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ABSTRACT

n-YEAR ROLL-FORWARD RESERVE FINANCING OF SOCIAL SECURITY

by
Cecil Nesbitt, Andrew Cohen, David Gallers, Brett Houghton, Angela Myler, Paul Weiss

The first section (1.1) of Part 1 of this paper discusses the concept of n-year roll-forward reserve financing, and contrasts it with classical pension funding. This section also indicates characteristics of systems to which n-year roll-forward reserve financing could be applied. The Old-Age, Survivors and Disability Insurance (OASDI) has actually operated on such roll-forward reserve financing, with a relatively low number for n, for over 50 years. For early years, such financing was called "partial reserve funding"; when n approximated zero, the financing was considered to be "pay-as-you-go"; with a contingency reserve approximating the forthcoming year's outgo, the financing was termed "current-cost". We now attempt to clarify the concept by discussing a reserve fund equivalent at the end of each year to the projected outgo for the following n years.

Choice of n will depend on balancing the degree of reserve fund build-up with the need for participants to have confidence in Social Security, and with the need of Congress to have time to act when the financing needs adjustment.

The Appendix indicates the basic data kindly provided to us by the Office of the Actuary, Social Security Administration, and upon which our study is based. The primary data we used was the projected taxable payroll for the years 1993-2067, the total annual outgo in each year of that period for benefits and administration, and the interest rates assumed for those years. These 75-year projections are based on three sets of assumptions developed by the Office of the Actuary, namely

Alternative I ----- optimistic set
Alternative II ---- intermediate set, reflecting the best judgement of the Trustees and their advisers
Alternative III --- pessimistic set

The projections based thereon are presented and discussed in the 1993 Trustees' Report for OASDI.

In Section 1.2, we discuss the problems arising from the present financing for OASDI, and its components OASI and DI. In each case there is projected a greater or lesser degree of build-up of trust funds, followed in most cases by rapid depletion. Such financing has been described as "roller-coaster funding". The problem is particularly immediate for DI.

Section 1.3 indicates how to solve the "roller-coaster funding" by n -year roll-forward reserve financing. One then encounters the problem of massive trust funds, and what is needed to handle them responsibly, and to control their build-up.

Part 2 presents an overview of our computations by means of graphs of the required reserves under n -year roll-forward financing for OASDI, OASI and DI, and graphs of the required annual income, exclusive of interest, expressed as percents of taxable payroll. These graphs emphasize the underlying exponential growth characteristics of the actuarial estimates, and lay a foundation for exploring means of stabilizing the funding at reasonable levels.

Part 3 deals with various actuarial aspects of n -year roll-forward reserve financing. Section 3.1 reviews the basic mathematical formulas. Section 3.2 gives an introduction to gain-and-loss analysis that could be used as the experience unfolds, and considers amortization of any deficit or surplus by means of a level percent of an increasing payroll. A problem in regard to such amortization is indicated in Section 3.3. The last section of this part, Section 3.4, lists references to studies of deviations from projected costs.

The Office of the Actuary also supplied us with copies of the Trustees' Reports for Hospital Insurance (HI) and for Supplementary Medical Insurance (SMI). From these and related information, we have projected 1-year roll-forward reserve financing of HI for the years 1993-2001. Our findings appear in Part 4.

In Part 5, we plan to summarize our conclusions from this study of the applicability of n -year roll-forward reserve financing for Social Security, and to indicate further questions to be studied. One important question concerns the ramifications of long-term amortization by means of a level percent of an increasing payroll. This may affect our views of Social Security financing.

The Appendix organizes the various tables that were computed in our study, and gives examples thereof. If you wish to study some or all of the detailed tables, contact the first author at the Department of Mathematics, University of Michigan, Ann Arbor, MI 48109-1003.

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n-Year Roll-Forward Reserve Financing of Social Security

by
Cecil Nesbitt, Andrew Cohen, David Gallers, Brett Houghton, Angela Myler, Paul Weiss

I. INTRODUCTION

This paper is a follow-up of [12, 1991 and 13, forthcoming] where some initial ideas of roll-forward reserve funding are explored and developed. As in last year's paper, this paper reports on the work of actuarial students and teachers for an 8-week study period coordinated with the National Science Foundation program of Research Experience for Undergraduate grants. The work was facilitated by a supply of the 1993 Reports of the Board of Trustees for Old-Age, Survivors and Disability Insurance (OASDI), and for the Medicare programs, Hospital Insurance (HI) and Supplementary Medical Insurance (SMI). Both the Office of the Actuary, Social Security Administration, and the National Academy of Social Insurance provided such reports. In addition, Richard Foster, Deputy Chief Actuary for OASDI, furnished a comprehensive set of computer printouts that led to the tables and graphs of the Trustees' Reports. These printouts aided us greatly to explore our financing concepts in close relation to the current actuarial projections for Social Security.

We begin with

1.1 n-Year Roll-Forward Reserve Financing vs. Classical Pension Funding Methods. Classical pension funding methods begin with a census of the active and the retired members of the pension system, including information about years of service, salaries, and prospective benefits. For this **closed group** of members there is next determined the **actuarial present value** (apv) of the future benefits that will be paid to members of the closed group over their remaining lifetimes. To finance this apv, there is considered to be available the fund on hand and the apv of the stream of **annual normal costs** that the funding method calls for in regard to members of the closed group. The excess of the apv of future benefits over the apv of future normal costs for these members is called the **accrued actuarial liability**, the **supplementary present value**, etc. We shall use the former term. The accrued actuarial liability serves as a measuring rod of the system's financial status. The excess of such accrued liability over the fund on hand is the **unfunded accrued actuarial liability**. The usual requirements are that the sponsor must finance the annual normal cost and amortize the unfunded actuarial liability. Adaptation to the emerging experience, and the changing composition of the closed group of current members is provided by periodic, often annual, actuarial valuations of the pension system.

In the calculation of these pension funding numbers, rates of interest, of survivorship, of disability, and of retirement are employed. Extensive computations are required but have been facilitated greatly by modern computing equipment. The computations may be carried through on an individual or on some aggregate basis.

One intrinsic difference between n-year roll-forward financing and classical pension funding is that for the former we consider the year-by-year stream of outgo for benefits and administrative costs for the **open** group of covered members, and the stream of income, exclusive of interest, required to meet such outgo. Necessarily, one starts with the current covered population and projects the growth year-by-year of the various categories of active members and beneficiaries. The balancing of income and outgo may be on a very short-range pay-as-you-go basis where income is almost immediately expended for outgo. Alternatively, the objective may be to always have on hand the value, discounted under interest, of the next n years' outgoes. If that objective is obtained by the end of the calendar year z, the fund on hand is theoretically sufficient to provide the outgo of years $z+1$, $z+2, \ldots, z+n$. Once such objective is reached, it may be maintained by requiring that the income for year $z+1$ be equivalent to the projected outgo for year $z+n+1$ (since outgo for year $z+1$ is already provided for by the fund at end of year z), and the process rolls forward to the end of year $z+1$. This is the essence of n-year roll-forward reserve financing.

Under classical pension funding, there may seem to be a high degree of individual equity under which the normal costs of an individual are accumulated in the pension fund for many years before being expended in benefits for that individual [see 6, 1979]. However, in a dynamic economic environment, such preservation of individual equity may be more apparent than real.

Under n-year roll-forward reserve financing, the emphasis is on building only sufficient reserve funds to maintain public confidence that benefits will be paid when due, and to provide sufficient time for the sponsor to make adjustments when experience so requires. For Social Security systems, n may well be less than 5 (even 1 or 2), for other mature public employee systems, n may be as large as 15 or 20. The primary aim will not be individual equity; instead, social adequacy will be paramount, and the fund will be kept only large enough to provide that benefits will be paid, and experience deviations can be accommodated.

In classical pension funding, the actuarial analysis produces a series of large and mystifying apv's and annual normal costs. In n-year roll-forward reserve financing, much actuarial analysis [see 1, 1987] is devoted to projecting the income and outgo year-by-year streams. These can be communicated readily to the sponsors of the benefit system.

Thereafter, it is a rather straight-forward problem in compound interest mathematics to calculate the required income and the required reserve fund to achieve the desired balance of the streams.

1.2 Roller-Coaster Financing of OASDI, OASI, and DI. Figure 1.2.1, Projected Trust Fund for OASDI under Current Law, portrays the build-up and eventual depletion of the OASDI Trust Fund if future experience follows Alternative I, II, or III assumptions. Under Alternative I, the Trust fund is projected to increase steadily to at least year 2065, and depletion does not occur. Under Alternative II, the intermediate set of assumptions, the maximum fund of $4.97 trillion is projected for year 2024, and depletion is projected in year 2036. Under Alternative III, the pessimistic set, the maximum is projected for year 2007, and depletion for year 2017. By now the term "roller-coaster financing" should seem justified.

Corresponding information is given in Figure 1.2.2 for OASI, and in Figure 1.2.3 for DI. To the authors of this paper, it seems worthwhile to have information on trust fund growth for both the combined OASDI, and the two separate programs, OASI and DI. This should aid in understanding how the component systems are functioning.

1.3 A Mathematical Remedy for Roller-Coaster Funding. The current funding for OASDI and its components, is certainly not classical pension funding. It could, however, be converted into n-year roll-forward reserve financing by various means. For the present discussion, let us think in terms of the combined OASDI system, and its projections under Alternative II. Conversion could be accomplished by the following steps:

(1) For the selected value of n, await for the year z such that the OASDI Trust Fund at the end of year z approximates the required fund equal to the present value of the projected outgoes for the years $z+1$, $z+2$, ..., $z+n$. During those n years, the fund is expected, with interest, to provide the outgoes of those years.

(2) For the successive calendar years, $z+k$ (k=1,2,..., n) provide the required income equal to the present value of the outgo for the year $z+k+n$.

Thereby, the fund in year $z+k$ provides for the outgo of that year and is replenished by the income that will accumulate under interest to the outgo for year $z+k+n$. In this way, the reserve rolls forward and at the end of any year $z+k$ in the period (z, $z+n$) is equivalent to the present value of the outgoes for the years n in the period ($z+k+1$, $z+k+n$).

We have studied the required year-end reserves and required yearly incomes for OASDI financing by roll-forward reserves for n=1, 2, 3, or 4, and for calendar years in the interval (1993-2067). Also, we have on hand the projected income and outgo under present law. Thus, we have the means of studying feasible conversions to roll-forward reserve financing for n=1, 2, 3, or 4, by way of steps (1) and (2) indicated above.

In Part 3, there will be suggested another means of converting to roll-forward reserve financing that does not involve step (1).

Figure 1.2.1
OASDI Fund Balances

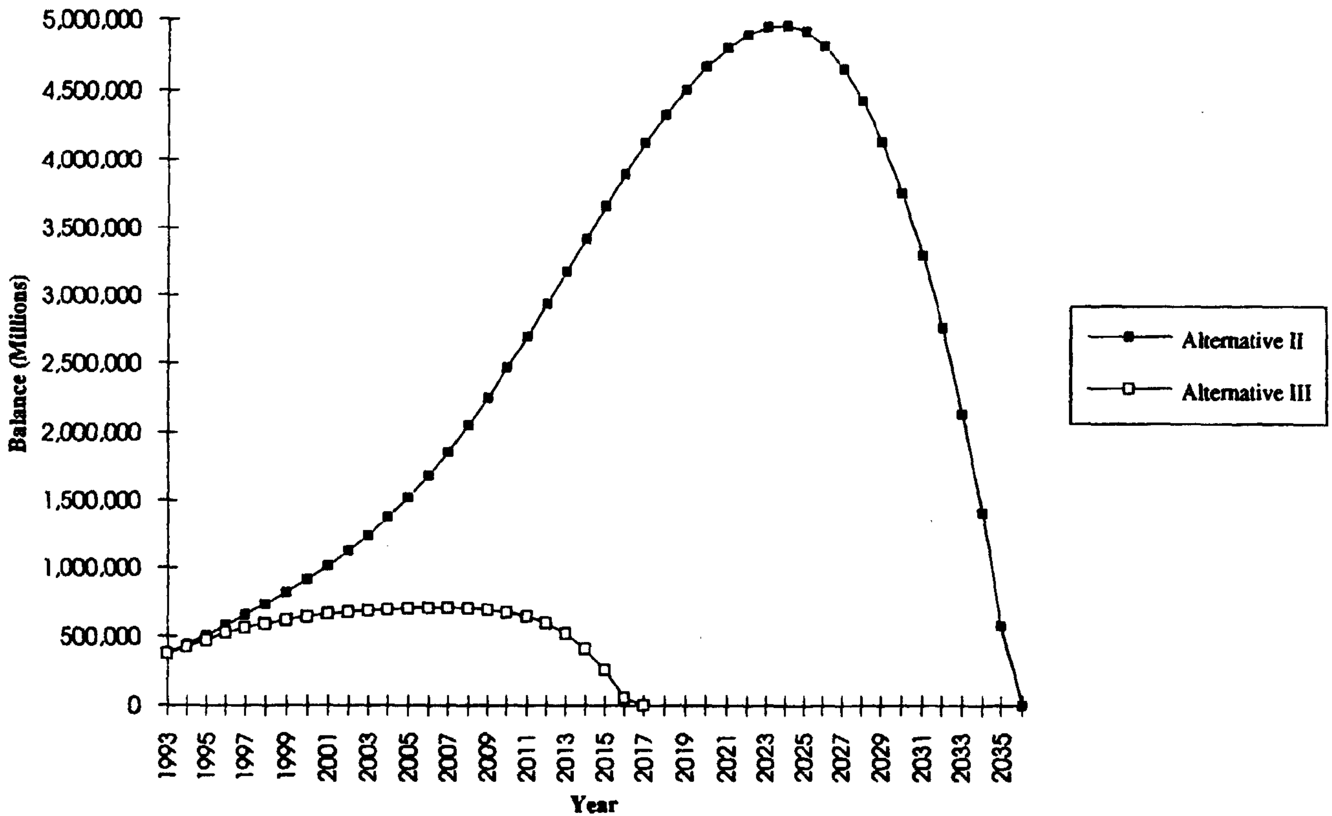


Figure 1.2.2
OASI Fund Balances

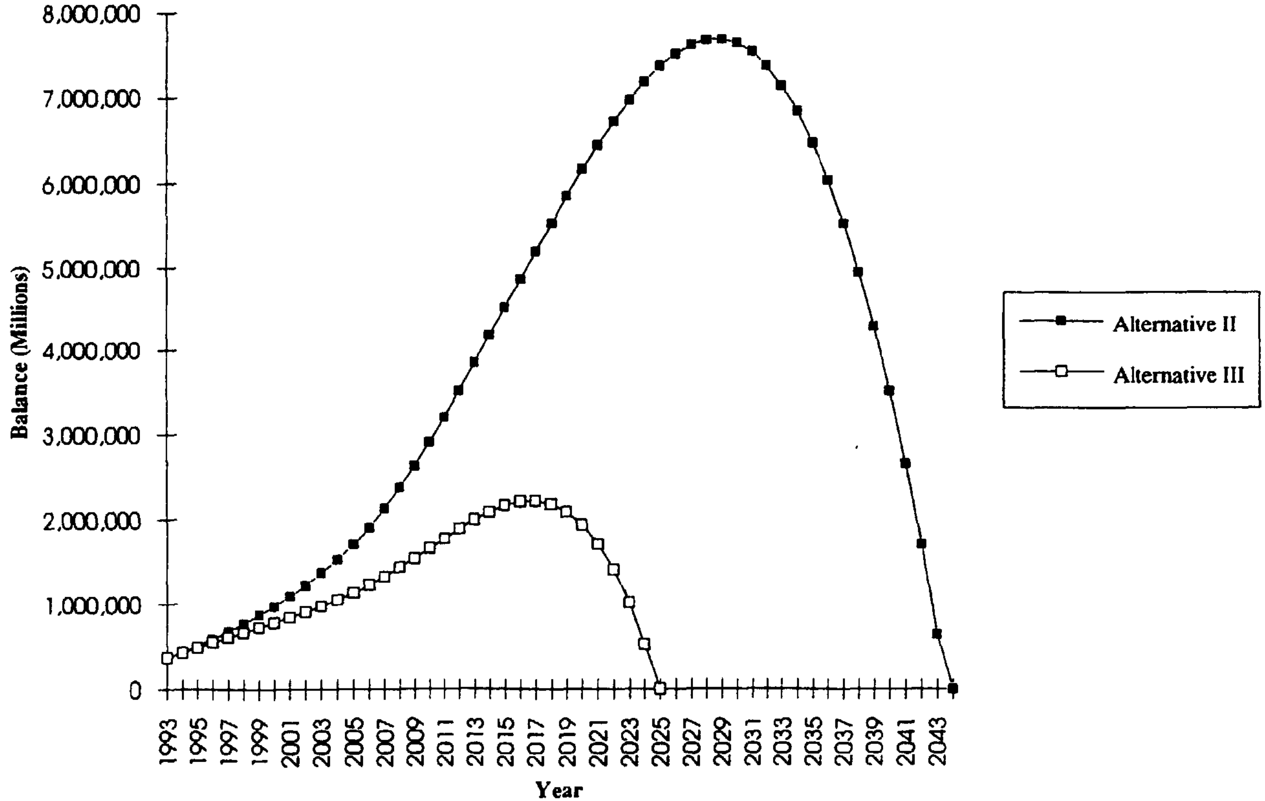
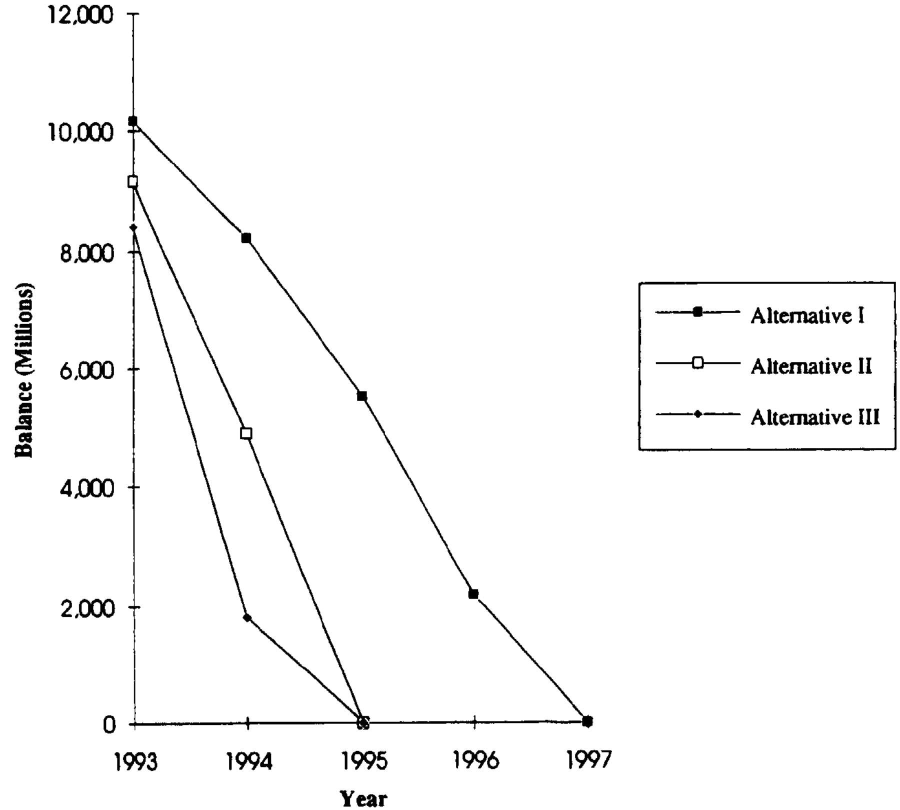


Figure 1.2.3
DI Fund Balances



II. OVERVIEW

By means of graphs, this part will give an overview of the n-year roll-forward reserve computations performed for OASDI, and its components OASI and DI. These were performed under the assumption of Alternative I, II and III and for n=1, 2, 3 and 4, resulting in 3×3×4=36 tables. Each table has columns of:

k to denote calendar year

O_k=outgo for benefits and administration in calendar year k

W_k=taxable payroll in calendar year k

${}_nI_k$=income exclusive of interest required in calendar year k to support n-year roll-forward reserves

${}_nA_{12/31/k}$=required fund at end of calendar year k under the discipline of n-year roll-forward reserves

$({}_nI_k/W_k)$=ratio of required income to taxable payroll, expressed as a percentage

Copies of some or all of these detailed tables may be obtained by a request to Cecil Nesbitt, Department of Mathematics, University of Michigan, Ann Arbor, MI 48109-1003.

2.1 Graphs for OASDI Figures 2.1.1 to 2.1.3 exhibit the required funds, ${}_nA_k$, for n=1, 2, 3 and 4 and for years 1993-2067, under Alternatives I, II and III respectively. These graphs indicate clearly the exponential growth of the required funds, the rate of growth varying directly with the magnitude of n. A basic problem in the next century will be how to stabilize, or at least moderate, such exponential growth.

Figures 2.1.4 to 2.1.6 exhibit the ratio of required income to taxable payroll, $({}_nI_k/W_k)$, expressed as a percentage for n=1, 2, 3 and 4, the years 1993-2067, and under Alternatives I, II and III respectively. Three observations stand out:

(1) The curves for n=1, 2, 3 and 4 are most distinct for Alternative I, and almost merge under Alternative III

(2) The curves for Alternatives I and II intertwine somewhat similarly in respect to the value of n.

(3) The Alternative I curves stabilize at about the 12% level, but those for Alternatives II and III have an increasing level at the longer duration.

2.2 Graphs for OASI These are given in Figures 2.2.1 to 2.2.6 and show somewhat similar features as those for OASDI, of which OASI is the dominant component.

2.3 Graphs for DI The required fund graphs, Figures 2.3.1 to 2.3.3 show exponential growth as was the case for OASDI and OASI. The ratio of required income to taxable payroll graphs, figures 2.3.4 to 2.3.6 are much flatter that those for OASDI and OASI, with only the graph for Alternative III retaining some increasing trend. The graphs are not differentiated much by the value of n.

Figure 2.1.1
Required Fund vs. Time For OASDI Under Alternative I

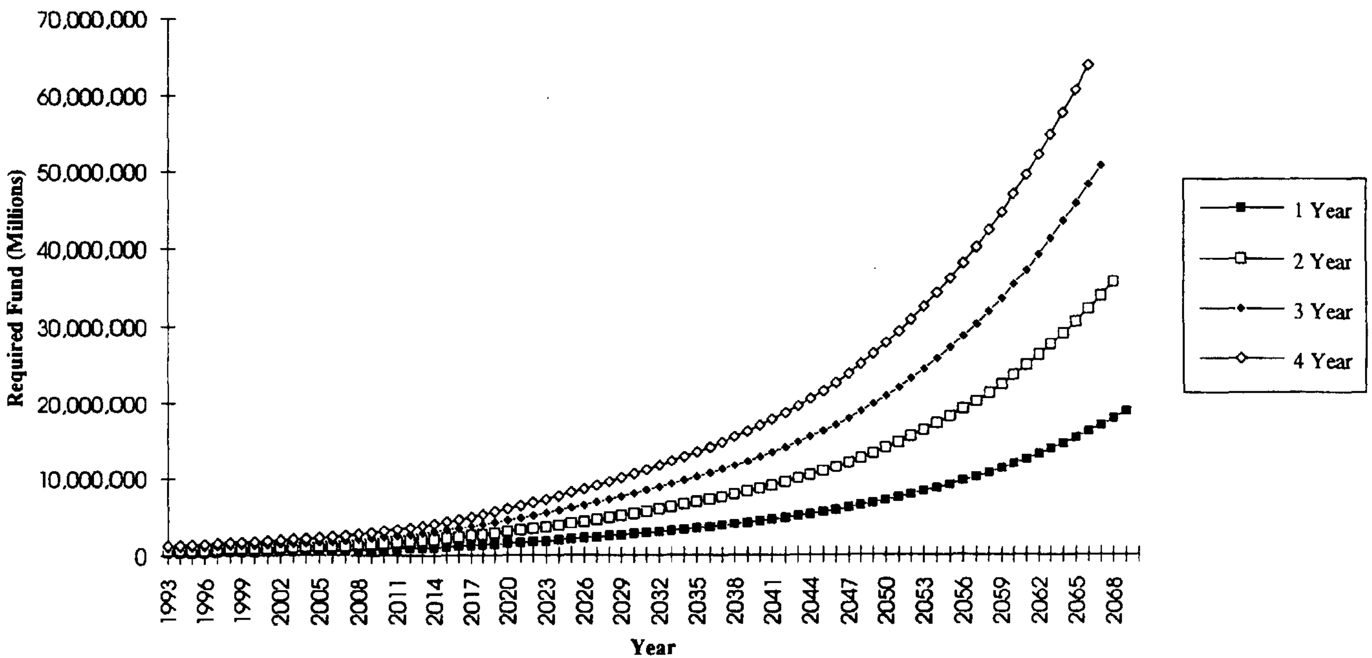


Figure 2.1.2
Required Fund vs. Time For OASDI Under Alternative II

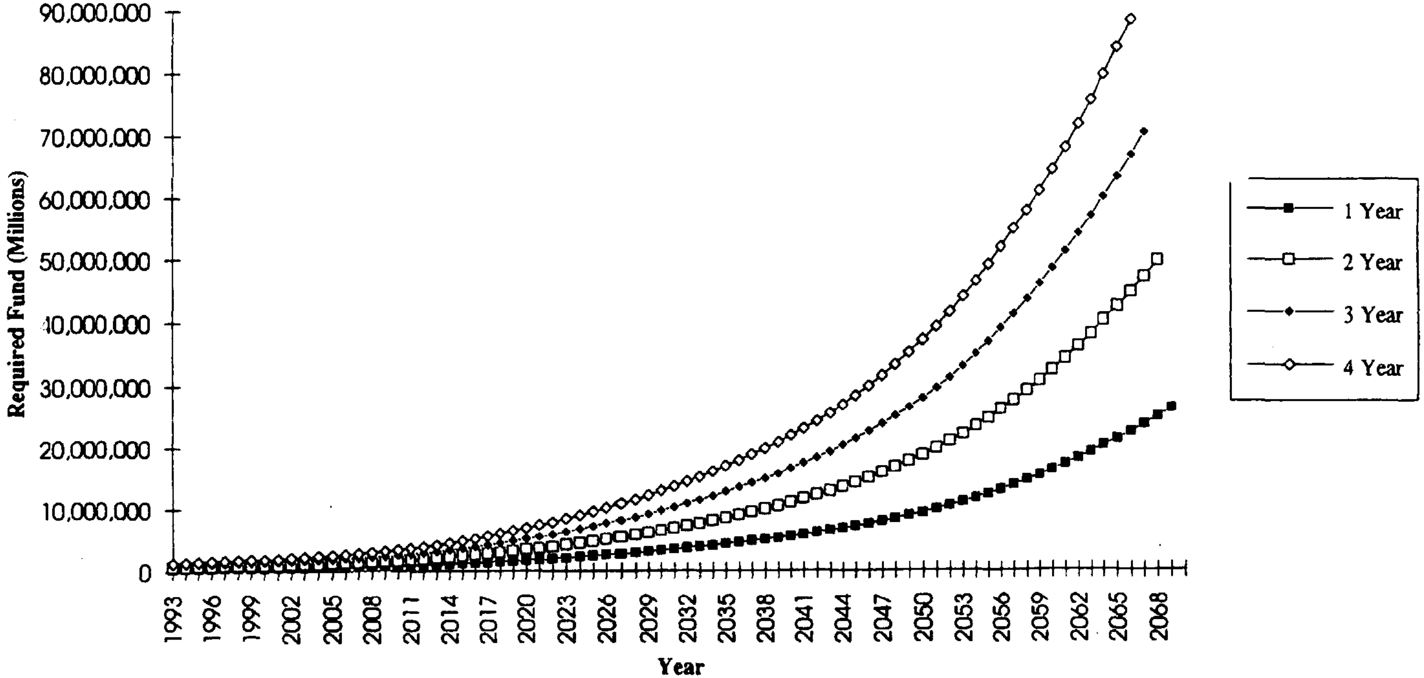


Figure 2.1.3
Required Fund vs. Time For OASDI Under Alternative III

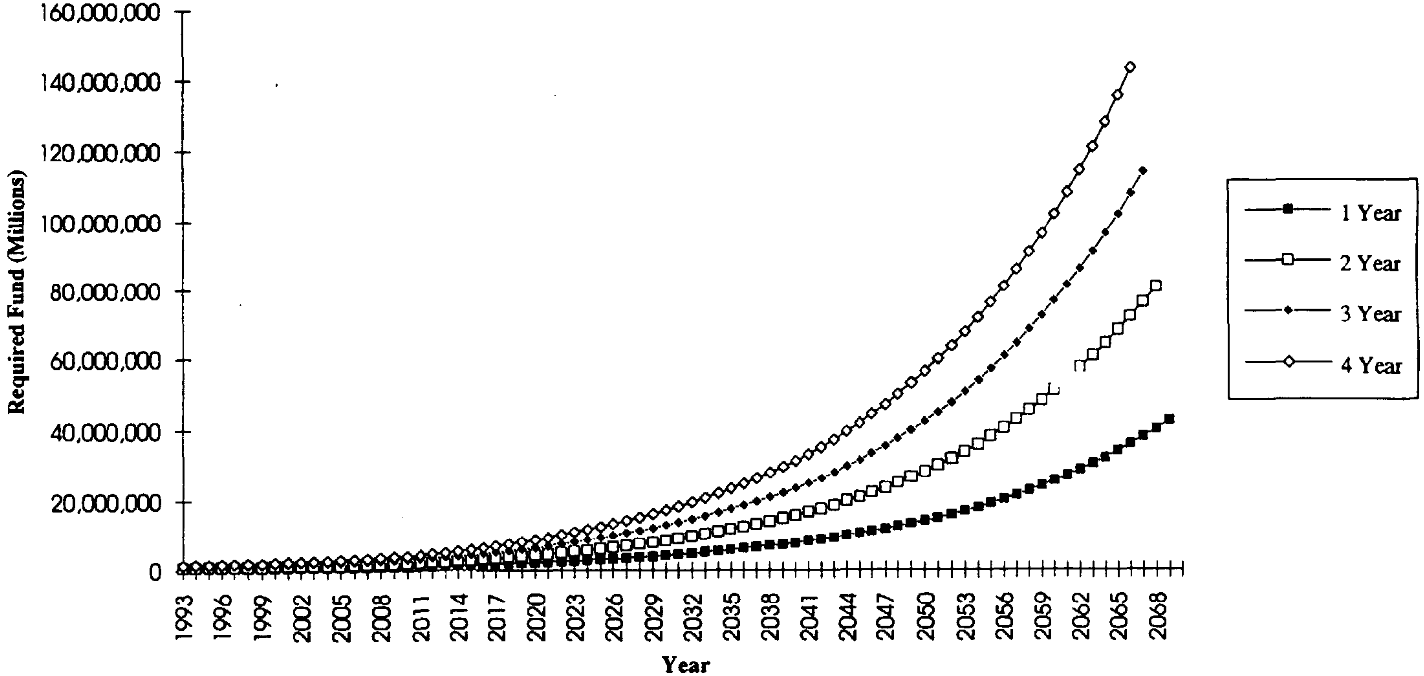


Figure 2.1.4
Cost Percentages vs. Time For OASDI Under Alternative I

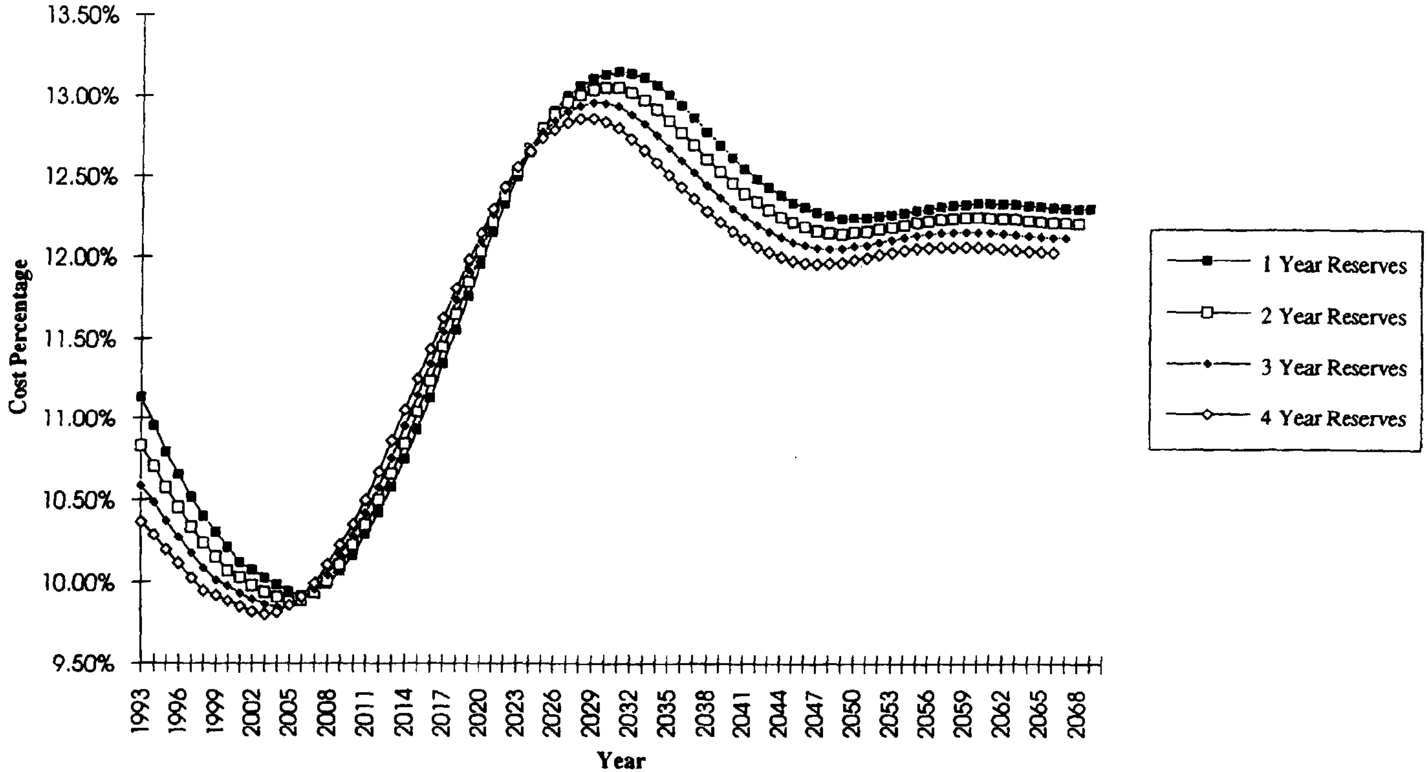


Figure 2.1.5
Cost Percentages vs. Time For OASDI Under Alternative II

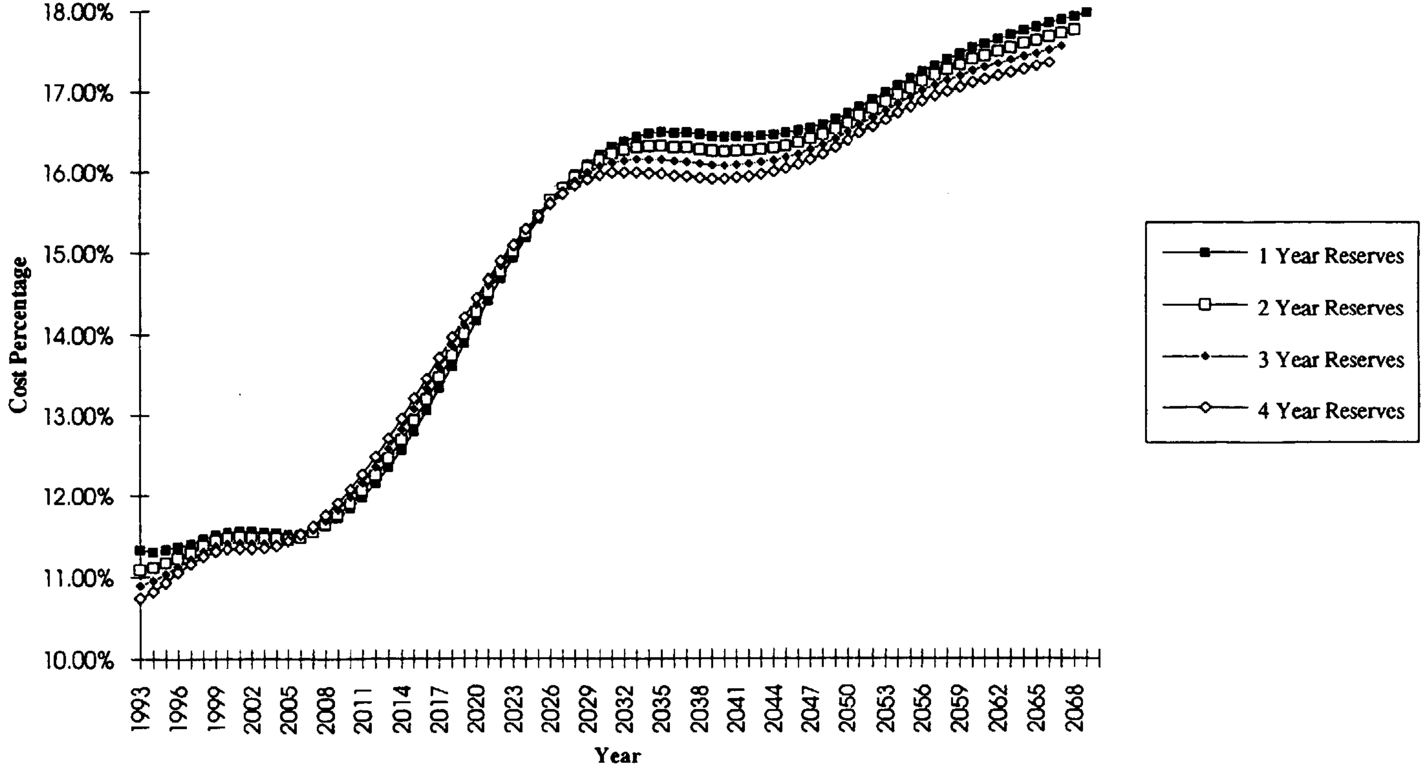


Figure 2.1.6
Cost Percentages vs. Time For OASDI Under Alternative III

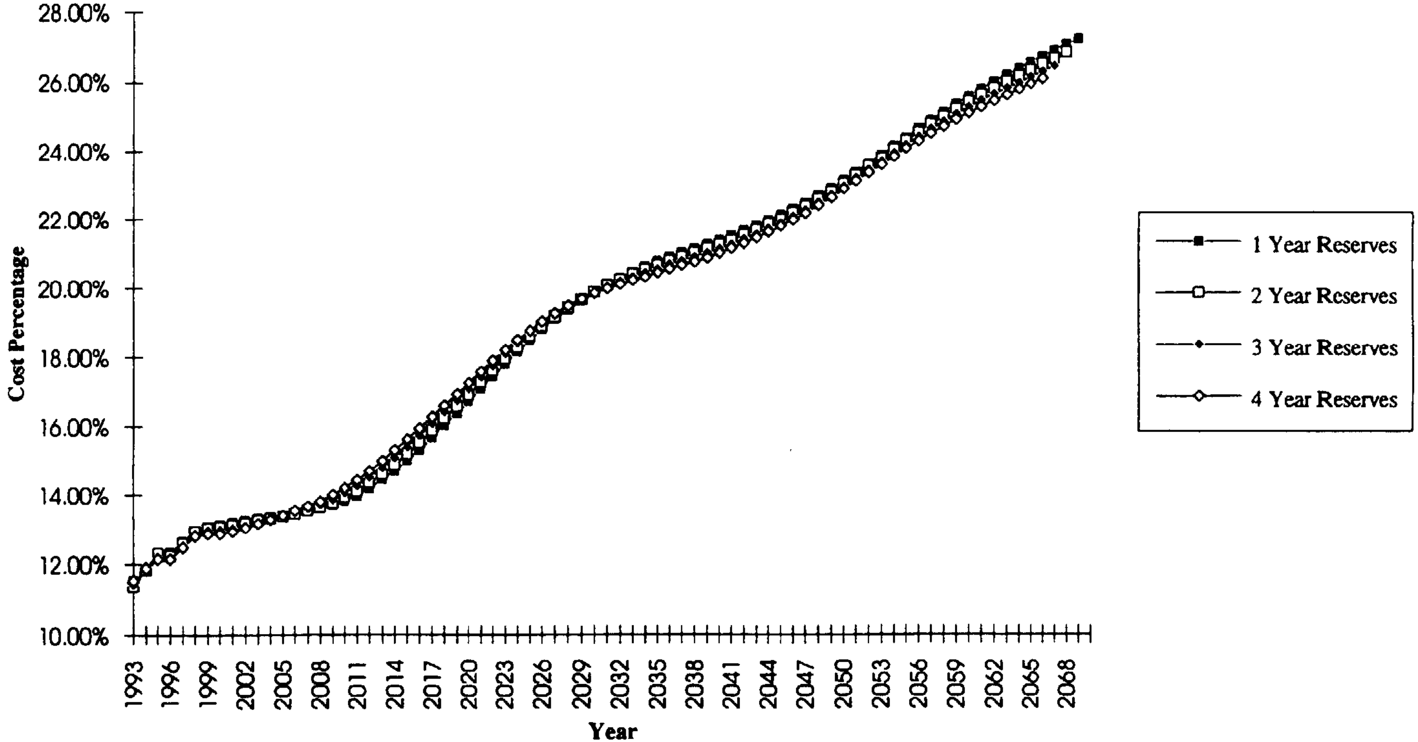


Figure 2.2.1
Required Fund vs. Time For OASI Under Alternative I

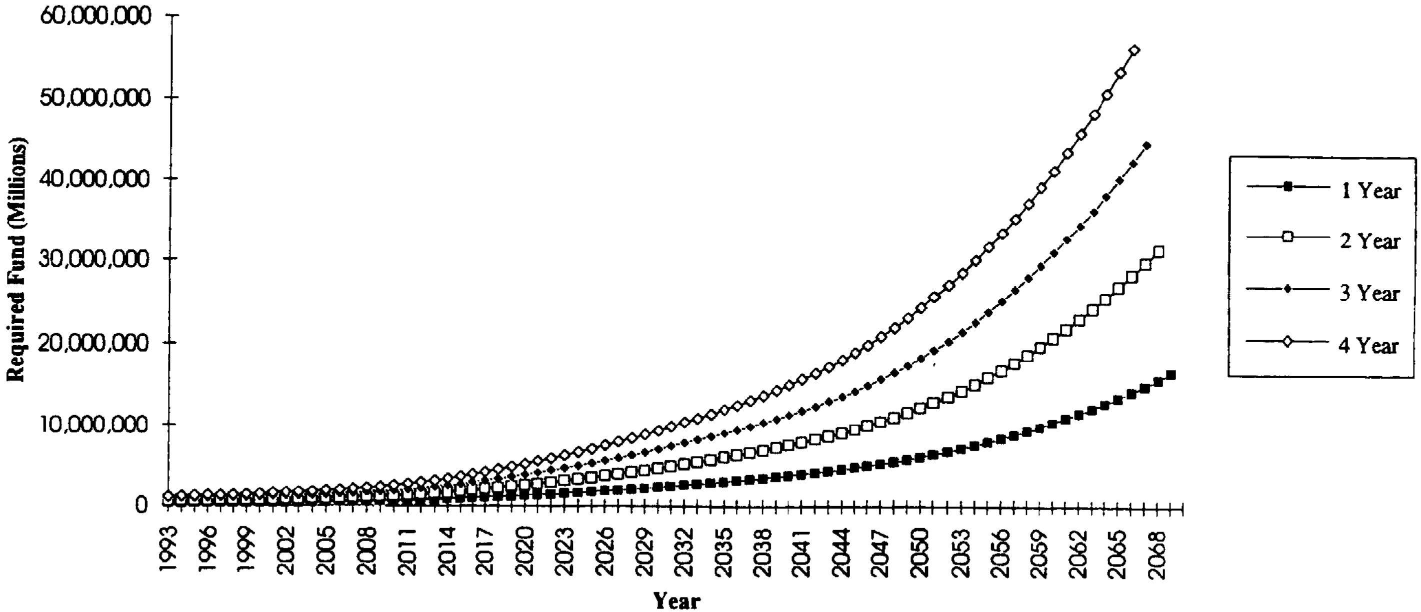


Figure 2.2.2
Required Fund vs. Time For OASI Under Alternative II

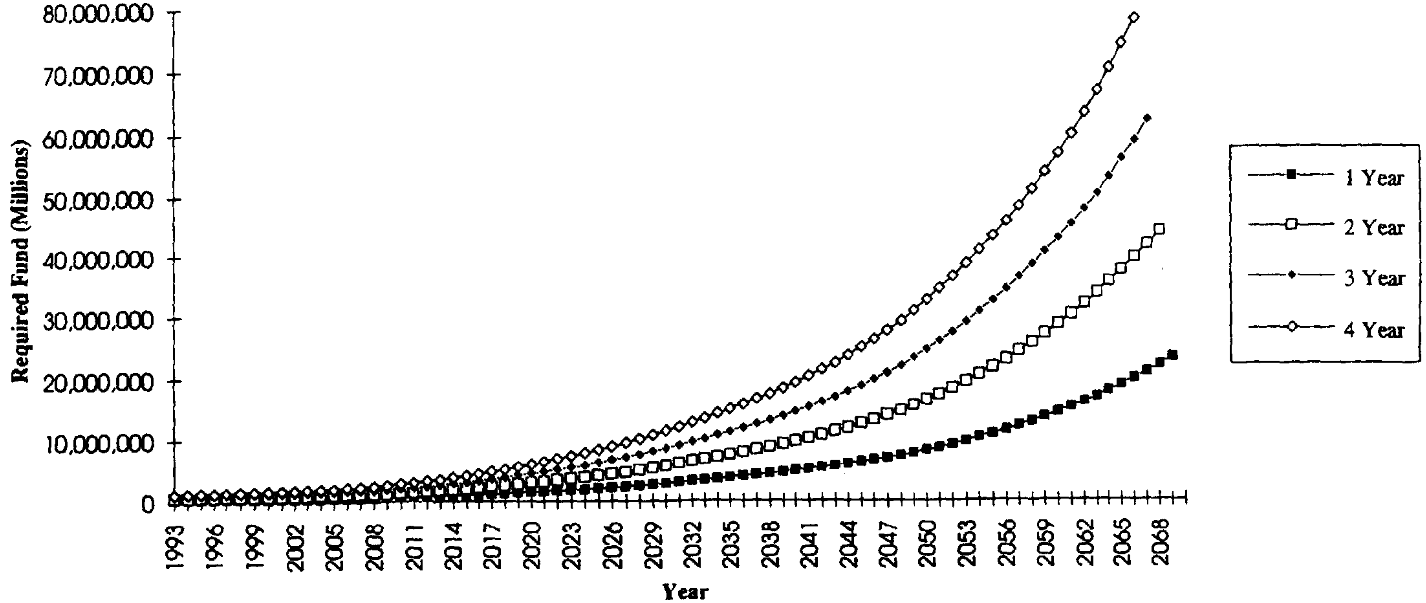


Figure 2.2.3
Required Fund vs. Time For OASI Under Alternative III

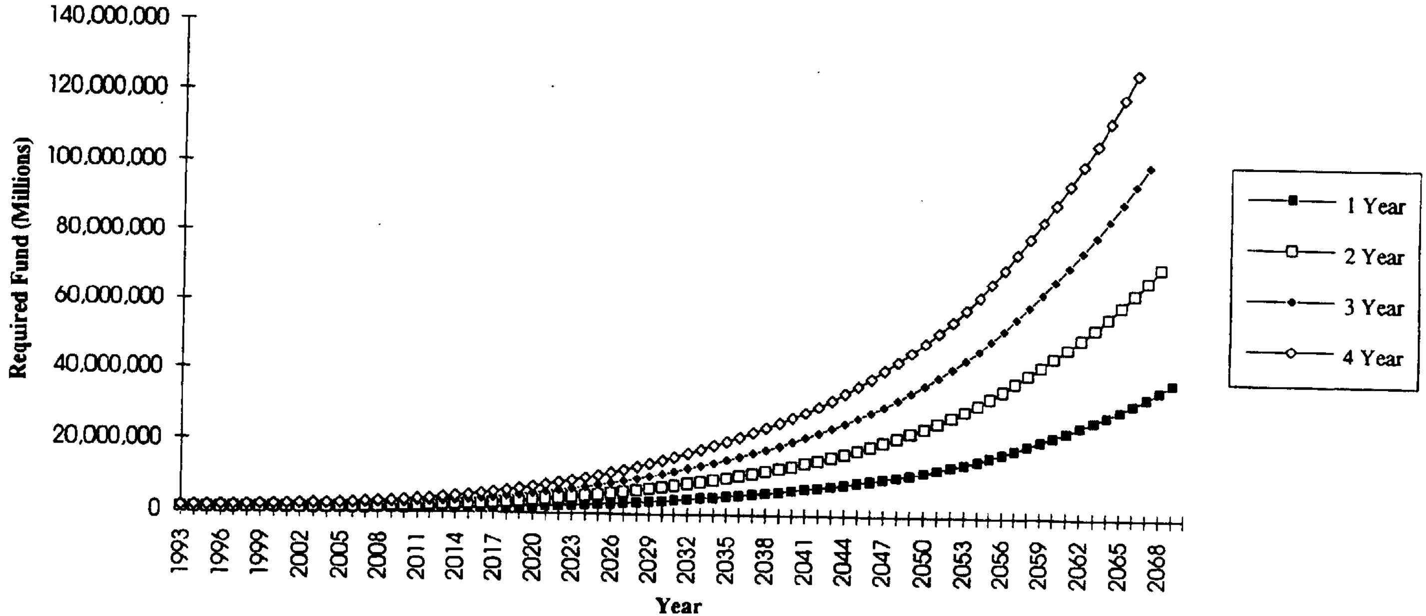


Figure 2.2.4
Cost Percentages vs. Time For OASI under Alternative I

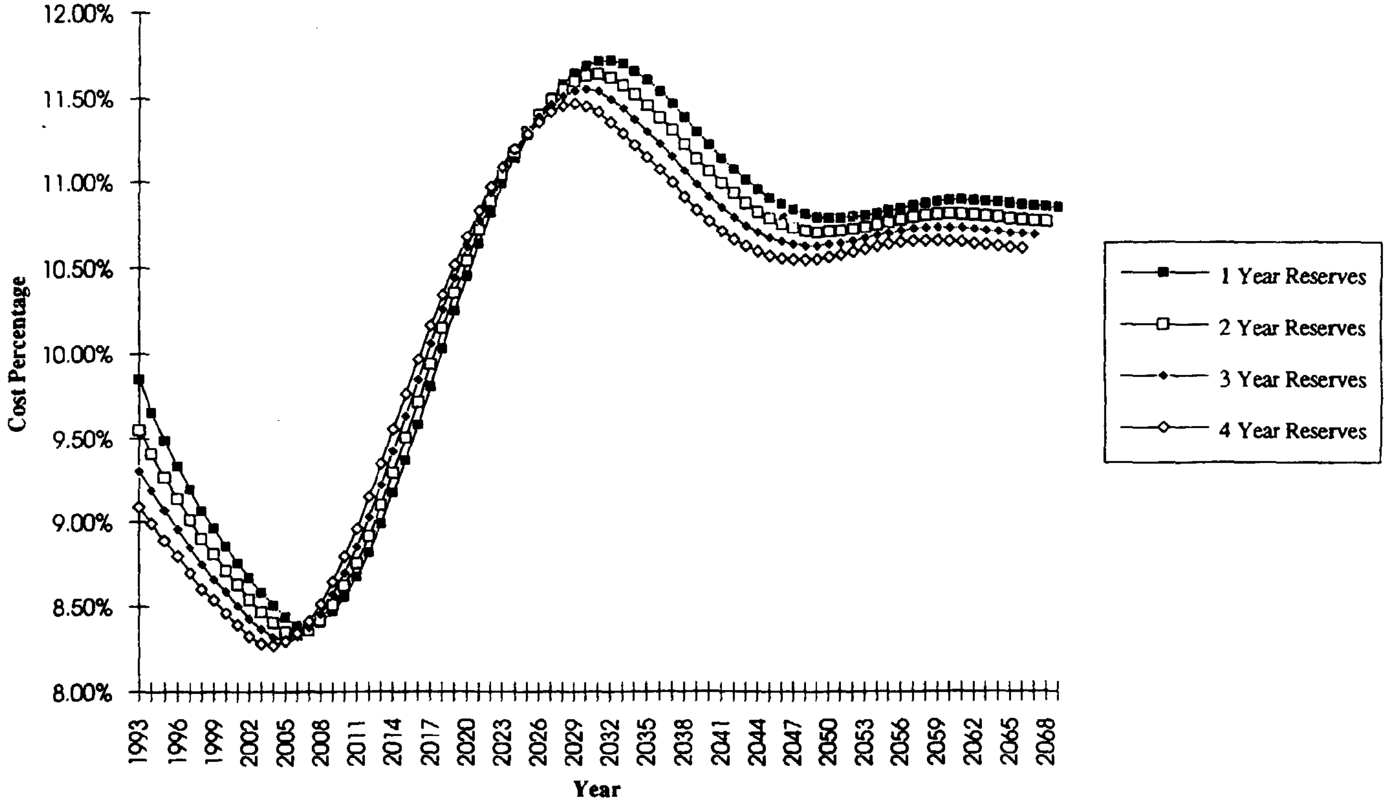


Figure 2.2.5
Cost Percentages vs. Time For OASI under Alternative II

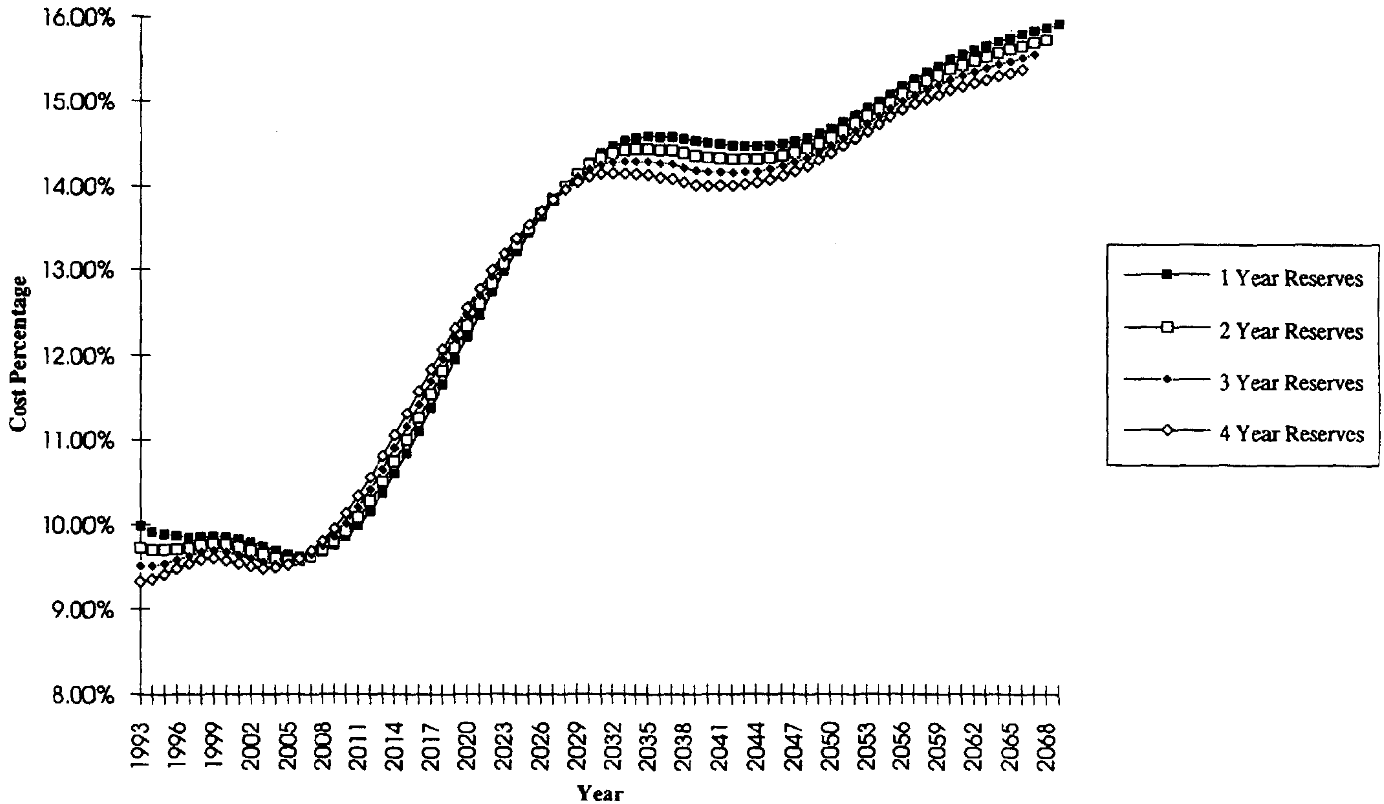


Figure 2.2.6
Cost Percentages vs. Time For OASI under Alternative III

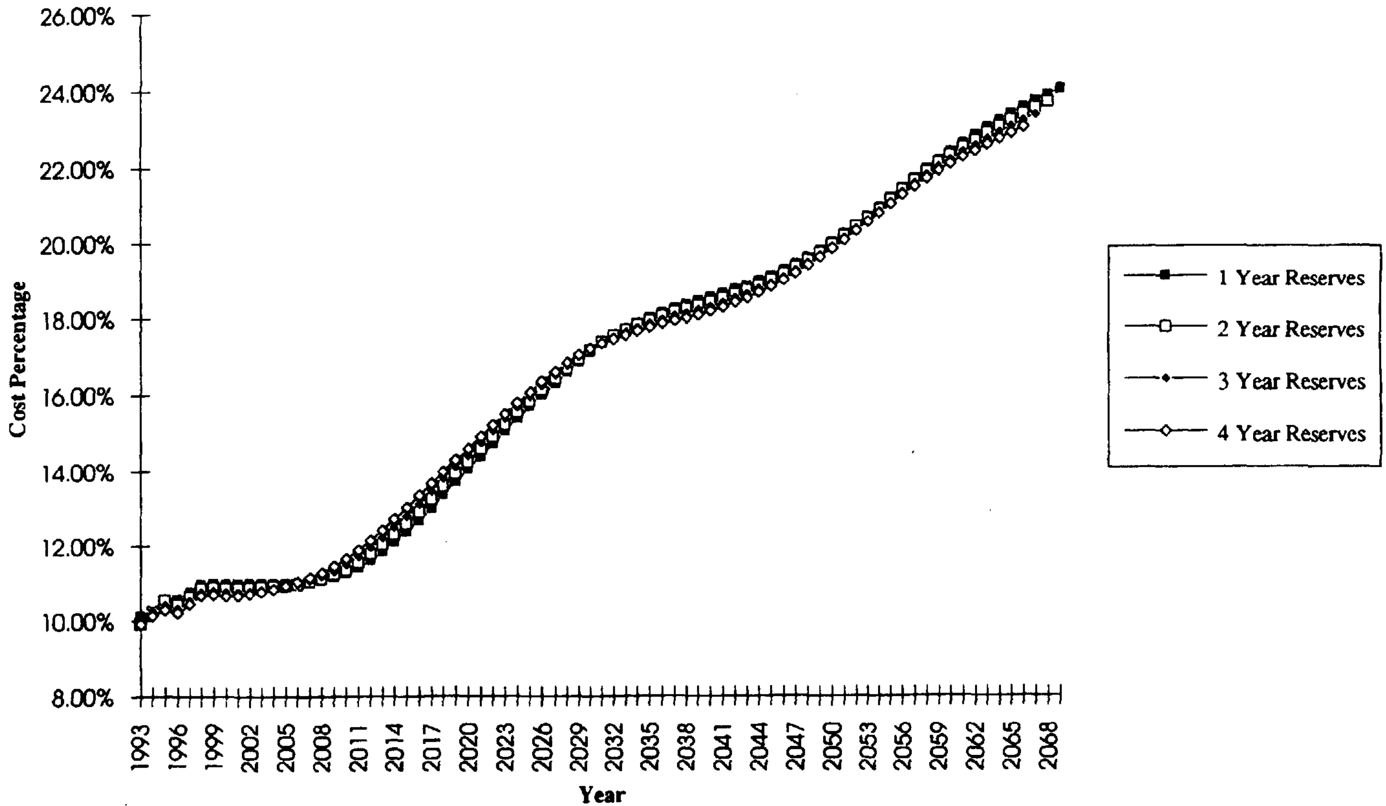


Figure 2.3.1
Required Fund vs. Time For DI Under Alternative I

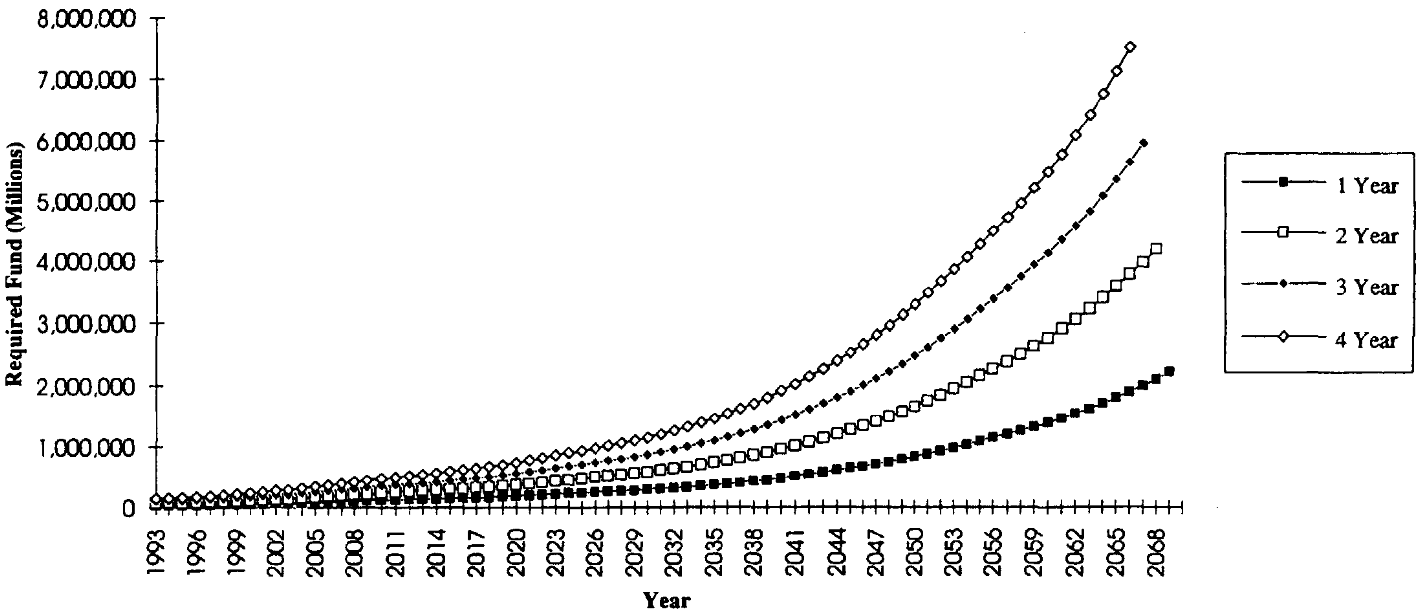


Figure 2.3.2
Required Fund vs. Time For DI Under Alternative II

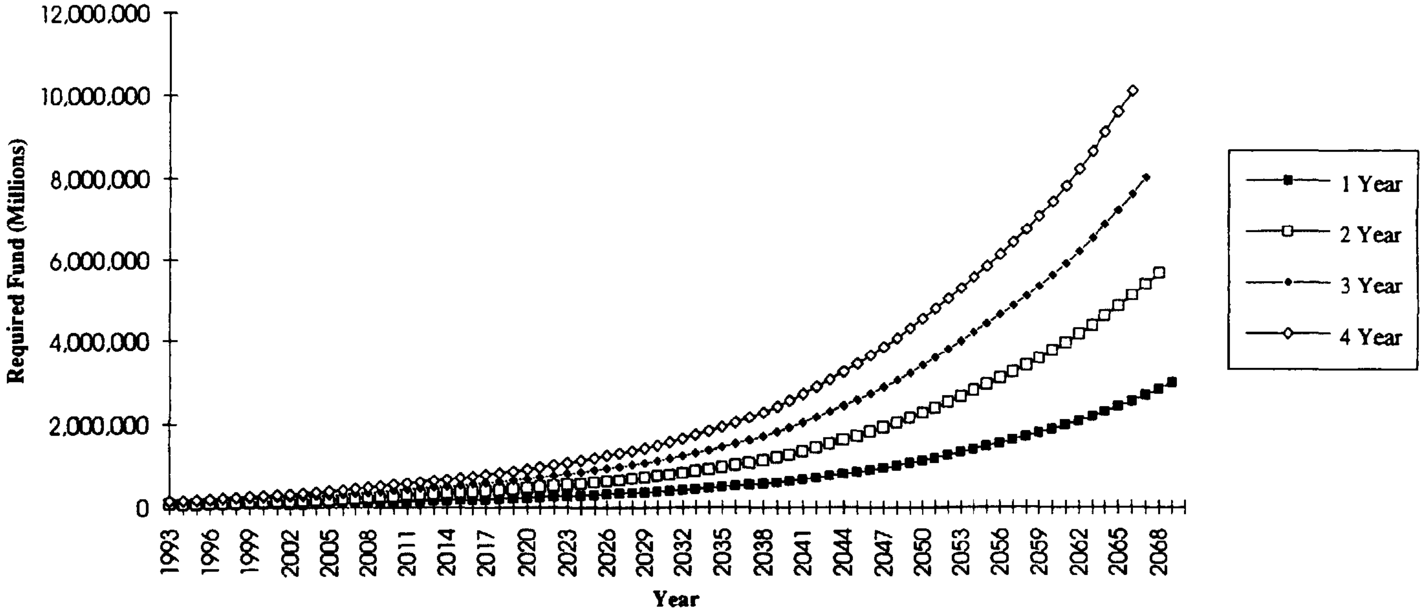


Figure 2.3.3
Required Fund vs. Time For DI Under Alternative III

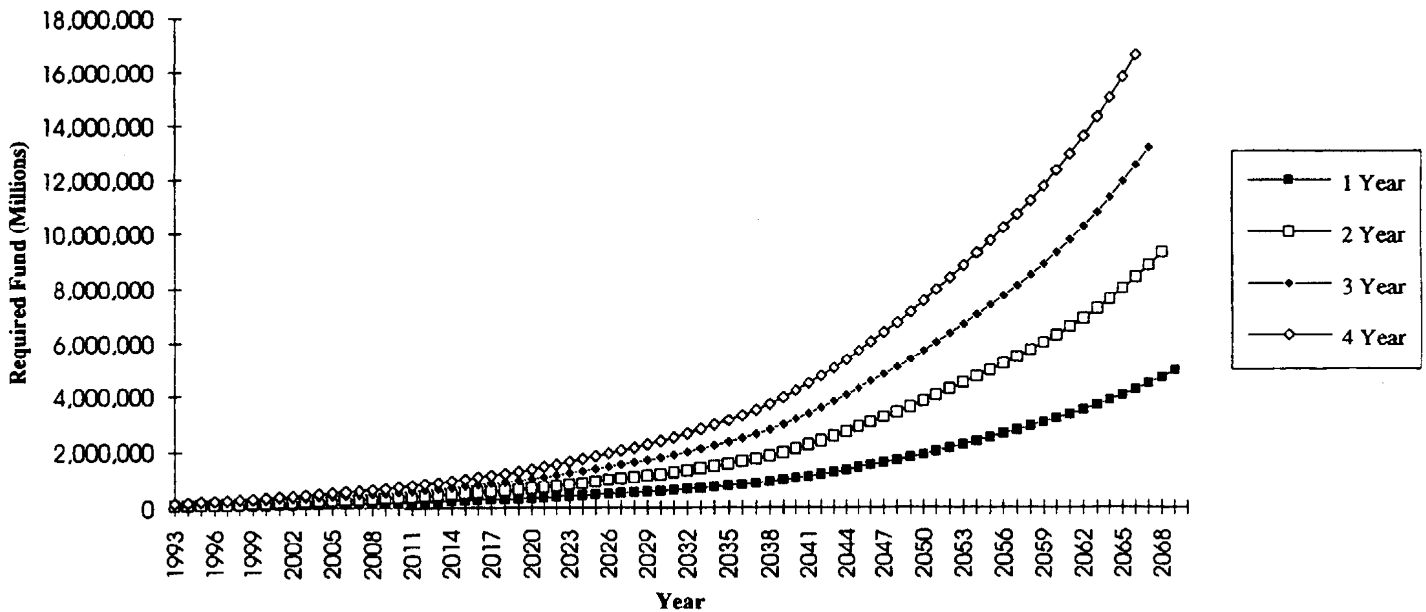


Figure 2.3.4
Cost Percentages vs. Time For DI under Alternative I

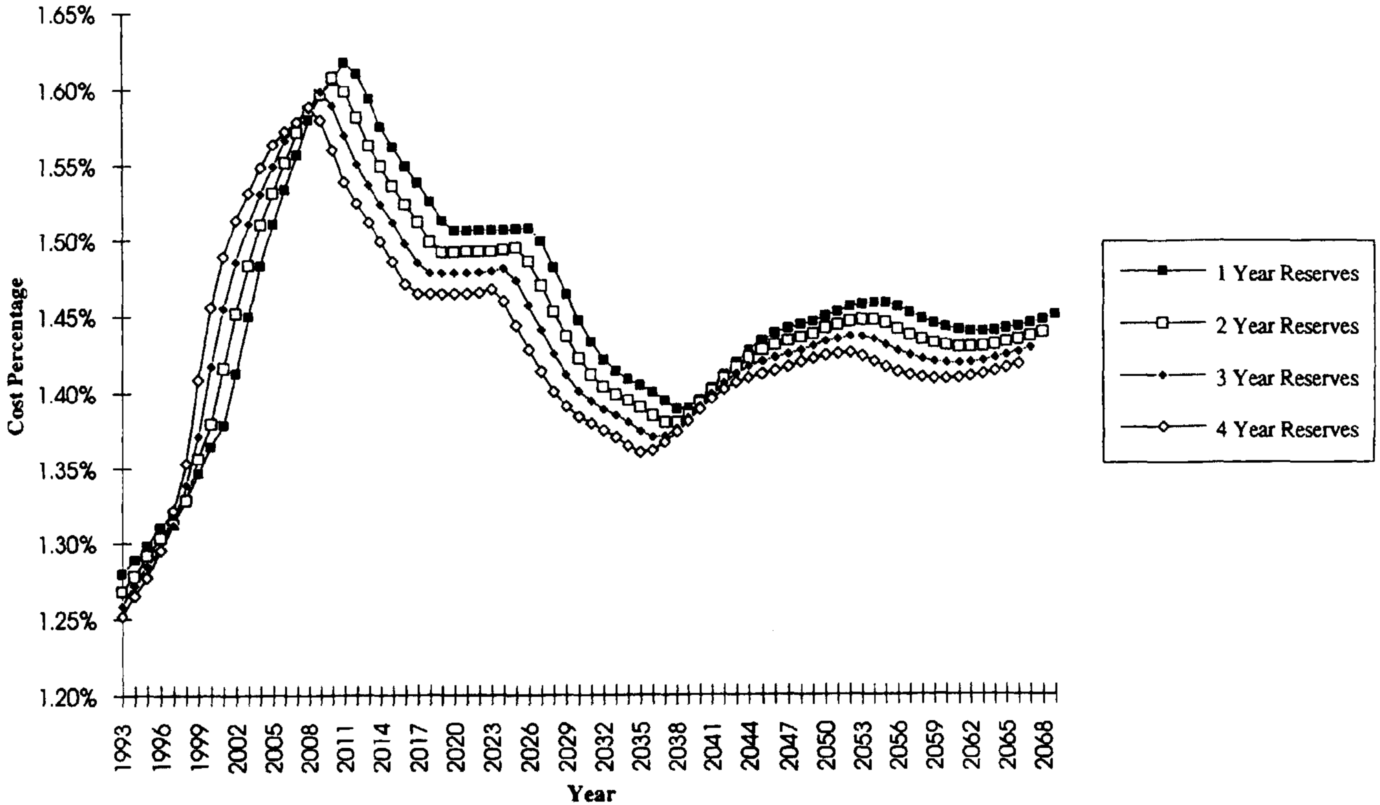


Figure 2.3.5
Cost Percentages vs. Time For DI under Alternative II

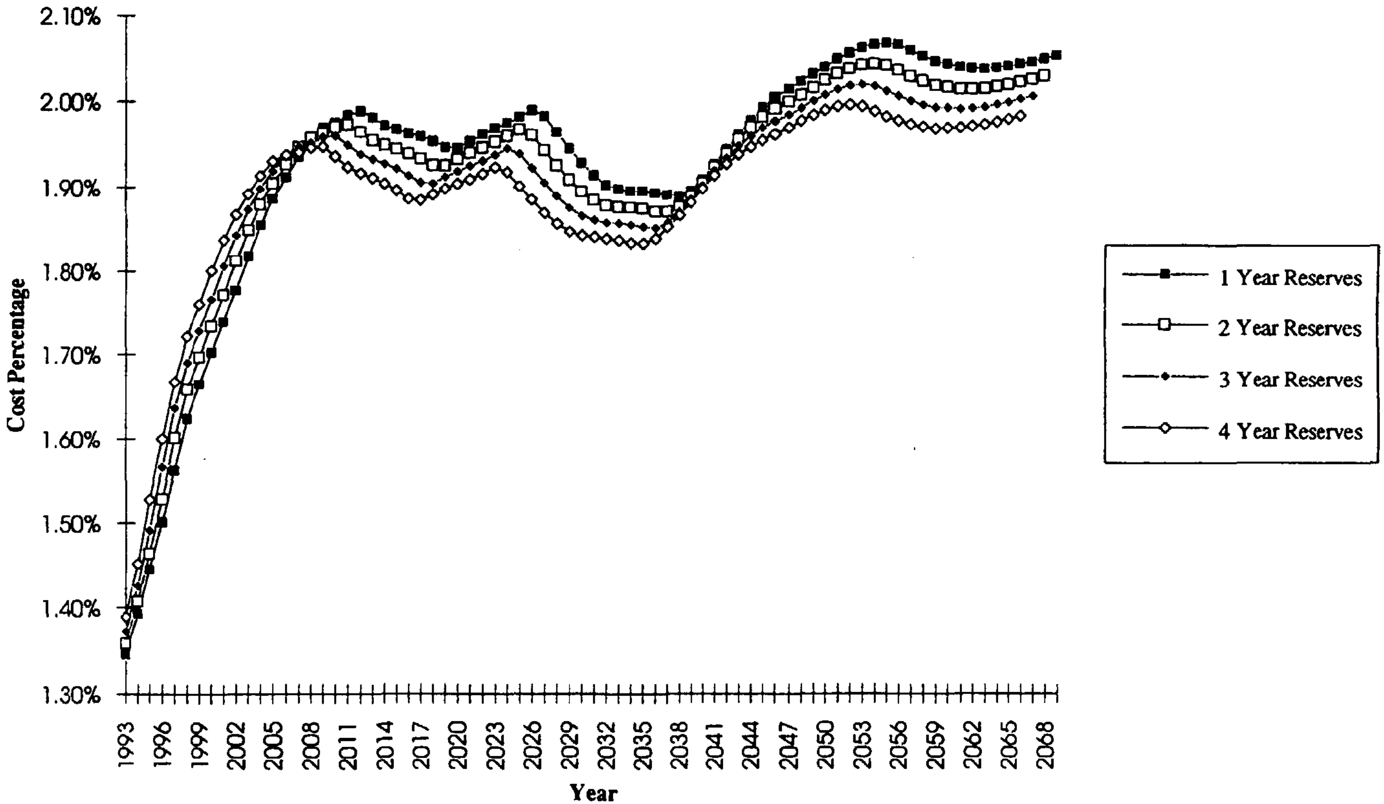
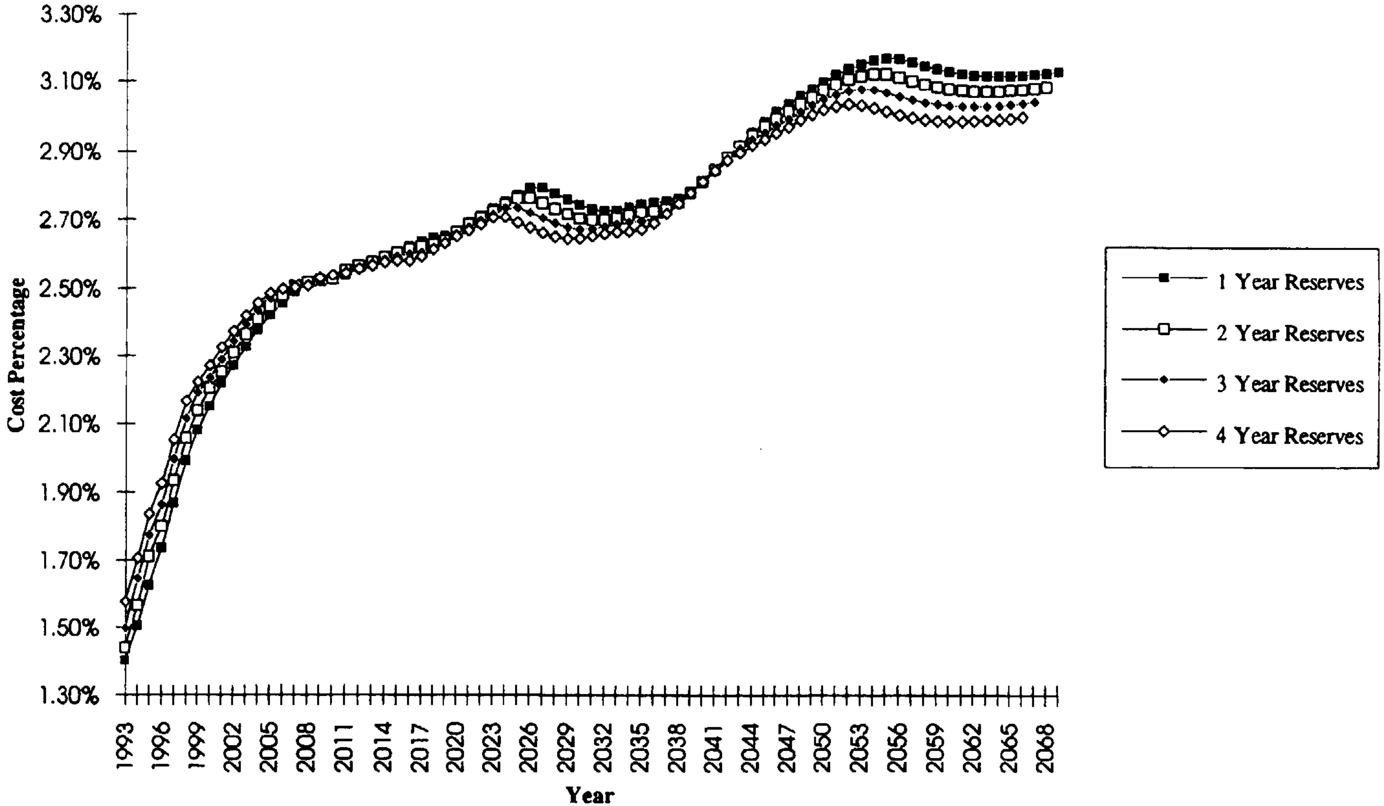


Figure 2.3.6
Cost Percentages vs. Time For DI under Alternative III



III. ACTUARIAL MATHEMATICS FOR *n*-YEAR ROLL-FORWARD RESERVES

This has already been treated in Sections 2 and 6 of [13, forthcoming]. In the present paper, notation has been re-stated in Part II. In Section 3.1, we restate the formulas we have used to compute ${}_nA_{12/31/k}$ and ${}_nI_k$.

3.1 Formulas For simplicity, we assume δ_k is constant at force δ, and leave it to the reader to make the easy extension to the more detailed formulas with variable forces. Then,

$$ {}_nA_{12/31/k} = O_{k+1}e^{-\delta/2} + O_{k+2}\,e^{-3\delta/2} + ... + O_{k+n}\,e^{-(2n-1)\delta/2} \tag{3.1.1}$$

If in the n-year interval O_{k+j} grows at the constant rate τ, that is

$$ O_{k+j} = O_{k+1}e^{(j-1)\tau} \tag{3.1.2}$$

then

$$ {}_nA_{12/31/k} = \ddot{a}_{\overline{n}|}O_{k+1}e^{-\delta/2} \tag{3.1.3}$$

where $\ddot{a}_{\overline{n}|}$ is calculated at force $(\delta-\tau)$, (or at the equivalent interest rate). If $\tau = \delta$, then $\ddot{a}_{\overline{n}|} = n$, and

$$ {}_nA_{12/31/k} = n \cdot O_{k+1}e^{-\delta/2} \tag{3.1.4}$$

which is larger than the quantity in (3.1.3) if $\delta > \tau$. If $\tau > \delta$, formula (3.1.3) becomes

$$ {}_nA_{12/31/k} = \ddot{s}_{\overline{n}|}O_{k+1}e^{-\delta/2} \tag{3.1.5}$$

which is larger still, as we would expect.

For ${}_nI_k$ we have the formula

$$ {}_nI_k = O_{k+n}e^{-n\delta} \tag{3.1.6}$$

In the simplified exponential case, with interest at a constant force δ, with outgo increasing at the constant force τ, and taxable payroll increasing at the constant force σ, we have

$$
\begin{aligned}
{}_nI_{k+j} / W_{k+j} &= O_{k+j+n}\, e^{-n\,\delta} / (W_k\, e^{j\sigma}) \\
&= (O_{k+n}\, e^{j\tau})\, e^{-n\delta} / (W_k\, e^{j\,\sigma}) \\
&= e^{(\tau-\sigma)j} ({}_nI_k \,/\, W_k) \qquad (3.1.7)
\end{aligned}
$$

[compare sec.6 of 13]. When τ and σ are approximately equal over a period of years, formula (3.1.7) may explain how ${}_nI_{k+j} / W_{k+j}$ varies.

3.2 Introduction to Gain and Loss Analysis For n-Year Roll-Forward Reserves We start with the recursion formula (6.10) of [13, forthcoming], namely

$$
{}_nA_{12/31/k-1} \cdot e^{\delta} + ({}_nI_k - O_k)e^{\delta/2} = {}_nA_{12/31/k}
$$

and rewrite it for year k+1, with force of interest δ_{k+1} (here denoted simply by δ) as

$$
{}_nA_{12/31/k} \cdot (1 + e^{\delta} - 1) + ({}_nI_{k+1} - O_{k+1})(1 + e^{\delta/2} - 1) = {}_nA_{12/31/k+1} \qquad (3.2.1)
$$

We designate the actual values on hand with a preceding superscript 'a', to get

$$
{}^a_nA_{12/31/k} \cdot (1 + e^{^a\delta} - 1) + ({}^a_nI_{k+1} - {}^aO_{k+1})(1 + e^{^a\delta/2} - 1) - {}^a_nM_{k+1} = {}^a_nA_{12/31/k+1} \qquad (3.2.2)
$$

where the term ${}^a_nM_{k+1}$ denotes the miscellaneous year-end losses, including those from modifications of the system that become effective in the year k+1.

Denote the difference ${}_nA_{12/31/k} - {}^a_nA_{12/31/k}$ by $D_{12/31/k}$ Then on subtracting (3.2.2) from (3.2.1) and rearranging, we have

$$
\begin{aligned}
D_{12/31/k} &+ ({}_nI_{k+1} - {}^a_nI_{k+1}) + ({}^aO_{k+1} - O_{k+1}) + [{}_nA_{12/31/k}(e^{\delta} - 1) - {}^a_nA_{12/31/k}(e^{^a\delta} - 1) + \\
&({}_nI_{k+1} - O_{k+1})(e^{\delta/2} - 1) - ({}^a_nI_{k+1} - {}^aO_{k+1})((e^{^a\delta/2} - 1)] + {}^a_nM_{k+1} = D_{12/31/k+1}
\end{aligned}
$$

(3.2.3)

This may be considered as

[loss from income deficit] + [loss from excess outgo] + [loss from interest] + ${}^a_nM_{k+1}$
= $D_{12/31/k+1}$ - $D_{12/31/k}$ =Total loss in year k+1 (3.2.4)

It is possible that [loss from interest] would be relatively small and could be absorbed into ${}^a_nM_{k+1}$. One might also want to analyze what were the principal components of [loss from income deficit] and [loss from excess outgo].

This section has introduced the idea that there exists a mathematical structure for computing gain-and-loss in respect to n-year roll-forward reserve financing. It also provides a means of proceeding toward a certain level of n-year roll-forward reserves without waiting for the current funding to produce that level. If 12/31/0 represents the time at which n-year roll-forward reserves are made effective, then the contribution rate for year 1 might be taken as

$$ {}_nI_1 / W_1 + (\text{amortization rate } r_0) \tag{3.2.5}$$

where

$$D_{12/31/0} = r_0[W_1e^{-\delta/2} + W_2e^{-3\delta/2} + \ldots + W_me^{-(2m-1)\delta/2}] \tag{3.2.6}$$

Here, $D_{12/31/0}$ is amortized by a level percentage of the increasing payrolls $W_1, W_2, \ldots, W_m$, of the next m years. The next section discusses a well-known difficulty that should be avoided if possible in such amortization.

3.3 Warning About Amortizing by a Level Percentage of an Increasing Payroll Without going into detail, we hope that it is clear to actuarial readers that if the amortization period of m years in (3.2.6) is extended too far, the amortization payment r_0W_1 in year one could be less than interest on $D_{12/31/0}$*. This situation arises because the amortization is by a level percent of increasing payroll. For instance, if the payroll is increasing at rate σ equal to the force of interest δ, then (3.2.6) becomes

$$D_{12/31/0} = r_0W_1 \cdot m \cdot e^{-\delta/2} \tag{3.3.1}$$

and

$$r_0W_1 = D_{12/31/0} \cdot e^{\delta/2}/m .$$

This is to be compared with interest on $D_{12/31/0}$ in year one, namely

$$D_{12/31/0} (e^{\delta} - 1)e^{-\delta/2}$$

(the last factor, $e^{-\delta/2}$, is to bring the interest calculated at the end of year one back to the middle of that year, and thus be comparable with $1/m$, the amortization factor by an annuity-due commencing as of that mid-date.) Then

*When we mention interest, we are always thinking in terms of the current year' dollars. We believe that is how the public will think.

$r_0 W_1$ < *interest in year-one on the deficit*

if

$$1/m < (e^{\delta} - 1)\, e^{-\delta}$$

or

$$1/m < 1 - e^{-\delta} = d, \text{ equivalent to } \delta \qquad (3.3.2)$$

This makes sense. It shows that the amortization factor $1/m$ provides less than the interest-in-advance (at rate d) on the deficit.

For δ=.062 this condition would hold if $m \geq 17$. Amortization over 17 or more years would, under the given conditions, produce less than interest on outstanding principal which then would increase, unless there was some other offsetting gain.

Also, if instead of amortization over a fixed period of m years, the process moves forward following the end of year one to the next m years, and so on in subsequent years., the net effect is that the amortization is always in a first-year stage, and may continue to provide less than interest on the deficit. The result may be to increase the deficit (or surplus) indefinitely.

We shall see in section 5.4 that this may raise a serious question about 'summarized cost rates' over a 75-year period.

3.4 Studies of Variance Here the word variance is used in its general sense, rather than in the special statistical sense. One study [5, 1982] was adapted from Actuarial Note 109, by Dwight K. Bartlett, III and Joseph A. Applebaum and was entitled, "Economic Forecasting: Effect of Errors on OASDI Fund Ratios". We quote the abstract:

> "A broad range of economic assumptions are used to project the future income and outgo of the Social Security system. The assumptions adopted by the Board of Trustees of the Old-Age and Survivors Insurance and Disability Insurance (OASDI) Trust Fund were rather consistently on the optimistic side of the actual experience that emerged. This article examines the experience of several key economic indicators during the 1970's. Acknowledging that forecasting such quantities is an inexact science at best, the authors present a formula for making estimates of OASDI fund ratios, given the necessary assumptions. The formula is used to project fund ratios from 1981 to 1986. It shows where the fund

would stand if forecasting errors were to continue at the magnitudes experienced in 1970-76"

Their Chart 1 exhibits that errors have relatively little effect as of the beginning of the second year, but in subsequent years there is a "snowball" effect and much larger deviations in the projection for the sixth year. (We assume, however, that a new projection at the end of the first year would recognize and correct for the original deviations.)

It may be well here to try to distinguish between "actuarial projections" and "economic forecasts". Each is an estimate based on a variety of assumptions. The actuary strives for complete accuracy in projecting the consequences of the assumptions, and in that sense the actuary's work is exact. The actuary fully expects to revise assumptions in the light of experience, and to re-project as necessary. On that basis, the actuary can work with projections that are reasonably in line with future experience. "Economic forecasts" strive to be the "best estimates", possibly in some probability sense, and may have a more difficult goal than "projections". So much for terminology.

This study [5] was followed by Alicia Munnell and Lynn Blais [8, 1984] who estimated that a range of 85 to 145 percent of annual expenditures would suffice for contingency purposes. In a 1990 memo, Richard Foster, after extensive computations of different economical scenarios, and with some allowance for non-economic adverse experience, concluded that 100 percent of outgo "represents a reasonable 'target ratio' for contingency purposes."

The 1993 OASDI Trustees Report has a section II.G on Long-Range Sensitivity Analyses. Some examples of "rules of thumb" emerging from these analyses are:

- "Each increase of 0.1 in the ultimate total fertility rate increases the long-range actuarial balance by about 0.15 of taxable payroll."
- "Each additional 10-percentage-point reduction in the age-sex adjusted death rate assumed to occur in 1997-2067, relative to the 36 percent reduction assumed for Alternative II, decreases the long-range actuarial balance by about 0.42 percent of taxable payroll."
- "Each additional group of 100,000 immigrants, relative to the 850,000 net immigration assumed for Alternative II, increases the long-range actuarial balance by about 0.05 percent of taxable payroll."
- "Each 0.5-percentage-point increase in the assumed real-interest rate increases the long-range actuarial balance by about 0.28 percent of taxable payroll."

Other factors considered are real-wage differential, Consumer Price Index, Disability incidence rates, and Disability termination rates.

n -Year roll-forward reserves can be one means of attaining financial equilibrium for the Social Security systems. Clearly, much more analysis of the future experience will be required.

IV. PRELIMINARY ESTIMATES FOR MEDICARE*

Among the data supplied to us by the Office of the Actuary was a Table 21 which, under Alternative II, estimated the HI tax income and total outgo. From this information we prepared TABLE 4a of 1-year roll-forward reserves for HI, and the required annual incomes for calendar years 1993-2001. By estimating the taxable payroll by: (HI Tax Income) / 0.029, we proceeded to include a column of ${}_1I_k \ / \ W_k$.

TABLE 4a
VALUES RELATED TO 1-YEAR ROLL-FORWARD RESERVES FOR HI, ALTERNATIVE II

k	O_k	W_k	${}_1I_k$	${}_1A_{12/31/k}$	$({}_1I_k/W_k)$
1993	95,600	3,000,000	99,526	103,588	3.32%
1994	107,600	3,200,000	111,506	115,824	3.48%
1995	120,100	3,300,000	123,586	128,149	3.75%
1996	132,700	3,500,000	135,173	139,974	3.86%
1997	144,800	3,700,000	147,667	152,759	3.99%
1998	157,900	3,900,000	161,445	166,879	4.14%
1999	172,400	4,100,000	176,494	182,333	4.30%
2000	188,300	4,400,000	192,899	199,212	4.38%
2001	205,700	4,600,000	210,509	217,365	4.58%

For SMI, we used Alternative II data from Table I.C.5 - Estimated Operations of the Supplementary Medical Insurance Trust Fund (Cash Basis) During Calendar Years 1992-2002. By means of the formulas

$$A_{12/31/k-1}e^{\delta_k} + ({}_1I_k - O_k)e^{\delta_k/2} = A_{12/31/k} \qquad (4.1)$$

starting with $A_{12/31/1992}$ = \$24.2 million and

$${}_1I_k = O_{k+1}e^{-(\delta_k+\delta_{k+1})/2} \qquad (4.2)$$

*For the estimates of Part IV, we used the forces of interest δ_k listed in Appendix Table 1 for OASDI, Alternative II.

we calculated our transitional TABLE 4b for SMI.

TABLE 4b
TRANSITIONAL 1-YEAR ROLL-FORWARD RESERVE FUNDING FOR SMI, ALTERNATIVE II

k	O_k	${}_1I_k$	${}_1A_{12/31/k}$
1993	56,900	60,678	30,147
1994	65,600	70,469	37,585
1995	75,900	80,559	45,243
1996	86,500	91,298	53,481
1997	97,800	103,152	62,770
1998	110,300	116,495	73,470
1999	124,400	131,785	86,040
2000	140,600	149,293	100,741
2001	159,200	169,233	117,770

V. SUMMARY AND CONCLUSIONS

In this part we draw together various themes of this paper which are surprisingly interrelated.

5.1 n-Year Roll-Forward Reserve Financing as a Solution for Roller-Coaster Financing In section 1.1 we contrasted n-year roll-forward reserve financing with classical pension funding, and noted it was particularly adaptable to Social Security financing which has a long record of its streams of taxable payroll, annual outgo for benefits and administration, and projections thereof for the next 75 years. The development of short- and long-range projections is a must for n-year roll-forward reserve financing, and the latter in turn provides a discipline for actuarial interpretation and guidance of the financing of Social Security systems.

Figure 5.1, Projected Trust Funds Under Current Law vs. Roll-Forward Reserve Financing, gives an overview of the situation as projected under Alternative II assumptions. Under current law, the fund is projected to increase to 4.97 trillion dollars by the year 2024 and to be depleted by the year 2036. The authors are agreed that n-year roll-forward reserve financing would solve the roller-coaster funding problem, but would entail the build-up of various levels of massive funds.

1-Year roll-forward financing could start soon with a substantial surplus over the reserve required by the funding system. It would entail the slowest build-up of reserve funds, but would be projected to require trillions to match the future yearly outgoes for OASDI. Since the fund would approximate one year's outgo, participants might worry about the strength of this guarantee. Also, if adjustments were necessary, Congress might have only a year to decide on modifications.

At the other extreme, 4-year roll-forward reserve financing would quadruple the required fund, but not the required annual income (See Appendix Tables OASDI.II.1-4). At any rate, the authors were inclined to dismiss this level of funding as beyond reach.

There remains 3-year roll-forward reserve financing, but this also seemed beyond practical reach even though current funding gets relatively close to the build-up required by 3-year roll-forward reserves.

The majority conclusion was that 2-year roll-forward reserve financing begun in the year 2001 was the most feasible system to choose. The objective would be to have, at the end of each year, a fund equivalent to the outgo for benefits and administration of the next two years. This could be arranged for in various ways [see 12,1991]. This would provide a reasonable guarantee to participants that "the money would be there" when their benefits become due. It would also provide Congress a two-year period to make adjustments that may be required in future years.

5.2 Computations and Graphs The tables indicated in the Appendix and the graphs prepared therefrom for part II, gave a basic foundation for our exploration of n-year roll-forward reserve financing of Social Security. They conveyed much information to us, and we hope they will do the same for readers of this paper.

5.3 Actuarial Aspects Part III of the paper reviews the actuarial mathematics of n-year roll-forward reserve financing, as presented to the 1992 Actuarial Research Conference at the University of Iowa. Borrowing from pension funding practice, this part goes on to discuss an introduction to gain and loss analysis for such financing, and how deficits or surpluses relative to required reserves might be amortized by a level percentage of the future increasing payrolls. This leads to a warning that the use of such amortization on a moving-term basis might, if the term is too long, entail the crediting (debiting) of less than interest on the surplus (deficit). This is so if one is thinking, as will the public, in terms of current dollars in future years (rather than in terms of some form of indexed dollars). This could lead to an increasing surplus or deficit in current dollars and, particularly in the deficit case, to a lack of confidence in the financing of the system.

Part III ends with the indication of some means that have been proposed to offset or to measure deviations from the expected financial requirements.

Part IV, in a very preliminary fashion as we await proposals for health insurance reform, discusses 1-year roll-forward reserve financing for HI, and a transitional funding for SMI.

5.4 A Questionable Financial Procedure This came up as we were drafting the paper and we were considering the key calculation of **summarized cost rate** [see 2, Glossary]. We prefer that the summarized cost rate be defined by the following equation:

$$\begin{aligned} &[e^{-\delta/2}O_1 + e^{-(1\frac{1}{2})\delta}O_2 + \ldots + e^{-(74\frac{1}{2})\delta}O_{75} + e^{-(75\frac{1}{2})\delta}O_{76}] - F_0 \\ &\quad = r_0[e^{-\delta/2}W_1 + e^{-(1\frac{1}{2})\delta}W_2 + \ldots + e^{-(74\frac{1}{2})\delta}W_{75}] \end{aligned} \tag{5.4.1}$$

where the subscripts k of O_k, W_k denote calendar years following a valuation time denoted by 0, and F_0 is the fund on hand at time 0. The OASDI Reports take F_0 as an addition to the present value of future income in calculating **summarized income rate**. At this stage, we dispense with summarized income rate and concentrate on our summarized cost rate as the key goal. To distinguish the left member of (5.4.1) from the procedure of the OASDI Reports, we label our expression as **net actuarial deficit**. Because we are considering a deficit, it seems natural to us to subtract F_0 from the actuarial present value of future outgoes.

Denoting the bracketed amount in the left side of (5.4.1) by A_0, we have

$$A_0 - F_0 = r_0 e^{-\delta/2}[W_1 + e^{-\delta}W_2 + \ldots + e^{-74\delta}W_{75}] \tag{5.4.2}$$

which indicates that r_0 could be interpreted as the level percentage rate of increasing payrolls, W_k, k=1,2,...75, which would amortize the net actuarial deficit, $A_0 - F_0$. This is reminiscent of the warning in section 3.3. In fact, if $W_k = W_1 e^{(k-1)\tau}$, then

$$A_0 - F_0 = r_0 e^{-\delta/2} W_1 \ddot{a}_{\overline{75}|\delta-\tau} \tag{5.4.3}$$

Then

$$r_0 W_1 \doteq (A_0 - F_0)e^{\delta/2} / \ddot{a}_{\overline{75}|\delta-\tau}$$

$$< (A_0 - F_0)(e^{\delta} - 1)e^{-\delta/2}$$ (that is, interest in year one on the deficit; see page 30)

~~if~~

if

$$\frac{1}{\ddot{a}_{\overline{75}|\delta-\tau}} < (e^{\delta}-1)e^{-\delta} = (1-e^{-\delta}) = d \tag{5.4.4}$$

As in section 3.3, if the amortization factor (here $\frac{1}{\ddot{a}_{\overline{75}|\delta-\tau}}$) is less than the interest-in-advance rate d equivalent to δ, the amortization process yields less than interest (in current year dollars) on the deficit $A_0 - F_0$. This is particularly true if the amortization term moves forward each year, so that the process is always in year one of amortization, and thereby may continue to take account of less than the interest on the net actuarial deficit.

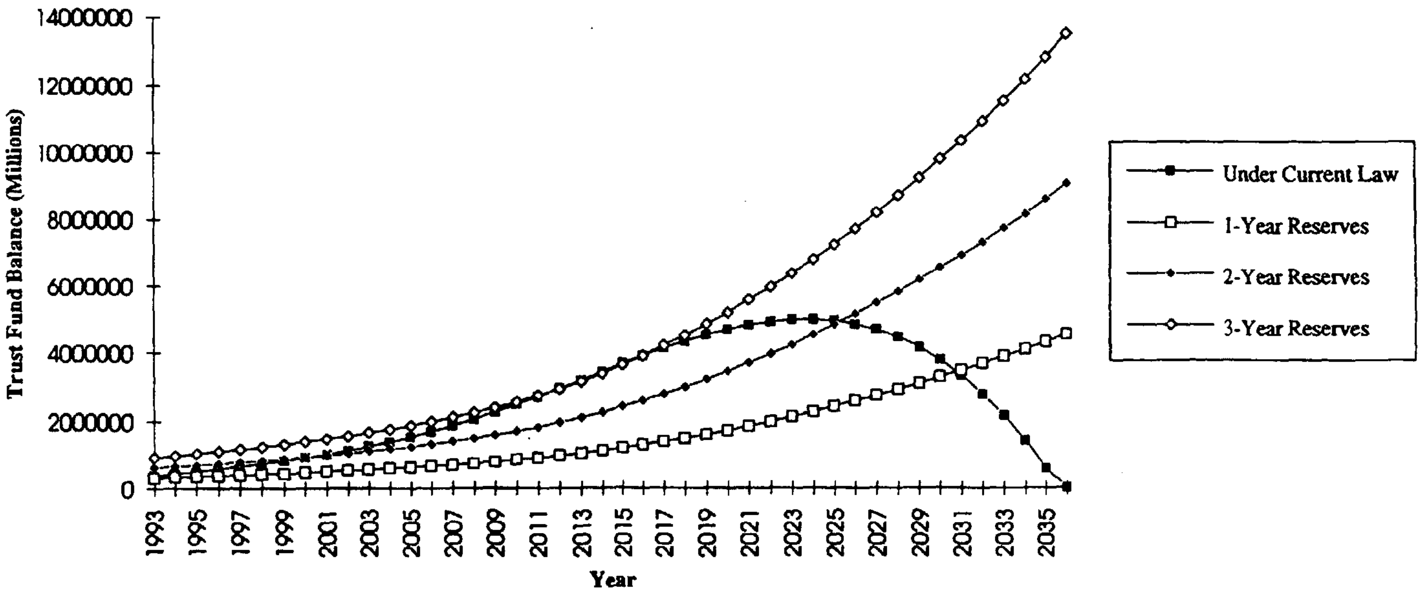
The idea of "actuarial deficit" is discussed by A. Haeworth Robertson in [14,p. 100] along the lines of the OASDI Reports without netting for F_0, the fund on hand. He does not go on to the effect of a moving amortization term. As in section 3.3, we have considered the effect of long-term amortization by a level percentage of an increasing payroll over a moving amortization term, and think that such amortization of an actuarial deficit (or surplus) is dubious, in that there is a continual non-accounting of interest (in current years' dollars). Being at the end of our study period, we have not examined fully this fairly large question, but present it to actuaries, and other interested parties, to consider. We note that even if the present summarized cost rate is abandoned, there still remains the fundamental question of how to finance the increases in the projected annual outgoes of OASDI.

One line of inquiry would build on the concept of 2-year roll-forward reserves. The net actuarial deficit would be defined as the

(present value of the projected outgoes for the next 10-15 years plus the present value of the 2-year target reserve at the end of the amortization period) - F_0 (5.4.5)

This would be amortized by a level percentage of the increasing payrolls projected for the next 10-15 years. Note that this involves a drastic reduction in the amortization period from 75 years to 10-15 years, thereby assuring that at least interest on the deficit is paid. The net actuarial deficit might also be defined as in (5.4.5) but with O_k replaced by ${}_2I_k$. One might also consider amortization over a fixed term of 10-15 years, or a moving term with a 10-15 year span. There are many directions to be explored to determine a 'better' or 'best' way to provide for 2-year roll-forward reserve financing.

Figure 5.1
Projected Trust Fund Under Current Law vs. Projected Trust Funds Required For n-year Roll-Forward Reserves for OASDI, Alternative II



VI. APPENDIX

A large number of tables for n-year roll-forward reserves were computed from the basic data supplied by the the Office of the Actuary. A key initial table was Table 1, appended hereto, of forces of interest calculated by the formula $\delta_k = \ln(1+i_k)$, where we interpreted the projected interest rates from the Office of the Actuary as effective annual rates.

A number of tables were basic to our study. They are coded as follows:

(System-OASDI, OASI, DI)-(Alternative-I, II, III)-(Reserve Period-n=1,2,3,4)

To illustrate the tables, we append the tables OASDI-II-1-OASDI-II-4. Besides indicating the format of the tables, these tables list the year-by-year taxable payroll and the annual outgo for the system and alternative in question, as provided by the Office of the Actuary. The tables next present the required income for the specified n, namely ${}_nI_k$, and the required reserve at the end of year k, namely, ${}_nA_{12/31/k}$. Finally, the ratio ${}_nI_k/W_k$ is shown, which represents the required rate to be applied to taxable payroll if the n-year roll-forward reserve financing is followed.

Since there are three alternatives and four reserve periods, there are twelve tables for the OASDI system and an equal number for each of the OASI and DI systems.

Data from these tables and additional data from the print-outs supplied by the Office of the Actuary were used in the preparation of Figures 1.2.1 - 1.2.3, 2.1.1 - 2.1.6, 2.2.1 - 2.2.6 and 2.3.1 - 2.3.6.

Some additional tables were calculated: tables 2-I - 2-III. Here the second digit refers to alternative assumed. These tables give various ratios for years 1993-2069, namely, W_{k+1} / W_k, O_{k+1} / O_k and $(O_{k+1} / O_k)/(W_{k+1} / W_k)$ for OASDI, OASI and DI.

Tables 3-I - 3-III. For the three alternatives these tables show adjusted end of year fund balances for OASI and DI trust funds resulting from proposed reallocation of 0.55% of taxable payroll from OASI to DI.

Some additional special tables were compiled for the separate systems:

Tables OASDI-S–I - OASDI-S–III. For years 2005-2035, these tables show for OASDI and alternatives I, II and III respectively, a listing of the ratios O_k / W_k, ${}_1I_k / W_k$, ${}_2I_k / W_k$, ${}_3I_k / W_k$ and ${}_4I_k / W_k$.

Tables OASI-S–I - OASI-S–III. Corresponding information for the OASI system.

Tables DI-S–I - DI-S–III. Corresponding information for the DI system.

Copies of some or all of the detailed tables of the appendix may be obtained by a request to Cecil Nesbitt, Department of Mathematics, University of Michigan, Ann Arbor, MI 48109-1003.

TABLE 1

ESTIMATED FORCES OF INTEREST δ_k FOR THE OASDI, OASI AND DI TRUST FUNDS BY CALENDAR YEARS 1993-2070

Year	OASDI I	OASDI II	OASDI III	OASI I	OASI II	OASI III	DI I	DI II	DI III
1993	7.98	8.00	8.02	7.97	7.98	8.01	8.32	8.40	8.50
1994	7.57	7.60	7.72	7.56	7.59	7.70	8.18	8.35	8.93
1995	7.20	7.25	7.52	7.19	7.24	7.49	7.97	7.96	8.69
1996	6.90	6.98	7.50	6.89	6.96	7.48	7.59	7.66	8.67
1997	6.68	6.78	7.55	6.67	6.74	7.53	7.34	7.42	7.36
1998	6.50	6.62	7.54	6.49	6.59	7.53	7.15	5.78	7.48
1999	6.36	6.51	7.48	6.35	6.48	7.45	6.99	5.90	7.28
2000	6.26	6.44	7.40	6.25	6.41	7.34	5.70	6.02	7.08
2001	6.20	6.41	7.32	6.19	6.38	7.24	5.57	6.09	6.95
2002	6.15	6.37	7.24	6.15	6.35	7.14	5.45	6.14	6.83
2003	6.10	6.34	7.08	6.10	6.32	6.98	5.54	6.15	6.74
2004	6.05	6.30	6.92	6.05	6.29	6.83	5.64	6.16	6.65
2005	6.01	6.27	6.75	6.01	6.26	6.68	5.73	6.17	6.55
2006	5.96	6.23	6.57	5.96	6.23	6.52	5.82	6.18	6.46
2007 and thereafter	5.91	6.20	6.37	5.91	6.20	6.37	5.91	6.20	6.37

TABLE OASDI-II-1
VARIOUS VALUES RELATED TO 1-YEAR ROLL FORWARD RESERVES FOR OASDI UNDER ALTERNATIVE II
(IN MILLIONS)

Year k	Outgo O_k	Taxable Payroll W_k	Income Req'd in Year k ${}_1I_k$	Fund at End of Year k ${}_1A_{12/31/k}$	$({}_1I_k/W_k)$
1993	309,056	2,657,782	301,277	313,572	11.34%
1994	325,717	2,816,613	318,627	330,967	11.31%
1995	343,185	2,975,169	337,164	349,610	11.33%
1996	362,027	3,137,427	356,790	369,462	11.37%
1997	382,201	3,309,436	377,427	390,441	11.40%
1998	403,581	3,487,656	399,723	413,175	11.46%
1999	426,845	3,683,498	423,961	437,988	11.51%
2000	452,321	3,896,924	449,729	464,446	11.54%
2001	479,573	4,127,497	476,821	492,350	11.55%
2002	508,284	4,375,089	505,481	521,840	11.55%
2003	538,647	4,637,306	535,320	552,561	11.54%
2004	570,244	4,914,494	566,751	584,888	11.53%
2005	603,515	5,210,554	600,060	619,170	11.52%
2006	638,761	5,519,214	635,723	655,838	11.52%
2007	676,487	5,843,923	674,958	696,209	11.55%
2008	718,130	6,182,589	718,542	741,166	11.62%
2009	764,502	6,539,237	765,932	790,048	11.71%
2010	814,923	6,913,901	817,725	843,472	11.83%
2011	870,029	7,306,842	874,685	902,225	11.97%
2012	930,632	7,714,695	937,035	966,538	12.15%
2013	996,970	8,138,828	1,005,138	1,036,785	12.35%
2014	1,069,429	8,585,486	1,078,831	1,112,799	12.57%
2015	1,147,836	9,053,008	1,159,143	1,195,640	12.80%
2016	1,233,285	9,544,079	1,246,289	1,285,529	13.06%
2017	1,326,005	10,049,345	1,339,762	1,381,945	13.33%
2018	1,425,456	10,581,047	1,439,902	1,485,238	13.61%
2019	1,532,001	11,136,123	1,546,711	1,595,410	13.89%
2020	1,645,642	11,720,146	1,659,768	1,712,027	14.16%
2021	1,765,931	12,334,616	1,779,392	1,835,417	14.43%
2022	1,893,206	12,977,626	1,906,185	1,966,202	14.69%
2023	2,028,109	13,652,132	2,039,774	2,103,998	14.94%
2024	2,170,243	14,360,633	2,180,605	2,249,263	15.18%
2025	2,320,082	15,111,464	2,328,904	2,402,231	15.41%
2026	2,477,866	15,896,751	2,483,610	2,561,808	15.62%
2027	2,642,468	16,732,006	2,643,758	2,726,998	15.80%
2028	2,812,859	17,615,584	2,810,291	2,898,775	15.95%
2029	2,990,044	18,548,864	2,983,512	3,077,449	16.08%
2030	3,174,344	19,534,776	3,165,146	3,264,802	16.20%

k	O_k	W_k	${}_1I_k$	${}_1A_{12/31/k}$	$({}_1I_k/W_k)$
2031	3,367,596	20,578,201	3,354,584	3,460,205	16.30%
2032	3,569,151	21,693,460	3,550,391	3,662,177	16.37%
2033	3,777,482	22,847,866	3,752,027	3,870,161	16.42%
2034	3,992,015	24,066,460	3,960,311	4,085,004	16.46%
2035	4,213,622	25,339,612	4,174,997	4,306,449	16.48%
2036	4,442,040	26,698,066	4,396,536	4,534,963	16.47%
2037	4,677,749	28,101,413	4,627,984	4,773,699	16.47%
2038	4,924,001	29,613,657	4,870,548	5,023,900	16.45%
2039	5,182,080	31,197,118	5,123,884	5,285,212	16.42%
2040	5,451,620	32,842,441	5,390,606	5,560,332	16.41%
2041	5,735,402	34,548,192	5,672,756	5,851,366	16.42%
2042	6,035,599	36,368,707	5,970,536	6,158,521	16.42%
2043	6,352,425	38,262,996	6,286,497	6,484,430	16.43%
2044	6,688,596	40,263,737	6,621,174	6,829,645	16.44%
2045	7,044,680	42,353,977	6,974,719	7,194,321	16.47%
2046	7,420,838	44,559,329	7,350,392	7,581,822	16.50%
2047	7,820,540	46,867,975	7,750,251	7,994,271	16.54%
2048	8,245,975	49,294,942	8,175,158	8,432,557	16.58%
2049	8,698,060	51,837,536	8,627,382	8,899,020	16.64%
2050	9,179,210	54,504,591	9,108,775	9,395,570	16.71%
2051	9,691,394	57,288,920	9,622,353	9,925,317	16.80%
2052	10,237,821	60,244,996	10,170,099	10,490,309	16.88%
2053	10,820,602	63,338,905	10,751,058	11,089,561	16.97%
2054	11,438,721	66,601,996	11,362,887	11,720,653	17.06%
2055	12,089,684	70,038,598	12,007,620	12,385,685	17.14%
2056	12,775,655	73,628,249	12,689,728	13,089,271	17.23%
2057	13,501,393	77,450,896	13,410,386	13,832,619	17.31%
2058	14,268,146	81,459,811	14,167,901	14,613,984	17.39%
2059	15,074,113	85,704,256	14,961,201	15,432,261	17.46%
2060	15,918,154	90,073,788	15,795,134	16,292,452	17.54%
2061	16,805,428	94,785,222	16,668,513	17,193,329	17.59%
2062	17,734,670	99,682,914	17,586,757	18,140,485	17.64%
2063	18,711,647	104,849,761	18,554,074	19,138,258	17.70%
2064	19,740,836	110,246,261	19,566,140	20,182,190	17.75%
2065	20,817,636	115,963,521	20,631,181	21,280,764	17.79%
2066	21,950,800	121,936,669	21,753,970	22,438,905	17.84%
2067	23,145,405	128,250,247	22,930,937	23,652,930	17.88%
2068	24,397,654	134,882,057	24,174,967	24,936,128	17.92%
2069	25,721,255	141,846,791	25,486,153	26,288,597	17.97%
2070	27,116,307	149,156,845			

TABLE OASDI-II-2
VARIOUS VALUES RELATED TO 2-YEAR ROLL FORWARD RESERVES FOR OASDI UNDER ALTERNATIVE II
(IN MILLIONS)

Year k	Outgo O_k	Taxable Payroll W_k	Income Req'd in Year k ${}_2I_k$	Fund at End of Year k ${}_2A_{12/31/k}$	$({}_2I_k/W_k)$
1993	309,056	2,657,782	294,718	620,318	11.09%
1994	325,717	2,816,613	313,036	656,128	11.11%
1995	343,185	2,975,169	332,286	694,163	11.17%
1996	362,027	3,137,427	352,333	734,308	11.23%
1997	382,201	3,309,436	373,819	777,150	11.30%
1998	403,581	3,487,656	397,022	823,558	11.38%
1999	426,845	3,683,498	421,532	873,467	11.44%
2000	452,321	3,896,924	447,148	926,227	11.47%
2001	479,573	4,127,497	474,191	981,985	11.49%
2002	508,284	4,375,089	502,359	1,040,456	11.48%
2003	538,647	4,637,306	532,041	1,101,738	11.47%
2004	570,244	4,914,494	563,507	1,166,429	11.47%
2005	603,515	5,210,554	597,207	1,235,396	11.46%
2006	638,761	5,519,214	634,286	1,310,193	11.49%
2007	676,487	5,843,923	675,346	1,392,819	11.56%
2008	718,130	6,182,589	719,887	1,483,719	11.64%
2009	764,502	6,539,237	768,566	1,582,813	11.75%
2010	814,923	6,913,901	822,102	1,691,458	11.89%
2011	870,029	7,306,842	880,703	1,810,658	12.05%
2012	930,632	7,714,695	944,712	1,940,995	12.25%
2013	996,970	8,138,828	1,013,975	2,082,686	12.46%
2014	1,069,429	8,585,486	1,089,459	2,236,560	12.69%
2015	1,147,836	9,053,008	1,171,366	2,403,887	12.94%
2016	1,233,285	9,544,079	1,259,219	2,584,396	13.19%
2017	1,326,005	10,049,345	1,353,339	2,777,894	13.47%
2018	1,425,456	10,581,047	1,453,727	2,984,736	13.74%
2019	1,532,001	11,136,123	1,559,988	3,204,515	14.01%
2020	1,645,642	11,720,146	1,672,420	3,437,104	14.27%
2021	1,765,931	12,334,616	1,791,591	3,683,417	14.52%
2022	1,893,206	12,977,626	1,917,149	3,943,713	14.77%
2023	2,028,109	13,652,132	2,049,514	4,218,041	15.01%
2024	2,170,243	14,360,633	2,188,897	4,507,078	15.24%
2025	2,320,082	15,111,464	2,334,303	4,810,030	15.45%
2026	2,477,866	15,896,751	2,484,823	5,124,867	15.63%
2027	2,642,468	16,732,006	2,641,345	5,451,507	15.79%
2028	2,812,859	17,615,584	2,804,151	5,791,216	15.92%
2029	2,990,044	18,548,864	2,974,866	6,145,981	16.04%
2030	3,174,344	19,534,776	3,152,916	6,516,989	16.14%

k	O_k	W_k	${}_2I_k$	${}_2A_{12/31/k}$	$({}_2I_k/W_k)$
2031	3,367,596	20,578,201	3,336,951	6,902,222	16.22%
2032	3,569,151	21,693,460	3,526,466	7,299,675	16.26%
2033	3,777,482	22,847,866	3,722,229	7,709,586	16.29%
2034	3,992,015	24,066,460	3,924,009	8,132,562	16.30%
2035	4,213,622	25,339,612	4,132,229	8,568,784	16.31%
2036	4,442,040	26,698,066	4,349,763	9,021,681	16.29%
2037	4,677,749	28,101,413	4,577,745	9,495,576	16.29%
2038	4,924,001	29,613,657	4,815,851	9,991,381	16.26%
2039	5,182,080	31,197,118	5,066,539	10,511,273	16.24%
2040	5,451,620	32,842,441	5,331,726	11,059,931	16.23%
2041	5,735,402	34,548,192	5,611,604	11,639,654	16.24%
2042	6,035,599	36,368,707	5,908,571	12,253,126	16.25%
2043	6,352,425	38,262,996	6,223,128	12,903,497	16.26%
2044	6,688,596	40,263,737	6,555,419	13,591,465	16.28%
2045	7,044,680	42,353,977	6,908,507	14,320,346	16.31%
2046	7,420,838	44,559,329	7,284,328	15,095,501	16.35%
2047	7,820,540	46,867,975	7,683,691	15,919,887	16.39%
2048	8,245,975	49,294,942	8,108,729	16,796,593	16.45%
2049	8,698,060	51,837,536	8,561,182	17,729,755	16.52%
2050	9,179,210	54,504,591	9,043,885	18,724,206	16.59%
2051	9,691,394	57,288,920	9,558,702	19,784,980	16.69%
2052	10,237,821	60,244,996	10,104,736	20,913,198	16.77%
2053	10,820,602	63,338,905	10,679,783	22,105,602	16.86%
2054	11,438,721	66,601,996	11,285,756	23,361,747	16.95%
2055	12,089,684	70,038,598	11,926,858	24,688,067	17.03%
2056	12,775,655	73,628,249	12,604,193	26,090,313	17.12%
2057	13,501,393	77,450,896	13,316,168	27,568,053	17.19%
2058	14,268,146	81,459,811	14,061,776	29,118,503	17.26%
2059	15,074,113	85,704,256	14,845,576	30,745,258	17.32%
2060	15,918,154	90,073,788	15,666,450	32,452,168	17.39%
2061	16,805,428	94,785,222	16,529,492	34,243,260	17.44%
2062	17,734,670	99,682,914	17,438,657	36,128,206	17.49%
2063	18,711,647	104,849,761	18,389,880	38,107,153	17.54%
2064	19,740,836	110,246,261	19,390,894	40,183,616	17.59%
2065	20,817,636	115,963,521	20,446,184	42,370,707	17.63%
2066	21,950,800	121,936,669	21,552,396	44,669,889	17.68%
2067	23,145,405	128,250,247	22,721,638	47,089,970	17.72%
2068	24,397,654	134,882,057	23,953,999	49,644,331	17.76%
2069	25,721,255	141,846,791			
2070	27,116,307	149,156,845			

TABLE OASDI-II-3
VARIOUS VALUES RELATED TO 3-YEAR ROLL FORWARD RESERVES FOR OASDI UNDER ALTERNATIVE II
(IN MILLIONS)

Year k	Outgo O_k	Taxable Payroll W_k	Income Req'd in Year k ${}_3I_k$	Fund at End of Year k ${}_3A_{12/31/k}$	$({}_3I_k/W_k)$
1993	309,056	2,657,782	289,547	921,682	10.89%
1994	325,717	2,816,613	308,508	976,584	10.95%
1995	343,185	2,975,169	328,136	1,034,412	11.03%
1996	362,027	3,137,427	348,965	1,095,667	11.12%
1997	382,201	3,309,436	371,293	1,161,245	11.22%
1998	403,581	3,487,656	394,747	1,231,591	11.32%
1999	426,845	3,683,498	419,113	1,306,447	11.38%
2000	452,321	3,896,924	444,682	1,385,461	11.41%
2001	479,573	4,127,497	471,262	1,468,596	11.42%
2002	508,284	4,375,089	499,282	1,555,896	11.41%
2003	538,647	4,637,306	528,996	1,647,772	11.41%
2004	570,244	4,914,494	560,827	1,745,203	11.41%
2005	603,515	5,210,554	595,857	1,850,229	11.44%
2006	638,761	5,519,214	634,651	1,964,924	11.50%
2007	676,487	5,843,923	676,609	2,090,731	11.58%
2008	718,130	6,182,589	722,362	2,228,825	11.68%
2009	764,502	6,539,237	772,679	2,379,820	11.82%
2010	814,923	6,913,901	827,758	2,545,278	11.97%
2011	870,029	7,306,842	887,919	2,726,533	12.15%
2012	930,632	7,714,695	953,018	2,924,019	12.35%
2013	996,970	8,138,828	1,023,964	3,138,890	12.58%
2014	1,069,429	8,585,486	1,100,947	3,372,171	12.82%
2015	1,147,836	9,053,008	1,183,518	3,624,669	13.07%
2016	1,233,285	9,544,079	1,271,980	3,896,425	13.33%
2017	1,326,005	10,049,345	1,366,333	4,187,247	13.60%
2018	1,425,456	10,581,047	1,466,206	4,497,106	13.86%
2019	1,532,001	11,136,123	1,571,879	4,825,885	14.12%
2020	1,645,642	11,720,146	1,683,885	5,174,007	14.37%
2021	1,765,931	12,334,616	1,801,895	5,542,046	14.61%
2022	1,893,206	12,977,626	1,926,303	5,930,667	14.84%
2023	2,028,109	13,652,132	2,057,307	6,340,123	15.07%
2024	2,170,243	14,360,633	2,193,971	6,770,128	15.28%
2025	2,320,082	15,111,464	2,335,443	7,219,005	15.45%
2026	2,477,866	15,896,751	2,482,555	7,685,586	15.62%
2027	2,642,468	16,732,006	2,635,574	8,170,063	15.75%
2028	2,812,859	17,615,584	2,796,026	8,675,277	15.87%
2029	2,990,044	18,548,864	2,963,372	9,202,656	15.98%
2030	3,174,344	19,534,776	3,136,344	9,752,082	16.06%

k	O_k	W_k	${}_3I_k$	${}_3A_{12/31/k}$	$({}_3I_k/W_k)$
2031	3,367,596	20,578,201	3,314,465	10,321,044	16.11%
2032	3,569,151	21,693,460	3,498,459	10,908,284	16.13%
2033	3,777,482	22,847,866	3,688,109	11,513,816	16.14%
2034	3,992,015	24,066,460	3,883,811	12,138,657	16.14%
2035	4,213,622	25,339,612	4,088,268	12,785,773	16.13%
2036	4,442,040	26,698,066	4,302,544	13,459,693	16.12%
2037	4,677,749	28,101,413	4,526,336	14,164,426	16.11%
2038	4,924,001	29,613,657	4,761,953	14,903,266	16.08%
2039	5,182,080	31,197,118	5,011,198	15,680,252	16.06%
2040	5,451,620	32,842,441	5,274,251	16,500,244	16.06%
2041	5,735,402	34,548,192	5,553,365	17,367,869	16.07%
2042	6,035,599	36,368,707	5,849,012	18,286,297	16.08%
2043	6,352,425	38,262,996	6,161,326	19,258,815	16.10%
2044	6,688,596	40,263,737	6,493,188	20,289,094	16.13%
2045	7,044,680	42,353,977	6,846,415	21,382,325	16.16%
2046	7,420,838	44,559,329	7,221,770	22,544,652	16.21%
2047	7,820,540	46,867,975	7,621,256	23,781,102	16.26%
2048	8,245,975	49,294,942	8,046,509	25,096,450	16.32%
2049	8,698,060	51,837,536	8,500,192	26,497,580	16.40%
2050	9,179,210	54,504,591	8,984,060	27,991,133	16.48%
2051	9,691,394	57,288,920	9,497,268	29,581,274	16.58%
2052	10,237,821	60,244,996	10,037,745	31,266,986	16.66%
2053	10,820,602	63,338,905	10,607,289	33,046,867	16.75%
2054	11,438,721	66,601,996	11,209,850	34,924,545	16.83%
2055	12,089,684	70,038,598	11,846,465	36,907,524	16.91%
2056	12,775,655	73,628,249	12,515,638	39,000,012	17.00%
2057	13,501,393	77,450,896	13,216,423	41,200,601	17.06%
2058	14,268,146	81,459,811	13,953,103	43,510,926	17.13%
2059	15,074,113	85,704,256	14,724,628	45,933,499	17.18%
2060	15,918,154	90,073,788	15,535,786	48,477,106	17.25%
2061	16,805,428	94,785,222	16,390,295	51,149,612	17.29%
2062	17,734,670	99,682,914	17,284,333	53,956,746	17.34%
2063	18,711,647	104,849,761	18,225,170	56,906,151	17.38%
2064	19,740,836	110,246,261	19,217,019	60,005,692	17.43%
2065	20,817,636	115,963,521	20,256,728	63,265,228	17.47%
2066	21,950,800	121,936,669	21,355,679	66,697,962	17.51%
2067	23,145,405	128,250,247	22,513,954	70,312,787	17.55%
2068	24,397,654	134,882,057			
2069	25,721,255	141,846,791			
2070	27,116,307	149,156,845			

TABLE OASDI-II-4
VARIOUS VALUES RELATED TO 4-YEAR ROLL FORWARD RESERVES FOR OASDI UNDER ALTERNATIVE II
(IN MILLIONS)

Year k	Outgo O_k	Taxable Payroll W_k	Income Req'd in Year k ${}_4I_k$	Fund at End of Year k ${}_4A_{12/31/k}$	$({}_4I_k/W_k)$
1993	309,056	2,657,782	285,359	1,218,686	10.74%
1994	325,717	2,816,613	304,654	1,293,038	10.82%
1995	343,185	2,975,169	324,999	1,371,408	10.92%
1996	362,027	3,137,427	346,607	1,454,584	11.05%
1997	382,201	3,309,436	369,166	1,543,141	11.15%
1998	403,581	3,487,656	392,482	1,637,281	11.25%
1999	426,845	3,683,498	416,802	1,737,038	11.32%
2000	452,321	3,896,924	441,936	1,841,859	11.34%
2001	479,573	4,127,497	468,376	1,952,226	11.35%
2002	508,284	4,375,089	496,424	2,068,386	11.35%
2003	538,647	4,637,306	526,480	2,191,209	11.35%
2004	570,244	4,914,494	559,560	2,322,670	11.39%
2005	603,515	5,210,554	596,199	2,465,415	11.44%
2006	638,761	5,519,214	635,838	2,620,880	11.52%
2007	676,487	5,843,923	678,936	2,791,044	11.62%
2008	718,130	6,182,589	726,228	2,977,918	11.75%
2009	764,502	6,539,237	777,995	3,182,311	11.90%
2010	814,923	6,913,901	834,540	3,406,093	12.07%
2011	870,029	7,306,842	895,725	3,650,460	12.26%
2012	930,632	7,714,695	962,406	3,916,727	12.47%
2013	996,970	8,138,828	1,034,761	4,206,231	12.71%
2014	1,069,429	8,585,486	1,112,369	4,519,563	12.96%
2015	1,147,836	9,053,008	1,195,512	4,857,822	13.21%
2016	1,233,285	9,544,079	1,284,193	5,221,051	13.46%
2017	1,326,005	10,049,345	1,378,062	5,608,698	13.71%
2018	1,425,456	10,581,047	1,477,382	6,021,004	13.96%
2019	1,532,001	11,136,123	1,582,655	6,458,371	14.21%
2020	1,645,642	11,720,146	1,693,571	6,920,901	14.45%
2021	1,765,931	12,334,616	1,810,499	7,409,549	14.68%
2022	1,893,206	12,977,626	1,933,627	7,925,175	14.90%
2023	2,028,109	13,652,132	2,062,076	8,467,125	15.10%
2024	2,170,243	14,360,633	2,195,042	9,034,282	15.29%
2025	2,320,082	15,111,464	2,333,311	9,625,781	15.44%
2026	2,477,866	15,896,751	2,477,131	10,240,711	15.58%
2027	2,642,468	16,732,006	2,627,937	10,880,742	15.71%
2028	2,812,859	17,615,584	2,785,222	11,548,193	15.81%
2029	2,990,044	18,548,864	2,947,796	12,243,264	15.89%
2030	3,174,344	19,534,776	3,115,209	12,965,375	15.95%

k	O_k	W_k	${}_4I_k$	${}_4A_{12/31/k}$	$({}_4I_k/W_k)$
2031	3,367,596	20,578,201	3,288,142	13,712,715	15.98%
2032	3,569,151	21,693,460	3,466,390	14,483,816	15.98%
2033	3,777,482	22,847,866	3,650,328	15,279,077	15.98%
2034	3,992,015	24,066,460	3,842,493	16,102,133	15.97%
2035	4,213,622	25,339,612	4,043,888	16,956,984	15.96%
2036	4,442,040	26,698,066	4,254,226	17,847,865	15.93%
2037	4,677,749	28,101,413	4,475,678	18,781,023	15.93%
2038	4,924,001	29,613,657	4,709,940	19,761,501	15.90%
2039	5,182,080	31,197,118	4,957,178	20,793,509	15.89%
2040	5,451,620	32,842,441	5,219,512	21,884,095	15.89%
2041	5,735,402	34,548,192	5,497,386	23,038,343	15.91%
2042	6,035,599	36,368,707	5,790,925	24,259,552	15.92%
2043	6,352,425	38,262,996	6,102,836	25,553,802	15.95%
2044	6,688,596	40,263,737	6,434,829	26,926,526	15.98%
2045	7,044,680	42,353,977	6,787,618	28,383,654	16.03%
2046	7,420,838	44,559,329	7,163,088	29,933,273	16.08%
2047	7,820,540	46,867,975	7,562,776	31,581,995	16.14%
2048	8,245,975	49,294,942	7,989,185	33,337,179	16.21%
2049	8,698,060	51,837,536	8,443,964	35,207,407	16.29%
2050	9,179,210	54,504,591	8,926,320	37,198,503	16.38%
2051	9,691,394	57,288,920	9,434,305	39,312,623	16.47%
2052	10,237,821	60,244,996	9,969,609	41,550,494	16.55%
2053	10,820,602	63,338,905	10,535,946	43,914,543	16.63%
2054	11,438,721	66,601,996	11,134,290	46,409,403	16.72%
2055	12,089,684	70,038,598	11,763,234	49,041,129	16.80%
2056	12,775,655	73,628,249	12,421,890	51,813,011	16.87%
2057	13,501,393	77,450,896	13,114,283	54,727,794	16.93%
2058	14,268,146	81,459,811	13,839,426	57,786,094	16.99%
2059	15,074,113	85,704,256	14,601,820	60,995,064	17.04%
2060	15,918,154	90,073,788	15,404,958	64,367,097	17.10%
2061	16,805,428	94,785,222	16,245,249	67,906,351	17.14%
2062	17,734,670	99,682,914	17,129,525	71,625,602	17.18%
2063	18,711,647	104,849,761	18,061,747	75,536,581	17.23%
2064	19,740,836	110,246,261	19,038,952	79,644,095	17.27%
2065	20,817,636	115,963,521	20,071,837	83,969,037	17.31%
2066	21,950,800	121,936,669	21,160,480	88,524,690	17.35%
2067	23,145,405	128,250,247			
2068	24,397,654	134,882,057			
2069	25,721,255	141,846,791			
2070	27,116,307	149,156,845			

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