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C-1 BOND RISK ANALYSIS

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A Generalized Approach to Risk Quantification

One concept of solvency-risk quantification is to maintain enough surplus to withstand a catastrophic event that is likely to occur only once every one hundred or so years. Surplus levels are then set deterministically by looking at historical experience or by providing for a specific scenario that is perceived to be near the tail of the risk spectrum. Another approach is to measure risk exposure in terms of the losses that can occur over a period of years, including the one-year catastrophe. Cash flow modeling is frequently used under this second approach.

These cash flow models often employ Monte Carlo techniques wherein the liability and/or asset flows are affected each period by random risk events and by outside influences such as economic trends and interest-rate changes. The required surplus for a given scenario (or trial) is defined in terms of the present value of modeled gain and loss deviations relative to expected cash flows. Ideally, these deviations take into account contract terms, pricing margins and reserve levels.

Required surplus for a given trial is calculated as the amount of initial surplus funds needed so that the accumulation (with interest) of this initial amount and subsequent cash flows will not become negative at any point throughout the modeling period. Obviously, this is a more stringent test than requiring non-negative surplus only at the end of the modeling period. This required initial surplus amount can then be divided by beginning assets or liabilities to get a surplus factor.

A risk-exposure distribution can be created by running an sufficient number of trials. Target-surplus factors can then be chosen from the distribution at whatever percentile provides the desired protection level.

The Basic C-1 Bond Model

An example of this generalized approach can be found in the model used for setting C-1 bond factors. Key assumptions for each bond rating class include the expected default rate, the percentage of principal loss upon default, and the effect of economic conditions on the default rate and principal loss.

Here is a simple explanation of how the basic model works. Each scenario for a portfolio begins by randomly generating a series of annual economic conditions over the length of the modeling period. Using Monte Carlo techniques, a given bond is tested for default each year where the default probability varies with that year's economic environment. If a default takes place, the principal loss amount is determined and the bond's salvage value reinvested in a like quality-asset. The process is repeated for every bond in the portfolio. The portfolio's annual net cash flows are then used to determine the surplus needs for that trial.

Another important assumption of the model is that dollars are being regularly set aside to fund future default losses. This funding could be considered a credit "risk premium", similar to the expected annual default loss. The derived surplus factors therefore will reflect the loss deviations over and above expected losses. This is described in more detail below.

A numerical example of this basic model is found in Exhibits 1.1 - 1.4 at the end of this report. Exhibits 1.1 - 1.3 are spreadsheet examples depicting how a surplus factor would be determined for portfolios of 1, 5, and 400 bonds for one economic scenario. In the examples shown, the modeling period is six years. At the top of Exhibit 1.4, multiple scenarios are depicted. The surplus factors from each of the scenarios can be organized to create a risk-exposure distribution, shown graphically also in Exhibit 1.4. In the graph, it can be seen from the upper-tail of this risk-exposure distribution that a surplus factor of about 3.1 percent is greater than the required surplus in 92 percent of the modeled scenarios. Alternatively, protection at the 95th percentile level can be seen to be about 3.6 percent.

Additional Model Sophistication

The model described above is relatively straight-forward. There were additional levels of sophistication that were considered before being satisfied that the model provided a sound measure of risk. These items were evaluated in terms of their potential effect on the results and the model's ability to realistically reflect their influence on risk.

1. Economic Influences - The influence of economic conditions on bond performance was incorporated into the model. Obviously, econometric dynamics and their influence on bond performance could be modeled with many levels of sophistication.

   In the model, annual economic environments are determined randomly for each trial using a transitional probability matrix. On an expected basis, the weakest economic environment (severe contraction) has a 7% probability, and a less severe contraction environment has a 13% chance of occurring. These probabilities vary depending on the prior year's economic condition. In these poor environments, the expected rate of default increases affecting the lower credit quality bonds the most. In addition, the expected principal loss on default is assumed to increase somewhat during poorer economic times. The other environments are expansionary with resulting lower expected default rates.

2. "Select and Ultimate" Default Rates - The default rate assumptions used in the model were based primarily on published corporate bond default studies from Moody's. The interpretation of these studies was enhanced after obtaining further data provided by Moody's to the C-1 subcommittee. The Moody's study was more useful than other studies for use in the model for a couple of reasons. First, and most important, the Moody's data permitted analysis of default experience based on a portfolio's current rating classification (rather than rating at issue). This is consistent with the information that will be available in the NAIC blank - the current rating composition of a portfolio is known, not it's rating composition at issue. The current rating of a bond is a better indicator of its future performance than its rating at issue. Second, the Moody's default rates were based on the number of debt issuers that default rather than on par amount of defaulted debt. Issuer default rates are consistent with the model's bond by bond testing for defaults.

   The Moody's data also permits default analysis of a cohort of bonds subsequent to the year in which the bonds in the cohort all had the same rating. "Select and ultimate" default rates were developed and used. This is one way of incorporating the potential for future rating quality changes into the model.

   A summary of the important assumptions related to each NAIC rating category follows. Assumptions include: the expected default rates by year from the beginning of the modeling period (the point at which a bond's actual rating was last known); the factor applied to the expected default rates when the economic conditions are least favorable; the range of principal losses on default (they vary around an expected level); and the bond size amount distribution.

ASSUMPTIONS BY BOND CATEGORY

	NAIC Bond Category						
	1			2	3	4	5
	AAA/Aaa (25%)#	AA/Aa (25%)#	A/A (50%)#	BBB/Baa	BB/Ba	B/B	CCC/Caa
				DEFAULT RATES			
YR. 1*	0.00%	0.02%	0.04%	0.25%	1.80%	7.50%	18.00%
2	0.00	0.04	0.12	0.40	3.20	6.50	13.00
3	0.02	0.12	0.20	0.50	3.00	5.50	10.00
4	0.04	0.16	0.24	0.55	2.80	4.50	8.00
5	0.06	0.20	0.28	0.60	2.50	4.00	[illegible].00
6-10	0.07	0.22	0.30	0.65	2.00	3.50	5.00

* Number of years since rating of bond was last known
\# Assumed distribution within Category 1.

ASSUMPTIONS BY BOND CATEGORY
(continued)

	NAIC Bond Category 1			2	3	4	5
	AAA/Aaa	AA/Aa	A/A	BBB/Baa	BB/Ba	B/B	CCC/Caa

% PRINCIPAL LOSS ON DEFAULT

	AAA/Aaa	AA/Aa	A/A	BBB/Baa	BB/Ba	B/B	CCC/Caa
Maximum	49%	58%	77%	85%	88%	94%	94%
Expected	30%	35%	45%	50%	55%	60%	60%
Minimum	15%	18%	22%	25%	30%	34%	34%

DEFAULT ADJUSTMENT FOR WORST ECONOMIC ENVIRONMENT
(Expected default rates times this factor)

	AAA/Aaa	AA/Aa	A/A	BBB/Baa	BB/Ba	B/B	CCC/Caa
Adjust.	1.1	1.2	1.3	1.4	1.75	2.0	2.25

DISTRIBUTION BY SIZE OF BOND

Amount (millions)	1	2	3	4	5
$50.00	1%				
35.00	3				
27.50	5				
22.50	4				
17.50	6	2%			
12.50	12	4	4%		
8.00	20	7	2	3%	3%
5.00	16	9	10	12	12
3.00	19	21	21	56	56
1.50	6	17	21	22	22
0.80	2	13	7	3	3
0.45	2	10	5	2	2
0.15	4	17	30	2	2
	----	----	----	----	----
	100%	100%	100%	100%	100%

3. Risk Premium Funding - The modeling incorporates an offsetting cash flow into the analysis. A credit "risk premium" is regularly set aside each year and amounts in excess of actual default losses accumulate in a reserve to fund future losses. The risk premium is based on the bond's current rating. The surplus factors therefore reflect the loss deviations over and above expected with the provision for expected losses covered by the risk premium. This fund acts like a buffer or first defense to calls on capital and surplus for default losses. Obviously, the greater the contributions that are made to the fund, the lower the needs are for surplus in excess of the fund. Integrating the risk premium offset into the surplus analysis permits studies of how surplus factors are affected by reserve funding contribution levels.

Early in our studies, we were setting this "risk premium" equal to the annual expected losses of the portfolio, although there was no statutory basis for this assumption. There is, however, a statutory basis for interpreting the annual contributions to the proposed Asset Valuation Reserve (AVR) as the "risk premium". We have attempted to coordinate with the NAIC AVR Task Force and build into our modeling the contributions that would develop from the AVR as we anticipate it will operate in its final (1993) form. The risk based capital (RBC) bond factors we are proposing are thus a function of the AVR contributions. To the extent that the final AVR proposal differs significantly from what we assumed it to be, the RBC factors technically would change as well. The RBC factors take into account only the future contributions to the AVR; the AVR's actual balance developed from past contributions is used elsewhere in the RBC formula as an additive adjustment to actual capital and surplus.

It should be noted that the AVR contributions are actually "cash flows" being generated from the spreads over Treasuries returned to buyers of assets that are not risk-free. It is assumed that spreads in excess of those needed for AVR contributions are required to fund the cost of liabilities, as well as related investment expenses and taxes (i.e., no profit pricing margins, if they exist, are reflected).

4. Tax Treatment - There is no right answer as to how to treat taxes for RBC needs analysis. Actual tax treatment will vary from company to company and by specific circumstances within a company. The decision on how to best reflect tax treatment in an RBC formula can vary as well by the purpose of the formula and its desired level of conservatism. The use of tax credits assumes that other assets or product lines are generating gains that at least offset the losses of the modeled risk.

   Several factors will influence whether or not tax credits are likely to be available when losses develop. These include the correlation and diversity of risk of other asset and product portfolios relative to the modeled risk, and whether the loss event will actually have been realized for tax purposes. Another consideration is that realized capital losses (C-1 risks) cannot be offset with operating gains, but only realized capital gains. The possibility of future changes to tax rates and/or tax bases further complicates tax treatment in the analysis.

   After considerable debate and discussion within the committee, for the bond modeling we assumed that 50% of the possible tax credit would be taken as an offset to the default loss. This recognizes that in many cases, the company is not likely to have offsetting capital gains permitting the tax credit. Also, to recognize that the realization of the loss for tax purposes (the sale of the defaulted bond) may often not occur immediately, the model assumed that the sale took place one year after default. Both the 50% assumption (relative to a full tax credit) and the deferral of sale result in higher RBC factors.

5. Loss Recoverability from Contractholders - The nature of the liability supported by the assets may permit some recovery of default losses through the application of a dividend formula change or through the reduction of crediting rates for accumulation-type products. This recoverability was considered and modeled in some of our analysis, but the recommended factors make <u>no</u> provision for these recoveries. Provision for this flexibility in the RBC formula could be considered (for par products) to be part of the dividend liability adjustment.

6. Public vs. Private Bonds - Differences in default and loss experience between public and private bonds, if known, could be recognized in the model. This was considered, but rejected for lack of reliable experience data. The Society of Actuaries is currently studying the credit risk of private placements and mortgages. This study may yield data that would suggest public and private bond RBC factor differentiation.

7. Other Concentrations of Risk - Other concentrations of risk such as by industry or underwriter may have some influence on the overall level of risk. We did not feel we could make reliable assumptions related to these concentrations, and the NAIC blank is not structured to permit such analysis. We viewed this as a second order risk parameter.

Bond Portfolio Parameters

As important as the model processes and related assumptions are to the measured risk of a bond portfolio, the true risk is derived from the makeup of the portfolio itself. Key parameters that will differentiate one portfolio from another are: the number of issuers, the distribution by bond amount, the rating class mix, and the bond maturity mix. Ideally, each company's portfolio could be used in the model and thus each of the key parameters would be captured. Realistically, that is not possible. However, to the extent possible, we attempted to construct a RBC bond component that recognized such differences in portfolios.

1. Number of Issuers - The measured risk will decrease as the default risk is spread over a larger number of issuers. For modeling purposes, portfolios of 400 bonds (issuers) were constructed for each rating category. Portfolios with 400 bonds in its largest category would likely have between 800 and 1,200 bonds in the total portfolio. A later section describes how we attempted to vary RBC requirements based on number of issuers.

2. Distribution by Bond Amount - A greater variance in the bond amounts within a portfolio (that is, where all bonds are not the same size) increases the risk. The portfolios we constructed had some variance by size although not extreme (some actual distributions were reviewed). The variance by size decreased as the credit risk increased. The relative size of a bond to others in the portfolio is the key, not its actual size.

   The increase in risk from having a concentration of a few very large issues could be measured if individual portfolios were modeled. As before, this in not possible. Instead, the RBC formula considers the increased risk resulting from size concentration by looking at the ten largest debt issuers (not necessarily bonds) and doubling the assigned RBC factor for those particular issuers.

3. Rating Class Mix - The overall risk of the bond portfolio varies as its mix by rating class changes. The NAIC blank provides six categories that captures much of the differences in a portfolio's credit quality. The RBC formula has factors applied to each category so that as the mix changes, the portfolios overall RBC requirements will change.

   At the end of 1990, nearly 60% of the industry's bonds assets were categorized as Category 1 bonds. This category is made up of AAA/Aaa, AA/Aa, and A/A rating classes. Our analysis shows significant relative differences in measured risk for these categories. Since this category constitutes such a large percentage of the bond portfolio, further breakdowns by rating class would result in better portfolio risk classification. For modeling purposes, the Category 1 mix by rating class was assumed to be AAA/Aaa = 25%, AA/Aa = 25%, and A/A = 50%. To the extent that a company's actual Category 1 bond mix is significantly different than this, the RBC factor should have been higher or lower.

4. Modeling Period - The modeling period, the period the bond is exposed to default, of ten years (a "duration" of 6 to 7 years) was based on the industry's average length of time to maturity (from Schedule D - Part 1A). If the portfolio is relatively well matched with the liabilities it is supporting (as was assumed), the default risk associated with the assets will expire concurrently with the expiration of the liability obligations (i.e., both assets and liabilities are zero).

   Another approach for RBC would be to use the maturity breakdowns in Schedule D - Part 1A to differentiate between portfolios that are invested longer or shorter. We experimented with developing RBC factors that varied by maturity. Although we felt that risk does vary with exposure period to default, we did not think the differences in risk warranted the additional complexity such a formula would require.

Protection Levels

Protection levels can be chosen from the risk-exposure distribution created by running the model a sufficient number of trials. The RBC factors for bond Categories 1-5 are at the 92nd percentile (that is, the factors are greater than or equal to 92 percent of the factors resulting from running 2,000 trials). As discussed earlier, factors were developed for each category separately using a portfolio of 400 bonds. If the largest category had 400 bonds, the total portfolio would likely have 800 to 1,200 bonds.

Portfolios of 1,200 bonds were constructed of a mix of rating categories with differing degrees of non-investment grade (Category 3 and below) bonds, ranging from 5 percent non-investment grade to 25 percent. The weighted RBC factor for the entire portfolio, derived by applying the 92nd percentile factors by category, was compared to the risk distribution factors developed from running the 1,200 bond portfolio through the model. In all cases, the weighted factor provided protection in excess of 96 percent when compared to the factor developed from actually modeling the portfolio. Therefore, the RBC factors used provide protection in excess of 96 percent on a total portfolio basis. The individual rating category factor level of 92 percent was chosen based on this overall total portfolio protection level. This higher level of protection in the 1,200 bond portfolios (relative to the 400 bond factors) largely results from spreading the risk across a greater number of bonds.

Adjustment Factors Based on Number of Issuers

Portfolios similar in makeup except for number of bonds (issuers) were constructed ranging in size from 50 bonds up to 2,400. The mix by rating category was based on the composite industry mix at the end of 1990. These different size portfolios were each modeled to create a risk distribution. Using the 1,200 bond portfolio as a baseline (since the RBC factors were developed based on this size portfolio), the 96th percentile factor from the other size portfolios were compared. The relative increase or decrease in risk could then be calculated for each of these portfolio sizes. The following adjustment factor table (in the nature of a tax table) was calibrated to approximate the change in risk across the entire portfolio based on number of issues.

Number of Issuers	Factor
First 50	= 2.5
next 50	= 1.3
next 300	= 1.0
above 400	= 0.9

Below are adjustment factors using this table for several portfolio sizes.

Number of Issuers	Adjustment Factor	Number of Issuers	Adjustment Factor
50	2.50	600	1.12
100	1.90	800	1.06
200	1.45	1,200	1.01
300	1.30	1,600	0.98
400	1.23	2,400	0.95

For example, a portfolio with 200 issuers would have a adjustment factor of 1.45 derived as follows:

$$\{(50 * 2.5) + (50 * 1.3) + (100 * 1.0)\} / 200 = 1.45$$

Although not possible to reflect company by company, the adjustment factors would be somewhat different if a company's actual distribution by category mix differed from that assumed.

Category 6 Bonds

Although the statement value of Category 6 bonds has already recognized loss of value upon default by being marked to market, these are still very risky assets subject to additional writedowns. The risk level of Category 6 bonds was viewed as similar to that of common stock which is also valued at market. The risky character of Category 6 bonds as well as its increased statement value fluctuation due to market valuation suggests a RBC factor of 30 percent.

Future Improvements to RBC Factor Development

In a future generation of RBC formula development, better information (for assumptions) and more refined modeling techniques would permit increased differentiation of bonds for RBC purposes. Several of these have been previously mentioned throughout the document. The following list is a summary of some items to consider in future RBC bond factor quantification.

- Further breakdowns of Category 1
- Breakdowns between public and private placement bonds
- Improved assumptions related to time to maturity (or call)
- Improved economic scenarios and bond influences
- Concentrations of risk such as by industry
- Integration of C-1 risk modeling with C-3 (and ultimately C-2)

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EXAMPLE OF THE BASIC C-1 BOND MODELING - #1

EXHIBIT 1.1

1 BOND PORTFOLIO (1 Trial)

Principal = $1,000

Expected Annual Default Rate = 2.0%

Expected Principal Loss upon Default = 50%

Expected Loss per Year = 1.0%

Modeling Period = 6 Years

		1	2	3	4	5	6	
Expected Default Rate		2.0%	1.6%	2.0%	2.8%	2.0%	2.0%	Varies by rating category and economic environment
Default Test		No	Yes	No	No	Yes	No	Did the bond default? - A random number is compared to the default rate
Principal Loss %		--	40%	--	--	50%	--	Actual model varies loss % randomly and by economic environment
Outstanding Principal EOY		1,000	600	600	600	300	300	Assumes bond is sold and proceeds are reinvested in a new bond for the remainder of the modeling period. The new bond itself could default (and in this example it does in year 5). The actual model defers realization of the loss and reinvestment of the proceeds until one year after default.
Cash Flows								
Principal Loss		--	(400)	--	--	(300)	--	In this basic example, taxes are ignored
Reserve Funding		10	8	6	6	4.5	3	In this basic example, "risk premium" is set aside each year to fund defaults set equal to the expected loss (1.0%) times the average outstanding principal.
NET CASH FLOWS		10	(392)	6	6	(296)	3	
PV Net Cash Flows		10	(344)	5	4	(201)	2	Discounted at 9%, assume flows are mid-year
Accumulation of PV Net C/Fs		10	(335)	(330)	(326)	(526)	(524)	
Minimum of Accum PV Net C/Fs = - $526 (in year 5)								Required Initial Surplus Fund such that this amount and the accumulation (with interest) of subsequent cash flows will not become negative at any point throughout the modeling period
Accumulated Surplus	526	584	227	254	283	0	3	Demonstration that if you start with $526 in surplus and accumulate with interest plus subsequent cash flows that fund never becomes negative (becomes $0 in year 5)

EXAMPLE OF THE BASIC C-1 BOND MODELING - #2

5 BOND PORTFOLIO (1 Trial)

Principal = varies by bond

Expected Annual Default Rate = 2.0%

Expected Principal Loss upon Default = 50%

Expected Loss per Year = 1.0%

Modeling Period = 6 Years

		1	2	3	4	5	6	
Expected Default Rate		2.0%	1.6%	2.0%	2.8%	2.0%	2.0%	Same environment for all bonds within the portfolio
NET CASH FLOWS								
Bond #1	$1,000	10	(392)	6	6	(296)	3	Only bond to default during modeling period, net cash flows are the same as shown in Exhibit 1.1
Bond #2	$2,000	20	20	20	20	20	20	
Bond #3	$5,000	50	50	50	50	50	50	For bonds 2-5, net cash flows are equal to the risk premium since none of these bonds defaulted in this trial
Bond #4	$2,000	20	20	20	20	20	20	
Bond #5	$3,000	30	30	30	30	30	30	
	$13,000							
NET CASH FLOWS		130	(272)	126	126	(176)	123	
PV Net Cash Flows		125	(239)	102	93	(119)	77	
Accumulation of PV Net C/Fs		125	(114)	(13)	80	(39)	37	

Minimum of Accum PV Net C/Fs = - $114 (in year 2)

Surplus Factor for this Trial = $114 / $13,000 = 0.88%

Accumulated Surplus	114	261	0	132	275	116	255	Even though there are losses beyond year two, positive cash flows in years 3, 4, and 5 more than offset loss in year 5

EXAMPLE OF THE BASIC C-1 BOND MODELING - #3

EXHIBIT 1.3

400 BOND PORTFOLIO (1 Trial)

Principal = varies by bond

Expected Annual Default Rate = 2.0%

Expected Principal Loss upon Default = 50%

Expected Loss per Year = 1.0%

Modeling Period = 6 Years

		1	2	3	4	5	6	
Expected Default Rate		2.0%	1.6%	2.0%	2.8%	2.0%	2.0%	Same environment for all bonds within the portfolio
NET CASH FLOWS								
Bond #1	$1,000	10	(392)	6	6	(296)	3	
.								
.								
Bond #5	$3,000	30	30	30	30	30	30	
.								
.								
Bond #400	$20,000	200	200	200	(9,000)	110	110	
	$901,850							
Net Cash Flows		(6,693)	(3,678)	578	4,921	(3,794)	3,183	
PV Net Cash Flows		(6,411)	(3,232)	466	3,640	(2,574)	1,981	
Accumulation of PV Net C/Fs		(6,411)	(9,643)	(9,177)	(5,537)	(8,112)	(6,130)	

Minimum of Accum PV Net C/Fs = - $9,643 (*in year 2*)

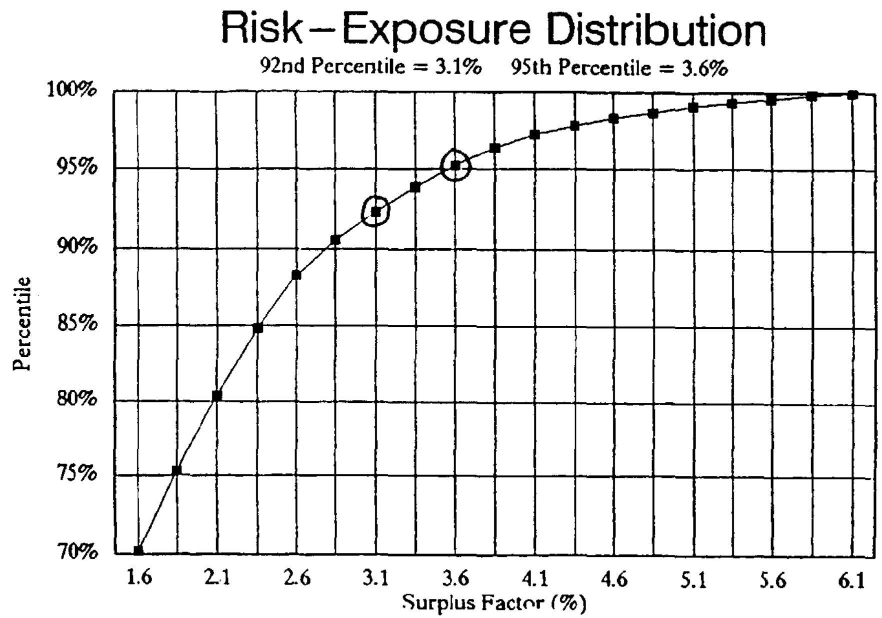
Surplus Factor for this Trial = $9,643 / $901,850 = 1.07%

		1	2	3	4	5	6
Accumulated Surplus	9,643	3,523	0	604	5,796	2,356	5,892

400 BOND PORTFOLIO (500 Trials)

Accumulation of PV Net C/Fs

Trial #1	(6,411)	(9,643)	(9,177)	(5,537)	(8,112)	([illegible],130)	Minimum = - $9,643	Surplus Factor = 1.07%
.								
Trial #27	(7,352)	(3,241)	4,545	4,389	3,097	(3,093)	Minimum = - $7,352	Surplus Factor = 0.86%
.								
Trial #500	7,702	2,229	4,044	2,883	4,430	776	Minimum = + $766	Surplus Factor = 0.00%



The amount of surplus required for defaults over and above the 1.0% annual contributions to a "default reserve"; shown at the upper-tail of the risk-exposure distribution.