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Computing Ruin Probabilities -- A Life-Table Approach*

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Abstract

Easy access computing power is harnessed to approximate ruin probabilities. The dynamics of the traditional insurance model are stated in the form of period to period constraints. This allows the development of a life-table with the survival probabilities for a given set of parameters. Given the spreadsheet approach of the model, sensitivity analysis to study the impact of different parameter values and loss variable distributions can be easily implemented.

Introduction

The traditional definition of surplus in an insurance process can be perceived as a simple accounting identity that relates the 'end of a period' surplus to the 'beginning of the period' surplus augmented by the premiums collected in the period and subject to the claims that are paid out during the period. Any insurance system so defined will be deemed to be solvent so long as the end of the period surplus is not less than zero. In order to simplify our presentation we will assume that all premiums are collected at the beginning of the period and that all claims are settled at the end of the period.

With such a definition of the surplus process we have an intuitively satisfying framework for defining the underlying insurance processes' ruin probability which in turn can be computed by deriving the cumulative claims distribution function. Solution techniques discussed in the literature are often based on approximations and/or simplifying assumptions about the claims distributions (see [1],[2], for example). Given the complexities of the convolution process even in the case of relatively simple and well-defined (and well-known) random processes (see [3] for a succinct presentation), approximating the ruin probabilities may be unavoidable except in simple text-book cases.

These commonly adopted solution strategies lead to two important questions about the practical uses of ruin probabilities: (1) can the convolution procedures be simplified so that ruin probabilities are readily available for decision-making? and (2) can the simplifying assumptions take into account the changes in the claims processes and in investment income? Our research was primarily focused on developing an applications package that would provide the answers to the above questions.

The Model

We were guided in our search by the simplicity and the succinct description of the surplus process. By relating the 'end of period' surplus to the 'beginning of (next) period' surplus, we could build the dynamics of the inter-relationship. Naturally, this increased the complexity of the claims process because we had to evaluate an 'n' fold convolution if we intended to study the dynamics of 'n' periods.

We decided to circumvent the difficulties of complex convolutions by simulation. This was suggested by the increasing capabilities of readily available spreadsheet programs. We have included an applications module to illustrate the simplicity of the methodology. The simulation approach also provided an answer to the other problem of incorporating changes in claims distribution and in investment income. We could parametrize the changes in these values.

The model was built in the form of the constraints of a forward looking dynamic programming problem. This gave us the flexibility of defining the time horizon based on the description of the underlying insurance system. We could relate it to the length of the tail of the claims process. For example, we could assume a five to seven year horizon to analyze auto policies or we could assume a 25 to 50 year horizon to analyze pension plans. We could also incorporate single premium payment at the beginning of the policy term like in many property and casualty products or a payment schedule that was in parallel with the policy term like in most life products. The model is developed in three steps.

THE MODEL

Used the Dynamic Programming Strategy of writing down a law of motion.

STEP 1

$$U_1 = (1 + r)U_0 + (1 + \theta)(1 + r)\mu - X_1$$

U_0 is the surplus at the end of period '0';
U_1 is the surplus at the end of period '1';
r is the rate of interest;
θ is the loading factor;
μ is the mean of the claim variable; and
X_1 is the period '1' claim variable.

Step 2

U_2 is developed as follows:

$$U_2 = (1 + r)U_1 + (1 + \theta)(1 + r)\mu - X_2$$

Substituting for U_1 we get:

$$U_2 = (1 + r)[(1 + r)U_0 + (1 + \theta)(1 + r)\mu - X_1] + (1 + \theta)(1 + r)\mu - X_2$$

$$= (1 + r)^2U_0 + (1 + \theta)(1 + r)(2 + r)\mu - [(1 + r)X_1 + X_2]$$

Step 3

This process can be extended and we get:

$$U_n = (1 + r)^nU_0 + (1 + \theta)(1 + r)^{(n-1)}(n + r)\mu - [(1 + r)^{(n-1)}X_1 + (1 + r)^{(n-2)}X_2 + \ldots + (1 + r)X_{n-1} + X_n]$$

Computation and Implementation

The computation of ruin probabilities is done in the form of a life-table. If the surplus variable is greater than or equal to zero at the end of the period then we can interpret that the system is solvent provided the system was solvent at the end of the previous period (or equivalently, at the beginning of the current period). In other words we have to verify whether the surplus is non-negative at time 'n' conditional on the event that it was non-negative at time 'n - 1'.

An examination of the right-hand side of the surplus equations will suggest that this probability can be obtained from evaluating the probability that a convolution of the loss variables is smaller than a given constant. The simulation experiment is designed to evaluate these probabilities.

The only major assumption needed to implement this computation is that the loss variables be independent from period to period. Under normal circumstances this need not be a stumbling block in most practical applications. For example, IBNR (incurred but not reported) claims from one period can be modeled directly as being part of the following periods' loss variables. In cases where catastrophic losses occur (like hurricane Andrew) the model is flexible enough to the introduction of a fixed charge in any period.

Implementing the simulation is quite straight forward in a spread sheet environment. Using random digits and representative loss distributions (that could be continuous or discrete) loss values are generated for the 'n' periods. The rest of the equation can be seen to be merely an accumulation calculus using the appropriate number of periods and discounting factors. The conditional probability that the surplus function is solvent at the end of a given period is obtained from the simulated values. A straight product rule is used to arrive at the probability that the system is solvent at the end of 'n' periods.

Given the structure of the experimental process it is easy to observe how the results from such an analysis can be used as a tool in decision-making. For example, one can study the impact of lowering the loading factor (in order to be competitive in a market?) on the required rate of return in order not to increase the probability of ruin. Or else, if the rate of return cannot be moved upward because of the current market forces then one can evaluate the additional initial surplus that may be needed to improve the solvency of the system. The rates of return can themselves be changed. Thus the experimental opportunities are plentiful for applications.

A simple application for an obviously hypothetical system has been appended. But given the simplicity of the method, the model can be easily adapted to represent different conditions.

1	2	3	4	5	6	7	8	9	10
0.822336	0.586983	0.292731	0.671814	0.889930	3.968388	2.798599	1.933440	1.491553	0.904570
0.007113	0.430266	0.934429	0.188109	0.054700	1.973177	1.744070	1.416328	0.438090	0.007824
0.548816	0.472430	0.950506	0.937758	0.339640	3.953611	3.285428	2.134245	1.076127	0.603697
0.945337	0.126883	0.148532	0.904139	0.336008	3.063233	2.479296	1.431961	1.166754	1.039871
0.280826	0.518327	0.375578	0.859562	0.290359	2.791378	2.273654	1.285538	0.827236	0.308909
0.848298	0.517253	0.344148	0.664736	0.068984	3.147060	2.798260	1.939568	1.450381	0.933128
0.343979	0.478779	0.100381	0.213768	0.578294	2.075775	1.361346	1.043253	0.857156	0.378377
0.254815	0.843920	0.105299	0.802820	0.383891	2.890737	2.278951	1.341937	1.124216	0.280296
0.675521	0.762349	0.753722	0.000135	0.063145	2.979015	2.650790	2.409687	1.505422	0.743073
0.481441	0.802838	0.812499	0.262682	0.285623	3.331152	2.768663	2.278165	1.332423	0.529586

(1) To simplify presentation, we have included a 5 - period horizon and shown only 10 loss realizations for each period.

(2) *Columns 1 – 5 are random realizations of the loss variable assuming a uniform [0,1] distribution.*

(3) Columns 6, 7, 8, 9, and 10 are respectively the 5 , 4, 3, 2 and 1 period accumulations of the loss variable.

(4) Interest rate was assumed to be 10%, risk-load was 0.25 and the initial surplus was).3.

(5) The survival probability is computed by counting the number of points that lie below the accumulated values of the inflows in each period.

References:

[1] Bowers, N.L et al Actuarial Mathematics, Society of Actuaries.

[2] Ramsey, C.L "A Practical Algorithm for Approximating the Probability of Ruin", Transactions of SOA Vol XLIV