tL}{n^2} - F_d$. Therefore, under free entry, the number of direct writers that would exist in equilibrium is

$$n^c = \sqrt{tL/F_d}. \tag{8}$$

Substituting for n^c and prices in equation (7), gives an equilibrium market size of

$$2\hat{\ell}^c = 2\ell^c|_{p_{dj}=p^c_d, n=n^c} = \frac{1}{n^c}. \quad \blacksquare \tag{9}$$

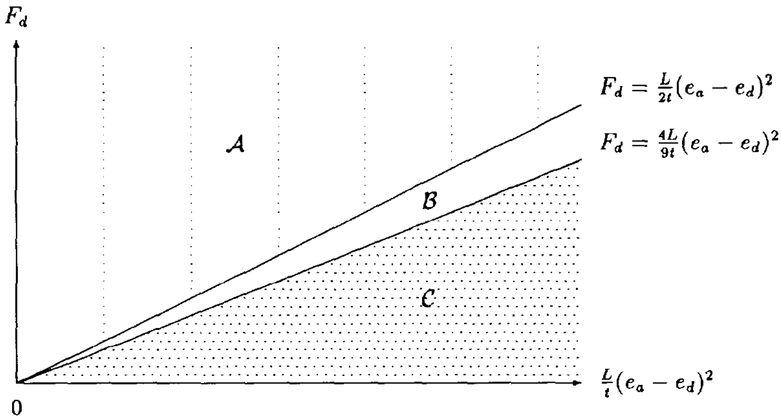

Figure 2: Equilibrium Characterised by Fixed Costs

3.4 Full Information Equilibrium

Given the above descriptions of consumer and insurer behaviours, one possible equilibrium in this model can now be derived. This equilibrium, characterised in Proposition 1, depends on the size of the fixed cost, F_d and its relation to the other exogenous variables in this model. Figure 2 illustrates the relationships. Before the equilibrium is discussed, it is useful to derive the boundaries for the regions $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ in Figure 2. Area $\mathcal{A}$ defines a region in which no direct writer could enter and earn non-negative profits. In region $\mathcal{B}$, non-negative profits can only be earned by direct writers acting as local monopolists, and in the area denoted as $\mathcal{C}$, non-negative profits are assured even if direct writers compete with each other in a Bertrand fashion.

To obtain the function defining the boundary between areas $\mathcal{A}$ and $\mathcal{B}$, consider the situation where the direct writer acts as a local monopolist. From Lemma 1, the direct writer earns a profit of $\Pi(p_d^m) = \frac{L}{2t}(e_a - e_d)^2 - F_d$. Therefore for a level of fixed cost greater than $\frac{L}{2t}(e_a - e_d)^2$, entry will not occur. The equation $F_d = \frac{L}{2t}(e_a - e_d)^2$ defines the boundary between regions $\mathcal{A}$ and $\mathcal{B}$ in Figure 2.

To derive the boundary between regions $\mathcal{B}$ and $\mathcal{C}$, it is necessary to examine the profits accruing to direct writers competing in a Bertrand fashion. From Lemma 2, n^c direct writers each earn zero profit if the marginal consumer prefers to purchase insurance from a direct writer instead of the agency writer. This will occur only if

$$p_d^c + \frac{t}{2n^c} \leq p_a.$$

Substituting for the equilibrium number of direct writers from equation (8) and for prices, p_a and p_d^c, yields

$$(e_a - e_d) \geq \frac{3t}{2n^c} \geq \frac{3}{2}\sqrt{\frac{tF_d}{L}} \tag{10}$$

and simplifying gives

$$F_d \leq \frac{4L}{9t}(e_a - e_d)^2. \tag{11}$$

Thus for fixed capital costs less than $\frac{4L}{9t}(e_a - e_d)^2$, insurers find it profitable to enter if they must compete with neighbouring direct writers. Therefore, it is possible to construct an equilibrium in which no agency writers exist and the distance between two adjacent direct writers is $\frac{1}{n^c} = \sqrt{F_d/tL}$. Equality in (11) defines the boundary between regions $\mathcal{B}$ and $\mathcal{C}$ in Figure 2.

To ensure the existence of agency writers in the situation where $F_d \leq \frac{4L}{9t}(e_a - e_d)^2$, restrictions on the number of direct writers existing in equilibrium are required. Proposition 1 characterises one possible equilibrium which supports both direct and agency writers.

Proposition 1 *For* $F_d \leq \frac{L}{2t}(e_a - e_d)^2$*, there exists a symmetric equilibrium in which all direct writers act as local monopolists and furthermore the number of direct writers in this equilibrium is* $n_m = \text{int}[\frac{t}{e_a - e_d}]$. *For* $F_d \leq \frac{4L}{9t}(e_a - e_d)^2$*, a sufficient condition so that this equilibrium exists is the number of direct writers is* $n_m \geq \frac{3t}{4(e_a - e_d)}$.

Proof: Consider first $F_d \in (\frac{4L}{9t}(e_a - e_d)^2, \frac{L}{2t}(e_a - e_d)^2]$. Since the city is of unit length, the number of direct writers in equilibrium, n_m must be such that

$$2\hat{\ell}^m(n_m + 1) > 1 \geq 2\hat{\ell}^m n_m,$$

where from equation (6), $\hat{\ell}^m = \frac{e_a - e_d}{2t}$. Solving for n_m gives $n_m = \text{int}[\frac{t}{e_a - e_d}]$.

For $F_d \leq \frac{4L}{9t}(e_a - e_d)^2$, the equilibrium characterised is one in which direct writers act as local monopolists and there is not sufficient space between any two adjacent direct writers so that another firm could enter and compete profitably in a Bertrand fashion with its two neighbours. Consider the situation depicted in Figure 3 where there exists a distance χ between adjacent monopoly markets.

For a direct writer not to be able to profitably enter between the two existing direct writers, it must be the case that $\chi + 2\hat{\ell}^m < 4\hat{\ell}^c$. Substituting for $\hat{\ell}^m$ from equation (6) and for $n^c = \frac{1}{2\hat{\ell}^c}$ from inequality (10) yields

$$\chi \leq 2\left[\frac{2(e_a - e_d)}{3t}\right] - \frac{e_a - e_d}{t} = \frac{e_a - e_d}{3t}.$$

Since the city is of unit length, the number of direct writers in equilibrium, n_m satisfies $n_m[\chi + 2\hat{\ell}^m] = 1$, and substituting for χ and $\hat{\ell}^m$ yields $n_m \geq \frac{3t}{4(e_a - e_d)}$.

Combining this with the previous definition of n_m, implies that for $F_d \leq \frac{4L}{9t}(e_a - e_d)^2$, an equilibrium supporting both agency and direct writers is possible only if $\frac{t}{e_a - e_d} \notin ((1\frac{1}{3}, 2) \cup (2\frac{2}{3}, 3))$.

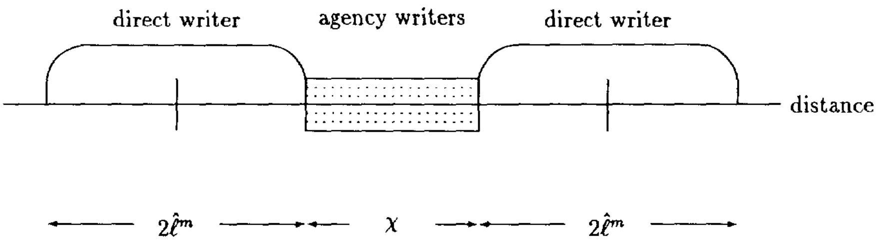

Figure 3: Spacing between Direct Writers

Therefore in equilibrium, there are n_m direct writers each earning non-negative profits and jointly providing insurance to $\frac{n_m L(e_a - e_d)}{t}$ consumers. Remaining consumers, who live between the captive markets of the direct writers, purchase insurance from the agency writers. Each direct writer charges a price of $p_d^m = \rho d + \frac{1}{2}(e_a + e_d)$ and earns a profit of $\Pi(p_d^m) = \frac{L}{2t}(e_a - e_d)^2 - F_d$. ■

Without the above characterisations of the number of direct writers, region $\mathcal{B}$ is the location of highest profit and as such operating in this area is preferred by the direct writers. Consider firms operating in an equilibrium defined by this region and suppose that the firms have the ability to manipulate $\frac{(e_a - e_d)^2}{t}$. That is, firms can change their commission structure to alter e_d or perhaps affect t, if t encompasses informational costs.[10] Since each direct writer earns a profit of $\Pi(p_d^m) = \frac{L}{2t}(e_a - e_d)^2 - F_d$, it has the incentive to decrease either e_d or t until

[10]The information costs of purchasing insurance is well accepted in current literature. Harrington (1984), in a survey paper on rate regulation, concludes that consumers' imperfect information about price and quality of insurance products prevents competition from eliminating excess profits of efficient producers. Berger (1988) implies that the existence of agency writers is in fact due to the lack of available information on direct writers and finally, Feldblum (1988) asserts that consumers' lack of information about insurers produces demand side imperfections which prevent perfect competition among only the most efficient producers of insurance.

the boundary condition $F_d = \frac{4L}{9t}(e_a - e_d)^2$ is met. Further increases in profits are possible only if $\frac{t}{e_a - e_d} \notin ((1\frac{1}{3}, 2) \cup (2\frac{2}{3}, 3))$.

If in equilibrium, $F_d < \frac{4L}{9t}(e_a - e_d)^2$ and direct writers act as local monopolists, they too have an incentive to decrease either e_d or t subject to the restriction that $\frac{t}{e_a - e_d} \notin ((1\frac{1}{3}, 2) \cup (2\frac{2}{3}, 3))$. If they decrease expenses too much, then it is possible that new direct writers will find it profitable to enter and the incumbents could be made worse off.

Exogenous changes in the number of consumers also affects the market structure. An increase in L has the same effect on direct writers as either a decrease in e_d or t. A large increase in L can eliminate all agency writers, whereas a large decrease in L can make the market unprofitable for any direct writers. For agency writers any increase in $\frac{L(e_a - e_d)^2}{t}$ is not desirable.

And finally, agency writers have an incentive to decrease their expenses. Although they would not earn higher profits if they reduced their expense margins since they operate in perfect competition, lower expenses increase their probability of survival.

The actions of the insurers in this section incorporated the simplifying assumption that only full insurance contracts were offered. The following section examines the rationality of this assumption.

3.5 Full Insurance Contracts

Because of the fixed cost involved with the purchase of insurance, the utility maximising individual would never purchase more than one policy, and as shown by Arrow (1965), Mossin (1968), Szpiro (1985), Borch (1990) and others, in the presence of an expense loading all insureds would prefer to purchase less than full insurance. Eisenhauer (1993) shows that full insurance may be purchased in the presence of an expense loading if the insurer and the consumer have differing estimates of the probability of loss.

Given a single insurance contract with a fixed loading, a risk neutral insurer would prefer to offer a full insurance contract since if the policy were earning non-negative expected profits on a partial insurance contract then it will also earn non-negative expected profits on the full insurance contract.

However in the presence of insurance offered from the agency writer and possible competition from adjacent direct writers, it is not obvious that offering full insurance contracts is profit maximising, since an insurer offering partial insurance contracts may be able to sell more insurance and thus earn higher profits. To show that full insurance contracts are sustainable in equilibrium, consider a situation in which one direct writer deviated and offered a partial insurance contract $I < d$ at a price p_I. It is assumed that only direct writers are potentially able to deviate from offering full insurance contracts.[11]

Proposition 2 *A direct writer offering partial insurance contracts would sell fewer policies at a lower price and thus earn less profit than if it had offered full insurance contracts and as such, only full insurance contracts will exist in equilibrium.*

Proof: Assume all direct writers are operating as local monopolists selling full insurance contracts, and one direct writer deviates, offering a policy with coverage of I which has an expected cost of $\rho I + e_d$.

The consumer who is indifferent between purchasing this contract from the deviating direct writer and purchasing full insurance from the agency writers is located at a distance

$$\ell^I = \frac{1}{t}[\rho d + e_a - p_I - \frac{1}{\alpha}\log(1 - \rho + \rho e^{\alpha(d-I)})]$$

from the deviating direct writer. Thus the demand for the direct writer's product then is $2L\ell^I$.

[11]This is not to say in real life that agency writers do not offer partial insurance contracts and in fact in Section 4, they will offer partial insurance contracts, but the depiction of the agency writers as a competitive fringe in this model does not allow for such strategic behaviour.

Solving the monopolist's profit maximisation problem yields a price of

$$p_I^m = \frac{1}{2}[\rho(d+I) + (e_a + e_d) - \frac{1}{\alpha}\log(1-\rho+\rho e^{\alpha(d-I)})],$$

and a profit of

$$\Pi(p_I^m) = \frac{L}{2t}[\rho(d-I) + (e_a - e_d) - \frac{1}{\alpha}\log(1-\rho+\rho e^{\alpha(d-I)})]^2 - F_d.$$

Lemma 3 $\rho(d-I) \leq \frac{1}{\alpha}\log(1-\rho+\rho e^{\alpha(d-I)})$.

Proof: See Appendix A. ■

From Lemma 1, a direct writer offering full insurance contracts could sell $\frac{L(e_a-e_d)}{t}$ policies at a price of $p_d^m = \rho d + \frac{1}{2}(e_a+e_d)$ and earn a profit of $\Pi(p_d^m) = \frac{L}{2t}(e_a-e_d)^2 - F_d$. Since from Lemma 3, $\rho(d-I) \leq \frac{1}{\alpha}\log(1-\rho+\rho e^{-\alpha(I-d)})$ therefore $2L\ell^I|_{p_I=p_I^m} < 2L\hat{\ell}^m$, $p_I^m < p_d^m$, and $\Pi(p_I^m) < \Pi(p_d^m)$.

Therefore, in equilibrium, partial insurance contracts will not be offered by any direct writer. ■

Therefore, using a spatial model, it is possible to characterise an equilibrium under which both direct writers and agency writers exist. However, it is a gross simplification to assume that all consumers are the same. Each consumer of insurance faces a different level of risk, and insurance companies may not wish to underwrite all risks, or may offer a menu of insurance contracts if price discrimination is not allowed. The next section derives an equilibrium where there are two types of consumers with differing loss probabilities.

4 Consumer Differentiation

This section embeds the Rothschild-Stiglitz insurance model within the spatial insurance framework. To examine this extension, consider two types of policyholders with differing loss probabilities as in the Rothschild-Stiglitz world. Furthermore assume that direct writers can differentiate between the risks but agency writers cannot. This situation arises from the standard agency model. Direct writer employees earn the majority of their income from salary and may have strong incentives not to approve bad risks. On the other hand, independent brokers work on a commission basis and have no incentive to refuse poor business.

Assume that existing legislation does not permit price discrimination and that firms are not constrained by common carrier requirements. Initially direct writers would only underwrite the better risks at the higher premium in an attempt to earn positive profits. Obviously, this would lead to undercutting by other direct writers until, in equilibrium, direct writers would price good risks competitively, and as in Section 3, only those good risks located "close enough" would purchase insurance from the direct writers. The ability of direct writers to discriminate between risk types and the threat of entry by new direct writers constrains the direct writers to underwriting the good risks only in equilibrium. Agency writers will underwrite all poor risks and those good risks that are too far from the direct writers.

Formally, there are two types of consumers: those with probability of loss ρ_g and those with a higher probability of loss ρ_b. Each consumer knows her risk type. Consumers with a lower probability of loss are referred to as good risks. As in Rothschild and Stiglitz, assume that the two consumer types are identical except for differing loss probabilities. For comparison with Section 3, assume that the good consumers have the same loss probability as the one type of consumer in the previous model, that is $\rho_g \equiv \rho$.

Suppose that there are N consumers in the market and the fraction of high risk consumers is λ. Again, for easy comparison with the model in the previous section, assume that $(1-\lambda)N \equiv L$. The two types of consumers are located uniformly about the circle. As in Section 3, only equilibria in which both direct and agency writers exist will be addressed.

The equilibrium obtained depends on the contracts offered by the agency writers, so the behaviour of these insurers will be discussed first.

4.1 Agency Writer Contracts

The definition of equilibria and the construction of equilibrium contracts follow Wilson (1977). There are two possible types of contracts that could exist in equilibrium: pooling and separating. The separating menu of contracts described by Wilson will be discussed first. Conditions when a pooling contract will be preferable to this menu, and such a pooling contract will then be presented. The notation is consistent with the previous section.[12]

Lemma 4 *If a separating equilibrium exists, the supporting menu of contracts consists of a full insurance contract at a price* $p_a^b = \rho^b d + e_a$ *and a partial insurance contract,* I^* *priced at* $p_a^g = \rho^g I^* + e_a$*, where* I^* *is the solution to*

$$e^{\alpha\rho^b d} = (1-\rho^b)e^{\alpha\rho^g I^*} + \rho^b e^{\alpha(d-(1-\rho^g)I^*)}.$$

Proof: This unique equilibrium set of contracts for the agency writers has been previously defined by Wilson. The menu consists of two policies: the first contract, which will be chosen by the high risk consumer, is a full insurance contract priced at the expected cost of insuring a high risk consumer and the second policy is a partial insurance contract priced at the expected cost of insuring a low risk consumer. The level of coverage is chosen so that a high risk consumer is indifferent between this partial insurance contract and the full insurance contract designed for the high risk type.

As such the two contracts offered are a full insurance contract at a price $p_a^b = \rho^b d + e_a$ and a partial insurance contract, I^*, priced at $p_a^g = \rho^g I^* + e_a$ where I^* satisfies $V_a^b(p_a^b) = V_a^b(p_a^g)$.

[12] Variable names will be differentiated only if results from both Sections 3 and 4 are being compared In this case variables defined in Section 3 will be pre-subscripted with a 3 and variable definitions arising from Section 4 will be pre-subscripted with a 4. For example, the variable label for the number of direct writers in equilibrium would be for Section 3, ${}_3n_m = \text{int}[\frac{t}{e_a - e_d}]$ and from Section 4, ${}_4n_m = \text{int}[\frac{t}{\kappa - \rho^g d + e_a - e_d}]$.

If a high risk consumer purchases a contract (p_a^i, I^i) from the agency writer, where $I^b = d$ and $I^g = I^*$, then she secures a utility of

$$V_a^b(p_a^i) = -e^{-\alpha W}[(1-\rho^b)e^{\alpha p_a^i} + \rho^b e^{\alpha(p_a^i - I^i + d)}]. \tag{12}$$

From (12), the level of partial insurance, I^*, offered to the good risks must satisfy

$$\begin{aligned} 0 &= V_a^b(p_a^b) - V_a^b(p_a^g) \\ &= -e^{-\alpha(W-e_a)}\left[e^{\alpha\rho^b d} - (1-\rho^b)e^{\alpha\rho^g I^*} - \rho^b e^{\alpha(\rho^g I^* + d - I)}\right], \end{aligned}$$

and rearranging terms gives

$$e^{\alpha\rho^b d} = (1-\rho^b)e^{\alpha\rho^g I^*} + \rho^b e^{\alpha(d-(1-\rho^g)I^*)} \tag{13}$$

It is straightforward to show that given these two insurance contracts, d and I^*, priced at p_a^b and p_a^g respectively, the good risks have no incentive to mimic the bad risks. ∎

Additionally, for the agency writers to exist in equilibrium it is necessary that

$$V_a^b(p_a^b) - V_o^b \propto (1-\rho^b) - e^{\alpha d}(e^{-\alpha((1-\rho^b)d - e_a)} - \rho^b) \geq 0.$$

From Section 3.2, Figure 1 illustrates the relationship between d, α, e_a and ρ^b for which the purchase of insurance is preferable to no insurance.

Wilson has shown that in the absence of a superior pooling contract, this separating set of contracts is unique and furthermore, it is Pareto optimal given the asymmetric information; there is no other separating contracts that are preferable to this one. Rothschild and Stiglitz note that such a pooling contract could exist if the costs of separating are too high or if the cost of pooling are small to the low risks, for example there are not many high risks, or the difference between loss probabilities is not great. Mathematically, the equations which identify the conditions under which a pooling equilibrium exists can be defined.

Lemma 5 *If λ satisfies*

$$(1-\rho^g)\left[e^{\alpha\rho^g I^*}-e^{\alpha\bar{\rho}I^p}\right]+\rho^g e^{\alpha d}\left[e^{-\alpha(1-\rho^g)I^*}-e^{-\alpha(1-\bar{\rho})I^p}\right]\geq 0$$

where I^p is the level of coverage offered in the pooling contract and $\bar{\rho}\equiv\lambda\rho^b+(1-\lambda)\rho^g$ is the pooled or average probability of loss, then the separating contracts characterised in Lemma 4 will not hold in equilibrium.

Proof: Suppose that there exists a pooling contract, (p_a^p, I^p), that breaks the equilibrium defined by the menu of screening contracts. For such a policy to exist, it must be the case that

$$p_a^p = (\lambda\rho^b+(1-\lambda)\rho^g)I^p+e_a=\bar{\rho}I^p+e_a \tag{14}$$

$$V_a^b(p_a^p) \geq V_a^b(p_a^b) \tag{15}$$

$$V_a^g(p_a^p) \geq V_a^g(p_a^g). \tag{16}$$

Equation (14) states that the price of the contract must equal the *ex-ante* expected average cost of an insurance policy; the insurer earns zero profits. Equations (15) and (16) confirm that both types of risks would prefer this pooling contract to the separating contracts defined in Proposition 3. Substituting for the prices of the separating contracts defined in Lemma 4 and for $e^{\alpha\rho^b d}$ from equation (13) in these two inequalities yields

$$(1-\rho^b)\left[e^{\alpha\rho^g I^*}-e^{\alpha\bar{\rho}I^p}\right]+\rho^b e^{\alpha d}\left[e^{-\alpha(1-\rho^g)I^*}-e^{-\alpha(1-\bar{\rho})I^p}\right] \geq 0$$

$$(1-\rho^g)\left[e^{\alpha\rho^g I^*}-e^{\alpha\bar{\rho}I^p}\right]+\rho^g e^{\alpha d}\left[e^{-\alpha(1-\rho^g)I^*}-e^{-\alpha(1-\bar{\rho})I^p}\right] \geq 0, \tag{17}$$

respectively. Since $\rho^b > \rho^g$, inequality (15) will be automatically satisfied if inequality (17) holds.

Since there exists a pooling contract which earns the insurer non-negative profits and is preferred to the separating contracts by both types of consumers, therefore the separating set of contracts will not exist in equilibrium. ∎

Economically, since the worse type is subsidised in the pooling contract, the high risk consumer would always prefer this pooling contract as long as the amount of coverage offered is not too much less than full insurance. In order for the pooling contract to be accepted by the good risks, it must be the case that $I^p > I^*$,and as such, the first term in the square brackets of inequality (17) is always negative.[13]

An upper bound on λ is given by

$$\lambda < \frac{(1-\rho^g)(I^p - I^*)}{(\rho^b - \rho^g)I^p}, \tag{18}$$

which can be found by setting the second square bracket in inequality (17) equal to zero.[14] The conditions under which inequality (18) is satisfied can be interpreted by Rothschild and Stiglitz's comments on the existence of a pooling contract. This inequality is likely to hold if the cost of separating, $I^p - I^*$, is high or if the proportion of high risks, λ, is small, or finally if the difference between loss probabilities, $\rho^b - \rho^g$, is small.

No pooling contract satisfies Wilson's **E1** criteria, which states that if a different set of policies is offered in addition to this pooling contract then this new set of policies cannot earn non-negative profits in aggregate with at least one policy earning positive profit. For every profitable pooling contract, it is possible for another agency writer to earn positive profits by offering a different pooling contract which is strictly preferred by the good risks and not desired by the bad types. The pooling contract characterised below does satisfy the less stringent **E2** criteria, which is referred to as Wilson's anticipatory equilibrium. This equilibrium condition requires that each policy offered earns non-negative profits and there cannot exist a new set of policies which earns non-negative profits in aggregate and strictly positive profits for at least one policy after all other unprofitable contracts have been

[13]For a pooling contract to exist, good consumers must receive higher utility than they would if they had purchased the separating contract. Since good risks subsidise high risk consumers in a pooling contract, it must be the case that $I^p > I^*$.

[14]This bound is not very tight, since it is necessary that the term in this bracket be strictly positive so that $e^{-\alpha(1-\rho^g)I^*} - e^{-\alpha(1-\bar{\rho})I^p} > \frac{(1-\rho^g)}{\rho^g} e^{-\alpha d} \left| \left[e^{\alpha\rho^g I^*} - e^{\alpha\bar{\rho} I^p} \right] \right|$.

withdrawn from the market. Wilson also shows that this pooling equilibrium is not unique and not necessarily Pareto optimal.

Lemma 6 *A pooling contract that satisfies Wilson's* **E2** *criteria is given by a level of coverage, I^p and a price $p_a^p = \bar{\rho} I^p + e_a$, where I^p is defined as*

$$I^p = d - \frac{1}{\alpha} \log \left[\frac{\bar{\rho}(1-\rho^g)}{(1-\bar{\rho})\rho^g} \right].$$

Proof: From Wilson, the pooling contract offered to both consumers is one that maximises the utility of the good consumer subject to equation (14). Solving the maximisation problem for the level of the indemnity gives

$$I^p = d - \frac{1}{\alpha} \log \left[\frac{\bar{\rho}(1-\rho^g)}{(1-\bar{\rho})\rho^g} \right]. \quad \blacksquare \tag{19}$$

In equation (19), it can be seen that the pooling contract offered is for less than full insurance (since $\rho^g < \bar{\rho}$). As λ increases, $\bar{\rho}$ increases and subsequently the indemnity is reduced, decreasing the utility to the better customer. Therefore the fewer the good customers, the worse off they are in a pooling equilibrium.

Since the contracts offered by the agency writer has been characterised, it is now possible to look at actions of the direct writers.

4.2 Asymmetric Information Equilibria

One possible equilibrium in which agency writers offer a separating menu of contracts will be discussed first. As in Section 3.4, only equilibria in which both direct writers and agency writers exist will be examined.

Proposition 3 *If* $(1-\rho^g)\left[e^{\alpha\rho^g I^*} - e^{\alpha\bar{\rho} I^p}\right] + \rho^g e^{\alpha d}\left[e^{-\alpha(1-\rho^g)I^*} - e^{-\alpha(1-\bar{\rho})I^p}\right] < 0$, *then a possible separating equilibrium is characterised by*

1. *A continuum of agency writers, each offering the menu of contracts defined in Lemma 4. Agency writers located within* $\hat{\ell}^m = \frac{1}{t}\left[\frac{1}{2}(\kappa - \rho^g d) + \frac{1}{2}(e_a - e_d)\right]$ *of a direct writer will not underwrite any good risks, where, for notational simplicity*

$$\kappa \equiv \frac{1}{\alpha}\log\left[(1-\rho^g)e^{\alpha\rho^g I^*} + \rho^g e^{\alpha(d-(1-\rho^g)I^*)}\right].$$

2. *For* $F_d \leq \frac{L}{2t}((\kappa - \rho^g d) + (e_a - e_d))^2$, $n_m = \text{int}[\frac{t}{\kappa - \rho^g d + e_a - e_d}]$ *direct writers acting as local monopolists serving the good risks only. Furthermore, if* $F_d \leq \frac{4L}{9t}((\kappa - \rho^g d) + (e_a - e_d))^2$, *this equilibrium is sustainable if* $n_m > \frac{3t}{4(\kappa - \rho^g d + e_a - e_d)}$. *Each direct writer offers full insurance at a price of* $p_d = \frac{1}{2}(\kappa + \rho^g d) + \frac{1}{2}(e_a + e_d)$ *and earns non-negative profits of* $\Pi(p_d^m) = \frac{L}{2t}((\kappa - \rho^g d) + (e_a - e_d))^2 - F_d$.

Proof: See Appendix A. ∎

The direct writer's optimisation problem is identical to that in Section 3, except that the quality of the insurance offered by the agency writers and the price p_a^g at which they sell it, differ.

The difference in the results of this model and the one presented in Section 3.4 is the term $\kappa - \rho^g d$ which appears in all of the direct writers' results, for example,

$$_4p_d^m = \frac{1}{2}(\rho^g d + \kappa) + \frac{1}{2}(e_a + e_d) = {}_3p_d^m + \frac{1}{2}(\kappa - \rho^g d)$$

and

$$_4q_d^m = \frac{L}{t}((\kappa - \rho^g d) + (e_a + e_d)) = {}_3q_d^m + \frac{L}{t}(\kappa - \rho^g d).$$

Rewriting the definition of κ as

$$-e^{\alpha\kappa} \equiv -(1-\rho^g)e^{\alpha\rho^g I^*} - \rho^g e^{\alpha(d-(1-\rho^g)I^*)},$$

it can be seen that the right side of the equation is the expected utility to the good consumer from purchasing insurance from the agency writer (ignoring the expense loading). Therefore κ can be viewed as the amount which provides the equivalent utility to the good risk as the purchase of partial insurance screening contract from the agency writer.

The amount $-e^{\alpha(\kappa-\rho^g d)}$ is the utility accruing to the good risk from the purchase of insurance from the agency writer due to the existence of the screening contracts.

Lemma 7 *Since* $\kappa \equiv \frac{1}{\alpha}\log\left[(1-\rho^g)e^{\alpha\rho^g I^*} + \rho^g e^{\alpha(d-(1-\rho^g)I^*)}\right] > \rho^g d$, *direct writers underwrite more policies at a higher price and thus earn higher profits than they would if there were only one type of consumer (or equivalently, if agency writers could price discriminate). It also follows that there are fewer direct writers operating in equilibrium.*

Proof : The calculation of the sign of $\kappa - \rho^g d$ can be found in Appendix A. Comparison of the quantities, ${}_3p_d^m$ and ${}_4p_d^m$, the prices, ${}_3q_d^m$ and ${}_4q_d^m$ and the profits, $\Pi({}_3p_d^m)$ and $\Pi({}_4p_d^m)$, completes the lemma. ∎

This result is not surprising. For the agency writers to effectively separate good consumers from the bad, they must offer the good consumers a contract that would not be acceptable to the high risk consumers. This contract earns the good consumers lower utility than the contract that would be offered by agency writers if they could distinguish between risk types, that is ${}_4V_a^g(p_a^g) < {}_3V_a(p_a)$. This reduction in the utility gained from the purchase of agency writers' product translates into an increased demand for the direct writers' products.

As outlined in Appendix A, the demand for the direct writer's product is given by

$$q_d^m = 2L\ell^m = \frac{2L}{t}(\kappa + e_a - p_d). \tag{20}$$

Comparing the demands from equations (4) and (20), it can be seen that the effect of the agency writers' screening contracts on the demand of the good consumers for the direct writers' product is to shift the entire demand curve upwards by a factor of $\kappa - \rho^g d$. The size of the shift is reflected in the price that the good risks are willing to pay for insurance from the direct writers.

The increased demand for the direct writers' product affects the structure of the industry. In Section 3, direct writers with a fixed cost in the range $(\frac{L}{2t}(e_a - e_d)^2, \frac{L}{2t}((\kappa - \rho^g d) + (e_a - e_d))^2]$, could not profitably sell insurance, but because of the effect of the screening contracts offered by the agency writers, direct writers now find it lucrative to enter the market. Similarly, for fixed costs in the range $(\frac{4L}{9t}(e_a - e_d)^2, \frac{4L}{9t}((\kappa - \rho^g d) + (e_a - e_d))^2]$, direct writers who were assured a monopoly market in Section 3, may now find themselves in Bertrand competition with neighbouring direct writers.

The necessity of the screening contract also affects the existence of the agency writers. In Section 3, if the agency writers could reduce their expenses to e_d, then they could capture the entire market and no direct writer could enter. Now even as agency expenses approach direct writer expenses, if fixed costs are less than $\frac{L}{2t}(\kappa - \rho^g d)$, direct writers will exist in equilibrium. In Figure 2, if $\frac{L}{t}$ remains constant, the effect of the screening contracts is to shift the curves separating the regions $\mathcal{A}, \mathcal{B}$ and $\mathcal{C}$ upwards. If $\frac{L}{t}$ is changing, but $(e_a - e_d)$ is constant, then the curves dividing the three regions in Figure 2 will become steeper because of the screening contracts.

The following lemma describes a possible symmetric equilibrium if the agency writers offer a pooling contract.

Lemma 8 *If the* $(1 - \rho^g)\left[e^{\alpha\rho^g I^*} - e^{\alpha\bar{\rho}I^p}\right] + \rho^g e^{\alpha d}\left[e^{-\alpha(1-\rho^g)I^*} - e^{-\alpha(1-\bar{\rho})I^p}\right] \geq 0$, *a possible pooling equilibrium is given by:*

1. *A continuum of agency writers located between the captive markets of the direct writers, earning zero profits and offering a partial insurance contract of* I^p *to both types of consumers at a price* $p_a^p = \bar{\rho}I^p + e_a$, *where* I^p *is defined in Lemma 6, and* $\bar{\rho} = \lambda\rho^b +$

$(1-\lambda)\rho^g$.

2. *An infinite number of agency writers located within the radius of each direct writer's market, offering a full insurance contract to the bad risks only, at its marginal cost of* $p_a^b = \rho^b d + e_a$. *These agency writers too earn zero profits.*

3. *For* $F_d \leq \frac{L}{2t}((\zeta - \rho^g d) + (e_a - e_d))^2$, $n_m = \text{int}[\frac{t}{\zeta - \rho^g d + e_a - e_d}]$ *direct writers acting as local monopolists serving the good risks only where* ζ *is defined as*

$$\zeta \equiv \frac{1}{\alpha} \log \left[(1-\rho^g) e^{\alpha \bar{\rho} I^p} + \rho^g e^{\alpha (d - (1-\bar{\rho}) I^p)} \right]$$

Each direct writer offers full insurance policies at a price of $p_d^m = \frac{1}{2}(\rho^g d + \zeta) + \frac{1}{2}(e_a + e_d)$ *and earns non-negative profits of* $\Pi(p_d^m) = \frac{L}{2t}((\zeta - \rho^g d) + (e_a - e_d))^2 - F_d$. *If* $F_d \leq \frac{4L}{9t}((\zeta - \rho^g d) + (e_a - e_d))^2$, *this equilibrium is sustainable if* $n_m > \frac{3t}{4(\zeta - \rho^g d + e_a - e_d)}$.

Proof: The description of the actions of the direct writers are the same as in Propositions 1 and 3 and so it will not be presented.

There are two differences between this equilibrium and the one given in Proposition 3. First, agency writers located within $\hat{\ell}^m = \frac{1}{t}\left[\frac{1}{2}(\zeta - \rho^g d) + \frac{1}{2}(e_a - e_d)\right]$ of a direct writer will not underwrite any good risks. Because of this, any pooling contract offered by these insurers will earn negative profits and as such will be withdrawn from the market. Since these agency writers will only underwrite the bad types, perfect competition requires that these risks be offered a full insurance contract at the marginal cost $p_a^b = \rho^b d + e_a$.

The second difference is the term $\zeta \equiv \frac{1}{\alpha} \log \left[(1-\rho^g) e^{\alpha \bar{\rho} I^p} + \rho^g e^{\alpha (d - (1-\bar{\rho}) I^p)} \right]$. In the pooling equilibrium, ζ has the same interpretation as κ in the separating equilibrium. By definition of the existence of the pooling equilibrium, it must be the case that $\zeta < \kappa$. But $\zeta > \rho^g d$, because the inability of the agency writers to separate good and bad risks makes the good risks worse off. ∎

A comparison of the profits earned by the direct writers in the full information and the asymmetric information models confirm that because agency writers cannot differentiate between the two types of consumers, direct writers earn informational rents.

Some low risk consumers are made better off by the existence of the direct writers. If agency writers offer a menu of contracts, the existence of the direct writers does not affect the welfare of the high risk consumers. Recall that if a pooling equilibrium exists, then the higher risks preferred the partial insurance pooling contract to the separating full insurance contract, therefore the existence of the direct writers makes these higher risk consumers worse off.

5 Future Work

Future work can be classified into three areas: technical details, welfare implications and empirical work.

Technical details involve the examination of the various underlying assumptions of the models. One conjecture in the paper is that insurance consumers have exponential utility functions. The sensitivity of the results of the paper to this functional form should be examined. The implications of the assumption that direct writers locate symmetrically about the circle need to be scrutinised, since it is possible that the existence of the agency writers in the above model depends heavily on these entry rules. Preliminary work suggests that the space of exogenous variables can be partitioned into three regions, and for variable combinations in one of those regions the existence of agency writers is assured.

The preceding sections have dealt with the normative issues of the economics of the structure of the insurance market. Future work will involve the calculation of welfare gains to consumers of admitting both agency and direct writers. From Section 3, one possible hypothesis to examine is that it may be welfare increasing or more cost effective for the social planner (or the insurance regulator) to abolish agency writers and to offer some sort travel subsidy to those risks located too far from the direct writers. A key assumption in the derivation of the models in Section 4 is the absence of a common carrier requirement for the direct writers. The welfare implications of such a requirement, or its absence, need to be addressed.

There are policy issues that also need to be addressed in future work. Joskow criticised insurance regulators for setting rates which support the inefficient technology of the agency writers. If in fact the predictions of this model are correct, then it would be in the high risk consumers' best interests to ensure the continued existence of the agency writers since they write business that would otherwise be uninsurable. The role of rate regulation in the insurance market can be examined within the framework presented in the paper.

The models in the Section 4 have many testable hypotheses which can be verified. If the model predictions are correct, than agency writers write more policies at differing deductible levels than direct writers, reflecting the existence of the screening contracts. One would also expect the average loss frequency and the variance of loss frequency between consumers to be lower for the direct writers, reflecting the homogeneity of the portfolio underwritten. If frequency data are not available, this conjecture can be tested using loss cost data under the assumption that there is no difference in the average size of loss between consumer types. Another hypothesis to be investigated is that consumers living in small communities that do not have any direct writer offices purchase insurance from agency writers. Unfortunately, it could be difficult to get data collected on this basis. In markets where insurance is compulsory, such as Private Passenger Third Party Liability, the reluctance of the direct writers to underwrite poor risks should be translated into a higher than average proportion of drivers underwritten by existing residual pools.

6 Conclusions

This paper provides an economic explanation for the co-existence of agency and direct property-casualty insurance companies. By exploiting the difference in accessibility between the two types of insurance distribution systems, a symmetric equilibrium is constructed in which direct writers act as local monopolists with intervals between their captive markets that are served by agency writers. In order to characterise the equilibrium, the model relies on a sequential entry rule and prohibitive relocation costs for the direct writers.

This model is extended to include two types of consumers, under the assumption that direct writers can differentiate between risk types but agency writers cannot. It is shown that the traditional Rothschild-Stiglitz results can be obtained in a model with two types of insurance companies with differing fixed and variable costs and consumers that have transportation costs. In this model the existence of the direct writers improves the utility of most of the good consumers and can actually decrease the utility of the high risk consumers if agency writers offer a pooling contract. The direct writers earn informational rents because the agency writers cannot distinguish between consumer types.

References

1993 General Insurance Register. Stone and Cox Limited, Toronto, (published annually).

Archibald, G., B.C. Eaton and R. Lipsey (1982). Address Models of Value Theory. *Discussion Paper No. 495, Institute for Economic Research*, Queen's University, Kingston.

Arrow, K.J. (1965). *Aspects of the Theory of Risk-Bearing*, Yrjö Jahnsson Foundation, Helsinki.

Barrese, J. and J. Nelson (1992). Independent and Exclusive Agency Insurers: A Reexamination of the Cost Differential. *Journal of Risk and Insurance* **59**, 375-397.

Berger, L.(1988). Word of Mouth Reputations in Auto Insurance Markets. *Journal of Behavior and Organization* **10**, 225-234.

Borch, K.H. (1990). *Economics of Insurance*, North-Holland, Amsterdam.

Clapp, J. (1985). Quantity Competition in Spatial Markets with Incomplete Information. *Quarterly Journal of Economics* **100**, 519-528.

Cummins J.D. and J. VanDerhei (1979). A Note on the Relative Efficiency of Property-Liability Distribution Systems. *Bell Journal of Economics and Management Science* **10**, 709-719.

Eaton, B.C. and M.H. Wooders (1985). Sophisticated Entry in a Model of Spatial Competition. *Rand Journal of Economics* **16**, 282-297.

Eisenhauer, J. (1993). Asymmetries and Household Insurance: A Note. *Insurance: Mathematics and Economics* **12**, 57-60.

Feldblum, S. (1988). How Competitive is the Property/Casualty Industry? *Best's Review Property/Casualty Insurance Edition* **88**, no. 10, 14-18,98-99.

Harrington, S. (1984). The Impact of Rate Regulation on Prices and Underwriting Results in the Property-Liability Insurance Industry: A Survey. *Journal of Risk and Insurance* **51**, 577-623.

Haubrich J.G. (1994). Risk Aversion, Performance Pay, and the Principal-Agent Problem. *Journal of Political Economy* **102**, 258-276.

Hoy, M. (1989). The Value of Screening Mechanisms under Alternative Insurance Possibilities. *Journal of Public Economics* **39**, 177-206.

Joskow, P.L. (1973). Cartels, Competition and Regulation in the Property-Liability Insurance Industry. *Bell Journal of Economics and Management Science* **4**, 375-425.

Kreps, D.L. and J.A. Scheinkman (1983). Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes. *Bell Journal of Economics* **14**, 326-337.

Mossin, J. (1968). Aspects of Rational Insurance Purchasing. *Journal of Political Economy* **76**, 553-568.

Quirin, G.D et al (1974). *Competition, Economic Efficiency and Profitability in the Canadian Property and Casualty Insurance Industry.* Insurance Bureau of Canada, Toronto.

Rothschild, M. and J. Stiglitz (1976). Equilibrium in Competitive Insurance Markets: An Essay in Economics of Imperfect Information. *Quarterly Journal of Economics* **90**, 629-650.

Salop, S. (1979). Monopolistic Competition with Outside Goods. *Bell Journal of Economics* **10**, 141-156.

Schlesinger, H. and E. Venezian (1986). Insurance Markets with Loss Prevention Activity: Profits, Market Structure and Consumer Welfare. *Rand Journal of Economics* **17**, 227-238.

Schlesinger, H. and J.-M. von der Schulenburg (1991). Search Costs, Switching Costs and Product Heterogeneity in an Insurance Market. *Journal of Risk and Insurance* **58**, 109-119.

Slade, M.E. (1993). Empirical Games: The Oligopoly Case. *Discussion Paper No. 93-31, Department of Economics*, University of British Columbia, Vancouver.

Sobti, R. (1988). Increasing Social Variability and Insurance Equilibrium. *Journal of Risk and Insurance* **55**, 509-517.

Spence, M. (1978). Product Differentiation and Performance in Insurance Markets. *Journal of Public Economics* **10**, 427-447.

Szpiro, G.G. (1985). Optimal Insurance Coverage. *Journal of Risk and Insurance* **52**, 704-710.

Van Cayseele, P. (1992). Current Problems in Insurance Economics. *Insurance: Mathematics and Economics* **10**, 239-248.

Wilson, C. (1977). A Model of Insurance Markets with Incomplete Information. *Journal of Economic Theory* **16**, 167-207.

Appendix A

Proof of Lemma 3

Show $\rho(d-I) - \frac{1}{\alpha}\log(1-\rho+\rho e^{\alpha(d-I)}) \leq 0$

Proof: Multiply both sides of the inequality by $\alpha > 0$ and exponentiate.

$$\begin{aligned}
&\rho(d-I) - \frac{1}{\alpha}\log(1-\rho+\rho e^{\alpha(d-I)}) \\
&\quad\propto\ e^{\alpha\rho(d-I)} - (1-\rho) - \rho e^{\alpha(d-I)} \\
&\quad=\ e^{\alpha(d-I)}[e^{-\alpha(1-\rho)(d-I)} - \rho] - (1-\rho)
\end{aligned}$$

Use the inequality $e^x > x+1$.

$$\begin{aligned}
&<\ (1+\alpha(d-I))[(1-\alpha(1-\rho)(d-I)) - \rho] - (1-\rho) \\
&=\ (1-\rho)[(1+\alpha(d-I))(1-\alpha(d-I)) - 1] \\
&=\ (1-\rho)[1-\alpha^2(d-I)^2 - 1] \\
&=\ -(1-\rho)\alpha^2(d-I)^2 \\
&<\ 0 \quad \blacksquare
\end{aligned}$$

Proof of Lemma 7

Show $\frac{1}{\alpha}\log\left[(1-\rho^g)e^{\alpha\rho^g I^*} + \rho^g e^{\alpha(d-(1-\rho^g)I^*)}\right] - \rho^g d > 0.$

Proof: Substitute for $e^{\alpha(d-(1-\rho^g)I^*)}$ from equation (13) to get

$$\begin{aligned}
&\frac{1}{\alpha}\log\left[(1-\rho^g)e^{\alpha\rho^g I^*} + \rho^g e^{\alpha(d-(1-\rho^g)I^*)}\right] - \rho^g d \\
&\quad=\ \frac{1}{\alpha}\log\left[\frac{1}{\rho^b}\left((\rho^b-\rho^g)e^{\alpha\rho^g I} + \rho^g e^{\alpha\rho^b d}\right)\right] - \rho^g d \\
&\quad\propto\ \log\left[\frac{1}{\rho^b}\left((\rho^b-\rho^g)e^{\alpha\rho^g I} + \rho^g e^{\alpha\rho^b d}\right)\right] - \alpha\rho^g d
\end{aligned}$$

$$
\begin{aligned}
&= \log\left[\frac{1}{\rho^b}\left((\rho^b - \rho^g)e^{\alpha\rho^g I} + \rho^g e^{\alpha\rho^b d}\right)\right] - \log(e^{\alpha\rho^g d}) \\
&= \log\left[\frac{1}{\rho^b}\left((\rho^b - \rho^g)e^{\alpha\rho^g(d-I)} + \rho^g e^{\alpha(\rho^b-\rho^g)d}\right)\right]
\end{aligned}
$$

Use the inequality $e^x > x + 1$.

$$
\begin{aligned}
&> \log\left[\frac{1}{\rho^b}\left((\rho^b - \rho^g)(1 + \rho^g(d - I)) + \rho^g(1 + \alpha(\rho^b - \rho^g)d)\right)\right] \\
&= \log\left[1 + \frac{(\rho^b - \rho^g)\alpha\rho^g I}{\rho^b}\right]
\end{aligned}
$$

and since $\dfrac{(\rho^b - \rho^g)\alpha\rho^g I}{\rho^b} > 0$

> 0 ∎

Proof of Proposition 3

Proof: The outline follows the logic of the proof of Proposition 1.

From Lemma 6, $(1-\rho^g)\left[e^{\alpha\rho^g I^*} - e^{\alpha\bar{\rho}I^p}\right] + \rho^g e^{\alpha d}\left[e^{-\alpha(1-\rho^g)I^*} - e^{-\alpha(1-\bar{\rho})I^p}\right] < 0$ will ensure that good risks have no incentive to pool; a separating contract will be preferable.

If the direct writer acts as a local monopolist, then the consumer who is indifferent between purchasing insurance from the agency writer and insurance from the direct writer is located at

$$
\ell^m = \frac{1}{t}(\kappa + e_a - p_d), \tag{21}
$$

where ℓ^m satisfies $V_a^g(p_a^g) = V_d(p_d)$.

From equation (21), the demand for the direct writer's product is given by

$$
q_d^m = 2L\ell^m = \frac{2L}{t}(\kappa + e_a - p_d). \tag{22}
$$

The cost to the direct writer of producing a policy for the good risk is $\rho^g I^* + e_d$ and solving the monopolist's profit maximisation function, given this demand and cost, yields a monopoly price of

$$p_d^m = \frac{1}{2}(\rho^g d + \kappa) + \frac{1}{2}(e_a + e_d) \tag{23}$$

and a profit of

$$\Pi(p_d^m) = \frac{L}{2t}((\kappa - \rho^g d) + (e_a - e_d))^2 - F_d. \tag{24}$$

Therefore no direct writer will enter the market if $F_d > \frac{L}{2t}((\kappa - \rho^g d) + (e_a - e_d))^2$.

If direct writers compete with each other in a Bertrand manner, the consumer that is indifferent between two adjacent direct writers is located at

$$\ell^c = \frac{1}{2t}(p_{dj} - p_{dj^+}) + \frac{1}{2n},$$

where p_{dj} is the price charged by the j^{th} direct writer. The supply faced by the direct writer located at j is given by $q_{dj}^c = \frac{L}{2t}(p_{dj} - p_{dj^-}) + \frac{L}{2t}(p_{dj} - p_{dj^+}) + \frac{L}{n}$, where j^- and j^+ are j's closest neighbours.

Using this demand function, the direct writer located at j will

$$\max_{p_{dj}} \Pi_j(p_{dj}) = (p_{dj} - \rho^g d - e_d)q_{dj}^c - F_d$$

taking p_a^g and p_{dk} for $k \neq j$ as given. Solving the direct writer's maximisation problem and appealing to the symmetry of the direct writers, yields an equilibrium price of $p_d^c = \rho^g d + e_d + \frac{t}{n}$.

Since the profit earned by each direct writer in the market is $\Pi(p_d^c) = \frac{t}{n} \cdot \frac{L}{n} - F_d$, the number of direct writers that would exist in a free entry equilibrium is

$$n^c = \sqrt{tL/F_d}. \tag{25}$$

For direct writers to earn non-negative profits, the marginal consumer must prefer to purchase insurance from a direct writer instead of the agency writer, or equivalently it must be the case that $V_d(p_d^c) \geq V_a^g(p_a^g)$. By rearranging terms and substituting for l^* from equation (13), this inequality simplifies to

$$\rho^g d + \frac{3t}{2n^c} \leq e_a - e_d + \kappa.$$

Combining this inequality with the equilibrium number of direct writers from equation (25) implies that for a direct writer to earn non-negative profits under Bertrand competition, it is necessary that

$$F_d \leq \frac{4L}{9t}((\kappa - \rho^g d) + (e_a - e_d))^2.$$

For a fixed cost higher than this amount, a direct writer could not survive if it had to compete with neighbouring direct writers.

The resulting equilibrium can be characterised by the relationship of fixed costs to the other variables. For $F_d \in (\frac{4L}{9t}((\kappa - \rho^g d) + (e_a - e_d))^2, \frac{L}{t}((\kappa - \rho^g d) + (e_a - e_d))^2]$, no other constraints are necessary to ensure the co-existence of both agency and direct writers in equilibrium, whereas for $F_d \leq \frac{4L}{9t}((\kappa - \rho^g d) + (e_a - e_d))^2$, restrictions on the number of direct writers is sufficient to ensure existence of the agency writers.

The size of the monopolist's market is, from equation (21),

$$2\hat{\ell}^m = 2\ell^m|_{p_d = p_d^m} = \frac{1}{t}[(\kappa - \rho^g d) + (e_a - e_d)].$$

Since the number of direct writers that can be supported by the market is n_m, where n_m is defined by

$$2\hat{\ell}^m(n_m + 1) > 1 \geq 2\hat{\ell}^m n_m,$$

therefore, in equilibrium, there are $n_m = \text{int}[\frac{t}{\kappa - \rho^g d + e_a - e_d}]$ direct writers each acting as local monopolists. Agency writers located within

$$\hat{\ell}^m = \frac{1}{t}\left[\frac{1}{2}(\kappa - \rho^g d) + \frac{1}{2}(e_a - e_d)\right]$$

of a direct writer will not underwrite any good risks.

For $F_d \leq \frac{4L}{9t}((\kappa - \rho^g d) + (e_a - e_d))^2$, further restrictions are necessary to ensure that agency writers exist in equilibrium. As in Section 3.4, for fixed costs in this range, the equilibrium constructed is one in which direct writers act as local monopolists, and there is insufficient space between any two direct writers so that another direct writer could enter and make non-negative profits. For this to occur, it must be the case that $\chi + 2\hat{\ell}^m < 4\hat{\ell}^c$, where χ, the distance between the endpoints of the captive market of two adjacent direct writers, is illustrated in Figure 3. Substituting for the optimal market sizes, $\hat{\ell}^m$ and $4\hat{\ell}^c$, yields

$$\chi \leq 2\left[\frac{2((\kappa - \rho^g d) + (e_a - e_d))}{3t}\right] - \frac{e_a - e_d}{t} = \frac{(\kappa - \rho^g d) + (e_a - e_d)}{3t}.$$

Therefore, the number of direct writers in equilibrium, n_m, satisfies $n_m[\chi + 2\hat{\ell}^m] = 1$, and substituting for χ and $\hat{\ell}^m$ yields $n_m \geq \frac{3t}{4((\kappa - \rho^g d) + (e_a - e_d))}$.

From equations (23) and (24), in equilibrium each direct writer sells full insurance contracts to the good risk only at a price $p_d^m = \frac{1}{2}(\rho^g d + \kappa) + \frac{1}{2}(e_a + e_d)$, and earns a profit of $\Pi(p_d^m) = \frac{L}{2t}((\kappa - \rho^g d) + (e_a - e_d))^2 - F_d$. Analogous to the result in Section 3.4, if $F_d \in (\frac{4L}{9t}(e_a - e_d)^2, \frac{L}{2t}(e_a - e_d)^2]$, an equilibrium supporting both agency and direct writers is possible only if $\frac{t}{(\kappa - \rho^g d) + (e_a - e_d)} \notin ((1\frac{1}{3}, 2) \cup (2\frac{2}{3}, 3))$. ■