ACTUARIAL RESEARCH CLEARING HOUSE
1996 VOL. 1

"Analyzing Accident Benefit Data Using Tweedie's Compound Poisson Model"

Mary Kelly

Bent Jorgenson

Following Jorgensen and Paes De Souza (1994), Tweedie's compound Poisson distribution is fitted to private passenger automobile accident benefit data. The Tweedie process is a three parameter distribution which corresponds to a Poisson process for claim frequency with gamma distributed claim sizes. Two of the parameters can be fit within the generalized linear model framework and the third parameter can be fit using maximum likelihood. Exposure, aggregate claim sizes and claim counts are collected by accident month over one accident year for over eight hundred liability limit/territory/rate class combinations. The resulting parameter estimates can be used for setting rating variable differentials in the rate making process.

Analysing Accident Benefit data Using Tweedie's Compound Poisson Model

Mary V. Kelly
Faculty of Commerce and Business Administration
University of British Columbia

Bent Jørgensen
Department of Statistics
University of British Columbia

Actuarial Research Conference 1995

Outline of Presentation

- Background theory.
- The Tweedie distribution as a model for aggregate claims.
- Previous analysis.
- Data set.
- Analysis of data.
- Conclusion.

Exponential Dispersion Models

Let Y be a r.v. with pdf

$$p(y;\theta,\sigma^2) = a(\sigma^2;y)\mathrm{e}^{(y\theta-\kappa(\theta))/\sigma^2}$$

$Y \sim ED(\mu,\sigma^2)$

$\mathbb{E}[Y] = \mu = \kappa'(\theta)$ and $\mathrm{Var}(Y) = \sigma^2 V(\mu)$.

$\kappa(\theta)$ "cumulant generator":
cgf is $\mathcal{K}(s;\theta) = \kappa(\theta+s) - \kappa(\theta)$.

Tweedie Distributions

Member of ED family with $V(\mu) = \mu^p$.

Denoted by $Y \sim ED^{(p)}(\mu,\sigma^2)$.

Closed w.r.t scale transformations:

$$cED^{(p)}(\mu,\sigma^2) \sim ED^{(p)}(c\mu, c^{2-p}\sigma^2)$$

Common Tweedie Distributions

$p = 0$	Normal
$p = 1$	Poisson
$p = 2$	Gamma
$p = 3$	Inverse Gaussian
$1 < p < 2$	Compound Poisson

p = 1.5

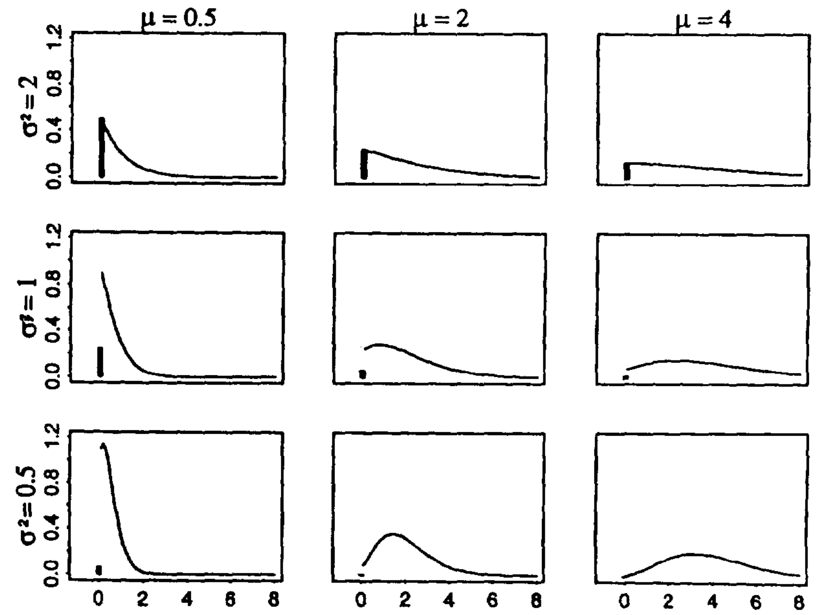

Figure 4.5. *Some compound Poisson Tweedie density functions—note positive probability in zero*

Aggregate Claims Model

- $Z(w) = \sum_{i=1}^{N(w)} Z_i$ "aggregate claims dist'n".
- $N(w)$ is Poisson variable and Z_i are gamma.
- w is measure of exposure.
- Let $Y(w) = Z(w)/w$ be claim rate per exposure.
- $Y(w) \sim ED^{(p)}(\mu, \frac{\sigma^2}{w})$ for $1 < p < 2$.
- $\mu = \mathbb{E}[Y(w)]$ is a function of rating variables: $\log(\mu_i) = \sum_{j=1}^{k} x_{ij}\beta_j$.
- $\left(\frac{p-2}{p-1}\right)^{-.5}$ is coefficient of variation of the Z_i.
  - $p \to 1$: constant size of loss.
  - $p \to 2$: high variation in losses.
  - $p = 1.5$: exponential claim size.

Previous Analysis

- Jørgensen and Paes De Souza (SAJ 1994).
- Brazillian p.p. auto collision insurance data.
- 4440 policies in 458 groups; 323 (67%) groups with no claims.
- 2 differences between this data set and the one analysed here:
  - definition of exposure.
  - short tailed distribution.

Data Set

- P.P. auto weekly benefit data for A.Y. 1992.
- Collected by:

accident month	12 months
rate class	6 classes
liability limit	9 limits
territory	15 territories

- 8000 observations
- Total earned exposure: 1 783 352.2 car years.
- Total number of claims: 12 758.
- Total losses \$ 21 995 830 with average loss per cell of \$2749.
- 85% of cells had no claims.

Hypotheses

- model is appropriate for data.
- liability limit should be insignificant.
- no interactions should be significant.

Analysis

- Reduce number of territories from 15 to 9.
- Reduce number of rate classes from 6 to 4.
- Reduce number of liability limit groups from 9 to 8.
- Examine models with interactions of various orders.

Results

- Main effects, 2 way interactions and 3 way interaction are all significant.
- estimate of p is 1.867
- estimate of σ^2 is 1405
- 288 parameters.

Interpretation

Hard!!!

RC1

	Terr1	Terr2	Terr3	Terr4	Terr5
Liab1	1.00	0.00	0.02	0.53	0.09
Liab2	0.40	0.00	0.00	0.00	0.00
Liab3	0.54	0.00	0.00	0.16	0.07

RC2

	Terr1	Terr2	Terr3	Terr4	Terr5
Liab1	0.53	0.00	0.00	0.08	0.03
Liab2	0.28	0.00	0.00	0.00	0.00
Liab3	0.97	0.00	0.00	0.36	0.06

Differentials from Model with Interactions

RC1

	Terr1	Terr2	Terr3	Terr4	Terr5
Liab1	1.00	0.34	0.12	0.81	0.38
Liab2	0.24	0.08	0.03	0.20	0.09
Liab3	0.71	0.24	0.08	0.57	0.27

RC2

	Terr1	Terr2	Terr3	Terr4	Terr5
Liab1	1.08	0.36	0.13	0.87	0.41
Liab2	0.90	0.09	0.03	0.21	0.10
Liab3	0.97	0.26	0.09	0.62	0.29

Differentials from Model with No Interactions

Conclusions

- Tweedie model provides solution for data with mass point at zero.
- Maybe still too simple?
- Rating variable liability limit significant.
- All 2 and 3 way interactions significant.