## ACTUARIAL ISSUES IN <br> PREPAID TUITION CONTRACTS

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## ACTUARIAL ISSUES IN PREPAID TUITION CONTRACTS


#### Abstract

In 1987 Wyoming became the first state to sell contracts which promised to provide tuition in the future. Although such contracts had previously been sold by individual postsecondary schools, there had not been any widespread success in marketing these contracts. Although less than 1,000 contracts have been sold by Wyoming, several other states have begun selling such contracts and the amount of revenues collected from these contracts and the future benefits promised by these contracts have become substantial.

Because these contracts provide a benefit whose dollar value is uncertain at the time of sale, and because the contract prices are fixed at the time of sale, these contracts contain significant risks to the states and create a difficult challenge to the actuaries who opine on the adequacy of the trust funds to provide the benefits promised by these contracts.

This article discusses some of the implications and possible results of the method currently used by the major states to project tuition inflation - use of a level inflation rate in all future years. In addition, other possible methods of projecting tuition inflation are described - use of autoregressive techniques, use of economic scenarios and use of an econometric model.

Addressed briefly are issues relating to adverse selection when tuition rates vary within a state and issues relating to investment strategy.


## ACTUARIAL CAVEATS

There are several warnings the reader should be aware of in reading this article. The first two warnings relate to the data itself and the second two warnings relate to general considerations of quantitative methodology.

The college tuition data used in this article is the enrollment weighted US average of tuition and fees at 4 -year universities ${ }^{1}$ for in-state students. ${ }^{2}$ These amounts and rates of inflation are shown in Table 1. The inflation rates are also shown in Graph 1. The use of national averages may cause smoothing of state-to-state data and may obscure certain information.

The data is also limited - 25 years - for use in statistical analysis. For instance, in

## G R A P H 1



## TABLE 1

## US AVERAGE TUITION INFLATION 4 YEAR PUBLIC UNIVERSITIES


time series analysis a rule of thumb for the minimum data is 50 data points. I am using only half that many pieces of information. Therefore, any conclusions based on statistical methods will have only limited power of discrimination.

The third warning is that quantitative methods, such as are used in this article, should be used for the purpose of providing increased understanding of the process being studied rather than a substitute for critical thinking. Furthermore, any quantitative model should be constructed in order to allow the actuary to consider the extent of possible outcomes rather than to confine the outcomes being considered to those which seem possible based on one's own experience.

The fourth warning, and perhaps the most critical, is that the projection methods used in this article assume that no changes occur in the process of setting tuition - that is, the education process and the process of legislative appropriations do not change in any material aspect. Furthermore, projections of the sort discussed in this article implicitly assume that the success of the prepaid tuition programs will be accomplished without any corresponding changes or reactions from the other parties involved in postsecondary education - educators, university administrators, students, legislators, parents and grandparents. For instance, the following questions are not answered by, and cannot be answered by, actuarial analysis:

- If the beneficiaries of prepaid tuition contracts become a significant portion of the total student population in any state, will this change the ability of state university regents to raise tuition or present a moral hazard? Will the student market be able to bear more tuition inflation if a third party is paying for education?
- Will money used for purchasing prepaid tuition contracts supplant or supplement other savings? What will be the reaction of competing sources of savings for college education? Will their reaction be such that prepaid tuition plans cannot provide meaningful benefits at a price that will attract consumers?
- Will the use of prepaid tuition reduce the amount of aid in the form of grants and student loans? Will the existence of a prepaid tuition program cause complacency among legislators so that less appropriations are available for higher education?
- How will total enrollment be affected by a prepaid tuition program? Will it be higher than it would be in the absence of such a program, or will total enrollment remain approximately the same? If enrollment increases, how will this affect the capital spending budgets of universities?
- Will university administrators be supportive of the program or not? Will beneficiaries be given preferential treatment in the admissions process? If not, what is the economic impact of substantial numbers of beneficiaries who do not gain admission to college or university?
- Will Washington be supportive of prepaid tuition plans, or will these plans be seen as merely another source of tax revenue?

Seldom, if ever, is it appropriate to assume that a successful program will not cause any reaction by those affected by the program. Anticipating what those reactions are likely to be is a difficult task requiring "a wise and understanding heart" yet there are too few Solomons for the task.

Two quotes are provided for the reader's consideration:
Never confuse hard work with hard thinking.
James Watson, Nobel laureate and codiscoverer of DNA

Every man takes the limits of his own field of vision for the limits of the world. Arthur Schopenhauer

Caveat actuarius.

## DESCRIPTION OF PREPAID TUITION PLANS

In August of 1987 prepaid tuition contracts went on sale in Wyoming. Since that time Michigan (1988), ${ }^{3}$ Florida (1988), Ohio (1989), Alabama (1990) and Alaska (1991) have begun selling prepaid tuition contracts. As of March 1992 over 250,000 contracts have been sold and over $\$ 650$ million of contract revenues have been collected by these six states.

Benefits vary from state to state but may be divided into two main categories. The first category has benefits that pay college tuition or a specified number of
tuition credits as a proxy for tuition. Alabama, Florida, Michigan and Wyoming fall in this category. The second category has benefits that pay only the number of tuition credits purchased. The number of credits available for purchase is flexible - a small number of credits (for example, 1 credit), a large number of credits or anywhere in between. Alaska and Ohio fall in this category. Wyoming's contracts cover tuition, room and board. Florida offers contracts that cover dormitory costs as well as contracts that cover tuition. All of the other states have contracts which cover tuition (or tuition and fees) only.

Benefits are paid when the beneficiary attends a public postsecondary school in the state which issues the contract. If the beneficiary does not attend a public college or university in that state, the prepaid tuition program will provide either a refund of contract payments or an amount equivalent to what would have been paid to the beneficiary in normal circumstances. These refund benefits vary by state and by the specific circumstances of the beneficiary. There has been some discussion, but no decision, among the different state prepaid plans to make benefits "portable" - that is, if the beneficiary attends school in another state with a prepaid plan, the benefits received would be the same as if the student had attended college in the state which issued the contract.

Payment plans vary, but on the whole, can be split along the same lines as the benefits. States with benefits in the first category offer both lump sum and monthly installment payment options and have contract prices that vary by the age of the beneficiary. States with benefits in the second category offer only lump sum purchase prices (but with the number of credits purchased being chosen by the purchaser) and with no variation in price by age of the beneficiary. Prices of the contracts vary from having a discount versus current tuition, being on par with current tuition, to being at a premium versus current tuition. These differences seem to be a reflection of the different investment strategies and public policy justifications of the plans.

Currently no state distinguishes the price of the contract based on which school the beneficiary is anticipated to attend. This is not an issues in Alaska, Florida and Wyoming because there is only one school in Wyoming at which the contracts can be used, and Alaska and Florida have essentially no tuition differences between public postsecondary schools. The other states have tuition differences between universities and use an average tuition concept in setting contract prices.

Alaska also has a fundamental difference in its program compared to the other states - the prepaid tuition is part of the state university system rather than outside of the university system as it is in the other states.

There are other ancillary benefits under the program; however, the magnitude of these benefits is limited when compared to the basic guarantee of tuition.

## OVERVIEW OF THE ECONOMICS OF PUBLIC UNIVERSITY FINANCING

An understanding of how postsecondary schools obtain revenue and how they spend their revenues is helpful in understanding how tuition rates change. Graph $2^{4}$ shows the proportionate revenues by source. The graph of revenues is interesting because it shows that only about one-seventh of revenues is derived from tuition and fees while almost half of revenue is derived from state and local appropriations. The implication of this is that changes in levels of appropriations can leverage tuition increases at a rate much higher than the underlying increase in educational expenses. (A\&H insurance actuaries will recognize the analogy to deductible erosion.) Note that this leveraging can work both ways - an increase in the level of appropriations can result in a much smaller tuition increase than the increase in expenses.

This leveraging effect is important due to the political process behind state budgets. Typically education is not a high priority item during budget discussions. If a state is anticipating an overall revenue shortfall, postsecondary education appropriations are likely to receive short shrift. This appears especially important for those states that are constitutionally required to have a balanced budget. An alternate scenario is that appropriations for higher education can stagnate at a fixed dollar level for several years while costs of supporting and maintaining schools continues to rise.

Graph 3 shows the proportionate share of expenses. Expenses are more diverse than revenue, but they do contain one category that clearly has a plurality of the expenses - Instruction. Compensation for professors and instructors accounts for almost half of expenses. Two categories of day-to-day expenses - Institutional Support ${ }^{5}$ and Operations \& Maintenance - account for another one-fifth of expenses. Nearly two-thirds of expenses are related to wages and benefits. These expense components have increased faster on average than general inflation as measured by the CPI.

Another item affecting the economics of colleges is enrollment. Over the last few years enrollment has been increasing at about $2 \%$ per year. However, this national average hides state and regional variations due to internal migrations.

Continued increases in enrollment may not continue due to the demographic

## G R A P H 2

## Public University Sources of Revenue



Tuition \& Fees ( $14.2 \%$ )

## G R A P H 3

## Public University Expenditures


profile of the U.S. population. Birth rates have declined, and after the baby boomlet passes through college, national enrollment levels may decline. A similar demographic consideration is that sectors of the population which have a higher population growth rate than the general population, have historically had a lower college participation rate than the general population.

Although the effects of population shifts are obscured by the use of national averages, we can hypothesize that any significant population change that affects postsecondary enrollment will have a leveraging effect on tuition inflation. The argument for states with significant decreases in population is that a decline in enrollment represents an absolute decline in revenue which is not matched, at least in the short term, by a decrease in expenses. In order to offset the lost revenue, postsecondary schools may increase the unit revenue (tuition rates) in the belief (hope?) that enrollment is inelastic in the short run.

The arguments for states with significant increases in population are classified into two scenarios - rationing and growth of overhead. Under the rationing scenario, population growth outstrips the states' ability to construct new facilities, so that tuition increases are used to ration a limited amount of space available for students.

Under the overhead growth scenario, a state will have had the foresight to have expanded postsecondary facilities to meet population growth so that rationing is not necessary. However, the increased size of plant and equipment, required maintenance and teaching staff has resulted in expenditure increases that have grown faster than legislative appropriations. In order to meet the budgetary shortfall, tuition is raised disproportionately.

Exacerbating the second growth scenario is a university's desire for recognition. Academic recognition is achieved by having professors who publish research papers. In practice this means hiring professors away from other universities resulting in higher salary inflation and in more expenditure allocated to research.

Other observations that may provide some insight are based on comparison of the tuition information to general economic data. The average compound rates of increase for several economic indicators compared to tuition inflation are shown below.

|  | Average Growth |
| :--- | :---: |
| Rate |  |
|  | $1965-1989$ |
|  | $8.7 \%$ |
| Nominal GNP | 7.9 |
| TUITION INFLATION | 6.1 |
| Service Industry Wages | 5.6 |
| CPI | 3.0 |

General inflation exceeds tuition inflation only in 8 out of the $\mathbf{2 5}$ years:

| 1967 | 1975 |
| :--- | :--- |
| 1968 | 1978 |
| 1973 | 1979 |
| 1974 | 1980 |

There appears to be a weak connection between these inflation crossover years and the beginnings of recession. There is also a weak connection between the tuition inflation/real GNP spread and the economic cycle. Because tuition inflation occurs once a year for the most part, it is difficult to precisely compare it to quarterly GNP information, but if we compare the spread to change in annual GNP growth, we get the following:

Tultion Inflation<br>minus Real GNP Growth

All Years: $\quad$ 4.85\%

Years with real GNP growth less than 2\%: $\quad \mathbf{9 . 4 6 \%}$
All other years: $\quad \mathbf{3 . 3 9 \%}$
The first cut at a model of tuition inflation would then include an item that would capture the effects of state fiscal policy and budgeting processes. Because state revenues depend on the level of economic activity, a macroeconomic indicator, such as changes in the GNP may be helpful in explaining tuition inflation.

Because a majority of state university expenditures appear to be related to wages and general expenses, a third item in the model would be an item for general price level increases, such as changes in the CPI or a wage index.

A fourth item would be a demographic item to capture the pressure of population increases or the shock of population decreases on the education system. This item is especially hard to incorporate into measures based on national averages since population shifts between states cancel out at the national average level. Multivariate regression is used to determine if these qualitative expectations can be confirmed and quantified. Note that the regression models, although they may provide insight into the causes of tuition inflation, do not necessarily provide a viable methodology for projecting long term tuition inflation since we would need to project the independent regression variables by some means similar to a direct projection of tuition inflation.

The first regression model is based only on lagged independent variables. That is, each of the independent variables occurs prior to rather than concurrent with
the dependent variable. Stepwise regression is used so that independent variables which add only insignificant additional explanation are excluded. The regression formula is:

$$
\begin{aligned}
& \text { Tuition inflation }= .527 \times \text { change in the CPI in the prior calendar year } \\
&+.269 \times \text { change in the juvenile population two } \\
& \text { years previously } \\
&-.233 \times \text { change in the GNP two years previously } \\
&+.057
\end{aligned}
$$

The R-squared value for this regression is .359 and the adjusted R -squared value .252. The only independent variable that is significant at the $95 \%$ confidence level is the CPI component.

Although the amount of variation explained by the lagged regression is small, it is interesting to note that the form indicates that tuition inflation is influenced by general inflation that occurs in the preceding year, confirming our expectation regarding the connection between tuition inflation and general inflation.

Our expectation regarding the influence of demographic trends is also confirmed, though not as strongly, by the inclusion of the change in population under age 18. In performing the stepwise regression, this variable was chosen rather than lagged change in university enrollment.

Expectations concerning state government finances are only indirectly confirmed through the inclusion of the negative coefficient on the change in GNP variable. No variable directly including state government surpluses or expenditures was included in the stepwise regression; however, we can hypothesize that the general economy has an inverse effect on tuition inflation as it works itself through state appropriations. The better the economy is performing, the higher state revenues are, and thus, more money is available for appropriations for higher education.

The regression model based on concurrent statistics is shown below (note that the independent variables are based on calendar years while the dependent variable is based on academic year. This is true for the previous regression model as well.)

$$
\begin{aligned}
\text { Tuition inflation }= & .699 \times \text { change in service workers salary } \\
& +.778 \times \text { state budget surplus } \\
& +.473 \times \text { change in state and local expenditures } \\
& -.093
\end{aligned}
$$

The $R$-squared value of this regression is .523 and the adjusted $R$-squared value .443. All of the independent variables are significant at the $95 \%$ confidence level.

This regression model says that current changes in service worker salaries combined with current surpluses in state budgets have a direct impact on tuition increases. The positive coefficient on the state budget surplus seems to indicate that whatever changes currently increase budget surplus also increase tuition inflation. Presumably this means that one method of increasing budget surplus is to cut appropriations for higher education. The same may be true of the positive coefficient associated with expenditures. Note also that there is no concurrent independent demographic variable. Demographic factors have only a lagged effect.

It is also possible to construct a regression model with both lagged and concurrent independent variables. This model is shown below.

$$
\begin{aligned}
\text { Tuition inflation }= & .775 \times \text { change in service workers salary } \\
& +1.087 \times \text { state budget surplus } \\
& +.663 \times \text { change in public university enrollment in the } \\
& \text { prior year } \\
& -.085
\end{aligned}
$$

The R-squared value for this regression is .510 and the adjusted $R$-squared value .428. All of the independent variables are significant at the $95 \%$ confidence level.

These three regression models indicate that the different factors - demographic, inflationary, budgetary and macroeconomic - have a discernable effect on tuition inflation. The explanatory power of all the elements taken together is weak. Furthermore, a certain amount of collinearity, or dependence, exists between the independent variables. This can be seen by visual inspection of the correlation matrices ${ }^{6}$ where some of the non-diagonal elements have values much greater than zero. Traditional econometric techniques to deal with multicollinearity, such as ridge regression, were not used above since the purpose of the regressions was to determine if the presumed factors were significant. The effect of removing the multicollinearity would be to reduce the R -squared values for each of the regressions.

## IS THERE AN INFLATION EQUILIBRIUM?

Tuition inflation is shown graphically in Graph 1. The values for this time series can be considered as a realization of a random variable for tuition inflation. An important question is whether there is a fixed equilibrium rate of tuition inflation about which actual inflation varies. If there is an equilibrium rate, then the further actual values are from the equilibrium, the more likely that inflation will move toward the equilibrium in the succeeding time period. An alternate formulation that is equivalent on a practical level is that there is an equilibrium rate which is itself a random variable, but with a small variance.

The alternative to a fixed equilibrium rate is that there is no equilibrium. If tuition inflation increases, then there is no reason to believe that it will decrease in the future. Likewise, if tuition inflation decreases, there is no reason to believe that it will increase in the future. Although there may by constraints on how high or how low tuition inflation may be, there is no reason to believe that inflation will vary about any level.

All prepaid tuition plans currently make the assumption that tuition inflation does have a fixed equilibrium rate - either explicitly or implicitly - since all plans currently use the simple average method of projecting tuition inflation. Reasons for thinking that there is an equilibrium are discussed below.

As seen above, legislative appropriations make up the largest piece of public university revenue. These appropriations are subject to all the various political pressures that other public expenditures face. As seen above, there is reason to believe that these appropriations are subject to stagnation or to being cut. On the other hand, if public attention becomes focused on the state of higher education, especially if there has been double-digit tuition inflation, there will be more political pressure to provide increased appropriations. When this pressure becomes greater than competing political pressures, appropriations will increase and inflation will decrease.

To some extent, tuition inflation depends on changes in the GNP, changes in the CPI, changes in wage indices and other macroeconomic factors. These factors have shown a certain amount of long-term stability, although inflation has had some dramatic swings in general inflation over the past twenty years. To some extent, federal fiscal and monetary policy is meant to keep inflation and growth at acceptable levels. Although acceptable levels may change over time, such as with inflation, these acceptable levels act to keep the underlying determinants of tuition inflation varying about a level or slowly changing equilibrium.

Another reason for thinking that there may be an equilibrium rate of inflation comes from an examination of the ratio of public universities and colleges to the GNP. Caution needs to be used in reviewing these statistics since the expenditures include capital outlays as well as current expenditures. Ratios for several years are shown below.

| 1961 | $.9 \%$ |
| :--- | :--- |
| 1965 | 1.3 |
| 1969 | 1.7 |
| 1971 | 1.8 |
| 1975 | 1.9 |
| 1979 | 1.7 |


| 1981 | 1.7 |
| :--- | :--- |
| 1985 | 1.7 |
| 1989 | 1.8 |

After increasing in the early-to-mid 1960's, this ratio seems to have leveled off at about $1.8 \%$ of the GNP. There is nothing apparent in public university expenditures to cause one to think that inflation is unconstrained. A similar analysis of the trend in revenues received through appropriations and tuition is shown below to indicate the historical progression of revenue items.

## Proportion of Revenues From: ${ }^{7}$

| Year | Appropriations | Tuition |
| :---: | :---: | :---: |
| 1977 | $51.9 \%$ | $13.1 \%$ |
| 1979 | 50.8 | 12.5 |
| 1980 | 50.0 | 12.9 |
| 1981 | 49.5 | 13.5 |
| 1982 | 49.0 | 14.5 |
| 1983 | 48.4 | 14.9 |
| 1984 | 49.2 | 14.5 |
| 1985 | 48.7 | 14.5 |
| 1986 | 47.0 | 14.7 |

Here the picture indicates that there has been an upward pressure on inflation throughout the late 1970's up through the mid 1980's. If this represents a permanent shift in state financing, then there may be further inflationary pressures on tuition that have not yet worked through the education system. This could lead to a long term increase in the level of tuition inflation.

An empirical reason for believing that there is an equilibrium for tuition inflation comes from examination of Graph 1. No upward or downward trend in inflation over time is apparent. At this point it is instructive to review the autocorrelation coefficients and partial autocorrelation coefficients of the time series shown in Graph 1. These coefficients are shown in Graphs 4 and 5 respectively.

Although it is difficult to draw conclusions with a high degree of confidence form such a short time series, the sample autocorrelations and partial autocorrelations indicate an autoregressive process of low order. As will be shown in a later section, a second order autoregressive model - abbreviated as $A R(2)$ - can be fitted reasonably well to the data. An AR(2) process is one that reverts to an equilibrium process that has such a strong tendency toward equilibrium, that is, tuition inflation may be a higher order autoregressive process, that fact that an autoregressive model can be fitted without any memory shortening process such as differencing or polynomial de-trending indicates that it is stationary in the mean - that is, that it has a level equilibrium.

## G R A P H 4

Sample Autocorrelation Coefficients


Lines are approximate $95 \%$ significance levels

## G R A P H 5

Sample Partial Autocorrelation Coefficients


There is, then, evidence both for and against an inflation equilibrium. A review of historical inflation seems to indicate that there is an equilibrium while a review of the key revenue items indicates that there has been an erosion in the main source of revenue. The conclusion one draws seems to depend on one's view regarding whether there is a permanent shift in state financing away from postsecondary education, or whether the proportion of revenues obtained from appropriations will swing back towards the levels seen in the mid 1970's. Events subsequent to the statistics shown above point toward further deterioration of state appropriations, but this may be due to an extended economic recession rather than to a long term trend.

## PROJECTING TUITION INFLATION - Simple Average Methods

Perhaps the simplest method of projecting future tuition increases is to use the mean inflation rate of the sample for all future years. In this model, tuition N years from now is equal to:

Current Tuition $\times(1.0793) N$
An obvious extension of this method is to use the average plus an additional amount as a provision for adverse deviation. The choice of appropriate conservatism is not clear, and may be a public policy issue as well as an actuarial issue.

This technique was used by Alabama, Florida, Michigan and Ohio in their published valuations of Plan Trust Funds. The rates used were: ${ }^{8}$

Alabama: $\quad 7.1 \%$
Florida: : 7.5\%
Michigan: $7.6 \%$
Ohio : 8.2\%

Due to the cyclical nature of historical tuition data, the use of the average inflation rate (or the average plus loading for conservatism) for predicting future tuition increases will result in the contracts for some years being over-priced and contracts in other years being under-priced. As an example based on the information in Table 1, the anticipated tuition in 1989 calculated using the average $7.93 \%$ annual increase and then current tuition is shown below for several different projection years.

| Projection <br> Year | Anticipated Tuition <br> For 1989/90 | \% Difference <br> From Actual |
| :---: | :---: | :---: |
| 1968 | 1,870 | $-6.8 \%$ |
| 1971 | 2,076 | 3.5 |
| 1975 | 1,868 | -6.9 |
| 1983 | 2,029 | 1.1 |
| 1984 | 2,029 | 1.1 |

This illustration is based on the use of the average determined over the 25 -year historical period. If, instead, a cumulative average or rolling average determined over a shorter baseline had been used, the mis-estimation would have been different. A cumulative average will tend to exaggerate the mis-estimation compared to the $7.93 \%$ until a long baseline has been established. The use of a rolling average will also tend to exaggerate the mis-estimations compared to the 25-year average. The projections of tuition for 1989/90 shown previously are recast using a cumulative average and a 5 -year rolling average.

| Projection Year | Anticipated Tuition for 1989/90 |  | \% Difference From Actual |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Cumulative | 5-year | Cumulative | 5-year |
|  | Average | Average | Average | Average |
| 1968 | 1,296 | 1,296 | -35.4\% | 35.4\% |
| 1971 | 2,267 | 2,059 | 13.0\% | 2.6\% |
| 1975 | 1,705 | 1,466 | -15.0\% | -26.9\% |
| 1983 | 2,036 | 2,346 | 1.5\% | 16.9\% |
| 1984 | 2,035 | 2,287 | 1.4\% | 14.0\% |

The reader should be aware that the projections in this paper of inflation in the 1965-1989 period are based on averages derived from the same period. Projections beyond the historical period are addressed later in the paper.

Although the year to year mis-estimation of tuition exists, the issue of whether the use of this methodology over several years of contract sales will produce prices that are adequate in the aggregate is still unaddressed. The answer depends not only on actual tuition increases, but on the mix of contracts that a Plan sells from year to year. To illustrate what could happen, Table 2 shows the results of several years of sales making the following assumptions:

- There are no expenses or taxes,
- $7.93 \%$ tuition inflation is assumed from then current tuition,
- every year an equal, but arbitrary number of contracts are sold,
- contracts pay one year's tuition,
- contracts are sold only for tuition 5, 6, 7, 8,9 and 10 years into the future,

TABLE 2
MODEL OFFICE ILLUSTRATION OF THE EFFECTS OF TREND PRICING LEVEL SALES

| Academic <br> Year <br> Beginning | Actual <br> Tuition | Aggregate <br> Tuition <br> Anticpated <br> In Pricing | Margin of <br> Pricing to <br> Actual |
| :--- | :---: | :---: | ---: |
| 1969 | 427 | 436 |  |
| 1970 | 478 | 475 | $2.19 \%$ |
| 1971 | 526 | 517 | $-0.65 \%$ |
| 1972 | 566 | 553 | $-1.64 \%$ |
| 1973 | 581 | 588 | $-2.34 \%$ |
| 1974 | 599 | 633 | $1.15 \%$ |
| 1975 | 642 | 685 | $5.64 \%$ |
|  |  |  | $6.64 \%$ |
| 1976 | 689 | 741 |  |
| 1977 | 776 | 799 | $7.57 \%$ |
| 1978 | 840 | 863 | $11.58 \%$ |
| 1979 | 915 | 932 | $10.99 \%$ |
| 1980 | 1,042 | 1,061 | $9.10 \%$ |
|  | 1,164 | 1,122 | $1.84 \%$ |
| 1981 | 1,284 | 1,182 | $-3.61 \%$ |
| 1982 | 1,386 | 1,257 | $-7.92 \%$ |
| 1983 | 1,536 | 1,349 | $-9.32 \%$ |
| 1984 |  |  | $-12.19 \%$ |
| 1985 | 1,651 | 1,462 | $-11.43 \%$ |
|  | 1,726 | 1,597 | $-7.50 \%$ |
| 1986 | 1,846 | 1,753 | $-5.06 \%$ |
| 1987 | 2,006 | 1,930 | $-3.78 \%$ |
| 1988 |  |  |  |

## TABLE 3

MODEL OFFICE ILLUSTRATION OF THE EFFECTS OF TREND PRICING SALES HIGHER AFTER HIGH INFLATION

| Academic <br> Year | Actual <br> Beginning | Tuition | Aggregate <br> Tuition <br> Anticipated <br> In Pricing |
| :--- | :---: | :---: | ---: | | Margin of <br> Pricing to <br> Actual |
| :---: |
| 1969 |

- there is a uniform distribution of sales in every year of the type of contract sold, and
- contracts are sold from 1964 to 1984.

For 1964 for example, an equal number of contracts will be sold that pay tuition in 1969, 1970, 1971, 1972, 1973 and 1974. In 1965, the contracts sold will pay tuition in years 1970-1975. And in 1984, the contracts sold will pay tuition in years 1989 - 1994.

The results indicated in Table 2 show that there is no clear trend towards either pricing sufficiency or pricing deficiency. The amount of aggregate pricing margin waxes and wanes depending on whether contracts were sold in years of low inflation or years of high inflation.

The reader may want to know what results will be obtained if sales vary by year. An illustrative example of a different sale assumption is shown in Table 3. Here I have assumed that in every year following tuition inflation in excess of the 7.93\% average, sales are 25\% greater than the base level assumed in Table 2. For all other years, sales are at the base level. Because of the heavier weighting of sales in years where projected inflation exceeds the trend, there are more overpriced contracts. As expected, the aggregate margins for this scenario are greater than the margins shown in Table 2. If one had hypothesized relatively more sales in years of low tuition inflation, the opposite results would have occurred - that is, more years in which the projection approach produced deficient results.

Another variation on the simple average approach is shown in Table 4. Here, the sales assumptions are the same as in Table 2; however, the projected tuition is equal to the average loaded by $5 \%$. That is, rather than using $7.93 \%$ expected inflation, the pricing anticipated inflation at $8.32 \%$. Again, as expected, aggregate margins are higher than the Table 2 scenario. It may be of some interest to note that elimination of negative margins through loading of the inflation rate requires the use of $\mathbf{9 . 8 3 \%}$ projected inflation - a loading of $\mathbf{2 4 \%}$.

Slightly different results are obtained if, instead of selling contracts every year for 5 years to 10 years into the future, we extend the sales to 5 years to 14 years into the future. In other words, we diversify across time. The analogous results to Tables 2, 3 and 4 are shown in Tables 5,6 and 7. Compared to the earlier results, these Tables indicate results where the aggregate margins reach higher peaks and, for the most part, less negative troughs. The elimination of negative margins at all years through loading the inflation rate requires the use of $9.07 \%$ projected inflation-a loading of $14 \%$. this is only about $60 \%$ of the loading required in the scenario noted above.

## TABLE 4

MODEL OFFICE ILLUSTRATION OF THE EFFECTS OF TREND PRICING INFLATION PROJECTIONS LOADED BY 5\%

LEVEL SALES

| Academic <br> Year <br> Beginning | Actual <br> Tuition | Aggregate <br> Tuition <br> Anticipated <br> In Pricing | Margin of <br> Pricing to <br> Actual |
| :--- | :---: | :---: | :---: |
| 1969 |  |  |  |
| 1970 | 427 | 444 | $4.08 \%$ |
|  | 478 | 485 | $1.37 \%$ |
| 1971 | 526 | 529 | $0.55 \%$ |
| 1972 | 566 | 566 | $0.02 \%$ |
| 1973 | 581 | 603 | $3.79 \%$ |
| 1974 | 599 | 650 | $8.60 \%$ |
| 1975 | 642 | 704 | $9.62 \%$ |
|  |  |  |  |
| 1976 | 689 | 762 | $10.56 \%$ |
| 1977 | 736 | 821 | $11.59 \%$ |
| 1978 | 777 | 887 | $14.17 \%$ |
| 1979 | 840 | 958 | $14.10 \%$ |
| 1980 | 915 | 1,026 | $12.16 \%$ |
|  |  |  |  |
| 1981 | 1,042 | 1,091 | $4.71 \%$ |
| 1982 | 1,164 | 1,154 | $-0.90 \%$ |
| 1983 | 1,284 | 1,215 | $-5.34 \%$ |
| 1984 | 1,386 | 1,292 | $-6.78 \%$ |
| 1985 | 1,536 | 1,386 | $-9.73 \%$ |
|  |  |  |  |
| 1986 | 1,651 | 1,503 | $-8.96 \%$ |
| 1987 | 1,726 | 1,641 | $-4.93 \%$ |
| 1988 | 1,846 | 1,801 | $-2.44 \%$ |
| 1989 | 2,006 | 1,983 | $-1.13 \%$ |

TABLE 5
MODEL OFICE ILLUSTRATION OF THE EFFECTS OF TREND PRICING LEVEL SALES - YEARS 5 TO 14

| Academic <br> Year | Actual <br> Teginning | Aggregate <br> Tuition <br> Anticipated <br> In Pricing | Margin of <br> Pricing to <br> Actual |
| :--- | :---: | :---: | ---: |
| 1969 | 427 | 436 |  |
| 1970 | 478 | 475 | $2.19 \%$ |
| 1971 | 526 | 517 | $-0.65 \%$ |
| 1972 | 566 | 553 | $-1.64 \%$ |
| 1973 | 581 | 588 | $-2.34 \%$ |
| 1974 | 599 | 633 | $1.15 \%$ |
| 1975 | 642 | 685 | $5.64 \%$ |
|  |  |  | $6.75 \%$ |
| 1976 | 689 | 743 |  |
| 1977 | 736 | 805 | $7.91 \%$ |
| 1978 | 777 | 867 | $9.42 \%$ |
| 1979 | 840 | 930 | $11.62 \%$ |
| 1980 | 915 | 995 | $10.74 \%$ |
|  |  |  | $8.77 \%$ |
| 1981 | 1,042 | 1,062 | $1.92 \%$ |
| 1982 | 1,164 | 1,139 | $-2.14 \%$ |
| 1983 | 1,284 | 1,225 | $-4.61 \%$ |
| 1984 | 1,386 | 1,311 | $-5.43 \%$ |
| 1985 | 1,536 | 1,399 | $-8.95 \%$ |
|  |  |  |  |
| 1986 | 1,651 | 1,497 | $-9.34 \%$ |
| 1987 | 1,726 | 1,608 | $-6.82 \%$ |
| 1988 | 1,846 | 1,741 | $-5.67 \%$ |
| 1989 | 2,006 | 1,894 | $-5.58 \%$ |

TABLE 6
MODEL OFFICE ILLUSTRATION OF THE EFFECTS OF TREND PRICING SALES HIGHER AFTER INFLATION SALES - YEARS 5 TO 14

| Academic Year Beginning | Actual Tuition | Aggregate Tuition Anticipated In Pricing | Margin of Pricing to Actual |
| :---: | :---: | :---: | :---: |
| 1969 | 427 | 436 | 2.19\% |
| 1970 | 478 | 475 | -0.65\% |
| 1971 | 526 | 561 | 6.71\% |
| 1972 | 566 | 622 | 9.86\% |
| 1973 | 581 | 647 | 11.41\% |
| 1974 | 599 | 686 | 14.59\% |
| 1975 | 642 | 760 | 18.37\% |
| 1976 | 689 | 838 | 21.62\% |
| 1977 | 736 | 919 | 24.87\% |
| 1978 | 777 | 978 | 25.84\% |
| 1979 | 840 | 1,049 | 24.93\% |
| 1980 | 915 | 1,124 | 22.83\% |
| 1981 | 1,042 | 1,173 | 12.53\% |
| 1982 | 1,164 | 1,230 | 5.64\% |
| 1983 | 1,284 | 1,323 | 3.00\% |
| 1984 | 1,386 | 1,416 | 2.19\% |
| 1985 | 1,536 | 1,508 | -1.79\% |
| 1986 | 1,651 | 1,612 | -2.34\% |
| 1987 | 1,726 | 1,731 | 0.29\% |
| 1988 | 1,846 | 1,921 | 4.05\% |
| 1989 | 2,006 | 2,139 | 6.61\% |

TABLE 7
MODEL OFFICE ILLUSTRATION OF THE EFFECTS OF TREND PRICING INFLATION PROJECTION LOADED BY 5\%

SALES - YEARS 5 TO 14

| Academic Year Beginning | Actual Tuition | Aggregate Tuition Anticipated In Pricing | Margin of Pricing to Actual |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
| 1969 | 427 | 444 | 4.08\% |
| 1970 | 478 | 485 | 1.37\% |
| 1971 | 526 | 529 | 0.55\% |
| 1972 | 566 | 566 | 0.02\% |
| 1973 | 581 | 603 | 3.79\% |
| 1974 | 599 | 650 | 8.60\% |
| 1975 | 642 | 706 | 9.94\% |
| 1976 | 689 | 767 | 11.32\% |
| 1977 | 736 | 832 | 13.09\% |
| 1978 | 777 | 898 | 15.58\% |
| 1979 | 840 | 963 | 14.63\% |
| 1980 | 915 | 1,031 | 12.65\% |
| 1981 | 1,042 | 1,100 | 5.55\% |
| 1982 | 1,164 | 1,180 | 1.35\% |
| 1983 | 1,284 | 1,269 | -1.19\% |
| 1984 | 1,386 | 1358 | -2.02\% |
| 1985 | 1,536 | 1449 | -5.67\% |
| 1986 | 1,651 | 1,550 | -6.09\% |
| 1987 | 1,726 | 1,665 | -3.52\% |
| 1988 | 1,846 | 1,803 | -2.35\% |
| 1989 | 2,006 | 1,961 | -2.26\% |

So far, all projections have used the formula shown above, where future inflation is projected from the most current tuition rates. Another variation that could be used is to project tuition as the highest value obtained by using the average inflation rate applied to current and past tuition amounts. This variation is more conservative than the original methodology because we always project from historical peaks in tuition rather than projecting from both peaks and valleys. The anticipated tuition in 1989 calculated using this variation in projecting increases in current tuition is shown below for several different years. . As can be seen, anticipated inflation is uniformly conservative for every projection year.

| Projection <br> Year | Anticipated Tuition <br> For 1989/90 | \% Difference <br> From Actual |
| :---: | :---: | :---: |
|  | 2,080 |  |
| 1968 | 2,080 | $3.7 \%$ |
| 1971 | 2,080 | $3.7 \%$ |
| 1975 | 2,080 | $3.7 \%$ |
| 1983 | 2,080 | $3.7 \%$ |
| 1984 |  |  |

The use of a simple average tuition inflation rate in projecting future tuition can give varying results depending on which variation one uses. As we saw in the table immediately above, the variation based on projecting from the highest point in the cycle seems to produce results which are consistently pessimistic. Most of the other variations that were considered produced results which fluctuated from pessimistic to optimistic depending on which point in the inflation cycle one was projecting from. The exceptions to this are the projections from current tuition using a "loaded" simple average, provided the loading is large enough.

## PROJECTING TUITION INFLATION - Autoregressive Methods

The simple average method may be adequate at capturing the equilibrium that we have hypothesized, but is not able to capture the cyclical component of tuition inflation, resulting in either under-projecting or over-projecting inflation, depending where in the inflation cycle we project from.
A method that recognizes that both the equilibrium and the cyclical elements is the autoregressive time series method. A review of Graphs 4 and 5 indicated that an AR(2) model may be appropriate for modeling the tuition inflation; however, the large value of the partial autocorrelation at lag 7 means that tuition inflation may have "memory" of events as long as seven years back. It is difficult to imagine what sort of elements would cause a memory this long, although

## TABLE 8

MODEL OFFICE ILLUSTRATION USE OF AR(2) PRICING MODEL

| Academic Year Beginning | Actual Tuition | Aggregate Tuition Anticipated In Pricing | Margin of Pricing to Actual |
| :---: | :---: | :---: | :---: |
| 1969 | 427 | 430 | 0.81\% |
| 1970 | 478 | 480 | 0.33\% |
| 1971 | 526 | 532 | 1.10\% |
| 1972 | 566 | 564 | -0.31\% |
| 1973 | 581 | 587 | 1.03\% |
| 1974 | 599 | 631 | 5.35\% |
| 1975 | 642 | 680 | 5.98\% |
| 1976 | 689 | 751 | 9.03\% |
| 1977 | 736 | 828 | 12.47\% |
| 1978 | 777 | 900 | 15.81\% |
| 1979 | 840 | 960 | 14.32\% |
| 1980 | 915 | 1,002 | 9.48\% |
| 1981 | 1,042 | 1,063 | 2.00\% |
| 1982 | 1,164 | 1,138 | 2.28\% |
| 1983 | 1,284 | 1,242 | -3.26\% |
| 1984 | 1,386 | 1,350 | -2.56\% |
| 1985 | 1,536 | 1,457 | -5.14\% |
| 1986 | 1,651 | 1,552 | -5.99\% |
| 1987 | 1,726 | 1,623 | -5.97\% |
| 1988 | 1,846 | 1,745 | -5.49\% |
| 1989 | 2,006 | 1,886 | -5.97\% |

## TABLE 9

MODEL OFFICE ILLUSTRATION USE OF MA(1) PRICING MODEL

| Academic <br> Year <br> Beginning | Actual <br> Tuition | Aggregate <br> Tgition <br> Andipated <br> In Pricing | Margin of <br> Pricing to <br> Acual |
| :--- | :---: | :---: | :---: |
| 1969 | 427 | 437 | $2.42 \%$ |
| 1970 | 478 | 478 | 0.06 |
|  |  |  |  |
| 1971 | 526 | 521 | -0.87 |
| 1972 | 566 | 552 | -2.55 |
| 1973 | 581 | 587 | 0.95 |
| 1974 | 599 | 635 | 6.08 |
| 1975 | 642 | 689 | 7.26 |
|  |  |  |  |
| 1976 | 689 | 748 | 8.56 |
| 1977 | 736 | 809 | 9.98 |
| 1978 | 777 | 870 | 11.95 |
| 1979 | 840 | 932 | 10.93 |
| 1980 | 915 | 996 | 8.88 |
|  |  |  |  |
| 1981 | 1,042 | 1,062 | 1.91 |
| 1982 | 1,164 | 1,143 | -1.82 |
| 1983 | 1,284 | 1,228 | -4.35 |
| 1984 | 1,586 | 1,311 | -5.40 |
| 1985 |  | 1,399 | -8.95 |
|  |  |  |  |
| 1986 | 1,651 | 1,500 | -9.15 |
| 1987 | 1,726 | 1,613 | -6.54 |
| 1988 | 1,846 | 1,753 | -5.04 |
| 1989 | 2,006 | 1,908 | -4.89 |

seven years is approximately the length of an average economic cycle. It is also nearly the length of two gubernatorial terms for most states. Another interpretation is that the value at lag 7 is spurious and is due to the brevity of the time series.

Two time series models are shown below. One is an autoregressive model and the other is a moving average model. Each of them captures almost;most the same amount of information regarding the time series but has slightly different results. The first is an AR(2) model treated as if there is seasonality over seven years and the second is a first order moving average - MA(1) model. ${ }^{9}$

1) $X(t)=.404 * X(t-1)-.314 * X(t-2)+e(t)$
2) $X(t)=.574^{*} e(t-1)+e(t)$

Where $X(t)$ is the value of the time series at time $t$ minus the mean of the time series. For equation $1, X(t)$ is the value of the de-seasonalized time series. The value $e(t)$ is the residual term at time $t$.

The model office results of using a time series projection of future tuition inflation are shown in Tables 8 and 9 for the AR(2) model and the MA(1) model respectively, corresponding to Table 5 for the simple average method - that is, level sales for 10 years.

In comparing the new tables to Table 5, results are, on the whole more economically favorable using autoregressive methods than using a simple average method. Both the AR(2) model and the MA(1) model appear to have less negative aggregate margins than the simple average model. In order to compare the two models, the average of the margins for all years is compared below. Some of the years shown do not contain 10 years' worth of contract sales, so they might be considered as not yet having reached inforce equilibrium. Because of this, the comparison below also shows the averages from the different models only for years which have contained 10 years' worth of contract sales the years 1978 to 1989 .

Pricing
Model
Simple Average
AR(2)
MA(1)

Average Aggregate Margins
All Years
1978-1989 Only
.62\%
1.94
.92
$-1.29 \%$
.41
$-1.04$

The results show that of the three models under consideration, the AR(2) is the only one which produces positive margins on average for years in which the model office has reached its sales equilibrium. This result is due to the incorporation of the seasonality component in the tuition inflation projections which allows for significant variation from the average inflation value.

The tentative conclusion from this analysis is that the use of strictly statistical methods of projection tuition inflation, such as the simple average method and autoregressive methods, should include some margin over the historical tuition inflation rate in order to counteract the underpricing of prepaid tuition contracts that results when average tuition is projected in years with inflation lower than the historical trend. As an alternate, the use of a statistical method that allow for significant variations above and below the average in future years. The use of a loaded simple average is simpler administratively and easier to explain legislatively; however, such a method may raise questions of equity between generations of contract purchasers since those who purchase in years of higher than average inflation will have relatively overpriced contracts while purchasers in years with lower than average inflation will have relatively underpriced contracts.

## PROJECTING TUITION INFLATION - Economic Scenarios

Both of the previous methods for projecting tuition inflation are empirical methods since they attempt to forecast tuition inflation based strictly on observed data rather than attempting to build an explanation of the tuition inflation process into the projection model. The projection model discussed in this section takes a partial step towards building an explanation.

The methodology of this model is based on some of the observations made in the Overview of the Economics of Public University Financing section. However, these observations are left in the vague form that "tuition inflation is somehow related to the economic cycle.

This method models the economic cycle and then fits a normal distribution to tuition inflation during the expansion and a different normal distribution during the recession. In other words, tuition inflation follows a white noise process during recession and a different white noise process during expansion. The rationale for modeling tuition inflation based on the economy is that the economy, as represented by change in the real GNP, has exhibited cyclicality as well as an apparent equilibrium of approximately $3 \%$.

## G R A P H 6



## G R A P H 7



G R A P H 8
Actual Inflation vs. Modeled Inflation


G R A P H 9
Actual Inflation vs. Modeled Inflation


$$
G R A P H \quad 10
$$



In this model, the length of the expansion is a multinomial distribution with possible values of between 4 years and 8 years. the recession is modeled as a multinomial distribution taking on values between 1 year and 3 years. the probabilities in this model are shown below.

|  | Probability <br> Number of Years |  |
| :---: | :---: | :---: |
|  | Expansion | Recession |
| 2 |  | .33 |
| 3 |  | .50 |
| 4 | .20 | .17 |
| 5 | .30 |  |
| 6 | .30 |  |
| 7 | .10 |  |
| 8 | .10 |  |

The two different normal distributions were set such that, if independence is assumed, the average one year mean and variance over the expected length of the economic cycle is approximately equal to the mean and variance of the full sample data. The normal distributions are as follows:

$$
\begin{array}{ll}
\text { Recession: } & \mathrm{N}(.0946, .001846) \\
\text { Expansion: } & \mathrm{N}(.0750, .000956)
\end{array}
$$

In order to illustrate what sort of projection this model produces, 5 paths generated by Monte Carlo techniques are shown in Graphs 6-10. For these paths, I assumed that the expansion phase of the economy was already three years old, then projected for 25 years. The actual times series is also shown for comparison.

Monte Carlo techniques were used to project inflation over the course of 25 years for 1000 different scenarios. As before, I assumed that the economic cycle was three years into expansion at the start of the projection. If this set of inflation values is used to project tuition from 1964, the following values are obtained:

| Projected | Actual | Projected Inflation Percentile |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Year | Tuition | 50th | 662/3's | 75th | 90 th |
| 1969 | 427 | 428 | 441 | 449 | 468 |
| 1974 | 599 | 634 | 660 | 677 | 718 |
| 1979 | 840 | 926 | 977 | 1,012 | 1,084 |
| 1984 | 1,386 | 1,363 | 1,442 | 1,491 | 1,618 |
| 1989 | 2,006 | 1,985 | 2,126 | 2,206 | 2,413 |

In order to demonstrate how the econometric model would work over an extended number of years, Table 10 illustrates model office results on the same basis as Table 5 - level sales for future years 5-14. The projected values for tuition were based on the 50 th percentile results of projecting inflation using 1000 Monte Carlo for each projection year.

The results shown in Table 10 are not noticeably better than the results in Table 5. In fact, they are slightly worse, as can be seen by reviewing the average margins.

| Pricing <br> Model | Average Aggregate Margins <br> All Years | $1978-1989$ Only |
| :--- | :---: | :---: |
| Simple Average | $.62 \%$ | $-1.29 \%$ |
| Economic Scenarios | -.04 | -1.58 |

The tentative conclusion to be drawn from this is that the use of economic scenarios suffers from the same shortcomings as the other models investigated so far - it does not make sufficient allowance for the year to year volatility of tuition inflation.

## PROJECTING TUITION INFLATION - Econometric Models

An econometric model (typically a system of difference of differential equations) is an attempt to abstract significant causal items based on relationships suggested by economic theory. The advantage of a good econometric model is that it focuses the user's attention on the perceived dynamic elements underlying the economic process in question. Such a model will alert the user to items in the process which can cause discontinuities in the observed time series - something which strictly statistical models are unable to do.

More research remains to be done in developing an econometric model of tuition inflation; however, the form of the equations of such a model is given below along with comments for the equations.

In the equations below, $\mathrm{B}($.$) represents a parameter to be solved for. These$ parameters are presented as invariant across time; however, it is easy to see how this model could be extended such that $B($.) would be function of time.

1) Revenues $(t)=B(1)^{*}$ Expenditures $(t)$

This is the equation that defines what tuition rates will be for year $t$, since revenues are forced to be proportional to expenditures.

TABLE 10
MODEL OFFICE ILLUSTRATION
USE OF ECONOMIC SCENARIOS PRICING MODEL

| Academic Year Beginning | Actual <br> Tuition | Aggregate Tuition Anticipated In Pricing | Margin of Pricing to Actual |
| :---: | :---: | :---: | :---: |
| 1969 | 427 | 428 | 0.26\% |
| 1970 | 478 | 467 | -2.40 |
| 1971 | 526 | 509 | -3.19 |
| 1972 | 566 | 547 | -3.42 |
| 1973 | 581 | 581 | 0.07 |
| 1974 | 599 | 626 | 4.58 |
| 1975 | 642 | 681 | 6.12 |
| 1976 | 689 | 740 | 7.40 |
| 1977 | 736 | 800 | 8.73 |
| 1978 | 777 | 861 | 10.85 |
| 1979 | 840 | 924 | 9.97 |
| 1980 | 915 | 992 | 8.36 |
| 1981 | 1,042 | 1,062 | 1.89 |
| 1982 | 1,164 | 1,136 | -2.36 |
| 1983 | 1,284 | 1,223 | -4.77 |
| 1984 | 1,386 | 1,309 | -5.57 |
| 1985 | 1,536 | 1,392 | -9.41 |
| 1986 | 1,651 | 1,489 | -9.81 |
| 1987 | 1,726 | 1,605 | - 7.03 |
| 1988 | 1,846 | 1,745 | - 5.47 |
| 1989 | 2,006 | 1,893 | -5.63 |

2) 

$\Delta$ Revenues $(t)=\Delta$ Appropriations $(t)+\Delta$ Tuition $(t)$
The change in revenues in any year is equal to the change in appropriations and the change in tuition revenue. All other sources of revenue are treated as having immaterial changes.
3) $\Delta$ Appropriations $(t)=B(2)^{*}$ Appropriations $(t-1)+$

B(3)* $\Delta$ Budget Surplus(t-1)
The change in appropriations in any year depends on the level of appropriations in the prior year and on the change in the budget surplus in the prior year.
4) $\Delta$ Tuition $(t)=\Delta$ Enrollment( $t$ *Tuition Rate( $(t-1)+$ $\Delta$ Tuition Rate(t)*Enrollment(t-1)

The change in tuition revenue in an year is equal to the increase in tuition rates times prior enrollment plus increase in enrollment times prior tuition rates. Second order effects are ignored in this equation.
5) $\Delta$ Enrollment $(t)=B(4)^{*}$ Enrollment $(t-1)+B(5)^{*} \Delta$ Juvenile Population $(t-Y)$

The change in enrollment is equal to prior years enrollment times a factor plus lagged change in population times a factor. " Y " represents the appropriate lag in the effect of population change on enrollment change.
6) $\Delta$ Surplus $(t)=B(5) *$ Real GNP(t-2) + $B(6) * \triangle$ Population $(t-1)+$ B(7)"Surplus(t-1)

This equation defines surplus to depend on a lagged economic factor, a lagged population factor and prior year's surplus. Prior year's surplus could cover any miscellaneous change such as tax laws.
7) $\Delta^{\text {Expenditures }(t)}=\% \Delta$ Salaries $(t) * S$ Salaries $(t-1)+B(8)^{*} \Delta$ Enrollment

University expenditure changes are defined to be a function of prior years salaries and change in enrollment.
8) \% $\Delta$ Salaries $(t)=B(9) * \% \Delta C P I(t-1)+B(10) * \% \Delta$ Real GNP $(t-1)$

The relative change in salaries is assumed to depend on lagged changes in the general price index and on lagged changes in the GNP.

This model requires, as exongeous factors, projections of GNP, population, juvenile population and CPI. If these are not readily available, or if the confidence intervals, at whatever significance level is appropriate, are deemed to be too wide, the actuary may decide that the use of methods other then econometric forecasting are appropriate.

## PROJECTING TUITION INFLATION - How well do the models work?

The real test of a projection model is not how well it reproduces historical data, but how well it predicts future inflation. In order to provide some indication of how well the different methodologies work, a comparison of four different projection results are compared against tuition inflation for academic years beginning in 1990 and in 1991. Information regarding actual tuition increases in these years is derived from the State of Washington Higher Education Coordinating Board. Their data is not the same as used by the National Center for Education Statistics; however, the national average tuition inflation from both sources compare reasonably well.

| 1990 |  | 1991 |  |
| :---: | :---: | :---: | :---: |
| Actual |  | Actual |  |
| Annual | 9.54\% | Annual | 11.52\% |
| Cumulative | 9.54\% | Cumulative | 22.16\% |
| Simple Average |  | Simple Average |  |
| Annual | 7.93\% | Annual | 7.93\% |
| Cumulative | 7.93\% | Curnulative | 16.49\% |
| MA(1) Model |  | MA(1) Model |  |
| Annual | 8.33\% | Annual | 7.97\% |
| Cumulative | 8.33\% | Cumulative | 16.96\% |
| AR(2) Model |  | AR(2) Model |  |
| Annual | 11.37\% | Annual | 9.09\% |
| Cumulative | 11.37\% | Cumulative | 21.49\% |
| Economic-50th \% |  | Economic-50th\% |  |
| Annual | 9.46\% | Annual | 9.46\% |
| Cumulative | 9.46\% | Cumulative | 18.91\% |

Of the four models, the AR(2) model and the Economic Scenario model come closest to actual. The Simple Average and the MA(1) models do not provide any sensitivity to inflation in excess of the long-term average and have a 500 basis point deficit after two years compared to actual tuition inflation.

Of all the models tested, the $\operatorname{AR}(2)$ appears to perform the best in providing reasonable to conservative projections of tuition. Although the Economic Scenario methodology is responsive to changes in the economic cycle, this method may not provide sufficient responsiveness over the long-term at the 50th percentile level, based on review of the results in Table 10. The other two methodologies, Simple Average and MA(1) do not provide reasonable results over the long-term without the use of explicit conservatism.

## ANTI-SELECTION

Michigan, Ohio and Alabama all have prices that vary significantly between postsecondary institutions. This presents another dilemma to the actuary who must project future tuition - what will be the pattern of college enrollment among contract beneficiaries, and will the existence of prepaid tuition contracts alter the current pattern of enrollment?

In order to give some indication of how sensitive results might be to antiselection, a simple model ${ }^{10}$ based on the assumption that the choice among prepaid tuition beneficiaries of more expensive and prestigious universities over local and regional universities is proportional to the relative difference between tuition at the local/regional colleges and tuition at the more prestigious schools.

Because data was readily available, I have used Michigan in my example. The results of this analysis apply to any state which has significant tuition differences between public postsecondary schools which is not reflected in prepaid contract prices.

The sources of information were the article "Social Responsibility, Actuarial Assumptions, and Wealth Redistributions: Lessons About Public Policy From a Prepaid Tuition Program" by Dr. J.S. Lehman, and the book College Costs 1989-90, published by the National Center for Education Statistics. I assumed that the University of Michigan at Ann Arbor and Michigan State University were the prestigious universities whose enrollment would benefit from prepaid tuition contracts at the expense of other Michigan public universities and colleges. An argument could be made to include Wayne State University as one of the prestigious universities; however, I did not include Wayne State in order to keep my model simple.

The increase in average tuition due to anti-selection by prepaid tuition beneficiaries was approximately $.9 \%$ for every $1 \%$ of price sensitivity. In other words, if the constant of proportionality in the anti-selection model is $1 \%$, the average tuition for prepaid tuition beneficiaries will be $.9 \%$ higher than the average for other students. Although these numbers will vary from state to state,
they do indicate that such an analysis is worthwhile in order to determine the possible magnitude of anti-selection.

Because there is not enough information available to determine the extent of anti-selection, this issue should be weighed carefully. An additional $10 \%$ cost due to anti-selection is certainly a possibility - the only question is determining how likely such an additional cost is. If these costs are expected to be significant, they can be provided for through various contract pricing mechanisms, such as an explicit additional amount in the projection of future tuition, or implicitly through higher tuition inflation, or through lower investment yields.

## INVESTMENT CONSIDERATIONS

Inflation is only one of the two critical assumptions in determining the financial adequacy of prepaid tuition plans. The other critical assumption is the yield on assets used to support the obligations of the prepaid plan. In fact, the spread between tuition inflation and asset yield is probably more critical than the absolute level of either inflation or asset yield. Even so, asset yield has to be addressed separately from inflation since the spread is not an independent item, but depends on the investment philosophy of the prepaid plan.

The spread, tuition inflation and asset yield for the four main prepaid plans are shown to demonstrate the range of thinking regarding possible spreads. All yields are shown on a pre-tax basis for comparison; however, the reader should note that currently the Michigan plan is subject to Federal Income Tax while the other plans either are pursuing private letter rulings or have made the presumption that they are not subject to FIT.

| STATE | INFLATION | YIELD | SPREAD |
| :--- | :---: | :---: | :--- |
| Alabama | $7.10 \%$ | $9.30 \%$ | 220 basis points |
| Florida | $7.50 \%$ | Treasury spot rates | Varies |
| Michigan | $7.30 \%$ | $9.75 \%$ | 245 basis points |
| Ohio | $8.20 \%$ | $10.00 \%$ | 180 basis points |

The difference in spreads between plans appears to be due to the different assets held by each of the plans. Based on the most current financial statements of these plans, the asset mix is as follows:

| ASSET CLASS | ALA | FLA | MICHIGAN | OHIO ${ }^{11}$ |
| :--- | :--- | :--- | :---: | :---: |
| Cash \& S/T | $20.3 \%$ | $32.9 \%$ | $1.0 \%$ | $.9 \%$ |
| Bonds | 60.7 | 66.6 | 44.5 | 99.1 |
| Preferred Stock |  |  | 54.5 |  |
| Common Stock | 19.0 |  |  |  |
| Other |  | .5 |  |  |

The difficulty with investment strategy for prepaid tuition plans is that the process of tuition inflation is not yet well understood (as discussed earlier in this article) so that no optimal strategy can be formulated. Even a satisfactory strategy is difficult to formulate in the face of our ignorance. Compounding the problem is the fact that no hedging of investment risk can be performed without significant basis risk since there are no public university tuition futures available.

An interesting digression to note here is that although the prepaid plans themselves do not have natural hedges available, colleges and universities have the ability to hedge since they provide the college education that tuition purchases. In fact, prepaid tuition plans appear to be naturally suited for colleges and universities to sell as a way to lock in future enrollment and guarantee their own tuition revenues. This appearance has not been matched by reality. A few postsecondary institutions have implemented prepaid tuition plans specific to their own institution, but none of these plans have had success in marketing their product.

One way to gain insight into a satisfactory investment strategy is to consider how the obligations of prepaid tuition plans resemble pension plan obligations and how these similarities imply investment strategy that is similar to pension plan investment strategy.

A defined benefit pension plan has benefits that are typically indexed to each participant's wage. The benefit to be paid is expected to increase at a rate somewhat faster than general inflation - similar to a prepaid tuition plan. Also, a pension plan typically has a long investment time horizon due to the long term nature of the assurances made by the plan - again similar to a prepaid tuition plan.

Pension investing goals are typically set to provide a long-term real rate of return - that is, a yield that beats inflation. This is usually accomplished through a mix of equities and long term high quality debt securities. Sometimes a small percent
of assets invested in speculative investments is also considered appropriate. The portion of assets in equities typically ranges from $50 \%$ to $60 \%$. Real estate has been thought of as the typical pension plan hedge against unanticipated inflation. However, the real estate market is currently (late 1991) in disarray. It may be 5 years, or more, before real estate is again considered an appropriate investment to hedge against inflation.

It is not clear that an equity strategy would be appropriate for prepaid tuition plans. Although equities offer returns that are somewhat connected to economic cycles, the timing of returns from equities may not coincide with the ups and downs of the tuition inflation cycle. A regression of tuition inflation against the December monthly averages of the S\&P 500 lagged one year for 1965 to 1989 indicates almost zero linear correlation. Although there may be distortions in using a monthly average and in only using December data for the S\&P 500, the regression results indicate the need for caution in the use of equities in the investment portfolio of a prepaid tuition plan.

More research is needed to determine the optimal investment strategy for prepaid tuition plans. Because there are no natural hedges available of tuition inflation, any solution that approaches the optimal solution will likely involve significant trade-offs between asset returns and matching of cash flows.

## SUMMARY

Tuition inflation at public universities has had an erratic pattern over the past quarter-century. Although it appears to be driven by economic forces, it is difficult to predict this inflation by a simple model, even a simple model that incorporates economic factors, because of the political element included in the single largest piece of university revenues - state appropriations. None of the models tested were perfect - they do not seem to give enough weight to future inflation above average when tuition inflation has been low. No doubt other models can, and will, be developed that will provide better predictions of tuition inflation.

Projecting tuition inflation for prepaid tuition contracts is made more difficult by the fact that most states have tuition rates that vary between institutions, yet prepaid tuition contracts do not distinguish between attendance at high priced colleges compared to moderately priced colleges. There is not yet enough information to determine if beneficiaries of these contracts will select against the different states by choosing the high priced colleges whenever possible, but testing through some simple models indicates that there is a potential for a significant increase in liability if such anti-selection occurs.

Because of the difficulty in projecting future tuition, and due to the possibility of anti-selection, a strategy of conservatism in determining prices for prepaid tuition contracts appears necessary unless the prepaid tuition plan has recourse to other state revenues.

The appropriate investment strategy for a prepaid tuition plan is one which is responsive to the economic cycle. The best combination of fixed income, equities, real estate and other investment categories is not clear since there are currently no natural hedges against tuition inflation.

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## NOTES TO THE ARTICLE

1) The National Center for Education Statistics uses the following definition for universities:

> An institution of higher education consisting of a liberal arts college, a diverse graduate program, and usually two or more professional schools or faculties and empowered to confer degrees in various fields of study.

Universities do not cover the entire universe of postsecondary institutions, but do seem to represent the most important portion of enrollment in prepaid tuition programs. Other postsecondary schools are typically classed as 4 -year schools. These other categories show tuition inflation patterns different from university tuition inflation. University tuition inflation is plotted against both other 4-year school tuition inflation and 2-year school tuition inflation in Graphs A and B.

These two other segments of postsecondary schools have both more volatility in tuition inflation and a higher average level of inflation over the 1965-1989 period. The conclusions reached for university tuition inflation do not apply to these other two segments. The lesson for prepaid tuition programs is that the different segments of postsecondary schools need to be analyzed separately.
2) Tuition rates are taken from the Digest of Education Statistics 1990 Edition, Table 281. Unless otherwise noted in the article, all tuition rates and tuition inflation rates are based on this data.

Note that I have used the geometric mean rather than the arithmetic mean for presentation. The geometric mean appears to be more consistent with the use of an average for compound inflation. However, the difference between the geometric mean and the arithmetic mean is small - 7.93\% versus $\mathbf{7 . 9 7 \%}$ respectively.
3) In September 1991, Michigan announced that no new contracts would be sold during the normal enrollment period. In December 1991, Michigan announced that no new contracts would be sold until economic conditions had improved. The reasons cited were higher than average tuition inflation and low yields on fixed income investments.
4) Information on revenues and expenditures is taken from the Digest of Education Statistics.
5) Institutional Support is defined as:

The category of higher education expenditures that includes day-to-day operational support for colleges, excluding expenditures for physical plan operations. Examples of institutional support include general administrative services, executive direction and planning, legal and fiscal operations, and community relations.
6) The correlation matrices of the regression parameters for all three regressions are shown below.

CASE 1. Regression_Against_Lagged Variables

|  | Change CPI | Change Population | Change GNP |
| :--- | :---: | :---: | :---: |
| Change CPI | 1.000000 | .243996 | .067411 |
| Change Population | .243996 | 1.000000 | .107511 |
| Change GNP | .067411 | .107511 | 1.000000 |

CASE 2-Regression_Against_Concurrent Variables

|  | Change Earnings | Surplus | Change Expendinmes |
| :--- | :---: | ---: | :---: |
| Change Earnings | 1.000000 | .446692 | .120345 |
| Surplus | .446692 | 1.000000 | .542585 |
| Change Expenditures | .120345 | .542585 | 1.000000 |

CASE 3-Regression_Against Concurrent_and Lagged Yariables

|  | Change Eaming | Surplus | Change Enrollment |
| :--- | :---: | ---: | :---: |
| Change Earnings | 1.000000 | .491908 | .292793 |
| Surplus | .491908 | 1.000000 | .815400 |
| Change Enrollment | .292793 | .815400 | 1.000000 |

7) Information on proportion of revenues and expenses relating to GNP are taken from the Digest of Education Statistics.
8) Comparison of Tuition Inflation - States with prepaid tuition plans

Academic
Year

| Beginning | Alabama | Elorida | Michigan | Ohic |
| :--- | ---: | ---: | ---: | ---: |
|  |  |  |  |  |
| 1973 | $0.00 \%$ | $0.00 \%$ | $22.41 \%$ | $0.00 \%$ |
| 1974 | $16.67 \%$ | $7.89 \%$ | $0.35 \%$ | $4.00 \%$ |
| 1975 | $0.00 \%$ | $11.06 \%$ | $5.73 \%$ | $3.85 \%$ |
| 1976 | $8.40 \%$ | $3.81 \%$ | $9.07 \%$ | $3.09 \%$ |
| 1977 | $0.00 \%$ | $0.00 \%$ | $9.33 \%$ | $9.58 \%$ |
| 1978 | $11.94 \%$ | $0.00 \%$ | $15.40 \%$ | $6.56 \%$ |
| 1979 | $5.96 \%$ | $0.00 \%$ | $10.37 \%$ | $3.08 \%$ |
| 1980 | $0.00 \%$ | $0.00 \%$ | $13.69 \%$ | $10.45 \%$ |
| 1981 | $29.93 \%$ | $6.91 \%$ | $19.22 \%$ | $24.32 \%$ |
| 1982 | $8.05 \%$ | $4.88 \%$ | $15.21 \%$ | $5.65 \%$ |
| 1983 | $6.89 \%$ | $0.38 \%$ | $5.46 \%$ | $6.79 \%$ |
| 1984 | $5.05 \%$ | $-6.89 \%$ | $3.94 \%$ | $5.39 \%$ |
| 1985 | $3.98 \%$ | $4.31 \%$ | $0.38 \%$ | $3.84 \%$ |
| 1986 | $3.99 \%$ | $4.77 \%$ | $14.24 \%$ | $0.00 \%$ |
| 1987 | $20.55 \%$ | $36.45 \%$ | $4.94 \%$ | $10.92 \%$ |
| 1988 | $4.45 \%$ | $4.69 \%$ | $12.09 \%$ | $7.94 \%$ |
| 1989 | $4.99 \%$ | $3.02 \%$ | $7.10 \%$ | $7.35 \%$ |
| 1990 | $4.99 \%$ | $12.47 \%$ | $8.63 \%$ | $6.99 \%$ |
| 1991 | $6.96 \%$ | $12.50 \%$ | $9.65 \%$ | $9.60 \%$ |

Geometric

| Mean | $7.27 \%$ | $5.27 \%$ | $9.70 \%$ | $6.69 \%$ |
| :--- | :--- | :--- | :--- | :--- |
| Standard | $7.67 \%$ | $8.94 \%$ | $5.92 \%$ | $5.29 \%$ |

These rates are derived from 1991-92 Tuition and Fee Rates - A National Comparison published by the State of Washington Higher Education Coordinating Board.
9) The quality of statistical models is usually judged by the character of the residuals. In order to provide a sense of the fitness of these models, several tests of randomness of the residuals are shown below. The residuals are shown in Graphs C and D for the AR(2) model and the MA(1) model respectively. Autocorrelation coefficients along with approximate $95 \%$ significance levels are shown in Graphs E and F for the AR(2) model and the MA(1) model respectively.

| Testof Randomness | AR(1) Model | MAdll Model | 95\% Significance Levels |
| :---: | :---: | :---: | :---: |
| Box-Pierce |  |  |  |
| Portmanteau Test | . 7785 |  | 7.815 (Chi-Square with 3 degrees of freedom |
| Box-Pierce |  |  |  |
| Portmanteau Test |  | 1.4758 | 9.488 (Chi-square with 4 degrees of freedom |
| Turning Points | 16 | 17 | (11.27, 19.39) |
| Difference-Sign | 13 | 13 | (9.06, 14.94) |
| Rank | 148 | 151 | (21.54, 278.46) |

The results of each of these tests indicate that the hypothesis that the residuals are randomly distributed from identical independent distributions is not rejected at the $95 \%$ significance level.
10) The model assumes that the proportion of expected enrollees at the regional/local universities who decide to enroll at UMAA or MSU is given by:

```
k * (UMAA tuition / Local & regional university tuition)
```

and
k* (MSU tuition / Local \& regional university tuition)
In this model, $k$ is assumed to be the same for UMAA and for MSU, although it is easy to see that the more general case allows $k$ to vary between MSU and UMAA.

Other models of anti-selection are, of course, possible. One possibility is to borrow from mortgage bankers the use of an arctangent function to describe mortgage prepayments. Arctangent functions in the context of prepaid tuition plans would result in anti-selection increasing at an increasing rate as differences in the tuition between the local/regional universities and the national level universities increase.
11) Ohio's asset mix as based on informal discussions with the Ohio Plan's consulting actuary. The asset description in the published financial information did not distinguish between bonds and stocks.


## G R A P H B



G R A P H C


## G R A P H D



G R A P H E - AR(2) Residual Autocorrelations


Lines are approximate $95 \%$ significance levels

## G R A P H F



Lines are approximate $95 \%$ significance levels

