Optimal Hedge Ratio and Hedge Efficiency: An Empirical Investigation of Hedging in Indian Derivatives Market

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Abstract

Risk is omnipresent, and hedging has been motivated by *the desire to reduce risk*. An essential feature of hedging is that the trader synchronizes his/her positions in two markets. One is generally the “cash" or "spot" market (the market for immediate delivery), while the other is the derivatives market (Johnson, 1960). Studies of hedging carried out in developed markets like the United States and Europe have finally arrived in emerging markets such as China, India, Brazil, Russia and many other Asian countries. Spyros (2005) has presented a brief account of contemporary studies in this area. Among emerging markets, India has a considerably large derivatives market supported by prudent risk management systems and a growing economy. However, hedging one’s stock position through futures and options in the Indian context is still the road less travelled. Even if it is done, the techniques used have been too naïve and primitive. This paper tries to explore Indian futures and options market as a market for hedging by equity holders. We have tried to examine optimal hedge ratio and hedge efficiency, and to provide empirical evidence from India.

An extensive literature review has enabled us to appreciate the advances in HKM (Herbst, Kare and Marshall, 1993) methodology in comparison to JSE (Johnson, 1960; Stein, 1961; and Ederington, 1979) methodology. JSE methodology has two limitations. It fails to follow basic tenets of econometrics. For example, residuals from JSE estimation of optimal hedge ratio are serially correlated. Therefore, a Box-Jenkins autoregressive, integrated moving average (ARIMA) technique should be used to estimate the minimum risk hedge to account for the serial correlation of error terms (Herbst, Kare and Caples, 1989). The JSE model fails to appreciate the fact that futures prices converge to their spot/cash market price on the maturity date. HKM methodology take cares of these two very basic but most serious lacunae in the JSE model. In case of option pricing, two problems exist in the Black-Scholes model.

Many studies have established that returns are not normally distributed as they were found to have fat tails. This implies that use of high frequency data will help in estimation of more accurate central measures. The second problem leads to long-range dependence. According to Efficient Market Hypothesis (EMH), all information is reflected in current asset prices, and hence it is reasonable to assume a Markovian process. However, this is only true when the market is efficient in strong form, which may not be valid in reality. Traders have been using long-term memory strategies to outperform the market. This motivated a series of research studies further purporting the existence of a non-Markovian process (Lo and MacKinlay, 1993). A few stochastic models have been developed that can produce quasi long-range dependence. However, these models are very complex as they use high-dimensional partial differential equations with variable coefficients.

Fractional Brownian motion (fBM) deals with the second problem while still assuming a Gaussian process. Nevertheless, it offers the promise of giving simple, tractable solutions to pricing financial options and presents a natural way of modelling long-range dependence, measured by Hurst parameter “H.” We have estimated the optimal hedge ratio based on HKM methodology using the JSE model as the benchmark for futures. To estimate the optimal hedge ratio for options, we have used fBM methodology with BSM (Black-Scholes model, 1973) as the

benchmark. We have estimated the returns on hedged positions to empirically validate the efficiency of optimal hedge ratios.

A study of hedging of Nifty (National Stock Exchange of India—NSE—50 Index) price risk through index futures and options is conducted using high frequency data (from 01.01.2002 to 28.03.2002). We find that estimates of optimal hedge ratio based on competing models (HKM in case of futures and fBM in case of options) are better than those estimated using benchmark models (JSE for futures and BSM for options, respectively). The results are statistically significant at 95 percent confidence level. However, the returns on hedged positions using the superior optimal hedge ratios are not significantly different. This is quite puzzling, and requires a plausible explanation.

1. Introduction

Risks are omnipresent and exist from time immemorial. In financial parlance, risk is any variation from an expected outcome. So, for an investor, risk includes an outcome when one may not receive the expected return (Stein, 1961). Traditionally, hedging has been motivated by *the desire to reduce risk* by taking a position *opposite* to the exposure. The quest for better hedge has been the motive for sophisticated risk management and hedging techniques. Derivatives are used as a tool to transfer risk, i.e., for hedgers (Bodla and Jindal, 2006) and, therefore, they are extensively used as hedging instruments worldwide, including emerging markets like Malaysian, Italian and Portuguese equity markets.

However, hedging one's stock position through futures and options is still the road less travelled in India. Even when it is done, the techniques used have been too naïve and primitive. Lack of suitable hedging models for the Indian market is a challenge to the risk management system of participants and regulators. It is also a deterrent for attaining greater market depth, and may severely affect the stability of Indian markets. Further, availability of high frequency data in the recent past will help validate such models empirically.

1.1 Motivation

Johnson (1960) has pointed out that hedgers prefer to hedge through the futures market as it is easier to square off and opt for cash settlement than taking actual delivery as is the case with the forward market, since the objective is to take advantage of relative price movements. The same is true for hedging with options. This study focuses on hedging price risk of equity index through index futures and options contracts. However, the models used have been too naïve and primitive and based on the assumption that the price movements are negatively correlated, and hence gains from one market offset the losses in the other. Even National Stock Exchange (NSE) of India Ltd., whose NCFM (NSE's Certification in Financial Markets) certification is mandatory for market participants, discusses naïve hedging only. This study is, therefore, an attempt to explore the Indian derivatives market for hedging by equity holders.

We reviewed the advances in HKM (Herbst, Kare and Marshall, 1993) methodology, and compared them with JSE (Johnson, 1960; Stein, 1961; and Ederington, 1979) methodology. We present a comparative study of HKM and JSE methodology for estimating optimal hedge ratio and hedge efficiency for futures. We propose to test JSE and HKM methodologies for estimating optimal hedge ratio and hedge efficiency using high frequency data from Indian financial futures market. Similarly, in the case of options, we compare Fractional Brownian motion (fBM) methodology with Black-Scholes model (BSM). We have estimated the returns on hedged positions to empirically validate the efficiency of optimal hedge ratios.

The paper is organized as follows. Section 2 covers a brief review of hedging and its evolution in chronological order followed by statement of hypotheses in Section 3. Results are discussed in Section 4, and conclusions are included in Section 5.

2. Review of Literature

Experts from different disciplines such as mathematics, statistics, economics, computer science, information technology and finance have contributed to the literature on derivatives. There are two main hypotheses to explain hedging. They are: (i) destabilizing force hypothesis; and (ii) market completion force/non-destabilization hypothesis. Destabilizing force hypothesis propounds that the derivatives market attracts highly levered and speculative participants due to lower trading costs, which creates artificial price bubbles and increases volatility in the spot market. Market completion force/non-destabilization hypothesis states that introduction of derivatives complements the spot market and improves information flow resulting in better investment choices for investors. It may bring more private information to the market and disseminate the same faster. Some studies suggest a possibility of speculators moving to the derivatives market from the spot market due to lower transaction costs and other benefits like cash settlement. This may lead to reduction in volatility.

Available evidence on financial futures can be divided into five areas: (i) Impact of (launch of) futures on spot market volatility (Shenbagaraman, 2002; Hetamsaria and Swain, 2003; Nagraj and Kotha, 2004; Thenmozhi and Thomas, 2004; Hetamsaria and Deb, 2004; Josi and Mukhopadhyay, 2004; Bodla and Jindal, 2006; Bagchi, 2006; Rao, 2007); (ii) Lead-lag relationship (reflected in price and non-price variables) between futures and spot market (Srivastava, 2003; Sah and Omkarnath, 2005; Praveen and Sudhakar, 2006; Mukherjee and Mishra, 2006; Gupta and Singh, 2006); (iii) Role of futures in price discovery (Sah and Kumar, 2006; Gupta and Singh, 2006; Kakati and Kakati, 2006); (iv) Impact of information and expiration effect on spot prices (Thenmozhi and Thomas, 2004; Barik and Supria, 2005; Mishra, Kanan and Mishra, 2006; Mukherjee and Mishra, 2007); and (v) Better forecasting methods for greater accuracy of derivatives prices (Ramasastri and Gangadaran, 2005; Shrinivas, Dulluri and Raghvan, 2006; Mitra, 2006).

2.1 Hedging with Futures

There is very little evidence of hedging in the Indian context. Lack of evidence on such a contemporary issue is surprising. There is evidence of hedging in different markets (Johnson, 1960, and Stein, 1961, in commodity market; Dale, 1981, and Herbst, Kare and Marshall, 1993, in foreign exchange market; Ederington, 1979, and Franckle, 1980, in fixed income securities market). The evidence on use of equity and equity derivatives as hedges is missing. Therefore, we have presented a review of literature from the commodity, foreign exchange and fixed income securities markets.

Hedge is used to reduce the risk associated with a cash position or an anticipated cash position. Keynes, in his "Treatise on Money" (1930), envisioned the futures market as an insurance scheme for hedgers, who pay premiums to speculators for taking their risk. The basic assumption here is that hedgers are generally long in cash market, and, therefore, they need to hedge their position by taking short position in the forward market or future market.

In general, for a position consisting of a number, "X_i" of physical units held in market "i," hedge may be defined as a position in market "j" of size "X_j^*" units such that the price risk of holding "Xi" and "X_j*" from time "t_1" to "t_2" is *minimized* (Johnson, 1960). Therefore, the hedge ratio could be defined as the number of "X_j*" units (of hedging instrument) in market "*j*" required to hedge one unit held in market "*i*" (cash position). So, a hedger would protect his position in physical/cash market by simultaneously selling a sufficient number of futures contracts. Once the underlying asset is sold, the futures position may be squared off by taking the equal and opposite position (long position, in this case) in the futures contract. Let "S_1" and "S_2" denote the spot prices, and "F_1" and "F_2" the prices of futures at "t_1" and "t_2" respectively. Then, hedge ratio (h) is defined as:

$$(S_2 - S_1) = (F_2 – F_1) . h$$
$$\Rightarrow \quad \mathbf{h = (S_2 - S_1) / (F_2 – F_1)} \tag{1}$$

If the change in spot price is equal to that of futures, i.e., if the price movements are parallel, the gain from one market offsets the loss in the other. Otherwise, he would be left with a residual capital gain or loss.

The hedger will take a total gain (loss) arising from price movements from "t_1" to "$t_{2,}$" equal to the positive (negative) value of x [$(S_2, - S_1) - (F_2 – F_1)$] for "x" units of inventory.

The hedge is perfectly effective if [$(S_2 - S_1) - (F_2 – F_1)$] is equal to 0.

$$\Rightarrow \quad (S_2 - S_1) = (F_2 – F_1)$$
$$\Rightarrow \quad h = 1$$

This indicates parallel shift in prices in cash and futures markets. This is one of the underlying assumptions of Keynes theory. This is a *naïve* approach to hedging.

However, Working (1960) has negated this assumption of parallel movement in prices of spot and futures. He argued that this assumption is false, and an improper standard to test the effectiveness of hedging. The effectiveness of hedging used with commodity storage depends on *inequalities* in the movements of spot and futures prices, and on reasonable predictability of such inequalities. This implies gains from hedging, if generalized, are:

$$\mathbf{R_h^* = (S_{t+1} – S_t) – h * (F_{t+1} – F_t)} \tag{2}$$

In the JSE methodology, spot prices are regressed on futures prices using ordinary least squares (OLS) method.

$$\mathbf{S = a + b. F + u} \tag{3}$$

Where "a" is the intercept term (expected to be zero), and "b" is the estimate of "h*".

There are limitations of this model as mentioned by Herbst, Kare and Marshall (1993). For example, residuals from JSE estimation of optimal hedge ratio are serially correlated and, therefore, a Box-Jenkins autoregressive integrated moving average (ARIMA) technique should be used to estimate the minimum risk hedge to account for the observed serial correlation

(Herbst, Kare and Caples, 1989). A commonly used alternative is first differences. The merits of levels versus differences are discussed, in the context of foreign currency hedging, by Hill and Schneeweis (1982). Another alternative is to specify the problem as minimizing the variance of returns on wealth. This leads to a regression of percent price changes, which is fairly clean.

Hedge ratio is estimated as first difference of prices. So, changes in spot price are regressed on changes in futures price.

$$\Delta S = a + b.\Delta F + u \qquad \textbf{(4)}$$

Where, terms "a" and "b" are constants, ΔS = S(t) - S(t-1) and ΔF = F(t,T) - F(t-1, T) and "u" represents the error term. The term "b" (slope of the line) is optimal hedge ratio (with minimum variance).

This was an improvement, though it retained some serious flaws. One of the limitations emerged from the assumptions of regression. Regression can be used when relationship between Explained Variable (S_t) and Explanatory Variable (F_t) is stable. This implies constant basis irrespective of time of observation. In reality, in a direct hedge, the basis must decline over the life of the futures contract and become zero at maturity. Franckle (1980), in his reply to Enderington (1979), drew attention to this point and suggested a modified hedge ratio that incorporates the declining basis. Castelino (1990) argued that regression based hedge ratios must be time dependent. However, he argued that time dependent hedge ratios cannot be of minimum variance. In tests with financial futures on short-term interest rates, he claimed superior results vis-à-vis JSE by accounting for time in the hedge ratio estimation. But his results had two limitations: (a) they are based on an arbitrage model for treasury bonds that is of limited applicability to hedges with other futures contracts, and (b) they implicitly rely on the stability of spot-futures relationship from the prior year into the year of the hedge. The problem of instability of hedge ratio was also addressed by others, such as Grammatikos and Saunders (1983) and Malliaris and Urrutia (1991a, 1991b). However, they did not address the problems arising from the exclusion of time.

Equation (4) suggests that the relationship is not stable but time-varying.

$F(t) = S(t)\, e^{rT}$

$\Rightarrow \quad S(t) = F(t)\, e^{-rT}$

Taking natural logarithm on both sides,

$$\Rightarrow \quad \mathbf{In[S(t)/F(t,T)] = -rT} \qquad \textbf{(5)}$$

Equation (12) can be estimated as:

$$\Rightarrow \quad \mathbf{ln[S(t)/F(t,T)] = a + dT + \mu_i} \qquad \textbf{(6)}$$

Where “a” is the intercept term (expected to be zero), and “d” (the slope), is the estimate of “r.” Once the coefficient of “T” in Equation (6) is estimated by regression, the optimal hedge ratio can be estimated as:

$$\Rightarrow \quad \mathbf{h^* = e^{dT}} \qquad \mathbf{(7)}$$

An important difference between the JSE hedge ratio and that defined by Equation (7) is that the latter can be revised daily once the estimate of full cost of carry is available (from a few trading days of a futures contract). The estimated hedge ratio “h*” will change daily depending on the term to expiration of the futures contract. The JSE hedge ratio “b,” on the other hand, is a constant estimated solely from the past data. Historical data may provide poor estimate of the minimum variance hedge ratio, especially when the spot-to-futures relationship is not stable.

2.2 Hedging with Options

The introduction of options has price effects, volatility effects, cross effects, announcement effects and persistence effects on the market for underlying shares (Detemple and Jorion, 1990). The evidence on options can be divided into five areas: (i) The effect of listing of options on volatility and liquidity (bid-ask spread) of underlying cash market (Trennepohl and Dukes, 1979; Skinner, 1989; Watt, Yadav and Draper, 1992; Chamberlain, Cheung and Kwan, 1993; Kumar, Sarin and Shastri, 1998; Chieng and Wang, 2002; Chaudhury and Elfakhani, 1997); (ii) The effect of option expiration on underlying cash market (Detemple and Jorion, 1990; Conrad, 1989; Corredor, Lechon and Santamaria, 2001); (iii) The lead-lag relationship between price (and non-price variables) of option and underlying spot market (Manaster and Rendleman, Jr., 1982; Easley, O’Hara and Srinivas, 1998); (iv) The role of options in price discovery in spot market (Bhuyan and Chaudhary, 2001; Bhuyan and Yan, 2002; Srivastava, 2003; Mukherjee and Mishra, 2007); and (v) The use of options as risk hedges.

Raina and Mukhopadhyay (2004) and Kakati (2005) are among the few who explored hedging though indirectly. For instance, Raina and Mukhopadhyay (2004) tried to minimize the risk of portfolio comprising equities, equity futures and equity options (European options only), in terms of value at risk (VaR). This study can help in design of portfolios of equity and equity futures or equity and equity options with minimum risk as measured by VaR for determination of hedge ratio. Kakati (2005) tried to show that Artificial Neural Network (ANN) based option pricing model is superior to Black-Scholes model. They also tried to show that, ANN models, if designed correctly, add value to option price forecasting. ANN methodology provides better results than those obtained using normal delta-hedging in the Indian options market. The Black–Scholes (BS) option pricing formula is based on arbitrage and explicitly provides a delta-based hedging strategy to replicate a plain put/call option assuming all risk can be mitigated from an option position via a continuously rebalanced delta hedge (Pellizzari, 2005). In practice, continuous rebalancing of a delta hedged portfolio is impossible, and, therefore, discretely adjusted delta hedging is augmented with gamma hedging, and sometimes even vega hedging (Dingler and Jarrow, 1997). This discretization may lower the effectiveness of delta hedging. Further, Merton (1989) showed that the inclusion of transaction costs, no matter how small, destroys the Black-Scholes (1973) continuous- time option pricing model completely.

Nonetheless, there have been attempts to improve this model, The BS pricing formula could be presented as:

$$C_{call} = SN(d_1) - Ke^{-r(T-t)}N(d_2)$$
$$C_{put} = Ke^{-r(T-t)}N(-d_2) - SN(-d_1)$$

Where:

$$d_1 = [\ln(S/K) + (r + \sigma^2/2)T]/\sigma\sqrt{T} \text{ ,}$$
$$d_2 = d_1 - \sigma\sqrt{T}$$

And, $OHR_{call} = N(d_1)$ and:

$$OHR_{put} = N(d_1) - 1 \quad \textbf{(8)}$$

Where C, S, K, r and (T-t) represent fair price/premium of the option, spot price of the underlying asset, strike price, risk-free rate of return, time to expiration respectively, and "N" is the standard normal cumulative distribution function. The optimal hedge ratio is $N(d_1)$ (for call options) and $[N(d_1) – 1]$ (for put options). The optimal hedge ratio is popularly known as Delta, and hedging strategies based on it are known as Delta Hedging. Delta of a call option is always positive (as it varies from '0' to '1') and Delta of a put option is always negative (as it varies from '0' to '-1'). Thus, the value of a call increases with an increase in the stock price while the value of a put decreases if the stock price increases.

Many studies have established that returns are not normally distributed as they were found to have fat tails. This implies that use of high frequency data will help in estimation of a more accurate central measure. The second problem leads to long-range dependence. According to Efficient Market Hypothesis (EMH), all information is reflected in current asset prices, and hence it is reasonable to assume a Markovian process. However, this is only true when the market is efficient in strong form, which may not be valid in reality. Traders have been using long-term memory strategies to outperform the market. This motivated a series of research studies further purporting the existence of a non-Markovian process (Lo & MacKinlay, 1993). A few stochastic models have been developed that can produce quasi long-range dependence. However, these models are very complex as they use high-dimensional partial differential equations with variable coefficients.

Fractional Brownian motion (fBM) deals with the second problem while still assuming a Gaussian process. Nevertheless, it offers the promise of giving simple, tractable solutions to pricing financial options and presents a natural way of modelling long-range dependence, measured by Hurst parameter "H". Razdan (2002) has estimated Hurst parameter (H) as 0.915 for Bombay Stock Exchange Sensitive Index (Sensex), and it is the basis of our estimation of h^*_{fBM} and Rh^*_{fBM}.

Using fBM , The European call price at $t \in [0,T]$ with strike price K, spot price S, risk free rate of return “r”, time to expiration (T-t), and estimated Hurst Parameter “H” is given by:

$$G_{Call} = SN(d_1) - Ke^{-r(T-t)}N(d_2)\text{, where } d_1 = \frac{\ln(\frac{S}{K}) + r(T-t) + \frac{\sigma^2}{2}(T^{2H} - t^{2H})}{\sigma\sqrt{T^{2H} - t^{2H}}} \text{ and}$$

$$d_2 = \frac{\ln(\frac{S}{K}) + r(T-t) - \frac{\sigma^2}{2}(T^{2H} - t^{2H})}{\sigma\sqrt{T^{2H} - t^{2H}}}$$

And,

$OHR_{call} = N(d_1)$ and:
$OHR_{put} = N(d_1) - 1$ **(9)**

This is also called fractional Black-Scholes formula. With generalized solutions the fractional pricing model is free of arbitrage and complete (Sottinen and Valkeila, 2003). One may further note that as H =1/2, set of equations (8) reduces to equation (9).

$\mathbf{R_h^* = (S_{t+1}-S_t) - h^* (O_{t+1}-O_t)}$ **(10)**

Where:

$O_t = K_i + C_i$ for i = 1,2,3,………..

3. Hypotheses

This study is an attempt to estimate hedge ratio and hedge efficiency. We have compared JSE and HKM methodologies for estimating optimal hedge ratio and hedge efficiency using high frequency data from Indian financial futures market from Jan. 1, 2002 to March 31, 2002. To estimate optimal hedge ratio for options, we have used fBM methodology with BSM as the benchmark, using high frequency data for all 13 strike prices. We have estimated the returns on hedged positions to validate the efficiency of optimal hedge ratios.

The model with the higher estimate of “R_h*” (in Equation 2) was considered better. The hypotheses for futures are:

1. H_0: There is no difference between mean optimal hedge ratio (OHR) based on JSE and HKM methodology.

H_1: Mean optimal hedge ratio (OHR) based on JSE methodology is greater than that based on HKM methodology.

$$H_0 : h^*_{JSE} = h^*_{HKM}$$

$$H_1 : h^*_{JSE} > h^*_{HKM}$$

2. H_0: There is no difference between mean returns based on JSE and HKM methodology.

H_1: Mean return based on HKM methodology is greater than that based on JSE methodology.

$$H_0: R_h^*{}_{JSE} = R_h^*{}_{HKM}$$

$$H_1: R_h^*{}_{HKM} > R_h^*{}_{JSE}$$

The hypotheses for options are:

1. H_0: There is no difference between mean optimal hedge ratio (OHR) based on BSM and that based on fBM.

H_1: Mean OHR based on BSM is greater than that based on fBM.

$$H_0 : h^*_{BSM} = h^*_{fBM}$$

$$H_1 : h^*_{BSM} > h^*_{fBM}$$

2. H_0: There is no difference between mean returns based on BSM and fBM.

H_1: Mean return based on fBM is greater than that based on BSM.

$$H_0: R_h^*{}_{BSM} = R_h^*{}_{fBM}$$

$$H_1: R_h^*{}_{BSM} > R_h^*{}_{fBM}$$

4. Results and Discussion

4.1 Futures

The daily weighted average prices are derived from high frequency data on Nifty index and its futures using Oracle 8i. The estimates of optimal hedge ratio using the two methods (JSE and HKM) h*, and the return (R_h*) are included in Table 1.

TABLE 1
Estimates of 'h*' and 'R_h*'

JSE	HKM	JSE	HKM	JSE	HKM	JSE	HKM
h*	**h***	**h***	**h***	**R_h***	**R_h***	**R_h***	**R_h***
1.062946786	1.004936809	1.062947	1.002386	-0.92049	-1.0017	3.249793	3.853783
1.062946786	1.004777177	1.062947	1.002227	-4.33728	-3.64266	4.40056	4.673644
1.062946786	1.004617571	1.062947	1.001749	-1.01609	0.198964	-1.84006	-1.90207
1.062946786	1.00445799	1.062947	1.00159	0.483177	1.271815	-4.06005	-4.87392
1.062946786	1.003979399	1.062947	1.001431	-0.72966	-0.60284	-1.02555	-0.65987
1.062946786	1.00381992	1.062947	1.001272	4.182337	4.184554	-0.26738	-0.0511
1.062946786	1.003660465	1.062947	1.001113	0.722432	0.068203	2.481399	2.761571
1.062946786	1.003501036	1.062947	1.000636	2.578327	2.30214	-2.83371	-1.68662
1.062946786	1.003341633	1.062947	1.000477	-7.07693	-5.82753	2.558119	3.087346
1.062946786	1.002863574	1.062947	1.000318	0.259149	-0.43045	-7.30266	-8.33077
1.062946786	1.002704272	1.062947	1.000159	-1.91998	-2.19596	13.6673	12.56617
1.062946786	1.002544995	1.062947	1.004458	-1.98562	-1.57135	-7.43262	-5.74562
1.062946786	1.002385743	1.062947	1.003979	2.885495	2.81004	3.056366	2.949569
1.062946786	1.002226516	1.062947	1.00382	0.334917	-0.09154	-0.93363	-1.6351
1.062946786	1.001748989	1.062947	1.00366	-0.74027	-0.63049	1.824657	2.589987
1.062946786	1.001589863	1.062947	1.003501	3.649791	3.33442	1.095284	1.258544
1.062946786	1.001430763	1.062947	1.003342	-2.07856	-2.02166	-3.14457	-3.88049
1.062946786	1.001271689	1.062947	1.002864	-2.31306	-2.74603	-9.36569	-9.89758
1.062946786	1.001112639	1.062947	1.002704	0.883455	0.589882	-1.33689	-1.18546
1.062946786	1.000635642	1.062947	1.002545	-0.86481	-1.06922	2.94034	2.85909
1.062946786	1.000476694	1.062947	1.002386	1.908962	1.29621	-3.33278	-2.56744
1.062946786	1.000317771	1.062947	1.002227	0.920893	1.640654	2.085977	2.364508
1.062946786	1.000158873	1.062947	1.001749	-2.17245	-1.73218	-5.182	-5.9162
1.062946786	1.00445799	1.062947	1.00159	0.032237	-0.24523	-1.43575	-1.62488
1.062946786	1.003979399	1.062947	1.001431	1.739985	1.497101	1.106439	0.762247
1.062946786	1.00381992	1.062947	1.001272	-2.24515	-0.7568	-0.90481	-1.27317
1.062946786	1.003660465	1.062947	1.001113	7.004007	7.724421	-0.73857	-1.50683
1.062946786	1.003501036	1.062947	1.000477	-1.08705	-0.86105	2.354733	2.135435
1.062946786	1.003341633	1.062947	1.000318	-0.97566	-0.04968	3.978126	4.05357
1.062946786	1.002863574	1.062947	1.000159	0.330696	0.0774		
1.062946786	1.002704272	**Mean h***	**Mean h***	-2.01443	-1.71104	**Mean R_h***	**Mean R_h***
1.062946786	1.002544995	1.063	1.002	0.243936	0.72082	-0.167	-0.103
		Variance	**Variance**			**Variance**	**Variance**
		0.000	0.000			12.773	12.482

t-Test: Two-Sample Assuming Equal Variances		
	h^*_{JSE}	h^*_{HKM}
Mean	1.062946893	1.002352284
Variance	0.00	0.00
Observations	60	60
Pooled Variance	0.00	
Hypothesized Mean Difference	0	
Df	118	
t Stat	335.6571571	
P(T<=t) one-tail	0.000000	
t Critical one-tail	1.657869523	
P(T<=t) two-tail	0.000000	
t Critical two-tail	1.980272226	

The null hypothesis is rejected. This means optimal hedge ratio estimated using JSE method is greater than that based on HKM methodology.

t-Test: Two-Sample Assuming Equal Variances		
	$R_h^*_{JSE}$	$R_h^*_{HKM}$
Mean	-0.15059029	-0.089868169
Variance	13.15131223	12.85612753
Observations	59	59
Pooled Variance	13.00371988	
Hypothesized Mean Difference	0	
Df	116	
t Stat	-0.09145851	
P(T<=t) one-tail	0.463643012	
t Critical one-tail	1.658095745	
P(T<=t) two-tail	0.927286023	
t Critical two-tail	1.980625937	

We don’t reject the null hypothesis. The mean returns estimated using JSE and HKM methodology are not statistically significantly different.

The results are encouraging in the case of estimates of optimal hedge ratio (OHR) for futures. OHR estimated using a superior method (like HKM) was better and statistically significant at 95 percent confidence level. However, Adjusted R^2 is very low (0.0676) in case of HKM methodology which is contrary to *a priori* expectations. It is expected to be significantly high due to the dual advantage vis-à-vis the benchmark model of JSE. This (HKM) model is also expected to provide ideal results about the error term, i.e., zero mean, constant variance and zero co-variance. Time to expiration is a significant variable in explaining futures price movement. This evidence requires a plausible explanation.

Optimal hedge ratio with futures based on HKM model is superior to that based on benchmark model. Therefore, returns on hedged positions using these ratios should be significantly higher. However, they are not significantly different.

4.2 Options

Optimal hedge ratio was estimated using equation (8) based on BSM and equation (9) based on fBM. We have used the yield on 364-Day Government of India Treasury Bills in the year 2002 (sourced from Reserve Bank of India Web site) as risk-free rate of return.

Estimation of (h*) and (R_h*) for call option with thirteen different strike prices and based on standard Black-Scholes Model and on Fractional Brownian Motion is included in Table 2.

TABLE 2

Strike Prices	920	940	960	980	1000	1020	1160	1060	1080	1100	1120	1140	1040
h^*_{BSM}	0.576381	0.576221	0.576064	0.575911	0.575761	0.575613	0.574655	0.575327	0.575188	0.575051	0.574917	0.574785	0.575469
	0.572775	0.572617	0.572463	0.572311	0.572163	0.572018	0.571074	0.571736	0.571599	0.571464	0.571332	0.571202	0.571875
	0.569294	0.569139	0.568987	0.568838	0.568692	0.568549	0.56762	0.568271	0.568136	0.568004	0.567873	0.567746	0.568409
	0.56595	0.565797	0.565647	0.565501	0.565357	0.565217	0.564302	0.564943	0.56481	0.56468	0.564552	0.564426	0.565079
	0.555926	0.555781	0.555639	0.5555	0.555364	0.55523	0.554363	0.554971	0.554845	0.554721	0.5546	0.55448	0.555099
	0.552682	0.55254	0.5524	0.552264	0.55213	0.551999	0.551149	0.551745	0.551621	0.5515	0.551381	0.551264	0.551871
	0.549515	0.549376	0.54924	0.549106	0.548975	0.548847	0.548721	0.548597	0.548476	0.548357	0.54824	0.548126	0.548013
	0.546325	0.546189	0.546055	0.545924	0.545796	0.54567	0.544854	0.545426	0.545308	0.545191	0.545077	0.544965	0.545547
	0.543252	0.543119	0.542988	0.54286	0.542735	0.542612	0.541814	0.542374	0.542258	0.542144	0.542032	0.541922	0.542492
	0.534571	0.534447	0.534326	0.534207	0.534091	0.533977	0.533237	0.533756	0.533648	0.533543	0.533439	0.533337	0.533865
	0.531735	0.531615	0.531497	0.531382	0.531269	0.531158	0.530438	0.530943	0.530838	0.530735	0.530634	0.530535	0.531049
	0.529005	0.528888	0.528774	0.528662	0.528552	0.528445	0.527746	0.528236	0.528134	0.528035	0.527937	0.527841	0.528339
	0.526415	0.526302	0.526191	0.526083	0.525976	0.525872	0.525195	0.52567	0.525571	0.525475	0.52538	0.525287	0.52577
	0.523889	0.523779	0.523672	0.523567	0.523465	0.523364	0.52271	0.523168	0.523073	0.52298	0.522888	0.522798	0.523265
	0.516786	0.516689	0.516594	0.516501	0.51641	0.516321	0.51574	0.516147	0.516063	0.51598	0.515899	0.515819	0.516233
	0.514623	0.51453	0.51444	0.514351	0.514264	0.514179	0.513626	0.514013	0.513933	0.513854	0.513777	0.5137	0.514095
	0.51255	0.512462	0.512376	0.512292	0.51221	0.512129	0.511604	0.511972	0.511895	0.511821	0.511747	0.511675	0.51205
	0.510588	0.510505	0.510424	0.510344	0.510267	0.51019	0.509695	0.510042	0.50997	0.5099	0.50983	0.509762	0.510116
	0.508711	0.508633	0.508557	0.508483	0.50841	0.508339	0.507876	0.5082	0.508133	0.508067	0.508002	0.507938	0.508269
	0.503941	0.503882	0.503825	0.503769	0.503714	0.50366	0.50331	0.503555	0.503504	0.503454	0.503405	0.503357	0.503607
	0.502643	0.502593	0.502543	0.502494	0.502447	0.5024	0.502097	0.502309	0.502265	0.502222	0.502179	0.502138	0.502354
	0.501522	0.501481	0.50144	0.5014	0.501362	0.501323	0.501076	0.501249	0.501213	0.501178	0.501143	0.501109	0.501286
	0.500654	0.500625	0.500596	0.500568	0.500541	0.500514	0.500338	0.500461	0.500436	0.500411	0.500386	0.500362	0.500487
	0.532597	0.532487	0.53238	0.532275	0.532172	0.532071	0.531445	0.531874	0.531779	0.531685	0.531594	0.531503	0.531941

Strike Prices	920	940	960	980	1000	1020	1160	1060	1080	1100	1120	1140	1040
h^*_{fBM}	0.503922	0.503863	0.503806	0.50375	0.503695	0.503641	0.50329	0.503536	0.503485	0.503435	0.503386	0.503338	0.503588
	0.503601	0.503544	0.503488	0.503434	0.50338	0.503328	0.502988	0.503226	0.503177	0.503128	0.503081	0.503034	0.503277
	0.503323	0.503268	0.503214	0.503161	0.503109	0.503059	0.502729	0.50296	0.502912	0.502865	0.502819	0.502774	0.503009
	0.50309	0.503036	0.502984	0.502933	0.502883	0.502834	0.502515	0.502738	0.502692	0.502646	0.502602	0.502558	0.502786
	0.502366	0.502317	0.50227	0.502224	0.502179	0.502135	0.501847	0.502049	0.502007	0.501966	0.501925	0.501886	0.502091
	0.502146	0.5021	0.502055	0.50201	0.501967	0.501924	0.501647	0.501841	0.501801	0.501761	0.501722	0.501684	0.501882
	0.501947	0.501902	0.501858	0.501816	0.501774	0.501733	0.501692	0.501653	0.501614	0.501576	0.501539	0.501502	0.501466
	0.501733	0.50169	0.501648	0.501607	0.501567	0.501527	0.501271	0.501451	0.501413	0.501377	0.501341	0.501306	0.501489
	0.501549	0.501508	0.501468	0.501428	0.50139	0.501352	0.501107	0.501279	0.501243	0.501208	0.501174	0.50114	0.501315
	0.501109	0.501073	0.501039	0.501004	0.500971	0.500938	0.500725	0.500874	0.500843	0.500813	0.500783	0.500754	0.500906
	0.500961	0.500927	0.500894	0.500862	0.50083	0.500799	0.500596	0.500738	0.500709	0.50068	0.500652	0.500624	0.500768
	0.500832	0.5008	0.500769	0.500739	0.500708	0.500679	0.500488	0.500622	0.500594	0.500567	0.50054	0.500514	0.50065
	0.500731	0.500701	0.500672	0.500643	0.500614	0.500587	0.500406	0.500533	0.500506	0.500481	0.500455	0.500431	0.500559
	0.500635	0.500606	0.500579	0.500552	0.500525	0.500499	0.50033	0.500448	0.500424	0.5004	0.500376	0.500353	0.500473
	0.500388	0.500366	0.500343	0.500322	0.5003	0.500279	0.500144	0.500239	0.500219	0.5002	0.500181	0.500162	0.500259
	0.500327	0.500306	0.500286	0.500266	0.500246	0.500227	0.500102	0.50019	0.500172	0.500154	0.500136	0.500119	0.500208
	0.50027	0.500251	0.500233	0.500214	0.500197	0.500179	0.500066	0.500146	0.500129	0.500113	0.500097	0.500082	0.500162
	0.500221	0.500204	0.500187	0.500171	0.500155	0.50014	0.500038	0.500109	0.500094	0.50008	0.500066	0.500052	0.500124
	0.500173	0.500158	0.500143	0.500129	0.500115	0.500101	0.500011	0.500074	0.500061	0.500048	0.500035	0.500023	0.500087
	0.500079	0.50007	0.500061	0.500053	0.500044	0.500036	0.499982	0.50002	0.500012	0.500004	0.499997	0.499989	0.500028
	0.500055	0.500048	0.500042	0.500035	0.500029	0.500022	0.499981	0.50001	0.500004	0.499998	0.499992	0.499986	0.500016
	0.500034	0.50003	0.500025	0.50002	0.500016	0.500011	0.499983	0.500003	0.499999	0.499995	0.499991	0.499987	0.500007
	0.500019	0.500016	0.500014	0.500011	0.500009	0.500007	0.499992	0.500002	0.5	0.499998	0.499996	0.499994	0.500004

Mean h^*_{BSM}	0.533	0.532	0.532	0.532	0.532	0.532	0.531	0.532	0.532	0.532	0.532	0.532	0.532
Mean h^*_{fBM}	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501

Strike Prices											
920	**940**	**960**	**980**	**1000**	**1020**	**1160**	**1060**	**1080**	**1100**	**1120**	**1140**
Rh* $_{BSM}$											
-2.40452	-26.025	-2.40452	-11.0178	-8.6411	-8.18191	-2.40452	-4.14832	-4.5766	-3.57603	-3.14726	-1.37636
-14.4509	9.601464	14.15302	3.798638	3.145802	2.834402	8.320901	3.405355	6.758527	6.673691	6.872824	6.390566
21.06569	21.06569	21.06569	21.06569	8.627833	12.44614	19.09064	14.17339	14.48566	16.66119	16.35169	17.65092
14.69917	14.69917	14.69917	14.69917	14.50479	13.39438	14.03393	13.42274	13.22883	13.11821	14.11684	14.94869
-22.7678	1.550153	-17.3696	-17.2268	-3.72269	-7.08864	-0.62688	-3.36038	-3.0283	-2.2552	0.19927	-0.73759
4.222073	-19.6758	4.496693	4.222073	8.998155	11.90593	37.22763	11.51842	12.64118	11.51522	8.772469	7.56564
18.52576	-18.3493	-22.716	6.493782	-10.9758	-7.1561	-40.0165	-8.685	-8.65824	-9.20392	-9.47683	-9.58613
-2.34695	-2.34695	15.57167	-15.3756	4.274422	2.400911	-1.10077	1.6395	-0.93708	-0.61209	2.721053	-2.18437
15.0265	15.0265	15.0265	15.0265	1.006605	2.878517	10.22737	4.031129	6.568174	8.623986	6.918227	12.30648
-11.9085	8.691548	-37.4204	-6.59471	-1.30972	1.370416	-7.37329	-5.40448	-4.50334	-6.33581	-7.55733	-7.66425
-6.77668	-6.77668	-6.77668	-6.77668	-6.61811	-13.118	-6.53919	-4.66374	-5.19228	-5.6414	-6.24874	-6.11688
5.285528	-18.2665	5.285528	5.285528	0.078362	-2.07668	3.97254	-3.15147	-2.01991	-1.36173	0.951143	2.422715
1.564609	-25.6719	1.564609	-14.666	6.877776	9.415068	3.18501	13.10047	10.40455	8.703285	6.66277	4.622979
-19.9919	40.46312	-7.07229	8.422749	-6.19439	-7.07229	-6.47918	-6.94325	-5.5499	-4.56978	-5.55038	-6.47909
1.161712	21.74293	1.161712	1.161712	-0.71535	2.19007	1.238756	-0.50883	-0.14882	0.313853	1.752555	1.624042
18.70244	-41.2592	-1.79957	-1.79957	-1.79957	-0.77531	-1.79957	2.808179	1.118237	0.836309	-0.75049	-1.36464
-1.09802	2.475514	9.110456	-25.0842	4.8721	4.25898	-1.09802	0.789137	1.987301	0.17673	-0.40975	-0.84314
-9.75585	-9.75585	-9.75585	-9.75585	-9.75585	-7.97666	-9.40033	-8.6378	-8.99365	-9.22237	-9.67965	-9.73045
-4.12452	-4.12452	-4.12452	-2.10945	1.920046	-4.12452	-4.12452	0.483009	-2.36226	-3.04209	-3.57078	-3.94835
-4.34292	18.27374	25.80965	30.83168	-3.33803	-4.34292	-4.34292	-6.87958	-3.94111	-4.14203	-4.24248	-4.34292
-31.0592	-2.47297	-8.49073	-8.49073	-7.43787	-8.49073	-8.49073	-4.00455	-6.46082	-7.93944	-8.34039	-8.41557
13.10584	-2.91415	4.09511	13.10584	7.624921	13.10584	13.10584	10.50344	13.10584	13.10584	13.10584	13.10584
-17.6686	-24.0489	14.10972	5.215965	1.422338	5.796878	16.60614	19.48735	23.926	21.82643	19.4506	17.84812

Strike Prices												
920	**940**	**960**	**980**	**1000**	**1020**	**1160**	**1060**	**1080**	**1100**	**1120**	**1140**	**1040**
Rh^*_{fBM}												
-2.40452	**-23.1757**	**-2.40452**	**-9.9812**	**-7.89137**	**-7.48814**	**-2.40452**	**-3.93937**	**-4.3166**	**-3.43594**	**-3.05853**	**-1.49906**	**-6.15394**
-11.812	**-11.8098**	**13.47884**	**4.32077**	**3.742605**	**3.466385**	**8.320901**	**3.970297**	**6.937893**	**6.862592**	**7.038713**	**6.611471**	**4.975892**
21.06569	**21.06569**	**21.06569**	**21.06569**	**10.00227**	**13.39748**	**19.30689**	**14.93229**	**15.20933**	**17.14505**	**16.86897**	**18.02522**	**11.81444**
14.69917	**14.69917**	**14.69917**	**14.69917**	**14.52341**	**13.51915**	**14.09695**	**13.54446**	**13.36885**	**13.26857**	**14.17215**	**14.92502**	**14.77448**
-20.5443	**-20.5422**	**-15.6452**	**-15.5182**	**-3.24363**	**-6.30496**	**-0.43135**	**-2.91623**	**-2.61479**	**-1.912**	**0.320934**	**-0.53184**	**-3.79489**
4.222073	**4.222073**	**4.473002**	**4.222073**	**8.587504**	**11.24633**	**34.39887**	**10.89406**	**11.92185**	**10.89303**	**8.384844**	**7.281235**	**-21.102**
16.11777	**16.11545**	**-21.7612**	**5.075624**	**-10.9758**	**-7.4651**	**-37.6935**	**-8.8697**	**-8.84479**	**-9.34632**	**-9.59711**	**-9.69746**	**22.32305**
-2.34695	**-2.34695**	**14.20148**	**-14.3812**	**3.770009**	**2.039884**	**-1.1944**	**1.337452**	**-1.04371**	**-0.74308**	**2.339027**	**-2.1966**	**3.468307**
15.0265	**15.0265**	**15.0265**	**15.0265**	**1.876008**	**3.630153**	**10.51997**	**4.708483**	**7.088127**	**9.01674**	**7.414591**	**12.47265**	**2.153212**
-11.9085	**-11.9085**	**-35.9514**	**-6.89991**	**-1.91698**	**0.611436**	**-7.62843**	**-5.77449**	**-4.92364**	**-6.65139**	**-7.80319**	**-7.90354**	**-3.04493**
-6.77668	**-6.77668**	**-6.77668**	**-6.77668**	**-6.62647**	**-12.7848**	**-6.55146**	**-4.77419**	**-5.2749**	**-5.70046**	**-6.27614**	**-6.15104**	**-7.77798**
5.285528	**5.285528**	**5.285528**	**5.285528**	**0.329446**	**-1.72268**	**4.034512**	**-2.74802**	**-1.67151**	**-1.04555**	**1.156771**	**2.558181**	**-8.73013**
1.564609	**1.564609**	**1.564609**	**-13.9525**	**6.644938**	**9.072093**	**3.115631**	**12.59949**	**10.02177**	**8.395063**	**6.443274**	**4.491672**	**19.03113**
-19.582	**-19.5814**	**-7.07229**	**7.937361**	**-6.22177**	**-7.07229**	**-6.49712**	**-6.94723**	**-5.59664**	**-4.64632**	**-5.59675**	**-6.4971**	**-7.07229**
1.161712	**1.161712**	**1.161712**	**1.161712**	**-0.66419**	**2.162166**	**1.236727**	**-0.4639**	**-0.11373**	**0.336458**	**1.736869**	**1.611819**	**-1.23929**
18.21123	**18.21048**	**-1.79957**	**-1.79957**	**-1.79957**	**-0.79921**	**-1.79957**	**2.701743**	**1.051169**	**0.776015**	**-0.77437**	**-1.3745**	**-1.79957**
-1.09802	**-1.09802**	**8.905728**	**-24.6061**	**4.753796**	**4.153446**	**-1.09802**	**0.752385**	**1.927552**	**0.15218**	**-0.42293**	**-0.84799**	**5.853707**
-9.75585	**-9.75585**	**-9.75585**	**-9.75585**	**-9.75585**	**-8.00549**	**-9.40584**	**-8.65568**	**-9.00575**	**-9.23079**	**-9.68084**	**-9.73084**	**-12.1063**
-4.12452	**-4.12452**	**-4.12452**	**-2.12431**	**1.876008**	**-4.12452**	**-4.12452**	**0.450659**	**-2.37448**	**-3.04951**	**-3.57453**	**-3.94953**	**3.975927**
-4.34292	**-4.34292**	**25.65958**	**30.65953**	**-3.34286**	**-4.34292**	**-4.34292**	**-6.86797**	**-3.94292**	**-4.14292**	**-4.24292**	**-4.34292**	**-5.59296**
-30.9923	**-30.9921**	**-8.49073**	**-8.49073**	**-7.4407**	**-8.49073**	**-8.49073**	**-4.01571**	**-6.46574**	**-7.94074**	**-8.34074**	**-8.41574**	**-2.74065**
13.10584	**13.10584**	**4.10559**	**13.10584**	**7.63074**	**13.10584**	**13.10584**	**10.50583**	**13.10584**	**13.10584**	**13.10584**	**13.10584**	**7.355788**

Mean $R_h^*_{BSM}$	**-17.669**	**-24.049**	**14.11**	**5.216**	**1.422**	**5.797**	**16.606**	**19.487**	**23.926**	**21.826**	**19.451**	**17.848**	**12.825**
Mean $R_h^*_{fBM}$	**-15.228**	**-35.998**	**15.845**	**8.274**	**3.858**	**7.803**	**16.474**	**20.425**	**24.443**	**22.107**	**19.614**	**17.945**	**14.571**

t-Test: Two-Sample Assuming Equal Variances		
	Mean h*$_{BSM}$	***Mean h*$_{fBM}$***
Mean	0.532	0.501
Variance	0.000	0.000
Observations	13.000	13.000
Pooled Variance	0.000	
Hypothesized Mean Difference	0.000	
Df	24.000	
t Stat	273.785	
P(T<=t) one-tail	0.000	
t Critical one-tail	1.711	
P(T<=t) two-tail	0.000	
t Critical two-tail	2.064	

The null hypothesis is rejected. This means optimal hedge ratio based on BSM is greater than that based on fBM methodology.

t-Test: Two-Sample Assuming Equal Variances		
	Mean R_h* BSM	***Mean R_h* fBM***
Mean	8.984	9.241
Variance	222.404	292.221
Observations	13.000	13.000
Pooled Variance	257.313	
Hypothesized Mean Difference	0.000	
Df	24.000	
t Stat	-0.041	
P(T<=t) one-tail	0.484	
t Critical one-tail	1.711	
P(T<=t) two-tail	0.968	
t Critical two-tail	2.064	

We don't reject the null hypothesis. The mean returns estimated using BSM and fBM methodology are not statistically significantly different.

The initial results are encouraging in the case of estimated optimal hedge ratio. OHR estimated using a superior method (fBM) was better and statistically significant at 95 percent confidence level. Therefore, returns on hedged positions using these ratios should be significantly higher. However, they are not significantly different.

5. Conclusions

Optimal hedge ratio was estimated from daily weighted average price (generated from high frequency data) of index and index futures (from 01.01.2002 to 28.03.2002) and (call) options for one month (from 01.01.2002 to 31.01.2002). The estimated ratios are significantly better than those based on benchmark models for both index futures and options. There is no significant difference between returns on hedged positions. This is contrary to *a priori* expectations, and requires a plausible explanation. We plan to estimate the optimal hedge ratios using high frequency data for a longer period. These models with suitable modification(s) may be used for hedging in Indian stock, commodity and foreign exchange markets.

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