ACTUARIAL RESEARCH CLEARING HOUSE
1996 VOL. 1

THE RISK-ADJUSTED PREMIUMS FOR LIFE INSURANCE AND ANNUITIES

Shaun Wang

E-mail: sswang@math.uwaterloo.ca

ABSTRACT

In the context of insurance economics, the PH-transform is justified through some basic postulates on market premiums. In this paper the PH-transform is applied to the present value random variable in life insurance and annuities. It provides an alternative to the commonly used 'equivalence principle' in life insurance and annuities premium calculation. The new pricing scheme has interesting implications such as (i) the relative loading for short-term life insurance decreases with age; (ii) more of a discount is given to long-term contracts, which will provide an incentive for loyalty from policy-holders; and (iii) the risk-adjusted cash value is less than the net premium cash value.

ACKNOWLEGEMENT: The author has benefited from many stimulating discussions with Harry Panjer. This research project is sponsored by a joint AERF/CKER Grant of the Society of Actuaries.

THE RISK-ADJUSTED PREMIUMS FOR LIFE INSURANCE AND ANNUITIES

Shaun Wang

1 INTRODUCTION

Traditional life contingencies interprets a life table as a deterministic survivorship summary resembling the negative growth rates in ecology and economics. Naturally the 'equivalence principle' (i.e., the average loss) is used in life insurance pricing. A safety margin is imposed implicitly by using conservative interest and mortality rates. For instance, a common practice is to add a percentage to the mortality rates.

Only until recently does life insurance mathematics take a probabilistic approach. The modern text of *Actuarial Mathematics* [1] starts with the assumption that time-until-death (T_x) is a continuous random variable. By recognizing that the present value of insurance benefits, Z, is a function of the time-until-death T_x, life contingencies become an integral part of risk theory. With this probabilistic approach, many rich random variable concepts come into play; for instance, survivor distribution, hazard rate, expected value, and variance.

It is recognized that the present value of insurance benefits, Z, is conceptually and mathematically the same as the loss variable in general insurance. Thus, the risk-loading can be imposed in the same fashion as in casualty insurance.

Recently, WANG [4] proposed a new premium calculation principle based on proportional hazard transforms, which can be applied to any loss distribution regardless of whether it is for casualty insurance or for life insurance and annuities.

This paper applies the proportional hazard transform method to the pricing of life insurance and annuities. It has the following practical implications:

1. Under today's pricing scheme, short-term insurance for younger ages are under-

priced, but overpriced for older ages.

2. With the new pricing theory, the risk-adjusted cash value is less than the net premium cash value. Thus the insurer and the continuing policies can better be guarded from losses in cases of massive withdrawals.

3. The new pricing theory gives more of a discount to long-term insurance contracts which, in turn, will provide an incentive for long-term commitment (loyalty) from policy-holders.

The **notations** will be mostly the same as in *Actuarial Mathematics* [1].

l_x	The number of survivors at age x
$T_x = T$	The time-until-death for a person of age x
${}_tp_x = 1 - {}_tq_x$	$\Pr\{T_x > t\}$
$\Phi(A)$	the risk-adjusted premium for a life insurance contract A

2 DETERMINISTIC VERSUS STOCHASTIC LIFE CONTINGENCIES

Consider the following short-term insurance contracts:

policy	age x at issue	term	mortality rate q_x	benefit
1	35	1 year	0.0016	100,000
2	70	1 year	0.0320	5,000

For simplicity, we ignore the interest rate discounting factor (i.e. $v = \frac{1}{1+i} = 1$).

2.1 Deterministic life contingencies

As a result of the deterministic view of life table, the 'equivalence principle' is used in premium calculation. The net premiums for both policies 1 and 2 are the same at

$$100,000 \times 0.0016 = 5,000 \times 0.032 = 160.$$

If, as in common practice, safety margins are imposed by adding a percentage (say 15%) to the mortality rates, both contracts will have the same risk-adjusted premium at 184.

2.2 Stochastic life contingencies

The standard deviations of losses for two contracts are

$$\sigma(Z_1) = 100,000\sqrt{(0.0016)(0.9984)} = 3996.8,$$
$$\sigma(Z_2) = 5,000\sqrt{(0.032)(0.968)} = 880.$$

If risk-loading is based on the standard deviation principle

$$\Phi(Z) = E(Z) + \alpha\sigma(Z), \quad \text{say } \alpha = 0.01, \tag{1}$$

the risk-loaded premium for policy 1 is

$$\Phi(Z_1) = 160 + 3996.8(0.01) = 200;$$

and the risk-loaded premium for policy 2 is

$$\Phi(Z_2) = 160 + 880(0.01) = 168.8.$$

2.3 A general comparison

Consider a general one-year term insurance for (x) with face amount b payable at the end of the year of death. Assume that the mortality rate is q_x and the effective interest is i. The present value of the insurance loss is

$$Z = \begin{cases} b/(1+i), & \text{if } T_x < 1, \\ 0, & \text{if } T_x > 1. \end{cases}$$

The standard deviation principle gives a risk-loading of

$$\alpha\sigma(Z) = \frac{\alpha b}{1+i}\sqrt{q_x(1-q_x)}.$$

Since

$$\frac{d\sigma(Z)}{dq_x} = \frac{b}{1+i}\frac{1-2q_x}{\sqrt{q_x(1-q_x)}},$$

the risk-loading increases with age provided that $q_x \leq 0.5$ as in most practical cases.

The relative risk loading

$$\frac{\alpha\sigma(Z)}{E(Z)} = \alpha\sqrt{\frac{1-q_x}{q_x}}$$

is a decreasing function of q_x. This implies that the relative risk loading should decrease with age.

Now let us re-examine the traditional premium calculation method. An increase of 15% in the mortality rates results in a relative risk loading of 15%. This does not reflect the relative variability of the insurance contracts at different ages.

Based on the above analysis, the traditional 'equivalence principle' underprices short-term contracts for younger ages, but overprices for older ages. In other words, younger ages are subsidized while old ages are penalized.

3 A DEFINITION FOR A HIGHER RISK

We define a risk Z as a non-negative loss random variable. It is determined by its distribution function $F_Z(t) = \Pr\{Z \leq t\}$ or survivor function $S_Z(t) = \Pr\{Z > t\}$. For convenience, we do not distinguish between risk Z and its distribution function $F_Z(.)$.

As a basic requirement, an equitable pricing scheme should assign a higher premium for a higher risk. What is the definition for a higher risk then ? Fortunately, there is a generally agreed upon definition for higher risks (e.g. ROTHSCHILD and STIGLITZ [3]).

Definition 1 *Risk Z_1 is less risky than Z_2 (notation $Z_1 \prec Z_2$) if and only if any of the following equivalent conditions hold*

1. *For all increasing concave utility functions u,*

$$E[u(-Z_1)] \geq E[u(-Z_2)]$$

i.e., an ordering shared by all risk-averters.

2. *The net stop-loss premiums satisfy the inequality*

$$\int_x^\infty (y-x)dF_{Z_1}(y) \leq \int_x^\infty (y-x)dF_{Z_2}(y), \quad \textit{for all } x \geq 0.$$

By assuming that all prudent insurers are risk-averse, an equitable pricing should preserve this natural ordering of riskiness.

Remark: It is noted that the standard deviation principle does not preserve the natural ordering of risks and as such is not equitable. For example, consider an n-year term insurance with an increasing benefit

$$b_t = \begin{cases} (1+i)^t, & 0 \leq t \leq n, \\ 0 & n < t. \end{cases}$$

The present value of insurance benefits, Z, has a Bernoulli distribution with $\Pr\{Z = 1\} = {}_nq_x$.

The risk-adjusted premium based on the standard deviation principle is

$$\Phi(Z) = q + \alpha\sqrt{q(1-q)}, \qquad \text{where } q = {}_nq_x.$$

One can easily verify that

$$\frac{d\Phi(Z)}{dq} = 1 + \frac{\alpha}{2}\frac{(1-2q)}{\sqrt{q(1-q)}}.$$

Since $\frac{d\Phi(Z)}{dq}|_{q=0.5} = 1$ and $\lim_{q\to 1}\frac{d\Phi(Z)}{dq} = -\infty$, after some point in time the risk-adjusted premium decreases as the term of the contract increases. Obviously the underlying risk increases as the term n increases. Thus the natural ordering of risks is violated by the standard deviation principle.

4 AN EQUITABLE RISK/REWARD RELATIONSHIP

Definition 2 *The* **proportional hazard (PH)** *transform with index $\rho \geq 1$ is defined as a mapping*

$$\Pi_\rho : Z \mapsto \Pi_\rho(Z)$$

such that

$$S_{\Pi_\rho(Z)}(t) = S_Z(t)^{\frac{1}{\rho}} \qquad (\rho \geq 1). \tag{2}$$

When Z is continuous, the PH transform (2) is equivalent to a proportional decrease in the hazard rate $\mu_Z(t) = -\frac{d}{dt}\log S_Z(t)$:

$$\mu_{\Pi_\rho(Z)}(t) = \frac{1}{\rho}\mu_Z(t), \quad t \geq 0, \rho \geq 1.$$

In terms of density functions

$$f_{\Pi_\rho(Z)}(t) = [\frac{1}{\rho}S_Z(t)^{\frac{1}{\rho}-1}] f_Z(t),$$

where the increasing weight function $\frac{1}{\rho}S_Z(t)^{\frac{1}{\rho}-1}$ gives more weight to larger losses. The expected value of the transformed variable $\Pi_\rho(Z)$ contains a risk-premium.

Definition 3 *For a given risk Z, the* **risk-adjusted premium** *is defined as the expected value of the transformed variable $\Pi_\rho(Z)$:*

$$\pi_\rho(Z) = E[\Pi_\rho(Z)] = \int_0^\infty S_Z(t)^{\frac{1}{\rho}}dt, \qquad \rho \geq 1. \tag{3}$$

It is shown in WANG [4] that $\pi_\rho(Z)$ $(\rho \geq 1)$ has the following desirable properties:

1. $E(Z) \leq \pi_\rho(Z) \leq \max(Z)$.

2. $\pi_\rho(a\,Z + b) = a\,\pi_\rho(Z) + b, \quad a \geq 0.$
   $\pi_\rho(aZ) = a\pi_\rho(Z)$ (scale invariant or positive homogeneous);
   $\pi_\rho(Z + b) = \pi_\rho(Z) + b$ (translation invariant);
   $\pi_\rho(b) = b$ (no unjustified loading).

3. Sub-additivity: for any two risks Z_1 and Z_2, be they independent or not,

   $$\pi_\rho(Z_1 + Z_2) \leq \pi_\rho(Z_1) + \pi_\rho(Z_2).$$

   (Otherwise one can purchase separate insurance for risks Z_1 and Z_2.)

Theorem 1 *The PH-transform principle (3) is equitable in the sense that it preserves the natural ordering of risks:*

$$Z_1 \prec Z_2 \Longrightarrow \pi_\rho(Z_1) < \pi_\rho(Z_2).$$

Proof: See WANG [5].

A function g is weakly increasing if g is non-decreasing but non-constant.

Lemma 1 *If g is weakly increasing, then g is continuous on the whole range* **except** *at most* **countably** *many* **(e.c.)** *points.*

Lemma 2 *For any non-negative random variable Z,*

$$E(Z) = \int_0^\infty S_Z(t)dt.$$

Furthermore, if $S_{Z_1} = S_{Z_2}$ except at most countably many points, then $E(Z_1) = E(Z_2)$.

Theorem 2 *If $Z = h(T)$ and h is a weakly deceasing function, then*

$$\pi_\rho(Z) = E[h(T^*)], \quad \textit{where } F_{T^*}(t) = [F_T(t)]^{\frac{1}{\rho}}.$$

Proof: Since $Z = h(T)$ and h is weakly decreasing, we have

$$S_{h(T)}(t) = \Pr\{h(T) > t\} = \Pr\{T < h^{-1}(t)\} = F_T(h^{-1}(t)), \text{ e.c.}$$

$$\pi_\rho(Z) = \int_0^\infty [S_{h(T)}(t)]^{\frac{1}{\rho}} dt = \int_0^\infty F_T(y)^{\frac{1}{\rho}} dh(y), \qquad \text{with } t = h(y).$$

Similarly,

$$E(h(T^*)) = \int_0^\infty F_{T^*}(y)dh(y)$$

Thus, when $F_{T^*}(t) = F_T(t)^{\frac{1}{\rho}}$ we have $\pi_\rho(Z) = E(h(T^*))$. □

5 LEVEL BENEFIT INSURANCE

We assume that the force of interest $\delta(t)$ is deterministic, and the discounting function is $v_t = \exp[-\int_0^t \delta(s)ds]$. In the special case of constant force of interest $\delta(t) = \delta$ and $v_t = e^{-\delta t}$.

An insurance contract is defined by the benefit function b_T. The loss variable, as a function of the time-until-death T, can be expressed as

$$Z = b_T v_T.$$

When the insurance benefit is fixed at a constant level, we call it level benefit insurance, which is a common form of life insurance contracts.

Lemma 3 *For a level benefit n-year term (or n-year endowment) insurance, the loss variable Z is a decreasing function of the time-until-death variable T.*

Remark:

- A whole life insurance can be viewed as a special n-year term insurance with $n = \omega - x$ (the limiting remaining future lifetime).
- The Lemma holds regardless whether the payment is made at the moment of death, or at the end of the period in which death occurs.

Theorem 3 *For a level benefit n-year term (or n-year endowment) insurance, the risk-adjusted premium with index $\rho \geq 1$ is*

$$\Phi(Z) = E(b_{T^*} v_{T^*}), \qquad \textit{with} \;\; F_{T^*}(t) = [F_T(t)]^{\frac{1}{\rho}}.$$

In other words, for a level benefit n-year term (or n-year endowment) insurance, the risk-adjusted premium is still the expected loss but under an adjusted time-until-death variable T^*. This method of pricing is equitable.

Remark: When calculating the risk-adjusted premium $\Phi(Z)$ for a level benefit n-year term (or n-year endowment) insurance, the adjusted time-until-death variable T^* does not depend on the deterministic interest rates. It is noted that the standard deviation principle lacks this nice property.

5.1 Term Insurance

For an n-year term insurance payable at the end of the year of death,

$$\begin{aligned} A^1_{x:\overline{n}|} &= \sum_{k=0}^{n-1} v^{k+1}\, {}_kp_x\, q_{x+k} \\ &= \sum_{k=0}^{n-1} v^{k+1}[{}_{k+1}q_x - {}_kq_x] \\ &= \sum_{k=1}^{n-1}(1-v)v^k\, {}_kq_x + v^n\, {}_nq_x. \end{aligned} \tag{4}$$

The risk-adjusted premium for an n-year term insurance can be calculated as

$$\begin{aligned} \Phi(A^1_{x:\overline{n}|}) &= \sum_{k=0}^{n-1} v^{k+1}[({}_{k+1}q_x)^{\frac{1}{\rho}} - ({}_kq_x)^{\frac{1}{\rho}}] \\ &= \sum_{k=1}^{n-1}(1-v)v^k\,({}_kq_x)^{\frac{1}{\rho}} + v^n\,({}_nq_x)^{\frac{1}{\rho}}. \end{aligned} \tag{5}$$

From (5) we can get a recursive method to evaluate $A^1_{x:\overline{n}|}$ as follows:

- Firstly,

$$\Phi(A^1_{x:\overline{1}|}) = v(q_x)^{\frac{1}{\rho}} = v(1-\frac{l_{x+1}}{l_x})^{\frac{1}{\rho}};$$

- For k from 2 to n apply the following recursion:

$$\Phi(A^1_{x:\overline{k}|}) = \Phi(A^1_{x:\overline{k-1}|}) + v^k[(1-\frac{l_{x+k}}{l_x})^{\frac{1}{\rho}} - (1-\frac{l_{x+k-1}}{l_x})^{\frac{1}{\rho}}]. \tag{6}$$

5.2 Endowment insurance

For an n-year endowment insurance payable at the end of the year of death,

$$A_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^{k+1}\, {}_kp_x\, q_{x+k} + v^n\, {}_np_x$$

$$= \sum_{k=1}^{n-1}(1-v)v^k\,{}_kq_x + v^n$$

The risk-adjusted premium for an n-year endowment insurance is

$$\Phi(A_{x:\overline{n}|}) = \sum_{k=1}^{n-1}(1-v)v^k\,({}_kq_x)^{\frac{1}{\rho}} + v^n.$$

Remark: When calculating the risk-adjusted premium, it is easier to work with the life table function l_x than the mortality rates q_x (noting that ${}_kq_x = 1 - l_{x+k}/l_x$). If the benefit is payable at the moment of death, effective approximations can be done by refining the life table in terms of a smaller time unit. For instance, one can interpolate the life table functions for ages at every month or every week.

6 AN EXAMPLE

Consider a person of age 50. Assume that the mortality rates are as given by the Illustrative Life Table in *Actuarial Mathematics* [1] which is based on the Makeham law for ages 13-110:

$$1000\mu_x = 0.7 + 0.05(10^{0.04})^x.$$

Assume that the face amount is 1,000 and the annual effective interest rate is a constant at 6%. Use $\rho = 1/0.95$.

By using recursion (6) one can easily calculate the risk-adjusted single premiums. Table 1 gives the risk-adjusted single premiums for k-year term insurance ($k = 1, 2, \cdots, 5$).

First assume that the person (50) chooses to buy 1-year term insurance and to renew every year. The 1-year term premiums for subsequent years are given in Table 2.

From Table 2 one can see that the 1-year term premium goes up with age every year. The expected present value of total (future) premium payments is

$$\sum_{k=0}^{4} {}_kp_{50}v^k\Phi(A^1_{50+k:\overline{1}|}) = 37.0708,$$

Table 1: RISK-ADJUSTED PREMIUMS FOR A PERSON OF AGE 50

Term	Single premium
1 year	$\Phi(A^1_{50:\overline{1}\rvert}) = 7.2176$
2-year	$\Phi(A^1_{50:\overline{2}\rvert}) = 14.0522$
3-year	$\Phi(A^1_{50:\overline{3}\rvert}) = 20.8152$
4-year	$\Phi(A^1_{50:\overline{4}\rvert}) = 27.5698$
5-year	$\Phi(A^1_{50:\overline{5}\rvert}) = 34.3458$

Table 2: RISK-ADJUSTED PREMIUMS FOR ONE-YEAR TERM INSURANCE

Age	Premium
50	$\Phi(A^1_{50:\overline{1}\rvert}) = 7.2176$
51	$\Phi(A^1_{51:\overline{1}\rvert}) = 7.7981$
52	$\Phi(A^1_{52:\overline{1}\rvert}) = 8.4316$
53	$\Phi(A^1_{53:\overline{1}\rvert}) = 9.1230$
54	$\Phi(A^1_{54:\overline{1}\rvert}) = 9.8774$

which is approximately 7.93% higher than the single premium $\Phi(A^1_{50:\overline{5}|}) = 34.3458$ for a 5-year term insurance.

Remark: The risk-adjusted premium provides a discount to multi-year contracts. This is reasonable since a person can only die once. Thus, the losses in each policy year are negatively correlated. By contrast, the traditional equivalence principle does not provide a discount to multi-year contracts.

6.1 Level premiums by annuity certain

If the risk-adjusted single premium $\Phi(A^1_{50:\overline{n}|})$ for an n-year term insurance is paid through n level installment payments commencing at the beginning of each policy year, then by assuming annuity certain, the annual level premium payment is

$$\phi(A^1_{50:\overline{n}|}) = \frac{\Phi(A^1_{50:\overline{n}|})}{\ddot{a}_{\overline{n}|}}.$$

Some values of $\phi(A^1_{50:\overline{n}|})$ are listed in Table 3.

Table 3: RISK-ADJUSTED LEVEL-CERTAIN PREMIUMS FOR A PERSION AT AGE 50

Term	Level certain premium
1-year	$\phi(A^1_{50:\overline{1}|}) = 7.2176$
2-year	$\phi(A^1_{50:\overline{2}|}) = 7.2308$
3-year	$\phi(A^1_{50:\overline{3}|}) = 7.3464$
4-year	$\phi(A^1_{50:\overline{4}|}) = 7.5060$
5-year	$\phi(A^1_{50:\overline{5}|}) = 7.6920$

7 THE RISK-ADJUSTED CASH VALUE

Consider the same person of age 50, denoted by (50). Assume that (50) bought a 5-year term insurance and is paying a level premium $\phi(A^1_{50:\overline{5}|}) = 7.6920$ at the beginning of each policy year. It is noted that $\phi(A^1_{50:\overline{5}|})$ is higher than all $\phi(A^1_{50:\overline{k}|})$ for $k = 1, 2, 3, 4$. In the k-th policy year ($k < 5$) (50) can stop paying premiums and declare his policy as a k-year term rather than a 5-year term. (50) is allowed to do so without owing money to the insurance company since he has been paying premiums higher than any k-year term level premiums ($k < 5$). In fact, (50) may be entitled to receive some cash value by declaring a shorter term contract.

For example, if (50) stops paying premiums after the third policy year, the cash-value at the end of the third year is the accumulated balance:

$$[\phi(A^1_{50:\overline{5}|}) - \phi(A^1_{50:\overline{3}|})](1.06^3 + 1.06^2 + 1.06) = 1.1665.$$

It is very interesting to compare this risk-adjusted cash value to the net premium cash value. One can easily check that the net level premiums are

$$P^1_{50:\overline{3}|} = 6.0171, \qquad P^1_{50:\overline{5}|} = 6.4780.$$

As such, the net premium cash-value is

$$[P^1_{50:\overline{5}|} - P^1_{50:\overline{3}|}](1.06^3 + 1.06^2 + 1.06) = 1.5555.$$

Remark: We have seen that the risk-adjusted cash-value is less than the net premium cash value. Thus the insurer and the continuing policies can better be guarded from losses in cases of massive withdrawals.

Remark: Note that the exposure is earned gradually as a time-evolution process. In practice, for long-term contracts, premiums are usually collected periodically as an annuity. Since the mortality rate generally increases with age, by collecting level premiums, the insurer earns premiums more quickly than exposure. Thus no extra risk is assumed by dividing a single premium into level periodic premiums. This argument suggests that an annuity-certain should be used in dividing a single risk-adjusted premium into level periodic premiums.

Remark: The allocation of expenses and the associated modified reserves can be done accordingly as in *Actuarial Mathematics* [1].

8 DEFERRED INSURANCE

Now consider an m-year deferred n-year term insurance.

The present value of insurance benefit is

$$Z_x = \begin{cases} 0, & \text{if } 0 \le T(x) < m, \\ v^{T(x)}, & \text{if } m \le T(x) \le m+n \\ 0, & \text{if } m+n < T(x). \end{cases}$$

One can verify that

$$Z_x = (v^m \, {}_m p_x) \, Z_{x+m}$$

where

$$Z_{x+m} = \begin{cases} v^{T(x+m)}, & \text{if } 0 \le T(x+m) \le n \\ 0, & \text{if } n < T(x+m). \end{cases}$$

From the equation

$$\Pr\{Z_x > y\} = {}_m p_x \Pr\{Z_{x+m} > y/v^m\}$$

we have

$$\Phi({}_{m|}A^1_{x:\overline{n}|}) = v^m({}_m p_x)^{\frac{1}{\rho}} \Phi(A^1_{x+m:\overline{n}|}).$$

9 INSURANCE PRICING: A GENERAL MODEL

Now we consider a more general insurance contract in which

1. The benefit varies with time of death and may depend on other factors (e.g. cause of death, place of death, etc).
2. The interest rates vary with time and follow some stochastic process.

It is well-known that interest rate is an important factor in pricing long-term contracts. Nevertheless, one can always use Monte Carlo simulation to generate various scenarios. Once the sample loss distribution is obtained by simulation, one can apply the PH-transform to this sample distribution to arrive at a price for the insurance contract.

Generally, unlike life insurance contracts, annuities are increasing functions of the future lifetime variable.

Theorem 4 *If $Y = g(T)$ and g is weakly increasing function, then*

$$\pi_\rho(Y) = E[g(\Pi_\rho(T))], \qquad \rho > 0.$$

Proof:

(i) Firstly

$$S_Y(t) = \Pr\{g(T) > t\} = S_T(g^{-1}(t)), \quad \textbf{e.c.}$$

(ii) Secondly

$$S_{g(\Pi_\rho(T))}(t) = \Pr\{\Pi_\rho(T) > g^{-1}(t)\} = [S_T(g^{-1}(t))]^{\frac{1}{\rho}}, \quad \textbf{e.c.}$$

(iii) Therefore, $[S_Y(t)]^{\frac{1}{\rho}} = S_{g(\Pi_\rho(T))}(t)$ **e.c.** From Lemma 2 we know that

$$\pi_\rho(Y) = E[g(\Pi_\rho(T))], \qquad \rho > 0. \quad \square$$

Theorem 5 *For a single premium annuity, the risk-adjusted premium can be calculated as the expected cost under an adjusted future lifetime variable $T^* = \Pi_\rho(T)$ $(\rho \geq 1)$, i.e., the proportional hazard transform of T.*

For example, the net single premium for an m-year deferred life annuity is

$$_{m|}\ddot{a}_x = \sum_{k=m}^{\omega-x-1} v^k \, _kp_x.$$

Given an index ρ, the risk-adjusted single premium is

$$\Phi(_{m|}\ddot{a}_x) = \sum_{k=m}^{\omega-x-1} v^k \, (_kp_x)^{\frac{1}{\rho}}.$$

Annuities with multiple decrements can be adjusted similarly by a proportional decrease in each single associated hazard rate.

11 CONCLUSIONS AND FUTURE RESERACH

In insurance pricing, equity is demonstrated through the risk/reward relationship: higher premium for higher risk. The new pricing theory of this paper has an inherent merit of equity.

The practical implications are rather interesting and need more discussion from practicing actuaries. Many people believe that insurance prices should be market driven. However, in today's life insurance market, the relative low prices for short-term contracts and the high premiums for long-term contracts are rooted in the actuarial 'equivalence principle'. As a result, many policy-holders are moving away from the overpriced long-term contracts to the underpriced short-term polices.

An individual, who may enjoy the low prices for short-term life insurance at younger ages, will probably pay back more when he/she gets older.

If some insurers take the lead to give more of a discount to long term contracts, policy-holders would realize the benefit of long-term commitment and thus choose long-term contracts. The loyalty from policy-holders, in return, will further reduce the transaction costs and commissions.

In *Actuarial Mathematics* [1], life contingencies is thought as an integral part of risk theory (a part of non-life insurance mathematics). KLING's [2] doctoral thesis also applied non-life techniques to life insurance problems. In WANG [4]), the PH-transform was firstly proposed to price casualty insurance layers based on considerations such as layer-additivity and increased relative loading at upper layers. The present paper shows that the PH-transform provides a unified approach to risk loads in both life and non-life insurance.

It is noted that insurance and investment have a dual relationship. It is possible to apply the PH transform to the pricing of bond default risk. However, one should be aware of the complications of diversifiable/non-diversifiable risks.

12 REFERENCES

1 BOWERS, N.L., GERBER, H.U., HICKMAN, J.C., JONES, D.A., and NESBITT, C.J. *Actuarial Mathematics.* Itasca, Ill.: Society of Actuaries, 1986.

2 KLING, B. *Life Insurance, A Non-Life Approach.* The Tinbergen Institute Research Series, no. 60. Amsterdam, 1993.

3 ROTHSCHILD, M., and STIGLITZ, J.E. "Increasing risk I: a definition", *Journal of Economic Theory* **2**, (1970), 225-243.

4 WANG, S. "Insurance Pricing And Increased Limits Ratemaking By Proportional Hazards Transforms", *Insurance: Mathematics and Economics*, **16**, (1995).

5 WANG, S. "Ordering of risks under PH transforms", The Institute of Insurance and Pension Research, University of Waterloo, Research Report, 1995-05.