ACTUARIAL RESEARCH CLEARING HOUSE
1993 VOL. 3

The Taylor Series Approximation for FAS 91 Adjustments

Steven P. Miller, F.S.A.

Abstract

Financial Accounting Standard Number 91 deals with the calculation of investment income and amortized cost for mortgage backed securities, as well as whole mortgages and callable bonds. Complying with the standard's treatment of mortgage backed securities has proven to be difficult, partly because it requires the company to keep a record of all past cash flows in order to calculate the accrual of discount or amortization of premium. The method described here uses a simple Taylor series to eliminate the storage of all past cashflows, while still providing a very accurate FAS 91 adjustment.

FAS 91

Like any bond in good standing, FAS 91 requires that the book value of a mortgage backed security (MBS) will be equal to the present value of all future cash flows that are expected to be paid to the bondholder, with the cash flows being discounted at the purchase yield. The difference between FAS 91 for MBS's and other accounting is that yield is adjusted throughout the life of the bond to reflect the actual incidence of cash flows. At each adjustment period, the yield at purchase is calculated using all past cash flows, and current projections of future cash flows. Since all past cash flows are used to calculate the current estimate of the yield, they must be stored so that the computer can access them every time is does a new yield calculation. Since many MBS's have monthly payments, the storage requirements can be huge. The calculation of the yield to maturity for every mortgage adds to the processing time as well. Many insurance companies have ignored the historic cash flows because complete compliance is impractical.

The Approximate Method

The method that I will now describe eliminates the need to keep historical cash flows while sacrificing only a small amount of accuracy. This is accomplished as follows:

1. Translate the original purchase yield into the force of interest by using the formula $\delta_0 = m \cdot ln(1+i_0^{(m)}/m)$, where interest is paid m times per year.

2. The present value, discounting at δ is
$$PV(\delta) = \sum_t e^{-\delta t} \cdot CF(t)$$

   The first derivative of the present value is
$$PV'(\delta) = -\sum_t t \cdot e^{-\delta t} \cdot CF(t)$$

   The second derivative is
$$PV''(\delta) = \sum_t t^2 \cdot e^{-\delta t} \cdot CF(t), \ etc.$$

3. Using a Taylor series, we can write the present values as:

$$PV(\delta)=PV(\delta_0)+(\delta-\delta_0)PV'(\delta_0)+\frac{(\delta-\delta_0)^2}{2!}PV''(\delta_0)+\cdots,\ or$$

$$PV(\delta)=D_0-(\delta-\delta_0)D_1+\frac{(\delta-\delta_0)^2}{2!}D_2-+\cdots,\ where$$

$$D_n=\sum_t t^n e^{-\delta_0 t}CF(t)$$

4. At time n, separate the present value into past and present pieces. (The variable t represents the time since purchase).

$$PV(t)=\sum_{t=0}^{n} e^{-\delta t}CF(t)+\sum_{t=n+1}^{\infty} e^{-\delta t}CF(t)$$

5. Approximate the first term by

$$\sum_{t=0}^{n} e^{-\delta t}CF(t)=PV^{Past}(\delta)\approx D_0-(\delta-\delta_0)D_1+\frac{(\delta-\delta_0)^2}{2!}D_2$$

6. D_0, D_1, and D_2 can be accumulated as time progresses by adding the appropriate term to each at the receipt of each cash flow.

Table 1 gives an example, where $\delta_0=.1$.

Table 1

Time	Cash Flow	Formula	D_0	D_1	D_2
0	0		0	0	0
.5	100	$0+.5^n\cdot 100\cdot e^{-.5\cdot .1}$	95.1229	47.5615	23.7807
1	250	$D_n+1^n\cdot 250\cdot e^{-1\cdot .1}$	321.3322	273.7708	249.9901
1.5	50	$D_n+1.5^n\cdot 50\cdot e^{-1.5\cdot .1}$	364.3675	338.3239	346.8197

To calculate $PV^{Past}(.12)$ at time .5, we use the formula

$$PV^{Past}(.12)\approx 364.3675-(.12-.1)\cdot 338.3239+\frac{(.12-.1)^2}{2}346.8197\approx 357.6704$$

The actual value is calculated as follows:

$$PV^{Past}(.12)=100e^{-.12\cdot .5}+250e^{-.12\cdot 1}+50e^{-.12\cdot 1.5}=357.6701$$

7. If we have a projection of future cash flows, we can calculate the present value of future cash flows at any rate in the usual way:

$$PV^{Future}(\delta)=\sum_{t=n} e^{-\delta t}CF(t)$$

Then the present value at issue of all past and future cash flows is
$$PV(\delta)=PV^{Past}(\delta)+PV^{Future}(\delta)$$

8. Solve for δ such that $PV(\delta)$ is equal to the purchase price.
9. The book value is equal to $PV^{Future}(\delta)$.
10. The yield is equal to $i^{(m)}=m\cdot(e^{\delta/m}-1)$.

We have now produced a book value that complies with FAS 91, while only storing 5 numbers to represent the past (D_0, D_1, D_2, δ_0 plus the original cost.) The accuracy can be increased by extending the Taylor series (and storing more numbers). Moreover, the number of calculations required to calculate the present values during the IRR calculation is decreased every payment period, so a reduction in calculation time is achieved.

Example

The Exposure Draft for for the proposed amendments to FASB Statements No. 65 and 91 contains a simple example of the "retrospective method" which is the accounting standard described above[1]. This example showed the accounting on a hypothetical security consisting of the interest on five $10,000 bonds paying 10% interest. One bond is expected to prepay after three years. The price of the security is $17,500, which would yield 10.35% under the initial assumptions. The prepayment is accelerated to the end of year 2, which reduces the realized yield to maturity to 8.58% The following table shows how the income would would be realized under the retrospective approach and under the Taylor series approximation. The yields, book values and investment income numbers have been carried out to two more significant digits in order to show the difference between the two approaches.

Year	Original Cash Flow Estimate	Actual Cash Flow	D_0	D_1	D_2	Estimated δ	Estimated Yield	Beginning Book Value	Ending Book Value	Approx. Invest. Income	Actual Inv. Income
1	5,000	5,000	4,531	4,531	4,531	9.8490%	10.3503%	17,500.00	14,311.31	1,811.31	1,811.31
2	5,000	5,000	8,637	12,743	20,955	9.8490%	10.3503%	14,311.31	10,201.98	890.67	890.63
3	5,000	4,000	11,613	21,673	47,745	8.2291%	8.5771%	10,201.98	7,077.04	875.06	875.05
4	4,000	4,000	14,311	32,463	90,906	8.2288%	8.5768%	7,077.04	3,684.04	607.00	607.02
5	4,000	4,000	16,755	44,685	152,018	8.2283%	8.5763%	3,684.04	0.00	315.96	315.99

[1] Financial Accounting Standards Board, *Financial Account Series* No. 107-B, Proposed Statement of Financial Accounting Standards, "Accounting for Investments with Prepayment Risk--an amendment of FASB Statements No. 65 and 91", paragraphs 40-45.