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**Capital Allocation by Possibilistic Linear Programming Approach**

**Zhen Huang**

**Otterbein College, Westerville, Ohio**

**Lijia Guo \***

**The Ohio State University, Columbus, Ohio**

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## I. Models

Consider  $N$  asset classes,  $S_1, S_2, \dots, S_N$ , the problem is to determine allocation weights  $x_1, x_2, \dots, x_N$ .

We first review the Mean-Variance Approach (see [3]):

Assume the rate of return,  $R_i$ , of asset  $S_i$  to be a random variable

$\mu_i$  = expected value of  $R_i$ ,

$\sigma_i$  = standard deviation of  $R_i$ ,

$\rho_{ij}$  = correlation between  $R_i$  and  $R_j$

$x_i$  := the weight for asset class  $S_i$

$i, j = 1, 2, \dots, N$ .

Then the return rate of the portfolio is

$$R_p = \sum_{i=1}^N x_i R_i$$

The expected return of the portfolio is

$$\mu_p = \sum_{i=1}^N x_i \mu_i$$

The variance of the portfolio is

$$\sigma_p^2 = \sum_{i=1}^N x_i^2 \sigma_i^2 + \sum_{i=1}^N \sum_{j=1}^N x_i x_j \sigma_i \sigma_j \rho_{ij}.$$

Allocation weights  $x_1, x_2, \dots, x_N$  are determined by quadratic programming techniques.

Method 1:

Fix the expected portfolio return  $\mu_p$  to a certain desired level  $\mu$  then determine the allocation weights  $x_1, x_2, \dots, x_N$  which minimize the risk level  $\sigma_p$  of the portfolio for this fixed  $\mu$ .

$$\text{Min} \quad \sigma_p = \sum_{i=1}^N x_i^2 \sigma_i^2 + \sum_{j=1}^N \sum_{i=1}^N x_i x_j \sigma_i \sigma_j \rho_{ij} \quad (1.1)$$

*Subject to*

$$\sum_{i=1}^N x_i \mu_i = \mu \quad (1.2)$$

$$\sum_{i=1}^N x_i = 1 \quad (1.3)$$

$$l_i \leq x_i \leq u_i \quad i = 1, 2, \dots, N \quad (1.4)$$

Where  $l_i$  and  $u_i$  are the lower bound and the upper bound on funds allocation to the  $i$ th asset class,  $i = 1, 2, \dots, N$ .

Method 2:

Fix the risk level  $\sigma_p$  of the portfolio to a tolerable level  $\sigma$ , then determine the allocation weights  $x_1, x_2, \dots, x_N$  which maximize the expected portfolio return  $\mu_p$  for this fixed  $\sigma$ .

$$\text{Max} \quad \mu_p = \sum_{i=1}^N x_i \mu_i \quad (2.1)$$

*Subject to*

$$\sum_{i=1}^N x_i^2 \sigma_i^2 + \sum_{i=1}^N \sum_{j=1}^N x_i x_j \sigma_i \sigma_j \rho_{ij} = \sigma^2 \quad (2.2)$$

$$\sum_{i=1}^N x_i = 1 \quad (2.3)$$

$$l_i \leq x_i \leq u_i \quad i = 1, 2, \dots, N \quad (2.4)$$

The above two methods are equivalent under the general assumption that asset with higher return rate has higher risk. They are conventionally called Mean-Variance Approach.

According to the portfolio theory, when the rate of return is considered having a probability distribution, we also consider maximizing skewness while consider maximizing mean return or minimizing variance (see [2])

If we take skewness into consideration, then Model (1) then becomes a multiple objective programming model (3):

$$\text{Min} \quad \sigma_p = \sum_{i=1}^N x_i^2 \sigma_i^2 + \sum_{i=1}^N \sum_{j=1}^N x_i x_j \sigma_i \sigma_j \rho_{ij} \quad (3.1)$$

$$\text{Max} \quad E[(R_p - \mu_p)^3] / \sigma_p^3 \quad (3.2)$$

*Subject to*

$$\sum_{i=1}^N x_i \mu_i = \mu \quad (3.3)$$

$$\sum_{i=1}^N x_i = 1 \quad (3.4)$$

$$l_i \leq x_i \leq u_i \quad i = 1, 2, \dots, N \quad (3.5)$$

And Model (2) becomes model (4):

$$\text{Max} \quad \mu_p = \sum_{i=1}^N x_i \mu_i \quad (4.1)$$

$$\text{Max} \quad E[(R_p - \mu_p)^3] / \sigma_p^3$$

Subject to

$$\sum_{i=1}^N x_i^2 \sigma_i^2 + \sum_{i=1}^N \sum_{j=1}^N x_i x_j \sigma_i \sigma_j \rho_{ij} = \sigma \quad (4.2)$$

$$\sum_{i=1}^N x_i = 1 \quad (4.3)$$

$$l_i \leq x_i \leq u_i \quad i = 1, 2, \dots, N \quad (4.4)$$

Note that

$$E[(R_p - \mu_p)^3] / \sigma_p^3 = \sum_{i=1}^N w_i^3 \sigma_{iii} + 3 \sum_{i,j=1}^N w_i^2 w_j \sigma_{ijj} + 6 \sum_{i,j,k=1}^N w_i w_j w_k \sigma_{ijk}$$

where  $\sigma_{ijk}$  is central co-moment. Obviously, models (3) and (4) are complicated multiple objective non-linear programming.

We now present the Possibilistic Linear Programming Approach to the model problem.

## 2. Possibilistic Distribution

For each asset  $S_i$ ,  $i = 1, 2, \dots, N$ , we describe the imprecise rate of return by

$$\tilde{r}_i = (r_i^p, r_i^m, r_i^o), \text{ where}$$

$r_i^p$  = the most pessimistic value for  $R_i$

$r_i^m$  = the most possible value for  $R_i$

$r_i^o$  = the most optimistic value for  $R_i$

We further assume that the imprecise rate of return  $\tilde{r}_i = (r_i^p, r_i^m, r_i^o)$  can be modeled by the possibility theory (see [10]) and has the triangular possibilistic distribution  $\pi_{\tilde{r}_i}(\cdot)$  as in Figure 1.

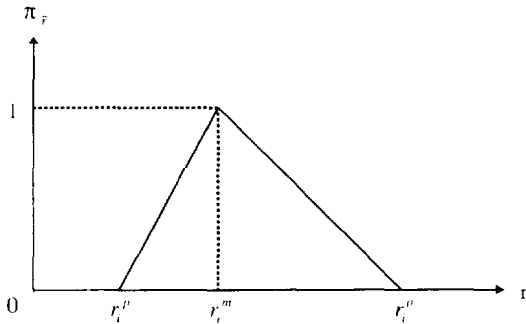


Figure 1. The triangular possibility distribution of  $\tilde{r}_i$ .

The capital asset location problem can be modeled as a possibilistic linear programming problem (5):

$$\text{Max} \quad \sum_{i=1}^N \tilde{r}_i x_i = \sum_{i=1}^N (r_i^p, r_i^m, r_i^o) x_i \quad (5.1)$$

Subject to

$$\sum_{i=1}^N x_i = 1 \quad (5.2)$$

$$l_i \leq x_i \leq u_i \quad i = 1, 2, \dots, N \quad (5.3)$$

Notice that the objective function is

$$\sum_{i=1}^N \tilde{r}_i x_i = \sum_{i=1}^N (r_i^p, r_i^m, r_i^o) x_i = \sum_{i=1}^N (r_i^p x_i, r_i^m x_i, r_i^o x_i) = (\sum_{i=1}^N r_i^p x_i, \sum_{i=1}^N r_i^m x_i, \sum_{i=1}^N r_i^o x_i)$$

therefore, the objective function is an imprecise rate of return for the portfolio  $\tilde{r} = (r^p, r^m, r^o)$ , where

$$r^p = \sum_{i=1}^N r_i^p x_i, \quad r^m = \sum_{i=1}^N r_i^m x_i \quad \text{and} \quad r^o = \sum_{i=1}^N r_i^o x_i,$$

with a triangular possibility distribution  $\pi_{\tilde{r}}(\cdot)$  in Figure 2.

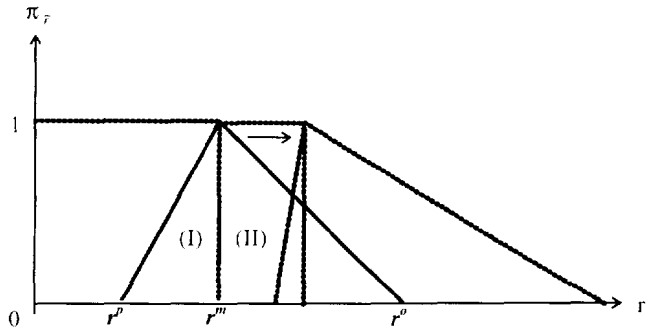


Figure 2. The triangular possibility distribution of  $\tilde{r}$

### 3. Method

In this paper, we suggest to use an auxiliary multiple-objective linear programming model (6) proposed by Lai and Hwang (see *"Possibilistic linear programming for managing interest rate risk"*, 1992) with our two additional control constraints.

$$\text{Max } z^{(1)} = \sum_{i=1}^N r_i^p x_i \quad (6.1)$$

$$\text{Min } z^{(2)} = \sum_{i=1}^N (r_i^m - r_i^p) x_i \quad (6.2)$$

$$\text{Max } z^{(3)} = \sum_{i=1}^N (r_i^u - r_i^m) x_i \quad (6.3)$$

Subject to

$$\beta_l \leq \sum_{i=1}^N r_i^p x_i \leq \beta_u \quad (6.4)$$

$$\gamma_l \leq \sum_{i=1}^N (r_i^m - r_i^p) x_i \leq \gamma_u \quad (6.5)$$

$$\sum_{i=1}^N x_i = 1 \quad (6.6)$$

$$l_i \leq x_i \leq u_i \quad i = 1, 2, \dots, N \quad (6.7)$$

By selecting parameters  $\beta_l$  and  $\beta_u$ , constraint (6.3) restricts the most possible rate of return of the portfolio to a desirable level. If we set  $\beta_l = \beta_u = \beta$  to be a constant,  $\gamma_l = \min_{i=1,2,\dots,N} \{r_i^m - r_i^p\}$  and  $\gamma_u = \max_{i=1,2,\dots,N} \{r_i^m - r_i^p\}$ , then objective (6.1) and constraint (6.5) both become inactive. In this case, model (6) becomes (6')

$$\text{Min } z^{(2)} = \sum_{i=1}^N (r_i^m - r_i^p) x_i$$



$$\text{Max } z^{(1)} = \sum_{i=1}^N (r_i^o - r_i^m) x_i$$

*Subject to*

$$\sum_{i=1}^N r_i^p x_i = \beta$$

$$\sum_{i=1}^N x_i = 1$$

$$l_i \leq x_i \leq u_i \quad i = 1, 2, \dots, N$$

It is interesting to note that model (6') is analogous to model (3).

By selecting parameters  $\gamma_l$  and  $\gamma_u$ , constraint (6.5) restricts the risk to a

tolerable interval. If we set  $\gamma_l = \gamma_u = \gamma$  to be a constant,  $\beta_l = \min_{i=1,2,\dots,N} \{r_i^m\}$  and

$\beta_u = \max_{i=1,2,\dots,N} \{r_i^m\}$ , then objective (6.2) and constrain (6.4) become inactive. In

this case, model (6) becomes (6'')

$$\text{Min } z^{(2)} = \sum_{i=1}^N (r_i^p - r_i^o) x_i$$

$$\text{Max } z^{(3)} = \sum_{i=1}^N (r_i^o - r_i^m) x_i$$

*Subject to*

$$\sum_{i=1}^N (r_i^m - r_i^o) x_i = \gamma$$

$$\sum_{i=1}^N x_i = 1$$

$$l_i \leq x_i \leq u_i \quad i = 1, 2, \dots, N$$

Model (6'') is analogous to model (4). The selection of parameters  $\beta_l$ ,  $\beta_u$ ,  $\gamma_l$ , and  $\gamma_u$  is based on the asset manager's experience and managerial judgment.

Multiple-objective linear programming (6) is then solve by using Zimmermann's fuzzy programming method with a normalization process as follows:

Let

$$\begin{aligned}
 z_{\min}^{(1)} &= \max_{x \in X} \sum_{i=1}^N r_i^m x_i & z_{\max}^{(1)} &= \min_{x \in X} \sum_{i=1}^N r_i^m x_i \\
 z_{\min}^{(2)} &= \min_{x \in X} \sum_{i=1}^N (r_i^m - r_i^u) x_i & z_{\max}^{(2)} &= \max_{x \in X} \sum_{i=1}^N (r_i^m - r_i^u) x_i \\
 z_{\min}^{(3)} &= \max_{x \in X} \sum_{i=1}^N (r_i^u - r_i^m) x_i & z_{\max}^{(3)} &= \min_{x \in X} \sum_{i=1}^N (r_i^u - r_i^m) x_i
 \end{aligned}$$

where  $X$  denotes the set of feasible solutions satisfying all the constraints in programming model (6). The linear membership function of these objective functions can now be computed (see Figure 3) as:

$$\mu_{z^{(1)}} = \begin{cases} 1 & \text{if } z^{(1)} \geq z_{\max}^{(1)} \\ (z^{(1)} - z_{\min}^{(1)}) / (z_{\max}^{(1)} - z_{\min}^{(1)}) & \text{if } z_{\min}^{(1)} < z^{(1)} < z_{\max}^{(1)} \\ 0 & \text{if } z^{(1)} \leq z_{\min}^{(1)} \end{cases}$$

$$\mu_{z^{(2)}} = \begin{cases} 1 & \text{if } z^{(2)} \leq z_{\min}^{(2)} \\ (z_{\max}^{(2)} - z^{(2)}) / (z_{\max}^{(2)} - z_{\min}^{(2)}) & \text{if } z_{\min}^{(2)} < z^{(2)} < z_{\max}^{(2)} \\ 0 & \text{if } z^{(2)} \geq z_{\max}^{(2)} \end{cases}$$

Definition of  $\mu_{z^{(3)}}$  is similar to  $\mu_{z^{(1)}}$ .

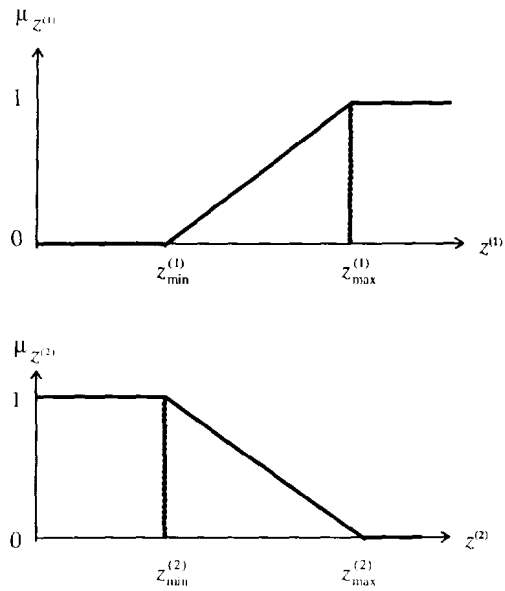


Figure 3 The linear membership function of objective functions (6.1) and (6.2)

Finally, the problem is solve by Zimmerman's equivalent single-objective linear programming model (7):

$$\begin{aligned}
 & \max \quad \lambda \\
 & \text{s.t.} \quad \mu_{z^{(1)}}(x) \geq \lambda \\
 & \quad \quad \mu_{z^{(2)}}(x) \geq \lambda
 \end{aligned}$$

$$\mu_{z^{(i)}}(x) \geq \lambda$$

$$x \in X$$

It is noted that model (7) actually uses Max-min principle which is

$$\max_{x \in X} \{ \min ( \mu_{z^{(1)}}(x), \mu_{z^{(2)}}(x), \mu_{z^{(3)}}(x) ) \}$$

The optimal solution of model (7) provides a satisfying solution under the strategy of minimizing the risk of lower rate of return, and maximizing the most possible value and the possibility of higher rate of return.

#### 4. Numerical Example

Now We consider a numerical example by using six asset classes with mean,  $\mu_i$ , and standard deviation,  $\sigma_i$ . We set  $r_i^l = \mu_i - 2\sigma_i$ ,  $r_i^m = \mu_i$ , and  $r_i^u = \mu_i + 3\sigma_i$ ,  $i = 1, 2, \dots, 6$ , with some small adjustment. The numerical data of our example is given in Table 1.

	$\mu$	$\sigma$	$r^l$	$r^m$	$r^u$	$r^m - r^l$	$r^u - r^m$
Stock 1	0.17	0.2	-0.23	0.17	0.8	0.4	0.63
Stock 2	0.15	0.185	-0.22	0.15	0.75	0.37	0.6
Bound 1	0.12	0.055	0.01	0.12	0.27	0.11	0.15
Bound 2	0.08	0.05	-0.02	0.08	0.2	0.1	0.12
Cash	0.06	0.005	0.05	0.06	0.09	0.01	0.03
T-bill	0.05	9.994	0.042	0.05	0.075	0.008	0.05

Table 1 Numerical data of example

We first solved the example by fixing the most possible rate of return of the portfolio at 22 different values while making constrain (6.4) inactive. The computational results are summarized in Table 2.

No.	$(\beta, \beta_0, \gamma, \gamma_0)$	Optimal solution X	$\tilde{r} = (r^l, r^m, r^u)$
1	(0.055, 0.055, 0.008, 0.4)	$x_5=0.5, x_6=0.5$	(0.046, 0.055, 0.0825)
2	(0.060, 0.060, 0.008, 0.4)	$x_2=0.0454, x_3=0.0152, x_4=0.4394, x_6=0.5$	(0.0331, 0.6, 0.1152)
3	(0.065, 0.065, 0.008, 0.4)	$x_2=0.0713, x_3=0.0597, x_4=0.3690, x_6=0.5$	(0.0244, 0.065, 0.1403)
4	(0.070, 0.070, 0.008, 0.4)	$x_2=0.0935, x_3=0.1098, x_4=0.2967, x_6=0.5$	(0.0164, 0.07, 0.164)

5	(0.075, 0.075, 0.008, 0.4)	$x_2=0.1157, x_1=0.1598, x_3=0.2245, x_6=0.5$	(0.0084, 0.075, 0.1876)
6	(0.080, 0.080, 0.008, 0.4)	$x_2=0.1381, x_3=0.2096, x_4=0.1524, x_6=0.5$	(0.0003, 0.08, 0.2113)
7	(0.085, 0.085, 0.008, 0.4)	$x_2=0.1605, x_1=0.2593, x_5=0.0802, x_6=0.5$	(-0.0077, 0.085, 0.2351)
8	(0.090, 0.090, 0.008, 0.4)	$x_2=0.1829, x_1=0.3089, x_4=0.0081, x_6=0.5$	(-0.0157, 0.09, 0.2588)
9	(0.095, 0.095, 0.008, 0.4)	$x_2=0.2310, x_4=0.3128, x_6=0.4562$	(-0.0285, 0.095, 0.2919)
10	(0.100, 0.100, 0.008, 0.4)	$x_2=0.2801, x_1=0.3142, x_6=0.4057$	(-0.0414, 0.1, 0.3253)
11	(0.105, 0.105, 0.008, 0.4)	$x_2=0.3250, x_1=0.3214, x_6=0.3536$	(-0.0534, 0.105, 0.357)
12	(0.110, 0.110, 0.008, 0.4)	$x_2=0.3701, x_1=0.3284, x_6=0.3015$	(-0.0655, 0.11, 0.3889)
13	(0.115, 0.115, 0.008, 0.4)	$x_2=0.4150, x_1=0.3357, x_6=0.2493$	(-0.0775, 0.115, 0.4206)
14	(0.120, 0.120, 0.008, 0.4)	$x_2=0.4601, x_1=0.3428, x_6=0.1972$	(-0.0895, 0.12, 0.4524)
15	(0.125, 0.125, 0.008, 0.4)	$x_1=0.0092, x_2=0.5, x_3=0.3415, x_6=0.1494$	(-0.1024, 0.125, 0.4858)
16	(0.130, 0.130, 0.008, 0.4)	$x_1=0.0504, x_2=0.5, x_3=0.3421, x_6=0.1074$	(-0.1137, 0.13, 0.5157)
17	(0.135, 0.135, 0.008, 0.4)	$x_1=0.0956, x_2=0.5, x_3=0.3361, x_6=0.0683$	(-0.1258, 0.135, 0.5473)
18	(0.140, 0.140, 0.008, 0.4)	$x_1=0.1410, x_2=0.5, x_3=0.3296, x_6=0.0293$	(-0.1379, 0.14, 0.579)
19	(0.145, 0.145, 0.008, 0.4)	$x_1=0.2204, x_2=0.4660, x_3=0.3136$	(-0.1501, 0.145, 0.6105)
20	(0.150, 0.150, 0.008, 0.4)	$x_1=0.3137, x_2=0.4771, x_3=0.2092$	(-0.175, 0.15, 0.6653)
21	(0.155, 0.155, 0.008, 0.4)	$x_1=0.4067, x_2=0.4888, x_3=0.1045$	(-0.2, 0.155, 0.7202)
22	(0.160, 0.160, 0.008, 0.4)	$x_1=0.5, x_2=0.5$	(-0.225, 0.16, 0.775)

Table 2 Solutions for different fixed the most possible return from portfolio

and

No.	$r^m$	$r^m - r^f$	$r^{**} - r^m$	85%	95%
1	0.055	0.0090	0.0275	(0.0537, 0.0591)	(0.0546, 0.0564)
2	0.060	0.0269	0.0552	(0.0560, 0.0683)	(0.0587, 0.0628)
3	0.065	0.0406	0.0753	(0.0590, 0.0763)	(0.0630, 0.0688)
4	0.070	0.0536	0.0940	(0.0620, 0.0841)	(0.0673, 0.0747)
5	0.075	0.0666	0.1126	(0.0650, 0.0919)	(0.0717, 0.0806)
6	0.080	0.0797	0.1313	(0.0680, 0.0997)	(0.0760, 0.0866)
7	0.085	0.0927	0.1501	(0.0711, 0.1075)	(0.0804, 0.0925)
8	0.090	0.1057	0.1688	(0.0741, 0.1153)	(0.0847, 0.0984)
9	0.095	0.1235	0.1969	(0.0765, 0.1245)	(0.0888, 0.1048)
10	0.100	0.1414	0.2253	(0.0788, 0.1338)	(0.0929, 0.1113)
11	0.105	0.1584	0.2520	(0.0812, 0.1433)	(0.0971, 0.1176)
12	0.110	0.1755	0.2789	(0.0837, 0.1518)	(0.1012, 0.1239)
13	0.115	0.1925	0.3056	(0.0861, 0.1608)	(0.1054, 0.1303)
14	0.120	0.2095	0.3324	(0.0886, 0.1699)	(0.1095, 0.1366)
15	0.125	0.2274	0.3608	(0.0909, 0.1791)	(0.1136, 0.1430)
16	0.130	0.2437	0.3857	(0.0934, 0.1879)	(0.1178, 0.1493)
17	0.135	0.2608	0.4123	(0.0959, 0.1968)	(0.1220, 0.1556)
18	0.140	0.2779	0.4390	(0.0983, 0.2059)	(0.1286, 0.1620)
19	0.145	0.2951	0.4655	(0.1007, 0.2148)	(0.1302, 0.1683)
20	0.150	0.3250	0.5153	(0.1013, 0.2273)	(0.1338, 0.1758)
21	0.155	0.3550	0.5647	(0.1018, 0.2397)	(0.1323, 0.1832)
22	0.160	0.3850	0.6150	(0.1023, 0.2523)	(0.1408, 0.1908)

Table 3 Solutions for different fixed the most possible return from portfolio

No.	$(\beta_L, \beta_M, \gamma_L, \gamma_M)$	Optimal solution X	$\tilde{r} = (r^p, r^m, r^u)$
1	(0.05, 0.17, 0.009, 0.009)	$x_4=0.5, x_6=0.5$	(0.046, 0.055, 0.0825)
2	(0.05, 0.17, 0.0269, 0.0269)	$x_2=0.0202, x_3=0.1038, x_5=0.5, x_6=0.376$	(0.0408, 0.0643, 0.1164)
3	(0.05, 0.17, 0.0406, 0.0406)	$x_2=0.0359, x_3=0.1824, x_5=0.5, x_6=0.2817$	(0.0308, 0.0714, 0.1423)
4	(0.05, 0.17, 0.0536, 0.0536)	$x_2=0.0506, x_3=0.2577, x_5=0.5, x_6=0.1917$	(0.0245, 0.0781, 0.1669)
5	(0.05, 0.17, 0.0666, 0.0666)	$x_2=0.0746, x_3=0.3001, x_5=0.5, x_6=0.1253$	(0.0169, 0.0835, 0.1914)
6	(0.05, 0.17, 0.0797, 0.0797)	$x_2=0.1061, x_3=0.3167, x_5=0.5, x_6=0.0773$	(0.0081, 0.0878, 0.2159)
7	(0.05, 0.17, 0.0927, 0.0927)	$x_2=0.1382, x_3=0.3301, x_5=0.5, x_6=0.0317$	(-0.0008, 0.0919, 0.2402)
8	(0.05, 0.17, 0.1057, 0.1057)	$x_2=0.1704, x_3=0.3435, x_5=0.4861$	(-0.0097, 0.0959, 0.2643)
9	(0.05, 0.17, 0.1235, 0.1235)	$x_2=0.2189, x_3=0.3469, x_5=0.4342$	(-0.0230, 0.1005, 0.2969)
10	(0.05, 0.17, 0.1414, 0.1414)	$x_2=0.2674, x_3=0.3514, x_5=0.3812$	(-0.0363, 0.1052, 0.3297)
11	(0.05, 0.17, 0.1584, 0.1584)	$x_2=0.3139, x_3=0.3538, x_5=0.3322$	(-0.0489, 0.1095, 0.3608)
12	(0.05, 0.17, 0.1755, 0.1755)	$x_2=0.3610, x_3=0.3552, x_5=0.2837$	(-0.0617, 0.1138, 0.3922)
13	(0.05, 0.17, 0.1925, 0.1925)	$x_2=0.4071, x_3=0.3596, x_5=0.2333$	(-0.0743, 0.1182, 0.4234)
14	(0.05, 0.17, 0.2095, 0.2095)	$x_2=0.4511, x_3=0.3710, x_5=0.1779$	(-0.0867, 0.1229, 0.4545)
15	(0.05, 0.17, 0.2274, 0.2274)	$x_2=0.4975, x_3=0.3831, x_5=0.1194$	(-0.0996, 0.1278, 0.4873)
16	(0.05, 0.17, 0.2437, 0.2437)	$x_1=0.0411, x_2=0.5, x_3=0.3765, x_5=0.0823$	(-0.1116, 0.1321, 0.5169)
17	(0.05, 0.17, 0.2608, 0.2608)	$x_1=0.0911, x_2=0.5, x_3=0.3529, x_5=0.0561$	(-0.1246, 0.1362, 0.5482)
18	(0.05, 0.17, 0.2779, 0.2779)	$x_1=0.1476, x_2=0.5, x_3=0.3034, x_5=0.049$	(-0.1385, 0.1394, 0.5794)
19	(0.05, 0.17, 0.2951, 0.2951)	$x_1=0.2042, x_2=0.5, x_3=0.2546, x_5=0.0412$	(-0.1524, 0.1427, 0.6108)
20	(0.05, 0.17, 0.325, 0.325)	$x_1=0.3025, x_2=0.5, x_3=0.1703, x_5=0.0272$	(-0.1765, 0.1485, 0.6654)
21	(0.05, 0.17, 0.355, 0.355)	$x_1=0.4013, x_2=0.5, x_3=0.0848, x_5=0.0139$	(-0.2008, 0.1542, 0.7202)
22	(0.05, 0.17, 0.385, 0.385)	$x_1=0.5, x_2=0.5$	(-0.225, 0.16, 0.775)

Table 4 Solutions for different fixed the risk for portfolio

and

No.	$r^m$	$r^m - r^p$	$r^u - r^m$	85%	95%
1	0.0550	0.0090	0.0275	(0.0537, 0.0541)	(0.0500, 0.0564)
2	0.0643	0.0235	0.0521	(0.0608, 0.0721)	(0.0631, 0.0669)
3	0.0714	0.0406	0.0709	(0.0653, 0.0820)	(0.0694, 0.0749)
4	0.0781	0.0536	0.0888	(0.0701, 0.0914)	(0.0754, 0.0825)
5	0.0835	0.0666	0.1079	(0.0735, 0.0997)	(0.0802, 0.0889)
6	0.0878	0.0797	0.1281	(0.0758, 0.1070)	(0.0838, 0.0942)
7	0.0919	0.0927	0.1483	(0.0780, 0.1142)	(0.0873, 0.0993)
8	0.0959	0.1056	0.1684	(0.0801, 0.1212)	(0.0906, 0.1043)
9	0.1005	0.1235	0.1964	(0.0820, 0.1300)	(0.0943, 0.1103)
10	0.1052	0.1414	0.2245	(0.0840, 0.1389)	(0.0981, 0.1164)
11	0.1095	0.1584	0.2513	(0.0857, 0.1472)	(0.1016, 0.1221)
12	0.1138	0.1755	0.2784	(0.0875, 0.1556)	(0.1050, 0.1277)
13	0.1182	0.1925	0.3052	(0.0893, 0.1640)	(0.1086, 0.1335)
14	0.1229	0.2096	0.3316	(0.0915, 0.1726)	(0.1124, 0.1395)
15	0.1278	0.2274	0.3595	(0.0937, 0.1817)	(0.1165, 0.1458)
16	0.1321	0.2437	0.3848	(0.0958, 0.1898)	(0.1199, 0.1513)

Fifth column in Table 3 gives all the most possible rates of return of the portfolio whose degree of occurrence is at least 0.85. This interval is called the acceptable event with degree of occurrence at least 0.85. Similarly, the last column in Table 3 gives the acceptable event with degree of occurrence at least 0.95. We observed that both risk ( $r^m - r^n$ ) and skewness ( $r'' - r'''$ ) increase as  $r^m$  increases, which is consistent with the fact that as  $r^m$  is pushed higher, more weight should be allocated to higher risk assets. We also observed that when  $r^m$  increases gradually, the weights are adjusted gradually, which shows our numerical results are stable.

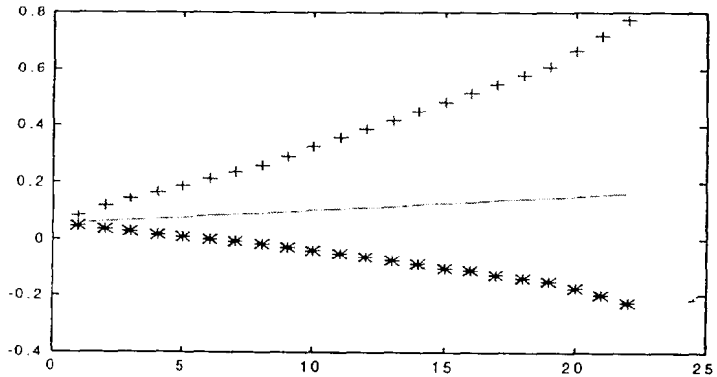


Figure 4

Secondly, we solved the example by fixing the risk ( $r^m - r^n$ ) of the portfolio at 22 different values while making constrain (6.3) inactive. The computational results are summarized in Table (4) and (5).

17	0.1362	0.2608	0.4120	(0.0971, 0.1980)	(0.1232, 0.1568)
18	0.1394	0.2779	0.4400	(0.0977, 0.2054)	(0.1255, 0.1614)
19	0.1427	0.2951	0.4681	(0.0984, 0.2129)	(0.1279, 0.1661)
20	0.1485	0.3250	0.5169	(0.0998, 0.2260)	(0.1323, 0.1743)
21	0.1542	0.3550	0.5660	(0.1100, 0.2391)	(0.1365, 0.1825)
22	0.160	0.3850	0.6150	(0.1023, 0.2523)	(0.1408, 0.1908)

Table 5 Solutions for different fixed the risk for portfolio

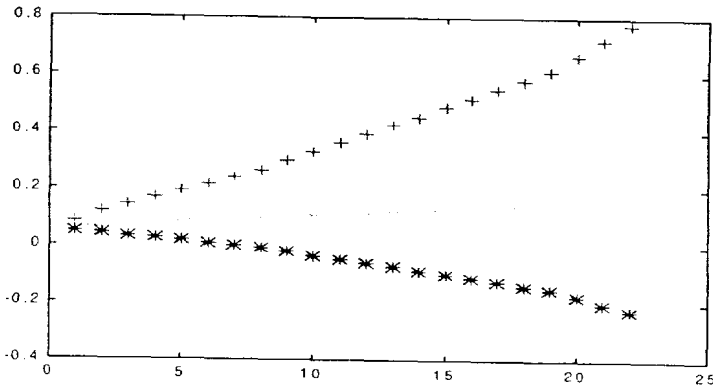


Figure 5

#### 4. Conclusion

Unlike our approach, traditional mean-variance method does not take skewness of the random rate of return into consideration. It only considers minimum variance for a given acceptable rate of return, which limited the probability of obtaining higher rate of return.

If the skewness,  $s^3$ , of return of return for the portfolio,  $R_p = \sum_{i=1}^N w_i R_i$ , is incorporated into model of traditional approach, say  $s^3 > k$ , then the resulting non-linear programming



model is not computational efficient since  $s^3$  which certainly is complicated. On the other hand, possibilistic programming provides more efficient techniques to solve the imprecision nature of rate of return and also preserves the original linear model. Besides, possibilistic distributions provide more flexible and meaningful representation of imprecision/uncertainty. Some problems still remain to be solved in the future research. In this study, we first obtained the most pessimistic value, the most probable value and the most optimistic value for the rate of return from mean and standard deviation. Secondly, we assume that possibility distributions are triangular. However, in practice, we should generate the most pessimistic value, the most probable value, the most optimistic value and the possibility distributions from decision makers' experience and managerial judgment and historical resources.

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