ACTUARIAL RESEARCH CLEARING HOUSE
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MAXIMUM AGES

by Paul Thomson

ABSTRACT

The mortality rates for white males in four of the last five U.S. Decennial Life Tables point to a maximum age of 116.63, when successively fitted with mathematical curves, the last of which contains a maximum age parameter. For white females, the results are less concentrated. Their maxima are also lower than for males, since their usually lower mortality rates cross over male rates at about age 100, exceeding them thereafter.

I. PROCESS AND RESULTS

The initial process was to fit a compound Gompertz formula to the upper ages of each Decennial Life Table and then to fit a Wittstein formula (1) to the resulting mortality rates. The compound Gompertz formula is taken from the paper by Tennenbein and Vanderhoof (2). Extra parameters therein permit recognition of the slight slackening of the increase in mortality rates after about age 85. The mathematics for these steps are given in the Appendix, and the published mortality rates used (3) are brought together here in Table 1. The resulting maximum ages are shown in Table 2. It is immediately apparent that the 1959-61 Tables are out of line while the maximum ages for the other four tables are surprisingly close, despite the large improvements in mortality over the 40 year period.

Table 1.
U.S. Decennial Life Table Mortality Rates
White Males *

Age	1939-41	1949-51	1959-61	1969-71	1979-81
70	.05454	.05027	.04871	.04916	.04148
75	.08313	.07499	.07066	.07231	.06146
80	.12471	.10993	.10732	.10466	.09099
85	.18104	.16304	.16039	.15033	.13507
90	.24894	.22890	.23601	.21344	.19058
92	.27760	.25766	.26973	.24152	.21864
			White Females *		
70	.04233	.03409	.02836	.02513	.02092
75	.06889	.05650	.04742	.04255	.03315
80	.10819	.09060	.08213	.07128	.05589
85	.16294	.13965	.13625	.11465	.09463
90	.23141	.20657	.22560	.17570	.14831
92	.26136	.23851	.26481	.20617	.17709

* The process was also applied to rates for non-whites but the results were so scattered that they were considered unreliable for this purpose.

Table 2.
Preliminary Maximum Ages

Table	White Males	White Females
1939-41	117.86	116.56
1949-51	115.54	114.69
1959-61	110.76	106.54
1969-71	115.61	114.08
1979-81	117.86	112.65

In the case of 1959-61 Tables, the likely reason for the disparity is that death rates of Civil War veterans were substituted at ages 95 and up (same rates for males and females, whites and non-whites). Grading lower Table ages to these rates altered the normal patterns. This result shows the sensitivity of the process described here. Consequently, the 1959-61 Tables are excluded from what follows.

Another point of interest is that maximum ages for females are lower than for males. This seems counterintuitive since female mortality is generally lower than for males. However, female rates increase at a faster pace, eventually overtaking male rates around age 100, according to this analysis.

Even though the Decennial Tables are derived from large population bases, some secular perturbations creep in. In order to minimize them, regression lines were found for each age used in the 8 remaining tables, allowing for the gaps of the excluded tables. The mortality rates on these lines were then used to reapply the process giving the maximum ages shown in Table 3.

Table 3.
Maximum Ages

Table	White Males	White Females	Cross-over Ages
1939-41	116.73	115.92	100.0
1949-51	116.86	115.53	101.2
1969-71	116.75	113.96	101.3
1979-81	116.45	112.68	100.8
Ave.	116.63	114.52	
Std.Dev.	.14	1.49	

The above maximum ages for males are now remarkably focused, sufficiently so, I believe, to lend some validity to the conceptual existence of a maximum age. I hasten to point out that this statistical result rests entirely on the assumption that male mortality at extreme ages follows a Wittstein formula, chosen here merely because it has a maximum age parameter and fits the entry data well.

While the male maxima are concentrated, those for females show a downward trend. This can be attributed to a rising trend in the Gompertz increase factor "c" for females in contrast to a fairly stable pattern for males as shown in Table 4.

Table 4.

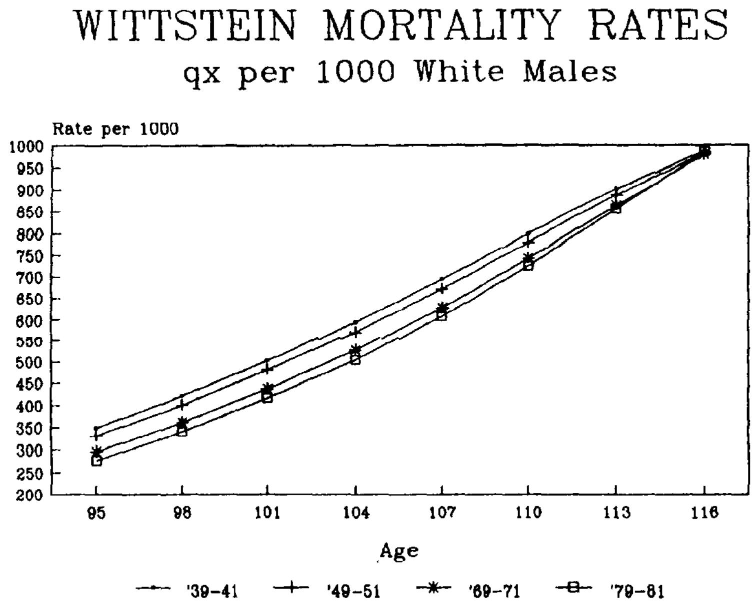
Table	Gompertz Factor "c" at Age 85 White Males	White Females
1939-41	1.0874	1.1000
1949-51	1.0864	1.1017
1969-71	1.0841	1.1073
1979-81	1.0828	1.1123

The graphs on the next page depict the derived Wittstein rates as they approach unity at maximum ages. (The banner-type titles are due to an exuberance factor embedded in my software). The graphic failure to converge, of the curves for the two latest female tables, is due entirely to trends implicit in the patterns of the published rates.

A question arises as to whether the result for males is merely an artifact generated by the process. If this were the case, however, all the female ages should also be concentrated, being derived by the same process.

II. GENERAL DISCUSSION

Whether the assumption that mortality at very old ages follows a Wittstein formula would be disproved in future decades is problematical since survivors to such advanced ages will be so scarce that statistical significance will lose out to the uncertainty principle. This principle holds, among other things, that where a measurement is impossible, or unreliable, one must fall back on a probable, or statistically determined, value. The question immediately arises of whether a biologically determined maximum age actually exists for the human species, as implied here for


WITTSTEIN MORTALITY RATES
qx per 1000 White Males
Rate per 1000
1000
950
900
850
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750
700
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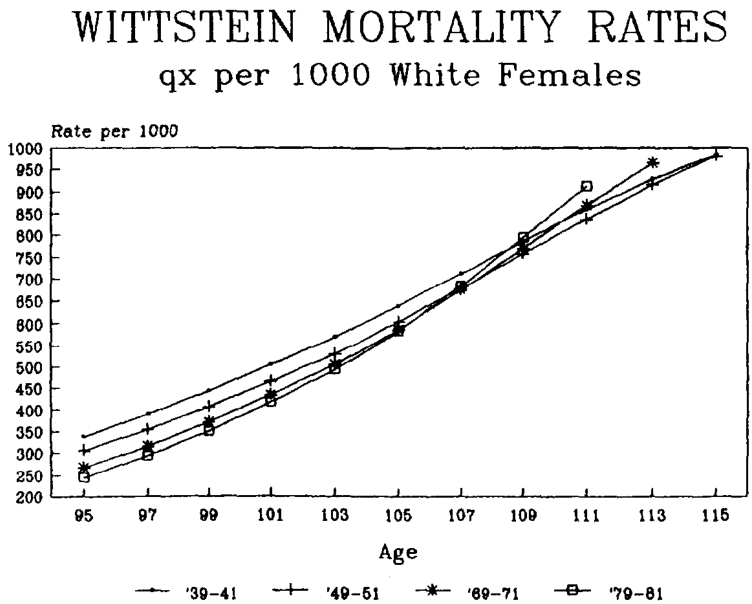
Age
'39-41
'49-51
'69-71
'79-81


WITTSTEIN MORTALITY RATES
qx per 1000 White Females
Rate per 1000
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Age
'39-41
'49-51
'69-71
'79-81

U.S. white males, at least. The answer will have to await progress of studies in other fields it would seem.

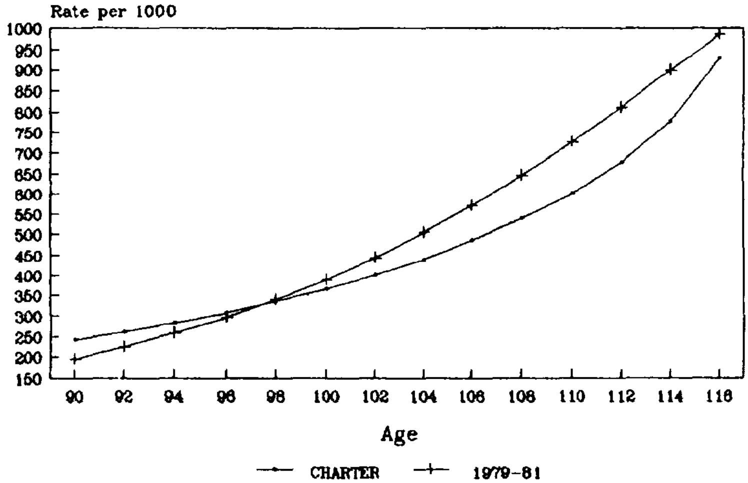
Scientists of other disciplines have studied the variations in life spans according to species, from insects on up, and have found rough correlations between typical body weights and life expectancies, measured in days [3]. Mortality rates of fruit flies, in particular, have shown adherence to the Gompertz formula at advanced ages, as well as slackening of rates at higher ages similar to that found in humans. These studies seem to point to the existence of species-specific maximum ages, actual as well as theoretical.

Studies related to smaller species involved following cohorts of subjects. In contrast, the Decennial Life Tables used here are in effect snapshots of populations composed of many different cohorts passing through particular points in time. The difference in approach is striking; the present study implies waiting ten years for each set of data points. If it were practical to follow large population cohorts, somewhat different results might emerge. But the convergence of mortality curves obtained above for U.S. white males, from populations heterogeneous as to cohort membership, seems to imply a common, inherent life-span limit independent of environments traversed. In this connection, the application of the process to English Life Table No. 8, reflecting English male experience around 1910, gave a maximum age of 116.67. This is apparently a serendipitous corroboration, since other tables weren't as cooperative.

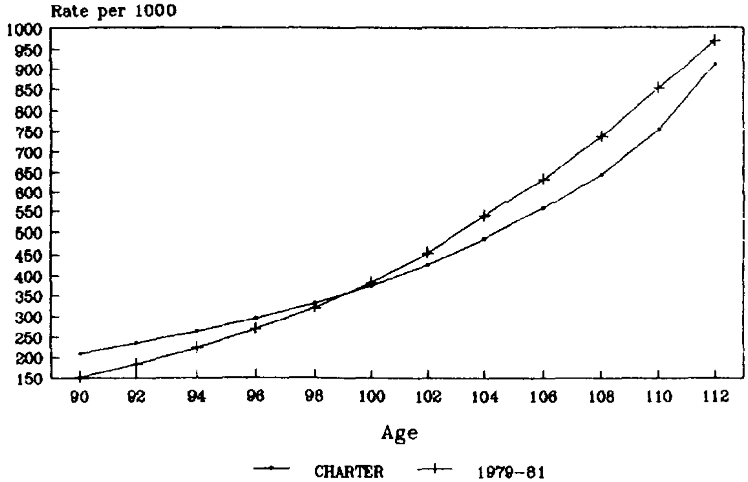
In a paper by Bayo and Faber [5] cohorts of individuals born in 1872-75 are studied as to mortality rates around age 100. Of the three sources of data used, the so-called "Charter" group is considered by the authors to be the most reliable as to ages and results. This group produces the highest graduated rates, the disparity increasing with age up to the highest age shown of 106. These rates are also higher than the corresponding rates of the 1979-81 Decennial Tables for white males and females. As a matter of interest, I fitted Wittstein curves to these Charter rates, assuming the same maximum ages given for 1979-81 rates in Table 3, i.e., 116.45 for males and 112.68 for females. The comparison is shown in the graphs on the next page, illustrating that, though the Charter rates are higher at first, their curves are flatter, cross over the 1979-81 curves, and then have to rise more steeply to reach the maximum ages used. This is thus another possible clue as to the levels of mortality rates at extreme ages, assuming the acceptability of these maximum ages. The authors' conjecture about a "sex crossover" around age 100 seems to be borne out in Table 3.

In conclusion, the convergence of mortality curves for males makes it seem more likely that, among all the complexities programmed into a living organism, there would be an element providing for a finite number of cell reproductions or divisions. Thus for those hardy few individuals who survive all the vicissitudes of life, there would be a final limiting age barrier. A new cause-of-death phrase such as "temporal

Wittstein Rates, White Males
Bayo Charter vs. 1979–81



Wittstein Rates, Wh. Females
Bayo Charter vs. 1979–81



expiry" might be added to our lexicon, though seldom, if ever, used.

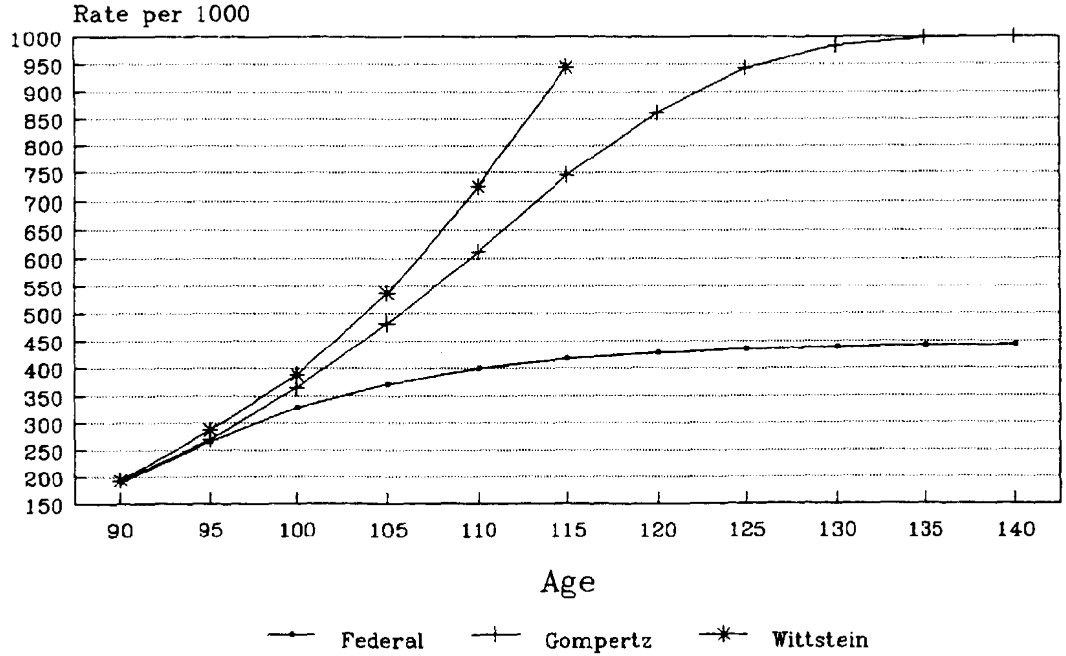
III. PRACTICAL CONSIDERATIONS AND DETAILS

1. The implied existence of maximum ages for humans and the Wittstein approach may be of some practical use for terminal ages of mortality tables. Traditionally, for insurance and annuity experience tables, arbitrary final ages have been set and available rates graded to unity at those points by means of cubic or other curves. Use of a Wittstein formula generates rates easily and would, at least, add some uniformity.

2. The U.S. Decennial Life Tables show life table functions for ages as high as 109, including the much-used complete expectation of life. According to the Methodology for the 1979-81 Tables, the latter is obtained by extending mortality rates up to age 132 on the assumption that the increase factor at each successive age is 90% of that for the preceding age. While this produces reasonable enough expectancies, it has the odd result that mortality rates become asymptotic at hypothetical extreme ages; i.e. the 1979-81 white male rate approaches indefinitely a level of about .45. The Gompertz formula, on the other hand, while directly producing a "force of mortality" exceeding unity at high ages, gives an associated "rate of mortality" that is asymptotic to a value of unity. These are compared with the corresponding Wittstein curve in the graph on the next page.

EXTREME AGE MORTALITY CURVES

3 Treatments of q(x) Extension



Based on 1979-81 White Males

(N.B. "Federal" in the legend refers to the Methodology approach.) The life expectancies produced by these three approaches are not all that different as shown in Table 5.

Table 5.
Complete Expectation of Life
1979-81 White Males Age 90

Published Table Value	3.83
Gompertz Formula only	3.72
Wittstein & Gompertz	3.67

3. Table 6. displays the mortality rates produced at the successive stages of the process described here and gives the chi-square factors for "goodness of fit" tests using the published rates as the "observed". Comparison with tabular values of the chi-square function indicates a very good fit in each case at the 99.5% level, while, at the 0.5% level, it is indicated that the fit is not too good to be true.

4. Tests were made of omitting the Gompertz step and going directly to the Wittstein formula. The results were not as satisfying and the female ages, especially, were more scattered.

5. Tests were also made of using other age vectors, but those specified here gave the best results; i.e. 70,75,80,85,90, and 92 for the entry ages and 65,70,75,80,85, and 90 for the transition to a Wittstein formula. The reason for stopping at age 92 is the warning given in the Decennial Life Tables that rates above that age are not reliable.

Table 6 White Males
Derived Mortality Rates and Chi-Square Factors

Age x	Tabular q(x)	Secular Trend q'(x)	Secular Trend C(sq)	Gompertz q''(x)	Gompertz C(sq)	Wittstein q'''(x)	Wittstein C(sq)
	1939-41						
70	0.05454	0.05431	0.00000	0.05457	0.00000	0.05500	0.00000
75	0.08313	0.08218	0.00001	0.08178	0.00002	0.08226	0.00001
80	0.12471	0.12212	0.00005	0.12167	0.00008	0.12117	0.00010
85	0.18104	0.17830	0.00004	0.17902	0.00002	0.17556	0.00017
90	0.24894	0.24690	0.00002	0.24688	0.00002	0.24976	0.00000
92	0.27760	0.27566	0.00001	0.27562	0.00001	0.28596	0.00024
Totals			0.00014		0.00015		0.00053
	1949-51						
70	0.05027	0.05159	0.00003	0.05176	0.00004	0.05206	0.00006
75	0.07499	0.07758	0.00009	0.07729	0.00007	0.07761	0.00009
80	0.10993	0.11485	0.00021	0.11462	0.00019	0.11415	0.00016
85	0.16304	0.16783	0.00014	0.16826	0.00016	0.16544	0.00003
90	0.22890	0.23368	0.00010	0.23366	0.00010	0.23588	0.00021
92	0.25766	0.26225	0.00008	0.26222	0.00008	0.27047	0.00061
Totals			0.00065		0.00064		0.00115
	1969-71						
70	0.04916	0.04615	0.00020	0.04615	0.00020	0.04632	0.00017
75	0.07231	0.06838	0.00023	0.06831	0.00023	0.06849	0.00021
80	0.10466	0.10031	0.00019	0.10054	0.00017	0.10033	0.00019
85	0.15033	0.14689	0.00008	0.14673	0.00009	0.14543	0.00017
90	0.21344	0.20724	0.00019	0.20726	0.00018	0.20834	0.00012
92	0.24152	0.23543	0.00016	0.23537	0.00016	0.23968	0.00001
Totals			0.00103		0.00103		0.00088
	1979-81						
70	0.04148	0.04343	0.00009	0.04334	0.00008	0.04342	0.00009
75	0.06146	0.06378	0.00008	0.06382	0.00009	0.06390	0.00009
80	0.09099	0.09304	0.00005	0.09351	0.00007	0.09338	0.00006
85	0.13507	0.13642	0.00001	0.13595	0.00001	0.13538	0.00000
90	0.19058	0.19402	0.00006	0.19407	0.00006	0.19450	0.00008
92	0.21864	0.22202	0.00005	0.22194	0.00005	0.22420	0.00014
Totals			0.00034		0.00035		0.00046

Chi-Square per 1000 Basis

	Secular Trend	Gompertz	Wittstein
1939-41:	0.14	0.15	0.53
1949-51:	0.65	0.64	1.15
1969-71:	1.03	1.03	0.88
1979-81:	0.34	0.35	0.46
Degrees of freedom	3	1	2
Table(99.5%)	12.80	7.88	10.60
" (00.5%)	0.07	0.00	0.01

Table 6 (cont'd) White Females
Derived Mortality Rates and Chi-Square Factors

Age x	Tabular q(x)	Secular Trend q'(x)	Secular Trend C(sq)	Gompertz q''(x)	Gompertz C(sq)	Wittstein q'''(x)	Wittstein C(sq)
	1939-41						
70	0.04233	0.04097	0.00005	0.04168	0.00001	0.04215	0.00000
75	0.06889	0.06736	0.00003	0.06626	0.00010	0.06680	0.00007
80	0.10819	0.10628	0.00003	0.10452	0.00013	0.10377	0.00019
85	0.16294	0.16029	0.00004	0.16288	0.00000	0.15774	0.00017
90	0.23141	0.22991	0.00001	0.22992	0.00001	0.23408	0.00003
92	0.26136	0.26096	0.00000	0.26090	0.00000	0.27212	0.00043
Totals			0.00017		0.00025		0.00088
	1949-51						
70	0.03409	0.03579	0.00008	0.03624	0.00013	0.03662	0.00017
75	0.05650	0.05882	0.00009	0.05815	0.00005	0.05859	0.00007
80	0.09060	0.09389	0.00012	0.09263	0.00004	0.09206	0.00002
85	0.13965	0.14413	0.00014	0.14593	0.00027	0.14181	0.00003
90	0.20657	0.21020	0.00006	0.21023	0.00006	0.21370	0.00024
92	0.23851	0.24087	0.00002	0.24079	0.00002	0.25010	0.00054
Totals			0.00051		0.00057		0.00108
	1969-71						
70	0.02513	0.02543	0.00000	0.02539	0.00000	0.02559	0.00001
75	0.04255	0.04174	0.00002	0.04192	0.00001	0.04216	0.00000
80	0.07128	0.06911	0.00007	0.06883	0.00009	0.06854	0.00011
85	0.11465	0.11181	0.00007	0.11196	0.00006	0.10975	0.00022
90	0.17570	0.17078	0.00014	0.17080	0.00014	0.17278	0.00005
92	0.20617	0.20069	0.00015	0.20062	0.00015	0.20606	0.00000
Totals			0.00045		0.00046		0.00039
	1979-81						
70	0.02092	0.02025	0.00002	0.01999	0.00004	0.02010	0.00003
75	0.03315	0.03320	0.00000	0.03380	0.00001	0.03394	0.00002
80	0.05589	0.05672	0.00001	0.05688	0.00002	0.05670	0.00001
85	0.09463	0.09565	0.00001	0.09492	0.00000	0.09358	0.00001
90	0.14831	0.15107	0.00005	0.15111	0.00005	0.15232	0.00011
92	0.17709	0.18060	0.00007	0.18052	0.00007	0.18426	0.00028
Totals			0.00016		0.00019		0.00046

Chi-Square per 1000 Basis

	Secular Trend	Gompertz	Wittstein
1939-41:	0.17	0.25	0.88
1949-51:	0.51	0.57	1.08
1969-71:	0.45	0.46	0.39
1979-81:	0.16	0.19	0.46
Degrees of freedom	3	1	2
Table(99.5%)	12.80	7.80	10.60
" (00.5%)	0.07	0.00	0.01

IV. ACKNOWLEDGEMENT

I hereby express my thanks to my daughter, Dr. Elinor T. Adman, Professor of Biological Structures at The University of Washington, for providing me with material regarding studies, in other fields, of mortality of various species. I am also grateful to her, and to my son, John C. Thomson Esq., for reading my early drafts of this paper, resulting in many helpful suggestions.

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The compound Gompertz formula used is:

$$\mu_x = B_b C_b^x \left[B_c C_c^x \right]^{s^t} \ldots\ldots\ldots\ldots \left\{ \begin{smallmatrix} s=9 \\ t=(x-85)\geq 0 \end{smallmatrix} \right\}$$

where $\mu_x =$ the force of mortality at age x

and $\mu_{x+.5} \cong -\ln(1-q_x)$

where $q_x =$ the rate of mortality

$$Y_x = \ln \mu_{x+.5} = \ln(-\ln(1-q_x))$$

$$= \ln B_b + (x+.5)\ln C_b + s^{(t+.5)} \ln B_c + s^{(t+.5)}(x+.5)\ln C_c$$

$$= \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3$$

where $\alpha_n =$ parameters to be found

$x_n =$ vectors of age $(x+.5), s^{t+.5}$, and their products,

and $Y_n =$ the vector derived from mortality rates.

Correlation coefficients $r_{12}, r_{13}, r_{23}, r_{14}, r_{24}, r_{34}$ are found and the factor C calculated, where $C^{-1} = 1 + 2r_{12}r_{13}r_{23} - r_{12}^2 - r_{13}^2 - r_{23}^2$, and applied to the matrix:

$$\begin{vmatrix} 1-r_{23}^2 & r_{13}r_{23}-r_{12} & r_{12}r_{23}-r_{13} \\ & 1-r_{13}^2 & r_{12}r_{13}-r_{23} \\ & & 1-r_{12} \end{vmatrix}$$

giving,

$$\begin{vmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{vmatrix} \text{ where } T_{21} = T_{12}, T_{31} = T_{13}, \text{ and } T_{32} = T_{23}$$

Multiplication of T_{ij} by r_{i4} gives $\hat{\alpha}_i$, and $\alpha_i = \dfrac{\sigma_y}{\sigma_{xi}} \hat{\alpha}_i$

The Wittstein formula is shown below. Values of q are taken from a mortality table. M represents the maximum age, to be determined, i. e. the age at which q equals one. a and n are parameters, also to be determined.

$$q_x = a^{-(M-x)^n} \quad \text{or} \quad \frac{1}{q_x} = a^{(M-x)^n}$$

$$\ln\ln\left(\frac{1}{q_x}\right) = n\ln(M-x) + \ln\ln a = f(q_x) \text{ , say}$$

$$\sum_1 = f(q_{65}) + f(q_{70}) = n[\ln(M-65) + \ln(M-70)] + 2\ln\ln a$$

$$\sum_2 = f(q_{75}) + f(q_{80}) = n[\ln(M-75) + \ln(M-80)] + 2\ln\ln a$$

$$\sum_3 = f(q_{85}) + f(q_{90}) = n[\ln(M-85) + \ln(M-90)] + 2\ln\ln a$$

$$\sum_1 - \sum_2 = \Delta_1 = n\left[\ln\left(\frac{M-65}{M-75}\right)\left(\frac{M-70}{M-80}\right)\right]$$

$$\sum_2 - \sum_3 = \Delta_2 = n\left[\ln\left(\frac{M-75}{M-85}\right)\left(\frac{M-80}{M-90}\right)\right]$$

$$k = \frac{\Delta_1}{\Delta_2} = \frac{\ln\left\{(M-65)(M-70)\Big/(M-75)(M-80)\right\}}{\ln\left\{(M-75)(M-80)\Big/(M-85)(M-90)\right\}}$$

Having found k from values of q at the ages indicated in the above derivation, a value of M can then be found which satisfies the equation by trial and error. The values of a and n can then be found by linear regression.

Appendix (continued)

Formula Parameter Values Obtained from Process

Decennial Table

	1939-41	1949-51	1969-71	1979-81
Gompertz		White Males		
B_b	0.0011029	0.0007947	0.0003928	0.0002631
B_c	0.1381796	0.1940777	0.4050270	0.6175639
C_b	1.0623974	1.0656500	1.0727236	1.0768325
C_c	1.0235547	1.0194740	1.0106156	1.0055444
B_a	0.0001524	0.0001542	0.0001591	0.0001625
C_a	1.0874218	1.0864025	1.0841112	1.0828029
$(B_a=B_b*B_c\ ;\ C_a=C_b*C_c)$				
Wittstein				
M	116.73	116.86	116.75	116.45
a	1.018281	1.021175	1.030429	1.038007
n	1.320341	1.286354	1.204157	1.154616
Gompertz		White Females		
B_b	0.0002688	0.0001659	0.0000475	0.0000221
B_c	0.1913634	0.2414142	0.4084806	0.5024919
C_b	1.0775890	1.0822893	1.0949608	1.1029269
C_c	1.0207852	1.0179027	1.0113140	1.0085479
B_a	0.0000514	0.0000401	0.0000194	0.0000111
C_a	1.0999869	1.1016653	1.1073493	1.1123545
Wittstein				
M	115.92	115.53	113.96	112.68
a	1.017315	1.021837	1.037959	1.052386
n	1.363282	1.317552	1.212948	1.155514

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