Coping with Longevity: The New German Annuity Valuation Table DAV 2004 R

Ulrich Pasdika
Gen Re
Cologne, Germany

and

Jürgen Wolff
MARC Life
Atlanta, Georgia

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1. Introduction

The German annuity market is dominated by two basic products, immediate annuities and deferred annuities. Immediate annuities are available for a single premium and the life-long benefit payment begins very shortly after the commencement date. Deferred annuities are normally regular level premium products that are taken out long before the benefit payment starts. However, the annuity rates applicable at the beginning of the benefit payment period are often already guaranteed at the outset. The policyholder usually has the option of annuitizing the policy at the end of the deferment period or receiving the corresponding lump sum.

Both products normally entail no or very little death benefit, e.g. the sum total of the premiums paid for a deferred annuity or a minimum guaranteed benefit payment period of five years for annuities in payment. With the limited death benefit for a deferred annuity, there is normally a positive reserve for the contract that is not paid to the beneficiaries in the event of death. It is therefore standard industry practice to take explicitly into account that a number of insured lives will die before the beginning of the benefit payment period. In a way, the surviving policyholders “inherit” the reserves of the deceased persons. This means that insurance companies are already running a longevity risk in the deferment period because fewer insured lives may actually die than was expected. For this reason, appropriate mortality rates are also needed for the deferment period.

German insurance companies are required by law to use prudent rates for the valuation of their business. It is explicitly forbidden to use only best estimate pricing assumptions, which in Germany are called "2nd order" mortality. Consideration needs to be afforded to any adverse current experience resulting from fluctuations and to changes to the underlying risk. The mortality rates, including appropriate provision for adverse deviations from the best estimate assumptions, are referred to as "1st order" mortality rates. It should be mentioned that valuation mortality tables are normally also used for pricing purposes in Germany.

Since the publication of DAV 1994 R, the most recent mortality table for annuity business by the German Actuarial Society (DAV) in 1994 (see [SS]), there has been an unprecedented annuity boom. In 2003, 46 percent of all the new individual regular premiums business was annuity business. At the end of 2003, 16 percent of all in-force contracts were annuities, compared with a mere 5 percent at the end of 1996. Annuities can be expected to remain one of the German life industry’s major products in the future.

With the increasing importance of annuity business, the legal valuation requirements and the long-term guarantees given in annuity business, insurance companies have a vested interest in using adequate mortality rates for pricing and evaluating annuity business. A committee of the DAV has thus examined the question as to whether or not a new revised mortality table is necessary for annuity business.

Munich Re and Gen Re have been collecting and analyzing insured lives data from individual annuity business for several years. Between 1995 and 2002, more than 20 life insurance companies contributed to this observation material, which was one of the main data sources for the committee's activities. Not only is this observation material, which encompasses 13.7 million years' exposure, much more voluminous than the data that was available for the 1994 table, it is also much more comprehensive with regard to contractual items such as annuity amounts and years lapsed since inception date or benefit payment commencement. Approximately 10 percent of the data relates to annuities in payment. Apart from the insured lives data, the committee was also able to analyze population mortality tables provided by the German Federal Statistical Office.

On the basis of the insured lives observation material and the population mortality tables, the committee was able to subject the industry's current standard DAV 1994 R table to intensive scrutiny. The results from the insured lives data and the mortality improvement pattern that became apparent from the population mortality tables both clearly indicate the need for a revised mortality table for annuity business. The German Actuarial Society has recommended using this table, called DAV 2004 R, for evaluating and pricing new business with effect from January 1, 2005. The German paper [DAV] is the official reference for the new DAV 2004 R table.

In the remaining part of this section, we first outline the basic DAV 2004 R derivation steps. In Section 2, we examine the various base tables. Section 3 focuses on the mortality improvement trend. Some international comparisons are contained in Section 4. The Appendix contains the table values of DAV 2004 R.

1.1 Generation Mortality Tables

Mortality tables for a fixed calendar year, such as the abridged mortality table published by the German Federal Statistical Office, cannot be used for calculating annuities, since they do not reflect the mortality improvement trend. This is why it has become standard international practice to use generation mortality tables for annuity business. Generation mortality tables are a combination of mortality rates per birth year and a trend assumption relating to the future mortality improvements. This approach has already been used for the two preceding German annuity mortality tables, 1987 R and DAV 1994 R.

The new DAV 2004 R table consists of the following components:

- 2nd order base table: a best estimate of the insured lives mortality rates in 1999,
- 1st order base table: the 2nd order base table reduced to take into account provisions for adverse deviation,
- 2nd order mortality trend: best estimate of the future mortality improvements, and
- 1st order mortality trend: the 2nd order trend increased to take into account provisions for adverse deviation.

1.2 Base Tables

There are separate base tables for the deferment period and for the benefit payment period in DAV 2004 R in order to reflect the annuitants' self-selection at the beginning of the benefit payment period. Only people who have a favorable opinion of their health and remaining life expectancy will purchase an immediate annuity or exercise their annuity option at the end of the deferment period. This self-selection works effectively, meaning that the gap between the mortality rates of annuitants and the general population is greatest in the first few years following the commencement of benefit payment. Therefore, the base table for the benefit payment period is graded not only in terms of the current age of the annuitant, but also in terms of the years lapsed since the start of the benefit payment.

The base table for the deferment period is not graded in terms of duration due to the fact that self-selection is much weaker in this case. For ages above 64, the base table for the deferment period is continued on the basis of the observation material relating to annuities in payment, because not very much insured lives data is available from deferred contracts for these ages. Thus, the base table for the deferment period is an aggregate table that also implicitly reflects the self-selection of the insureds at the beginning of the benefit payment period.

This aggregate base table may, under certain circumstances, also be used as a table for both the deferment and the benefit payment periods. Such a simplified approach may make sense if, for example, IT constraints exist. However, the appointed actuary needs to check if the aggregate base table is appropriate. For a company specializing in benefit commencement ages that are very different from the observation material's average, the aggregate table would not normally be an option.

Provisions for adverse deviation on the best estimate mortality rates have been determined for the purpose of affording consideration to the volatility risk and the level parameter risk.

1.3 Mortality Improvement Trend

The 2nd order trend is graded in terms of both the current age of the insured and the calendar year. It is likely that the trend reduction observed in other countries will also restrain mortality improvements in Germany in the future. A linear transition from the 2nd order initial trend to the 2nd order target trend is therefore assumed.

We have evidence (as explained in Section 3.2.3.1) that the mortality improvement trend is more pronounced in higher socioeconomic groups. For this reason, a flat-rate loading for insured persons is applied to the mortality improvements derived from the population tables. The 2nd order initial trend reflects the mortality improvements from 1990 to 1999, but increased by the loading for insured persons. The 2nd order target trend is 75 percent of the population trend in the period 1972 to 1999, plus the loading for insured persons.

The assumption of a linear trend reduction is omitted for the 1st order trend, which thus is only contingent upon age and gender. The 1st order trend is the 2nd order initial trend plus a flat-rate loading for the risk of change.

The following figure shows how the mortality rates of 65-year-old males have developed since 1970, starting from a level of 100 percent, according to the long-term trend assumption in the preceding table DAV 1994 R (which is applied to the mortality rates subsequent to 2000 in this table) and the population mortality tables for the former West Germany.

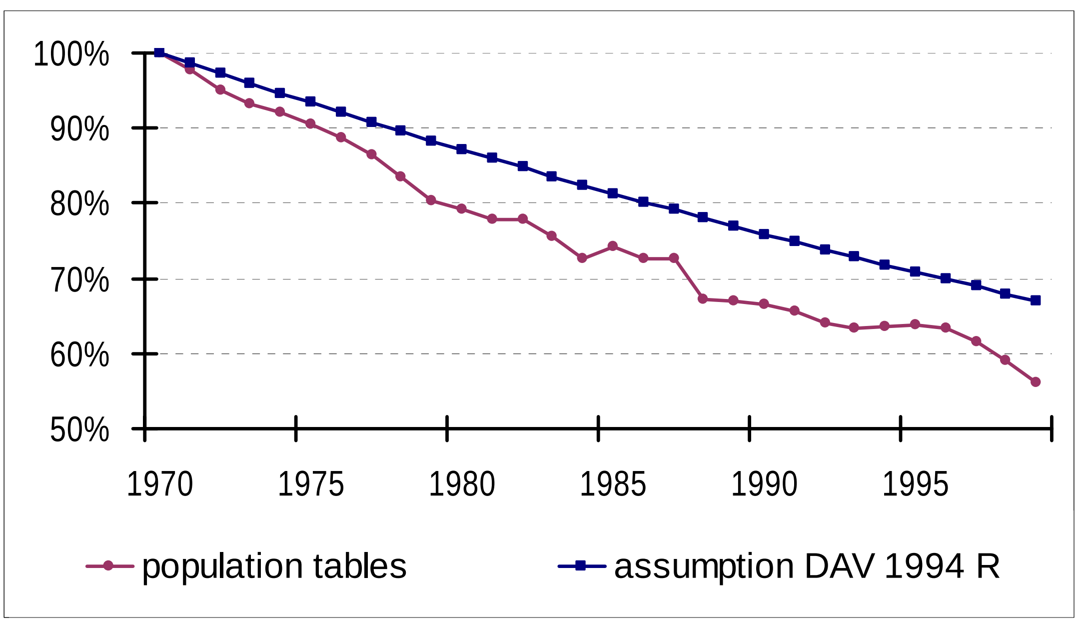


Figure 1. Development of mortality rates for 65-year-old males

DAV 1994 R's long-term trend was derived using all the available German population mortality tables from 1871 onward. An analysis of the most recent data reveals that the age profile of mortality improvements has undergone drastic change down the decades. In the more recent past, it is also possible to observe a major improvement in mortality at higher ages.

The long-term DAV 1994 R trend assumption does not appropriately reflect the mortality improvements in the last three decades of the twentieth century. In consequence, all mortality tables prior to the 1971/73 mortality table have been disregarded for the purpose of projecting the future mortality improvements of DAV 2004 R.

2. Base Tables

The insured lives data collected by Munich Re and Gen Re is used to derive base tables. The data contains information relating to the gender and the age of the insured life, the date of the first annuity payment and the amount of the insured annuity. Such detailed information has never previously been available for the purpose of deriving a new mortality table in Germany, and it facilitates the derivation of mortality rates graded not only in terms of the current age of the annuitant, but also in terms of the years lapsed since the start of the benefit payment.

2.1 Preparatory Steps

The data observation period is 1995 to 2002. If $E(t)$ denotes the entire insured lives exposure of calendar year t, the ratio

$$\frac{\sum_{t=1995}^{2002} t \cdot E(t)}{\sum_{t=1995}^{2002} E(t)}$$

gives the average observation material calendar year. This ratio yields an average of 1999.8. The mortality rate for a specific age x and calendar year t is applied to the policy year starting in t. On average, for a whole portfolio, the new policy year starts on July 1 and ends on June 30 in t+1. Therefore, the appropriate base year for a table derived from the observation material is the observation material's average calendar year minus half a year, which, after rounding, is 1999.

An analysis of the observation material demonstrates the existence of a strong relationship between mortality and annuity levels. Several annuity level phases were defined, and the mortality level in the different phases was measured against the overall mortality level. The results are shown in the following table.

Annual Annuity Amount (euros)	Males	Females
0 – 600	117%	111%
601 – 1200	110%	105%
1201 – 2000	101%	99%
2001 – 3500	90%	88%
3501 – 6000	89%	91%
> 6000	86%	91%
Aggregate	100%	100%

In the light of these results, mortality rates weighted by lives are not appropriate for annuity business. Accordingly, where possible, the mortality rates in the base table have been derived as mortality rates weighted by annuity level.

For some procedures, we need estimators for the mortality rate of the German population in 1999 that are consistent with the mid-term mortality improvement trend from 1972 to 1999. These estimators are derived from the 1971/73 to 1998/2000 population mortality tables for the former West Germany as follows:

Let $q_{x,t}^{pop}$ denote the mortality rates for $t = 1972, 1973, ..., 1999$ of an x-year-old according to the population mortality tables (table 1971/73 for $t = 1972$,…, table 1998/2000 for $t = 1999$). Then the logarithmic linear regression

$$\ln\left(\hat{q}_{x,t}^{pop}\right) = -\hat{F}^{pop}(x) \cdot t + B(x)$$

yields estimators $\hat{q}_{x,t}^{pop}$ for the population mortality in year t and $\hat{F}^{pop}(x)$ for the mortality improvement trend. The estimators $\hat{F}^{pop}(x)$ correspond to the crude mid-term mortality improvement trend that is used in Section 3.2.1.

2.2 The Selection Table for the Benefit Payment Period

The annuity payment period is divided into six selection phases by the number of years lapsed since the start of the benefit payment: 1st year,..., 5th year, 6th+ years ("ultimate"). Let $s = 1, ..., 6$ denote the selection phases and let $t = 1995, ..., 2002$ denote the calendar year. Then we define:

$T_{x,t}^{s}$ The sum total of the annual annuity amounts of the deceased persons in the observation material of Munich Re and Gen Re who died in year t and at age x,

$L_{x,t}^{s}$ The exposure of the insureds living in year t at age x in the observation material of Munich Re and Gen Re, weighted by the amount of the annuity,

q_{x}^{s} The graduated mortality rate of an x-year-old in selection phase s, based upon the average observation year 1999,

f^{1} The selection factor in selection phase 1,

f^{2-5} The selection factor in selection phases 2 to 5,

$q_{x}^{6}(crude)$ The crude mortality rate of an x-year-old in selection phase 6, based upon the average observation year 1999.

For the purpose of deriving the selection factors, in a first step we compare the mortality rates in the different selection phases with the population mortality. To this end, we calculate per selection phase s the ratio of the actual number of deceased persons to the number of expected deceased persons using the population mortality $\hat{q}_{x,1999}^{pop}$ defined in Section 2.1:

$$r_s = \frac{\sum_{t=1995}^{2002} \sum_{x=0}^{89} T_{x,t}^{s}}{\sum_{t=1995}^{2002} \sum_{x=0}^{89} \hat{q}_{x,1999}^{Bev} \cdot L_{x,t}^{s}}.$$

The following figure shows the various ratios.

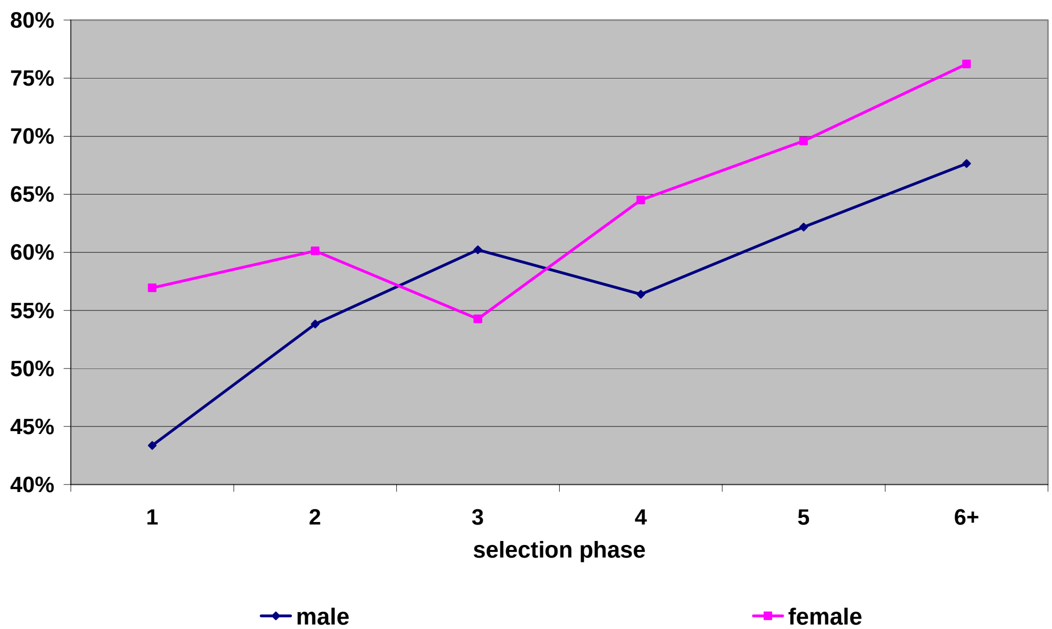


Figure 2. Mortality rates by selection phase relative to the population mortality

The following observations can be made:

- Male mortality in selection phase 1 is lower than in any other selection phase. Female mortality in selection phase 1 is lower than in selection phases 2, 4, 5 and 6+.
- Mortality in selection phase 6+ is highest, both for males and females.
- Mortality in selection phases 2 to 5 fluctuates.

The following model is thus adopted for the selection table:

- There is an ultimate mortality table for selection phase 6+.
- Mortality in selection phases 1 to 5 is a factor of the ultimate mortality table. This factor depends not on age, but on gender. There is a factor for selection phase 1 and a common factor for selection phases 2 to 5.

In order to determine appropriate selection factors, we first calculate crude ultimate mortality rates:

$$q_x^{ref}\left(crude\right) = \frac{\sum_{t=1995}^{2002} T_{x,t}^6}{\sum_{t=1995}^{2002} L_{x,t}^6}.$$

The crude mortality rates are graduated for the age band 60 to 99 using the Whittaker-Henderson method (see [KBLOZ] or [L]) with weight $L_{x,t}^6$ on the mortality rate at age x and weight 0.5 on the smoothness measured by second differences. The selection factors are then defined as the ratio of the actual number of deceased persons in the respective selection phase to the number of deceased persons that would be expected if the graduated ultimate mortality rates q_x^{ref} were applied to the exposure of insured lives in the respective selection phase:

$$f^1 = \frac{\sum_{t=1995}^{2002}\sum_{x=60}^{99} T_{x,t}^1}{\sum_{t=1995}^{2002}\sum_{x=60}^{99} q_x^{ref} \cdot L_{x,t}^1} \quad \text{and} \quad f^{2-5} = \frac{\sum_{s=2}^{5}\sum_{t=1995}^{2002}\sum_{x=60}^{99} T_{x,t}^s}{\sum_{s=2}^{5}\sum_{t=1995}^{2002}\sum_{x=60}^{99} q_x^{ref} \cdot L_{x,t}^s}.$$

We obtain the following values:

	Male	**Female**
f^1	0.670538	0.712823
f^{2-5}	0.876209	0.798230

In the absence of other available data, it is reasonable to use the graduated ultimate mortality rates q_x^{ref} for the purpose of deriving selection factors. However, the data in the ultimate selection phase is insufficient for deriving the final ultimate mortality rates for the new table. This is particularly the case for the age band 60 to 65 at which, in the case of the majority of policies, benefit payment commences. After adjusting the relationship between the number of deceased persons and the number of insured lives by means of the selection factors, we therefore use the data from all the selection phases to derive the ultimate mortality rates:

$$q_x^6(crude) = \frac{\sum_{s=1}^{6}\sum_{t=1995}^{2002} T_{x,t}^s}{f^1 \cdot \sum_{t=1995}^{2002} L_{x,t}^1 + f^{2-5} \cdot \sum_{s=2}^{5}\sum_{t=1995}^{2002} L_{x,t}^s + \sum_{t=1995}^{2002} L_{x,t}^6}.$$

The crude rates are again graduated using the Whittaker-Henderson method with weight $\sum_{s=1}^{6}\sum_{t=1995}^{2002} L_{x,t}^s$ on the mortality rate at age x and weight 0.5 on the smoothness measured by second differences. These graduated rates are the final best estimate ultimate mortality rates q_x^6.

We compared the ultimate mortality rates q_x^6, based upon the data from all the selection phases, with the ultimate mortality rates q_x^{ref}, which were derived using data from the ultimate selection phase only. It was found that for ages 70+, the differences between the two sets of mortality rates are negligible. More significant differences at younger ages are attributable to the small volume of ultimate data that exists and are also influenced by early retirees, who are not representative of the portfolio of all pensioners. Outside the age band 60 to 99, we have insufficient insured lives data, with the result that we are forced to extrapolate the ultimate mortality rates.

For the ages up to 59, we use the estimators $\hat{q}_{x,1999}^{pop}$ for the population mortality rates in the year 1999 defined in Section 2.1. We assume that the ratio of the ultimate mortality rate at age 60, q_{60}^6, to the estimator for the population mortality rate at age 60, $\hat{q}_{60,1999}^{pop}$, can be transferred to the younger ages. We thus define:

$$q_x^6 = \hat{q}_{x,1999}^{pop} \cdot \frac{q_x^6}{\hat{q}_{60,1999}^{pop}} = \hat{q}_{x,1999}^{pop} \cdot \begin{cases} 66.6\% & for\ men \\ 85.2\% & for\ women \end{cases}.$$

As the maximal age of the German population mortality tables is 89, we cannot use the population mortality to extrapolate at the oldest ages. Therefore, we follow the method set out in [TKV] to extrapolate the ultimate mortality rates at ages above 99. In [TKV] six extrapolation approaches are examined:

- The Gompertz model: $q_x = 1 - \exp(-\exp(a + bx))$
- The Quadratic model: $q_x = 1 - \exp(-\exp(a + bx + cx^2))$
- The Heligman and Pollard model: $q_x = \dfrac{a\exp(bx)}{1 + a\exp(bx)}$
- The Weibull model: $q_x = 1 - \exp(-a(x + \frac{1}{2})^b)$
- The Kannisto model: $q_x = 1 - \exp\left(-\left(\dfrac{a\exp(bx)}{1 + a\exp(bx)} + c\right)\right)$
- The Logistic model: $q_x = 1 - \exp\left(-\left(\dfrac{\beta\exp(bx)}{1 + \alpha\exp(bx)} + c\right)\right)$

We first fit all six models to the ultimate mortality rates q_x^6 at ages 85 to 95 and then compare the models with each other. The Maximum-Loglikelihood method is used to estimate the parameters. If the number of deceased persons at age x follows a binomial distribution with parameter q_x, then the probability of observing $\sum_{x \in X} T_x$ deceased out of $\sum_{x \in X} L_x$ living persons in age band X is given by:

$$P\left(\sum_{x \in X} T_x \ observed \ deceased\right) = \prod_x \binom{L_x}{T_x} q_x(\theta)^{T_x}(1 - q_x(\theta))^{L_x - T_x},$$

where θ is the array of parameters in a mortality model (for example, $\theta = (a,b)$).

In order to estimate the parameters, we maximize the loglikelihood

$$L(\theta) = \sum_x T_x \ln(q_x) + \sum_x (L_x - T_x)\ln(1 - q_x) + \sum_x \binom{L_x}{T_x}.$$

This leads to a non-linear system of equations $\partial L(\theta)/\partial\theta_i = 0$, and we use Newton's method to solve it. The age band 85 to 95 is used as calibration base, leaving the age band 96 to 99 to assess the accuracy of the calibration. We consider the following parameters for the purpose of comparing the models:

- The number of the expected deceased persons applying the extrapolated mortality rates to the actual number of deceased persons

$$\sum_{x=96}^{99} (\hat{q}_x L_x - T_x) \Big/ \sum_{x=96}^{99} \hat{q}_x L_x$$

- The loglikelihood without the constant term

$$\sum_{x=96}^{99} T_x \ln(\hat{q}_x) + \sum_{x=96}^{99} (L_x - T_x)\ln(1-\hat{q}_x)$$

- The chi-square statistic

$$\sum_{x=96}^{99} \frac{(T_x - L_x\hat{q}_x)^2}{L_x\hat{q}_x(1-\hat{q}_x)}$$

The following two tables show the results.

Males						
	(Expected-Actual) / Expected Deceased		**Loglikelihood**		**Chi-square Statistic**	
Model	**Value**	**Rank**	**Value**	**Rank**	**Value**	**Rank**
Gompertz	9.34%	(7)	-953.057	(7)	7.003	(7)
Quadratic	-0.55%	(2)	-949.504	(2)	29	(2)
Heligman & Pollard	7.29%	(6)	-951.557	(6)	4.075	(6)
Weibull	7.18%	(5)	-951.505	(5)	3.927	(5)
Kannisto with $c = 0$	4.07%	(4)	-950.078	(4)	1.166	(4)
Kannisto with $c \neq 0$	0.07%	(1)	-949.490	(1)	1	(1)
Logistic	-1.20%	(3)	-949.535	(3)	91	(3)

Females						
	(Expected-Actual) / Expected Deceased		**Loglikelihood**		**Chi-square Statistic**	
Model	**Value**	**Rank**	**Value**	**Value**	**Rank**	**Rang**
Gompertz	13.02%	(7)	-1.462.744	(7)	17.364	(7)
Quadratic	-1.00%	(1)	-1.453.831	(1)	120	(1)
Heligman & Pollard	11.43%	(6)	- 1.460.436	(6)	12.950	(6)
Weibull	10.98%	(5)	-1.459.898	(5)	11.916	(5)
Kannisto with $c = 0$	9.15%	(4)	- 1.457.819	(4)	7.913	(4)
Kannisto with $c \neq 0$	1.57%	(2)	- 1.453.879	(2)	214	(2)
Logistic	2.52%	(3)	- 1.454.067	(3)	587	(3)

With all three criteria, the Logistic, the Kannisto ($c \neq 0$) and the Quadratic model produce the best results. However, since mortality rates start decreasing at age *x=-b/2c* with the Quadratic model, it is disregarded. Given the actual ultimate mortality rates q_x^6 at ages 85 to 95, this point is reached much earlier than age 120, particularly for females.

For the purpose of deciding between the Logistic and the Kannisto model, we compare the extrapolated mortality rates with the Japanese 1999 mortality rates at ages 105 and 109.

	Males		**Females**	
	***x*=105**	***x*=109**	***x*=105**	***x*=109**
Japanese population	46.1%	52.2%	41.9%	49.6%
Logistic model	42.2%	50.0%	36.1%	43.8%
Kannisto model	40.9%	46.1%	33.4%	38.3%

As the level of the Kannisto mortality rates seems to be too low, we use the Logistic model for extrapolating the ultimate mortality rates at the ages 100 to 120 as follows:

- For males
$$q_x^6 = 1 - \exp\left(-\left(\frac{-0.7979071812 \cdot \exp(-0.006 \cdot x)}{1 - 2.4 \cdot \exp(-0.006 \cdot x)} - 0.9795414181\right)\right)$$
- For females
$$q_y^6 = 1 - \exp\left(-\left(\frac{-0.7415144956 \cdot \exp(-0.007 \cdot y)}{1 - 2.7 \cdot \exp(-0.007 \cdot y)} - 0.7587905969\right)\right).$$

2.3 The Aggregate Table for the Deferment Period

A table for the deferment period is also needed. Munich Re and Gen Re have collected details of more than 12 million years' exposure from more than 20 insurance companies in the observation period 1995 to 2002, which can be used to derive such a table.

The observation material shows that the selection effect in the deferment period is much weaker than in the annuity payment period. What is more, the impact of the selection effect on premiums and reserves at typical commencement ages between 30 and 40 is negligible. That is why an aggregate table is derived for the deferment period that is not graded in terms of the years lapsed since the commencement date.

The ages 20 to 64 account for more than 96 percent of the observation material relating to the deferment period. From age 65 onward, the observed exposure for annuities in payment strongly outweighs the exposure for deferred annuities, which rapidly becomes sparse at these ages. It is thus reasonable to revert to the observation material relating to annuities in payment for the ages 65+.

In a first step, we calculate crude mortality rates that are not graded separately in terms of the years lapsed since the commencement date or the start of the benefit payment:

- from the observation material relating to the deferment period

$$q_x^{def}(crude) = \frac{\sum_{s=1}^{6}\sum_{t=1995}^{2002} T_{x,t}^{s}}{\sum_{s=1}^{6}\sum_{t=1995}^{2002} T_{x,t}^{s}} \text{ and}$$

- from the observation material relating to the benefit payment period

$$q_x^{ann}(crude) = \frac{\sum_{s=1}^{6}\sum_{t=1995}^{2002} T_{x,t}^{s}}{\sum_{s=1}^{6}\sum_{t=1995}^{2002} T_{x,t}^{s}}.$$

Both sets of crude mortality rates are then graduated using the Whittaker-Henderson method with weight $L_{x,t}$ on the mortality rate at age x and weight 0.5 on the smoothness measured by second differences, the $q_x^{def}(crude)$ for the age band 20 to 70 and the $q_x^{ann}(crude)$ for the age band 60 to 99. The graduated rates q_x^{def} and q_x^{ann} are then put together at age 65 in order to obtain aggregate mortality rates q_x^{agg}:

$$q_x^{agg} = \begin{cases} q_x^{def} & 20 \le x \le 64 \\ q_x^{ann} & 65 \le x \le 99 \end{cases}.$$

q_{65}^{def} and q_{65}^{ann} differ by less than 1 percent, meaning that no additional graduating is needed.

For ages below 20, there is not enough insured lives observation material available. As with the approach we adopted for the selection table, we use the estimators $\hat{q}_{x,1999}^{pop}$ for the population mortality rates in the year 1999 defined in Section 2.1 instead. It seems plausible to assume that even at the youngest ages, insured lives' mortality rates in annuity policies will be slightly lower than the population mortality rate:

$$q_x^{agg} = 0.9 \cdot \hat{q}_{x,1999}^{pop} \qquad x < 20.$$

In order to be consistent with this approach at ages over 20, we minimize the mortality rates to 90 percent of the population mortality at any age. This minimization becomes effective at age 20 to 26 for males and at age 20 to 30 for females. As the ultimate mortality rate from the selection table, q_{99}^{6}, and the aggregate mortality rate at age 99, q_{99}^{agg}, are almost identical, the extrapolation for ages 100+ from the selection table can also be used for the aggregate table.

On the one hand, this aggregate table can be used as a table for deferred annuities in conjunction with the select table for the annuity payment period, and, on the other hand, it may also be used as an aggregate table for both the deferment period and the annuity payment period because for the typical ages of annuities in payment, it has been derived from annuitants' data, and the selection effect at the beginning of the payment period is also implicitly reflected in this data and thus in the table, even if there are no explicit selection factors.

2.4 Safety Margins

2.4.1 Margin for Volatility Risk

For the calculation of the margin for volatility risk, we use the following denotations:

q_x^{agg}	The best estimate aggregate mortality rates derived in Section 2.3,
s^α	The relative margin for volatility risk for confidence level $1-\alpha$,
L_x^M	The insured lives aged x in the model portfolio,
T_x	The random variable number of the deceased at age x in the model portfolio,
V_x	The mathematical reserve for an x-year-old's contract,
$u_{1-\alpha}$	The $(1-\alpha)$ quantile of the standard normal distribution.

The margin s^α is designed to reduce the volatility risk when the table is applied. The idea is to provide protection against a maximum loss at a defined confidence level. This basic concept has been widely used in German mortality and morbidity tables (cf. [L1], [SS] and more generally [P], [PS]). In the context of German annuity products, the most appropriate measurement for loss is the amount of the mathematical reserve that can be released in the event of death. If fewer insured lives die than was originally expected, then less mathematical reserve can be released, and the insurance company may suffer a loss. Thus, what is required is that with confidence $1-\alpha$, it is not possible for less mathematical reserve to be actually released for the deceased than was originally expected:

$$P\left(\sum_x T_x \cdot V_x \geq \sum_x \left(1-s^\alpha\right) \cdot q_x^{agg} \cdot L_x^M \cdot V_x\right) \geq 1-\alpha .$$

In order to solve the equation, an underlying model portfolio is defined as follows:

- The size of the annuity portfolio, 200,000 insured lives (50 percent male, 50 percent female), corresponds to the projected average size that German annuity portfolios will feature in a few years. The structure of the portfolio is geared to the observation material with constant annuity amounts (observation material average).

- It is assumed that 10 percent of the policies are annuities in payment, whereas 90 percent are deferred annuity policies.

- For all policies, benefit payment commences at age 65.

- There are no death benefits such as guaranteed periods of benefit payment or survivorship annuities.

We additionally assume that T_x is independently distributed. Given these assumptions, the Central Limit Theorem allows us to approximate $\sum_x T_x \cdot V_x$, the mathematical reserve that can be released at death by a normal random variable with

- expectation $\sum_x q_x^{agg} \cdot L_x^M \cdot V_x$, and

- variance $\sum_x Var[T_x \cdot V_x] = \sum_x Var[T_x] \cdot V_x^{\,2} = \sum_x q_x^{agg} \cdot \left(1 - q_x^{agg}\right) \cdot L_x^M \cdot V_x^{\,2}$.

This means that $s^\alpha = \dfrac{\sqrt{\sum_z q_x^{agg} \cdot (1 - q_x^{agg}) \cdot L_z^M \cdot V_z^2}}{\sum_z q_x^{agg} \cdot L_z^M \cdot V_z} \cdot u_{1-\alpha}$ is one solution to the above equation because

$$\alpha \geq P\left(\sum_x T_x \cdot V_x \leq \sum_x \left(1 - s^\alpha\right) \cdot q_x^{agg} \cdot L_x^M \cdot V_x \right) = P\left(\frac{\sum_x T_x \cdot V_x - q_x^{agg} \cdot L_x^M \cdot V_x}{\sqrt{\sum_x q_x^{agg} \cdot \left(1 - q_x^{agg}\right) \cdot L_x^M \cdot V_x^{\,2}}} \leq - \frac{\sum_x s^\alpha \cdot q_x^{agg} \cdot L_x^M \cdot V_x}{\sqrt{\sum_x q_x^{agg} \cdot \left(1 - q_x^{agg}\right) \cdot L_x^M \cdot V_x^{\,2}}} \right)$$

This equation holds if

$$u_{1-\alpha} = - \frac{\sum_x s^\alpha \cdot q_x^{agg} \cdot L_x^M \cdot V_x}{\sqrt{\sum_x q_x^{agg} \cdot \left(1 - q_x^{agg}\right) \cdot L_x^M \cdot V_x^{\,2}}} \text{ or if } s^\alpha = \frac{u_{1-\alpha} \sqrt{\sum_x q_x^{agg} \cdot \left(1 - q_x^{agg}\right) \cdot L_x^M \cdot V_x^{\,2}}}{\sum_x q_x^{agg} \cdot L_x^M \cdot V_x} .$$

The margin is calculated separately for males and females. For each sub-portfolio, the following quantile is used:

$$u_{1-\alpha^*} = \frac{1}{\sqrt{2}} \cdot u_{1-\alpha} .$$

If 1-α =95 percent, then 1-α^*=87.76 percent, and we obtain a margin for volatility risk of 6.26 percent for males and 7.22 percent for females on the basis of the best estimate aggregate mortality. The actual confidence level for the model portfolio using these deductions is 94.6 percent. The same confidence level is also attained on the basis of the best estimate selection mortality rates, meaning that we can use the same margin for volatility risk for the selection table. Even if it were the case that we needed to assume a specific model portfolio for this step, sensitivity calculations have demonstrated that the actual confidence level does not change significantly if the age structure, the proportion of male insured lives or the proportion of policies already in payment varies.

2.4.2 Margin for Level Parameter Risk

In order to derive the table, a certain model was postulated and parameters needed deriving. There are several sources of level parameter risk:

- Structural differences between the observation material and any actual portfolio to which the table is applied.
- A difference between the mortality levels at individual companies and that in the observation material.
- Structural differences in future new business, particularly due to changes to the political and taxation frameworks.
- The actual observation material having also been subject to statistical fluctuations.

A 10 percent flat-rate margin for level parameter risk is thus defined, meaning that there is a total deduction of 15.6 percent for males and 16.5 percent for females.

2.5 Depiction of the Results

The following figure shows the selection mortality rates q_x^6 and the aggregate mortality rates q_x^{agg} as a percentage of the estimators $\hat{q}_{x,1999}^{pop}$ for the population mortality rates in the year 1999 defined in Section 2.1. The strong degree of self-selection exercised by the insureds at ages of around 60 is clearly evident from the aggregate mortality rates.

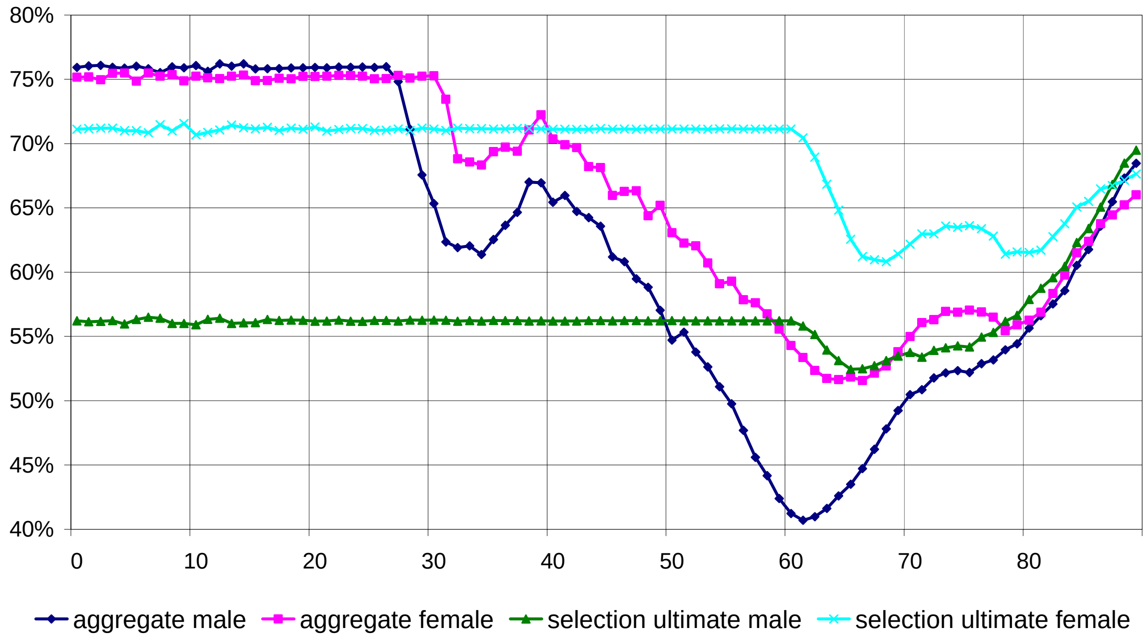


Figure 3. Comparison between the derived mortality rates and population mortality rates

3. Mortality Improvement

3.1 Model Choice

An age-dependent mortality improvement model was used for the previous German annuity table DAV 1994 R (see [SS]):

$$\frac{q_{x,t+1}}{q_{x,t}} = \exp\left(-F(x)\right),$$

with a trend function $F(x)$ depending on age x. In the following, this model is referred to as the "traditional model."

The cohort model of birth-year-dependent mortality improvement is defined in [W], Chapter 6.6:

$$\frac{q_{x,t+1}}{q_{x,t}} = \exp\left(-G(t+1-x)\right),$$

with a trend function $G(t+1-x)$ depending on birth year $t+1-x$. [W] contains studies of mortality data from England and Wales showing a cohort effect. For the purpose of choosing an appropriate model for mortality projections of the German population, the traditional model and the cohort model were examined, as well as the synthesis model stemming from a combination of the two:

$$\frac{q_{x,t+1}}{q_{x,t}} = \exp\left(-F(x)-G(t+1-x)\right).$$

These studies are described in Section 3.1.1. It transpires that the synthesis model is the best of these three models for the purpose of modeling the German mortality data from the past. However, Section 3.1.2 shows that the synthesis model is inappropriate for projection purposes. In Section 3.1.2, it is argued that the traditional model is more adequate for projecting DAV 2004 R mortalities than the cohort model.

3.1.1 Modeling Data from the Past

Mortality data from the past, $<\ln(q_{x,t})>$ for $t_0 \le t \le t_1$ and $x_0 \le x \le x_1$, are calibrated using the least squares method. For example, the trend function $F(x)$ of the traditional model is calculated by linear regression of $<\ln(q_{x,t})>$ for $t_0 \le t \le t_1$.

3.1.1.1 Likelihood Ratio Test

The traditional model can be interpreted as a special synthesis model case with all coefficients $G(t+1-x)$ restricted to 0. The cohort model can be interpreted as a special synthesis model case with all coefficients $F(x)$ restricted to 0.

The likelihood ratio test or F-test (see [K], Section 5.1.1 or [JHGLL]) is used in order to decide whether or not these restrictions to 0 are sensible. The test statistic L of the likelihood ratio test is defined by

$$L = (SSE_R - SSE_U) / SSE_U \cdot (T - K) / J ,$$

where

- SSE_R is the sum of the squared errors of the restricted model (traditional or cohort model),
- SSE_U is the sum of the squared errors of the unrestricted model (synthesis model),
- J is the number of coefficients set to 0 in the restricted model,
- T is the number of observations, and
- K is the number of coefficients of the unrestricted model.

This test statistic is F-distributed with J and $T-K$ degrees of freedom: $L \sim F(J,\ T-K)$. The null hypothesis H_0 must be rejected at a significance level of $\alpha \cdot 100\%$ if $L > F(J,\ T-K;\ \alpha)$ (upper α-quantile of the F-distribution).

The likelihood ratio test is applied to mortality data $q_{x,t}$ of 33 German population mortality tables from the period 1967 to 1999 for ages $20 \le x \le 89$. The likelihood ratio test produces the following results.

	H_0: the traditional model holds H_1: the synthesis model holds $T = 2310$ $K = 240$ $J = 100$ F(J, T-K; 0.05) = 1.25 F(J, T-K; 0.01) = 1.37	H_0: the cohort model holds H_1: the synthesis model holds $T = 2310$ $K = 240$ $J = 69$ F(J, T-K; 0.05) = 1.30 F(J, T-K; 0.01) = 1.45
Males $SSE_U = 2196.87$	$SSE_R = 7022.66$ $L = 45.47$	$SSE_R = 4181.51$ $L = 27.10$
Females $SSE_U = 2216.12$	$SSE_R = 4057.35$ $L = 17.20$	$SSE_R = 3855.54$ $L = 22.19$

The test statistic L clearly exceeds F(J, T-K; 0.01) in all cases. Therefore, the synthesis model seems to be the most appropriate of the three models for the purpose of modeling German mortality data from the past.

3.1.2 Projecting Mortality

Mortality rates $q_{x,t}$ are projected to years $t > 1999$ with the three models. The age-dependent trend functions $F(x)$ for ages $0 \le x \le 89$ and the cohort-dependent trend functions $G(t+1-x)$ for cohorts $1879 \le t+1-x \le 1999$ are determined by calibrating mortality data from the past, $q_{x,t}$ for $1967 \le t \le 1999$ and $0 \le x \le 89$. The cohort-dependent trend function G is extrapolated to cohorts later than 1999 by setting $G(t+1-x) := \frac{1}{10} \cdot \sum_{t+1-x=1990}^{1999} G(t+1-x)$ for $t+1-x \ge 2000$.

The following figure shows the projected mortality rate for males aged 89 as a percentage of the projected mortality rate for females aged 89.

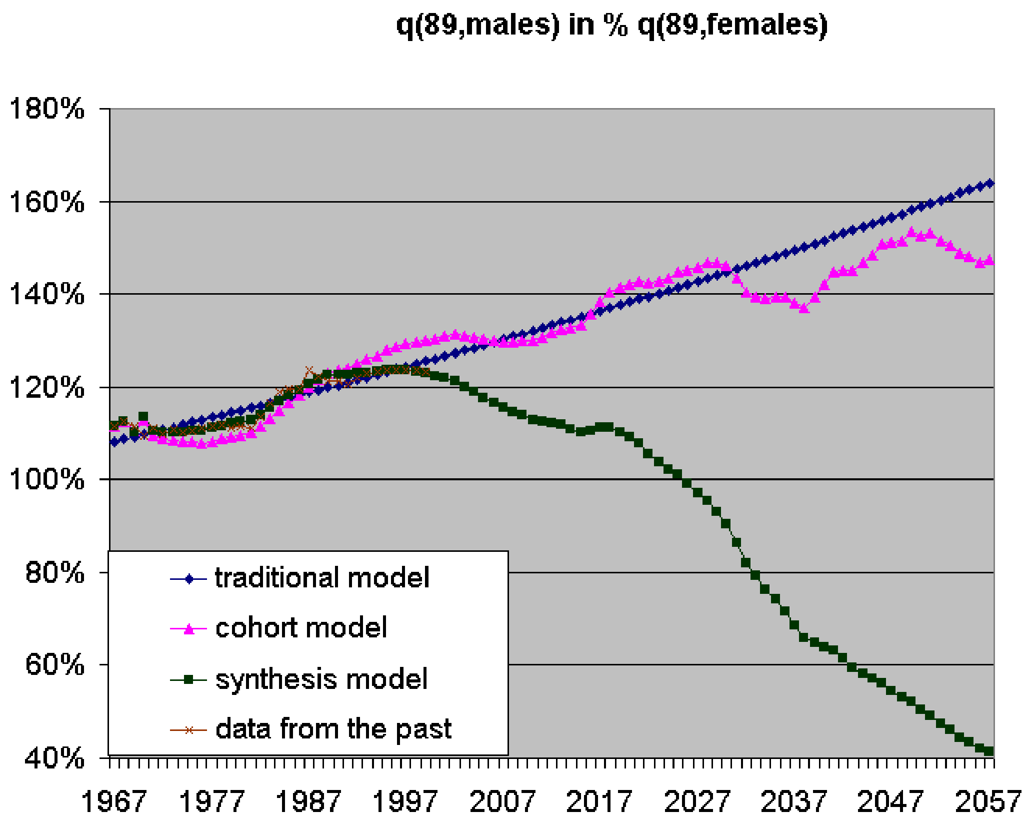


Figure 4. Males/females comparison

The synthesis model projects the mortality rate of males aged 89 to be less than 50 percent of the mortality rate of females aged 89 in years subsequent to 2050. This seems implausible for the following reason. Mortality improvement consists of an age-dependent component and a cohort-dependent component, according to the synthesis model. Fitting data from the past leads to trend functions with the following characteristics for males:

- Age-dependent mortality improvement is greater for older ages (for example, it is greater for ages from 70 to 89 years than for ages from 25 to 45 years).
- Cohort-dependent mortality improvement is greater for later cohorts (for example, it is greater for cohorts from 1970 to 1990 than for cohorts from 1925 to 1945).

Projecting mortality with these trend functions means that the high mortality improvement of old ages interacts with the high mortality improvement of late cohorts for males. The implausible effects thus result from the fact that, with regard to this synthesis model, in the future the two mortality improvement components (age-dependent and cohort-dependent) will interact for different age and birth year combinations than has been the case in the past. In consequence of this, in principle, the synthesis model seems unsuitable for mortality projections.

For the purpose of choosing between the traditional and cohort models for projections, the following should be taken into account. For the cohort model, the determination of the trend function $G(t+1-x)$ is increasingly uncertain for cohorts from 1970 onward due to the fact that the calibration is based on a decreasing number of observations (for cohorts from 2000 onward, it is even based wholly on extrapolations). However, cohorts from 1970 onward play an important role in the application of the new table DAV 2004 R. Therefore, the traditional model is chosen for projecting mortality.

3.2 2nd Order Mortality Improvement Trend

Mortality improvement trends for the population are examined in Section 3.2.1 for different periods. The reasons for assuming a reducing trend over time are stated in Section 3.2.2, which also includes a description of the linear trend reduction method. The 2nd order DAV 2004 R mortality improvement trend is defined in Section 3.2.3.

3.2.1 Mortality Improvement Trends for the Population

In order to study changes in population mortality improvement over the last decades, the following crude mortality improvement trends are considered:

- Short-term trend of 10 abbreviated population mortality tables for West Germany from St 1989/91 to St 1998/2000,
- Medium-term trend of 28 abbreviated population mortality tables for West Germany from St 1971/73 to St 1998/2000 (for 1986/88 the general population mortality table 1986/88 is used) and
- Long-term trend of 11 general population mortality tables from ADSt 1871/1880 to ADSt 1986/88 and the abbreviated population mortality table for West Germany St 1998/2000.

In December 1969 and January 1970, German population mortality was increased by an influenza epidemic. This could result in an incorrect assessment of the mortality improvement trend. Therefore, the medium-term trend is based on mortality tables from St 1971/73 onward.

Age-dependent mortality improvements are calculated according to the traditional model. The following two figures show the resulting annual mortality improvements.

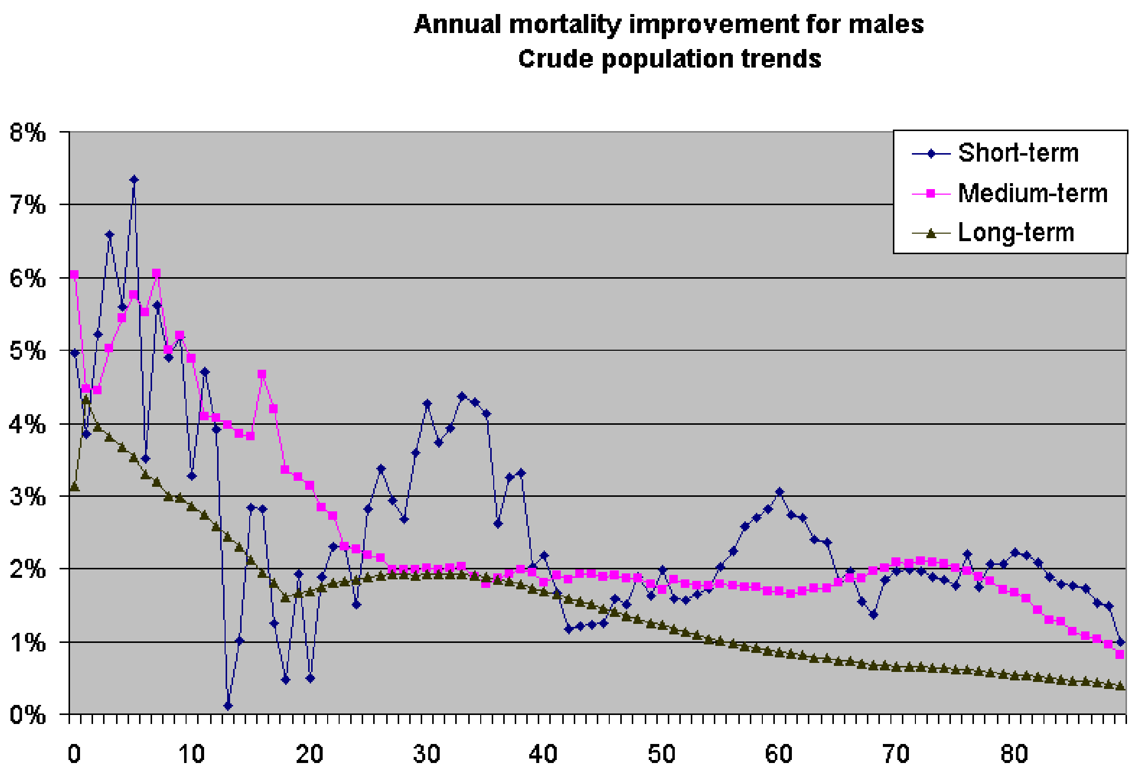


Figure 5. Annual mortality improvement of males

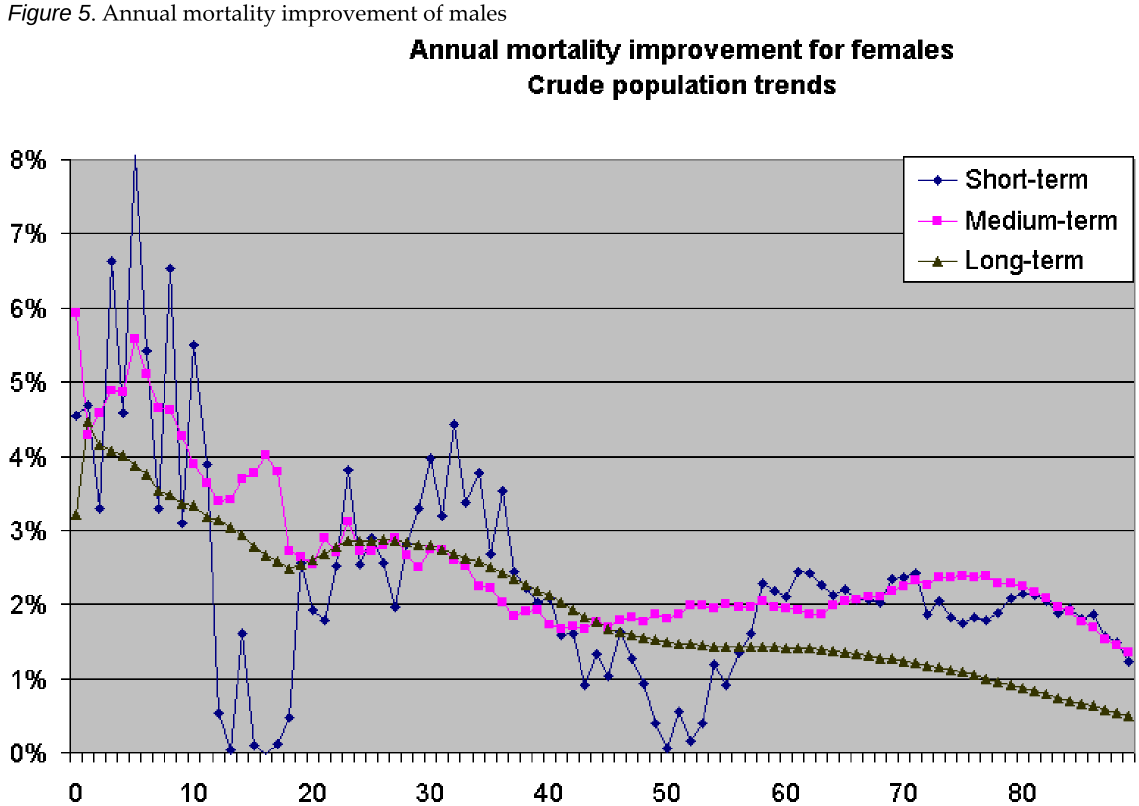


Figure 6. Annual mortality improvement of females

The annual mortality improvements are smoother for longer periods than for shorter periods.

In order to compare these trends, the arithmetic mean of annual mortality improvement for ages from 60 to 89 is considered.

	Males	**Females**
Short-term trend	1.97%	2.00%
Medium-term trend	1.67%	2.05%
Long-term trend	0.62%	1.04%

The long-term trend is significantly lower than the medium- and short-term trends. For males, the short-term trend is higher than the medium-term trend.

3.2.2 Linear Trend Reduction

In the previous section, it was noted that the long-term trend is significantly lower than the medium- and short-term trends. In Japan, where the mortality level is lower than in Germany, a reduction in the mortality improvement trend has been observed since 1970 (see Section 4.2). Given these findings, it seems inappropriate to use the high short-term trend for projecting mortality to the long-term future. Instead, a reduction in the mortality trend over time is used for projecting mortality. This is modeled by the linear trend reduction method, which was also used for the Austrian annuity table AVÖ 1996R (see [JLPS]).

The trend function depends on age and calendar year and is denoted by $F(x,t)$. The connection between trend function and mortality is given by

$$\frac{q_{x,t+1}}{q_{x,t}} = \exp\left(-F(x,t)\right).$$

The so-called "initial trend" $F_1(x)$ is used for the first years of the mortality projection. This initial trend is reduced linearly to the target trend $F_2(x)$ in a transition period. The target trend $F_2(x)$ is used after the transition period. This model is illustrated in the following figure, where the time $t = 1999$ corresponds to the start of the mortality projection.

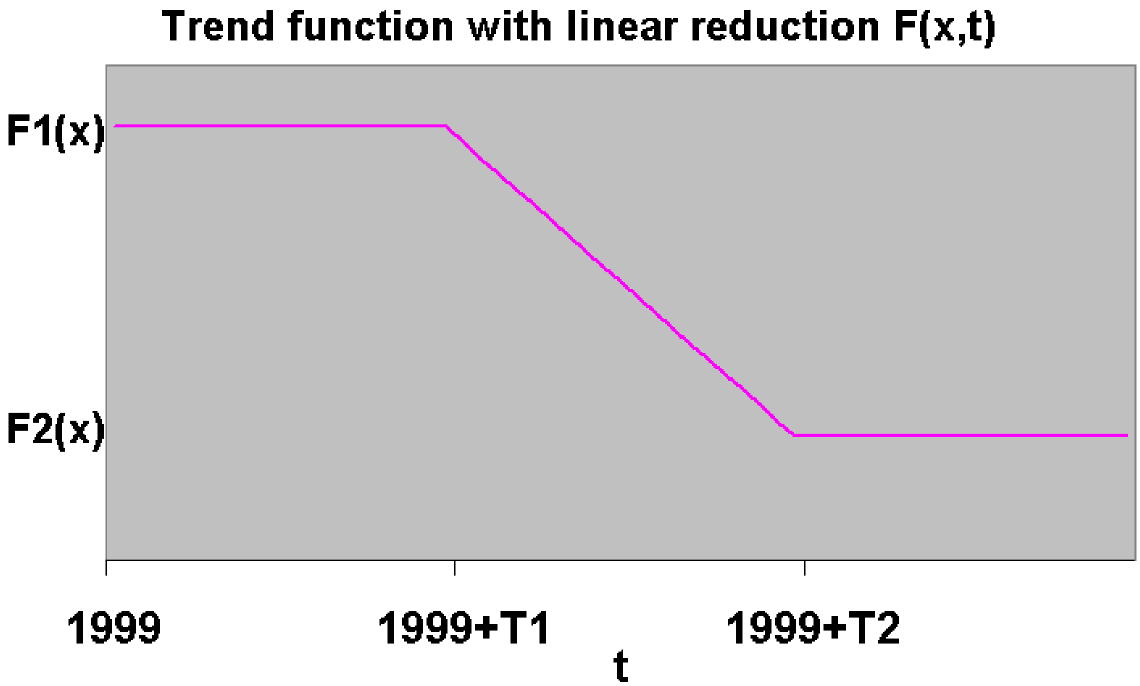


Figure 7. Linear trend reduction

The trend function $F(x,t)$ may be expressed by the formula

$$F(x,t)=\begin{cases} F_1(x) & 1999 \le t \le 1999+T_1 \\ F_1(x)\cdot\left(1-\dfrac{t-1999-T_1}{T_2-T_1}\right)+F_2(x)\cdot\dfrac{t-1999-T_1}{T_2-T_1} & 1999+T_1 \le t \le 1999+T_2 \\ F_2(x) & t \ge 1999+T_2 \end{cases}.$$

The following parameters need determining:

- Initial trend $F_1(x)$,
- Target trend $F_2(x)$,
- Period T_1 up to the commencement of the transition period and
- Period T_2 up to the end of the transition period.

3.2.3 Mortality Improvement Trend for Insured Persons

The initial trend is based on the level of the short-term trend. Due to the fact that it is smoother, the medium-term trend is used for defining the short-term trend. The crude medium-term trend function $F(x)$ is graduated using the Whittaker-Henderson method with weight 0.25 on the smoothness measured by second differences. The annual mortality improvement of the graduated medium-term trend for males is increased by 0.3 percent, which is the medium difference between short- and medium-term trends for ages from 60 to 89 years. For females, the graduated medium-term trend is used because this trend has approximately the same level as the short-term trend.

The loading for insured persons of 0.2 percent annual mortality improvements (see Section 3.2.3.1) is added to the initial trend. Finally, the trend is extrapolated for high ages to a level of 1 percent annual mortality improvement (see Section 3.2.3.2) and limited for low ages to a level of 3 percent annual mortality improvement (see Section 3.2.3.3). This defines the initial trend $F_1(x)$ for insured persons.

The target trend $F_2(x)$ for insured persons is defined as follows. The annual mortality improvement of the target trend $F_2(x)$ is 75 percent of the annual mortality improvement of the graduated (and for high ages, extrapolated, and for low ages, limited) medium-term trend, which was increased by the loading for insured persons but not by the medium difference between short- and medium-term trends for males.

The time parameters T_1 and T_2 need to be chosen appropriately, depending on the purpose for which the 2nd order mortality is used. Parameter combinations such as ($T_1 = 5$, $T_2 = 10$), ($T_1 = 10$, $T_2 = 15$) or ($T_1 = 15$, $T_2 = 25$) may be appropriate.

3.2.3.1 Loading for Insured Persons

The loading for insured persons is an adjustment for differences between population mortality improvement and mortality improvement of insured persons.

Various international studies (see, for example, [V]) and the results based on the Munich and Gen Re data and the German social insurance data have shown that the mortality improvement of insured persons is greater than mortality improvement of the population and the mortality improvement of upper socioeconomic groups is greater than mortality improvement of lower socioeconomic groups. Given that private annuities are mainly (especially if weighted by annuity amount) purchased by people belonging to upper socioeconomic groups, the second finding confirms that the mortality improvement of annuitants is greater than the mortality improvement of the population.

This difference is taken into consideration in the annuity tables of some countries (for example, the United Kingdom and Switzerland) by determining the mortality improvement trend based on insured persons' data rather than population data. In some cases, the difference between annuitant and population mortality improvement is fairly large. In Switzerland, for example, the annual mortality improvement of males aged 70 years is 1.33 percent for the population and 2.41 percent for annuitants (see [SVV]).

In Germany, there is not enough data on annuitant mortality to determine a mortality improvement trend based on the data of insured persons. Therefore, the loading for insured persons is derived from German social insurance data. The average annual mortality improvement for ages from 66 to 98 years is calculated for the total German social insurance collective ("total") and the subcollective encompassing white-collar workers ("white-collar").

German Social Insurance Ages 65 -98	Males			Females		
	Total (1)	White-collar (2)	(2) - (1)	Total (1)	White-collar (2)	(2) - (1)
1986 -2002	1.53%	1.76%	0.23%	1.58%	1.72%	0.14%

The mortality improvement of white-collar workers is 0.14 percent to 0.23 percent higher than for the total collective. Therefore, the loading for insured persons is defined as an increase in annual mortality improvement $1-\exp(-F(x,t))$ of 0.2 percent.

3.2.3.2 Trend Extrapolation for High Ages 90 to 120

The mortality improvement trend can only be determined directly from population data up to age 89. For ages 90 to 120, the trend needs extrapolating. For the purpose of obtaining an idea of a reasonable trend level for high ages, the trends for some other data were examined.

Graduating the mortality improvement trend based on German social insurance data for ages 66 to 98 results in an annual mortality improvement of approximately 1 percent at age 95 for both males and females. Based on data from Japan (see [RSJ]) for ages 100 to 104, annual mortality improvements of 0.82 percent (males) and 1.25 percent (females) respectively were calculated.

Therefore, annual mortality improvements of 1 percent for ages from 100 upward seem plausible. Annual mortality improvement is extrapolated for ages 90 to 120 with a polynomial of degree 2 in age band $90 \le x \le x_0$ and with 1 percent in age band $x_0 \le x \le 120$. The polynomial $p(x) = a \cdot x^2 + b \cdot x + c$ and the age x_0 are determined by the following conditions:

- $p(\mathrm{x}_0) = 1\%$,
- $p'(\mathrm{x}_0) = 0$,
- $p(89)$ and $p'(89)$ are determined by linear regression of annual mortality improvements for ages $80 \le x \le 89$.

With this extrapolation method, the annual mortality improvement is 1 percent for males from age 97 upward and 1 percent for females from age 99 upward.

3.2.3.3 Trend Limitation for Low Ages

For ages 0 to 22, the annual mortality improvement of the initial trend exceeds 3 percent. However, the short-term trend for these ages is lower. The short-term average annual mortality improvement for ages 0 to 22 is 1.74 percent for males and 1.85 percent for females, if the average is calculated by weighting the ages according to the age distribution of the model portfolio of German annuity data described in Section 2.4.1.

Therefore, the annual mortality improvement of the initial trend is limited to 3 percent and the annual mortality improvement of the target trend is limited to 2.25 percent. These limitations affect ages from 0 to 22 for both females and males. It should be noted that the trend limitation for low ages has virtually no effect on the calculation of premiums and reserves.

3.3 1st Order Mortality Improvement Trend

The fundamental risk in determining a mortality improvement trend is the principal uncertainty of estimating future mortality improvement based on data from the past. The main risks in estimating future mortality improvement are the model and trend parameter risks.

3.3.1 Model Risk Margin

A linear trend reduction is assumed for the 2nd order mortality improvement (see Section 3.2.2). The model risk particularly consists of the risk that mortality improvement will not decline in the future. For the purpose of making allowances for the model risk, a safety margin, which is defined by omitting a trend reduction assumption, is incorporated into the trend. This means that the initial trend defined in Section 3.2.3 is used for the whole future. It corresponds to a target trend increase $F_2(x)$ of at least 34 percent (namely, 34 percent for females and between 34 percent and 72 percent for males). The model risk margin results in an increase in the reserves for the model portfolio of approximately 2 percent for the parameter combination ($T_1 = 10$, $T_2 = 15$) and approximately 3 percent for the parameter combination ($T_1 = 5$, $T_2 = 10$).

3.3.2 Trend Parameter Risk Margin

An allowance was made for the risk of an increase in the mortality improvement trend by means of a safety margin of an additional 0.25 percent annual mortality improvement for all ages. This margin was determined using stress scenario considerations similar to those used in the Swiss annuitant table ER 2000 (see [K] and [SVV]). The consequences of certain stress scenarios for the reserves in 2005 are examined. The stress scenarios are defined by a trend function increase (including the model risk margin) of 50 percent for a period of 10 years[1]. For the model portfolio, these stress scenarios result in an average increase in reserves of approximately 2 percent. The risk of change margin of an additional 0.25 percent mortality improvement also results in an approximate 2 percent increase in reserves.

3.3.3 Other Risks

There is no explicit additional safety margin for other risks (for example, from the estimation of the trend parameters). It is assumed that the safety margins for the model and trend parameter risks implicitly make allowance for other risks.

4. International Comparisons

4.1 International Tables with Trends

4.1.1 Swiss Table ER 2000

Both the base Swiss annuity table ER 2000 (see [SVV]) and the trend function $F(x)$ for ages $x \geq 50$ were derived from mortality data on Swiss individual annuities from the period 1961 to 1995. Its trend is based on the traditional model $\frac{q(x,t+1)}{q(x,t)} = e^{-F(x)}$. Both the 2nd and 1st order trends are kept constant in the table ER 2000.

4.1.2 U.K. Table IA 92 mc

The "92" series of U.K. tables was published in 1999 (see [CMI1]). Again, both the base table and the age-dependent trend function $F(x,t)$ for table IA 92 by amount for immediate annuities were derived from mortality data on insured persons from the period 1955 to 1994. The age-dependent trend reduces over time:

[1] This means $F(x,t) = F_1(x)$ is replaced by $F(x,t) = \eta \cdot F_1(x)$ for $\tau - 5 \leq t < \tau + 5$ with $\eta = 150\%$. The center of the 10-year period is varied from 2005 to 2054.

$$\frac{q_{x,t}}{q_{x,0}} = \alpha(x) + [1-\alpha(x)].[1-f(x)]^{\frac{t}{20}}$$

with

$$\alpha(x) = \begin{cases} c & x < 60 \\ 1+(1-c).\dfrac{(x-110)}{50} & 60 \leq x \leq 110 \\ 1 & x > 110 \end{cases}$$

and

$$f(x) = \begin{cases} h & x < 60 \\ \dfrac{(110-x).h+(x-60).k}{50} & 60 \leq x \leq 110 \\ k & x > 110 \end{cases}$$

where $c = 0.13$, $h = 0.55$, and $k = 0.29$.

During the course of the last few decades, the mortality improvement trends of U.K. mortality tables have been increased several times. This is evident from the following chart from [I], p. 90, on projections of life expectancy for males aged 60 according to U.K. tables 1955, 1968, 1980 and 1992.

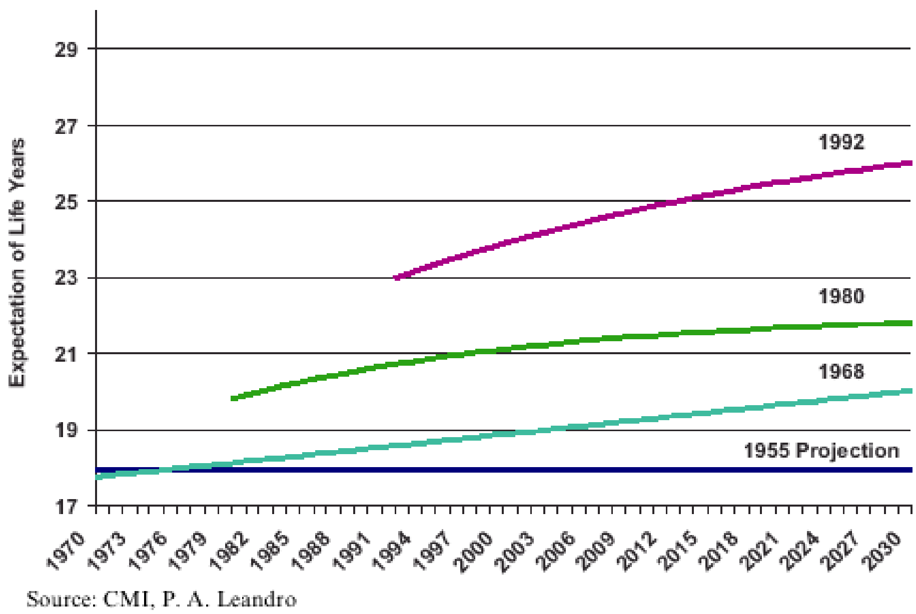


Figure R.2. Longer life, expectation of life for males aged 60

Figure 8. U.K. projections of life expectancy

In 2002, cohort-dependent mortality improvements were superimposed onto the IA 92 age-dependent mortality improvements (see [CMI2]). These cohort-dependent mortality improvements are used for years up to 2010, 2020 or 2040 (short, medium and long variants). The variant medium cohort (IA 92 mc) is used for comparisons in the following sections.

4.1.3 Austrian Table AVÖ 1996 R

The Austrian annuity table AVÖ 1996 R (see [JLPS]) is based on the traditional linear trend reduction model as described in Section 3.2.2. The trend function $F(x,t)$ of AVÖ 1996 R was derived from data on population mortality in Austria. The initial trend $F_1(x)$ is based on data from the period 1980 to 1995 and used for projections up to the year 2000. In the transitional period from 2000 to 2010, the trend is reduced to the target trend $F_2(x)$ on a linear basis. Data from the Austrian social insurance was used for the base table in AVÖ 1996 R.

4.2 International Observations on Trend Development

Based on population mortality data from the Berkeley Mortality Database, mortality improvements for several countries have been calculated for certain 20-year periods. The following table contains the average mortality improvement for ages 60 to 89.

Rolling Development of Mortality Improvement Over 20-year Periods

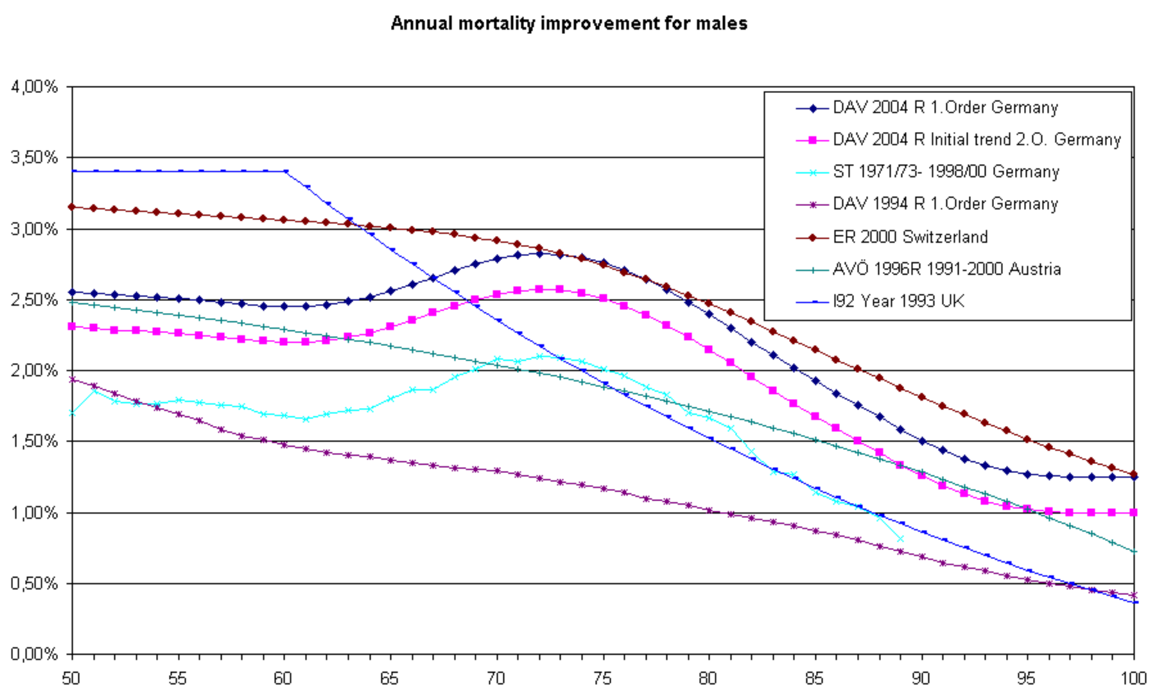
Average for ages 60-89	1960-1979	1965-1984	1970-1989	1975-1994	1980-1999
Males					
West Germany	0.0039	0.0130	0.0157	0.0167	0.0180
Denmark	0.0028	0.0030	0.0025	0.0039	0.0077
Japan	0.0228	0.0261	0.0254	0.0192	0.0153
France (1978-1997)	0.0089	0.0127	0.0156	0.0197	0.0198
UK (1979-1998)	0.0057	0.0087	0.0126	0.0167	0.0189
Italy	0.0035	0.0064	0.0113	0.0179	0.0203
Austria	0.0051	0.0107	0.0159	0.0193	0.0213
Sweden	0.0030	0.0048	0.0080	0.0134	0.0163
Switzerland	0.0118	0.0138	0.0146	0.0154	0.0182
USA	0.0075	0.0131	0.0136	0.0128	0.0132
Females					
West Germany	0.0132	0.0209	0.0222	0.0205	0.0198
Denmark	0.0177	0.0141	0.0070	0.0016	0.0012
Japan	0.0275	0.0320	0.0349	0.0326	0.0298
France (1978-1997)	0.0176	0.0205	0.0231	0.0248	0.0239
UK (1979-1998)	0.0094	0.0098	0.0117	0.0137	0.0141
Italy	0.0148	0.0171	0.0202	0.0240	0.0250
Austria	0.0099	0.0153	0.0198	0.0228	0.0247
Sweden	0.0186	0.0179	0.0158	0.0153	0.0146
Switzerland	0.0231	0.0254	0.0246	0.0210	0.0196
USA	0.0154	0.0175	0.0138	0.0089	0.0061

Data source: Berkeley Mortality Database - http://www.mortality.org/

The development of mortality improvement differs from country to country. However, a trend reduction can be observed in countries with high life expectancies such as Japan and Switzerland. For females, the rolling 20-year mortality improvement for ages 60 to 89 decreased in Switzerland from a maximum of 2.54 percent for the period 1965-84 to 1.96 percent for the period 1980-99. In Japan, it decreased from a maximum of 3.49 percent for the period 1970-89 to 2.98 percent for the period 1980-99. For males, the mortality improvement in Japan decreased from 2.61 percent for the period 1965-75 to 1.53 percent for the period 1980-99.

4.3 Comparison of Trends

The following four figures show the DAV 2004 R trends compared to the trends of the Swiss, U.K. and Austrian tables. The figures also include the 1st order trend of the previous German mortality table for annuity business and the medium-term trend of 28 German population mortality tables used to derive the DAV 2004 R trend ("ST 1971/73-1998/00 Germany"). The upper figures show annual mortality improvements $1-\exp(-F(x,t))$ at the beginning of the projections. In particular, $1-\exp(-F(x,t))$ refers to the year $t=1993$ for I 92 and to the years $1991 \le t \le 2000$ for AVÖ 1996 R. In the lower figures, $1-\exp(-F(x,t))$ refers to year $t=2030$ for I 92 and to the years $t \ge 2010$ for AVÖ 1996 R.



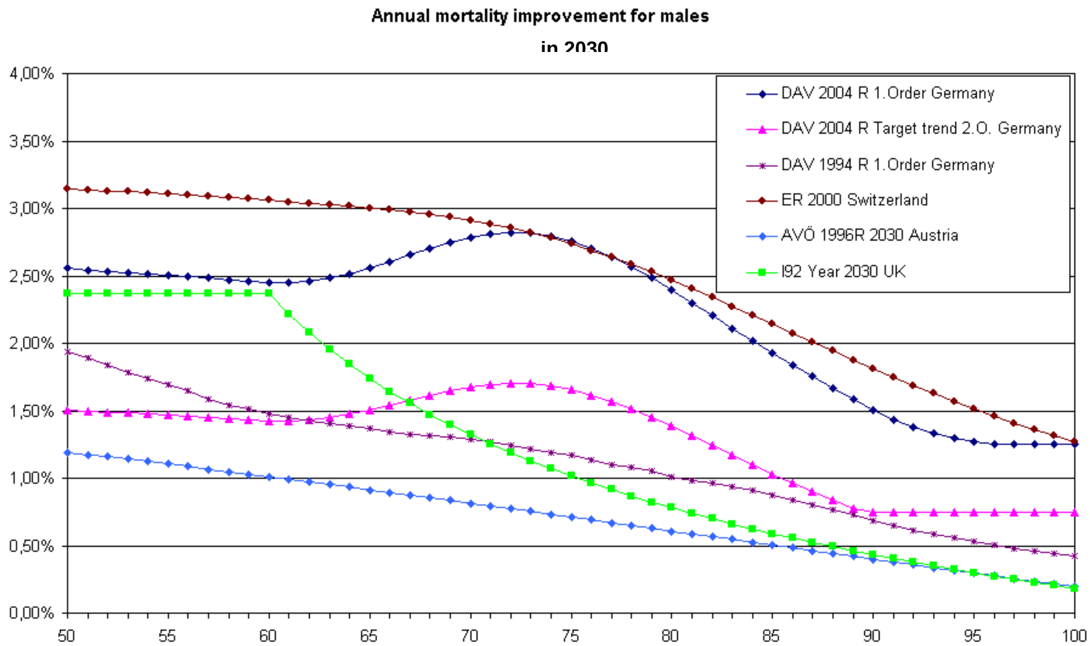


Figure 9. Annual mortality improvement, males

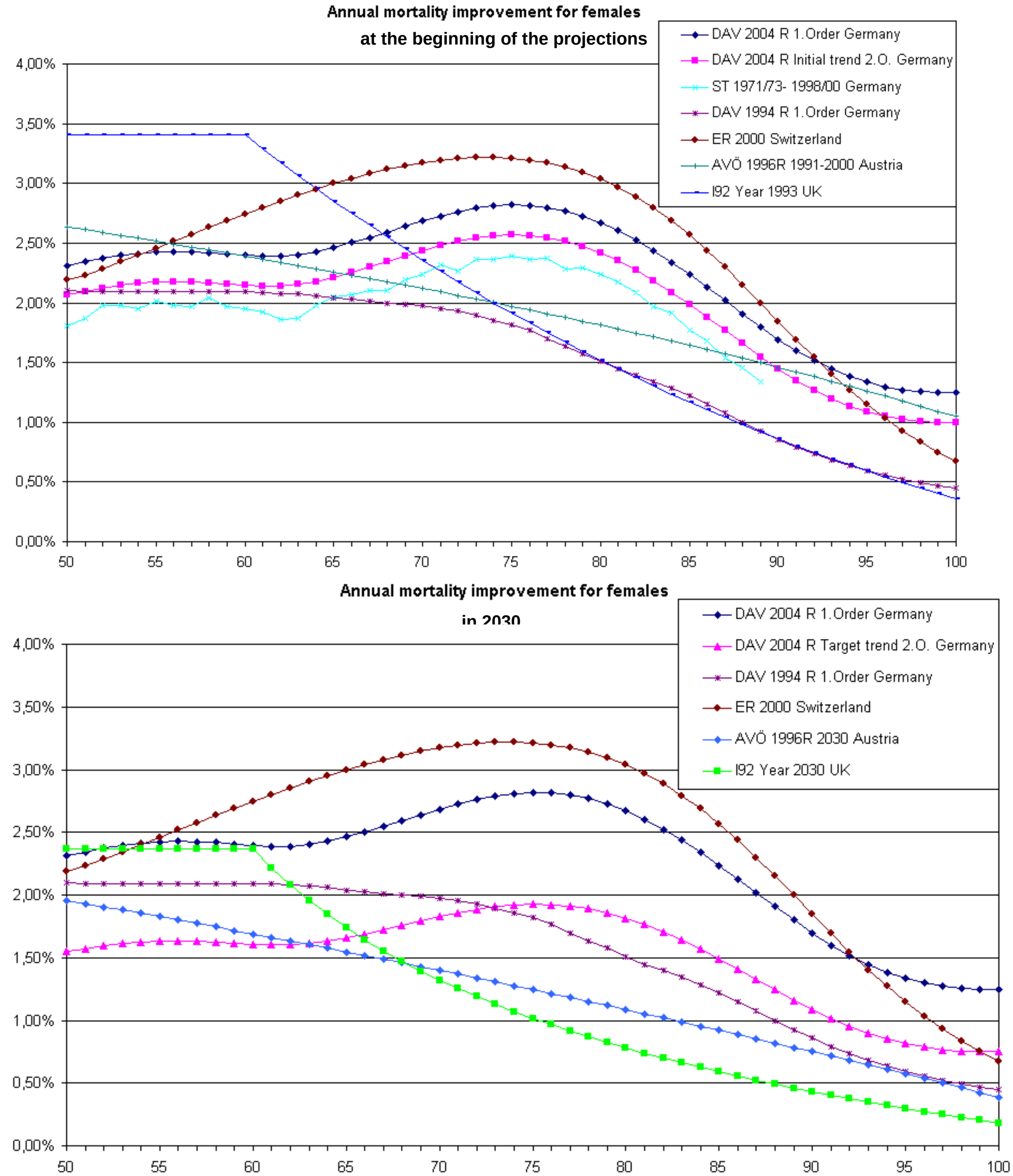


Figure 10. Annual mortality improvement, females

Annual Mortality Improvements of Various Trend Functions (in %, males)

	DAV 2004 R 1st Order Germany	DAV 2004 R Initial Trend 2nd Order Germany	DAV 2004 R Target Trend 2nd Order Germany	ST 1971/73- 1998/00 Germany	DAV 1994 R 1st Order Germany	ER 2000 Switzerland	AVÖ 1996R 2030 Austria	I92 Year 2030 UK
20	3.25%	3.00%	2.25%	3.14%	2.45%	2.19%	1.59%	2.37%
25	2.96%	2.71%	1.81%	2.18%	2.45%	2.19%	1.54%	2.37%
30	2.71%	2.46%	1.62%	2.00%	2.45%	2.19%	1.49%	2.37%
35	2.66%	2.41%	1.58%	1.79%	2.45%	2.19%	1.42%	2.37%
40	2.65%	2.40%	1.57%	1.80%	2.41%	2.19%	1.35%	2.37%
45	2.63%	2.38%	1.56%	1.88%	2.24%	2.85%	1.28%	2.37%
50	2.55%	2.30%	1.50%	1.70%	1.94%	3.15%	1.19%	2.37%
55	2.51%	2.26%	1.47%	1.79%	1.69%	3.11%	1.10%	2.37%
60	2.45%	2.20%	1.42%	1.69%	1.48%	3.06%	1.01%	2.37%
65	2.56%	2.31%	1.51%	1.80%	1.37%	3.00%	0.91%	1.74%
70	2.79%	2.54%	1.68%	2.08%	1.29%	2.91%	0.81%	1.32%
75	2.76%	2.51%	1.65%	2.01%	1.17%	2.74%	0.71%	1.02%
80	2.39%	2.14%	1.38%	1.66%	1.01%	2.47%	0.61%	0.78%
85	1.92%	1.67%	1.03%	1.14%	0.87%	2.14%	0.50%	0.59%
90	1.50%	1.25%	0.75%		0.68%	1.81%	0.40%	0.43%
95	1.27%	1.02%	0.75%		0.53%	1.52%	0.30%	0.30%
100	1.25%	1.00%	0.75%		0.42%	1.27%	0.19%	0.18%
Arithmetic mean								
60-	2.42%	2.17%	1.40%	1.67%	1.14%	2.64%	0.72%	1.16%

Annual mortality improvements of various trend functions (in %, females)

	DAV 2004 R 1st Order Germany	DAV 2004 R Initial Trend 2nd Order Germany	DAV 2004 R Target Trend 2nd Order Germany	ST 1971/73- 1998/00 Germany	DAV 1994 R 1st Order Germany	ER 2000 Switzerland	AVÖ 1996R 2030 Austria	I92 Year 2030 UK
20	3.25%	3.00%	2.25%	2.54%	3.20%	2.19%	2.64%	2.37%
25	3.23%	2.98%	2.23%	2.73%	3.20%	2.19%	2.54%	2.37%
30	3.10%	2.85%	2.14%	2.75%	3.19%	2.19%	2.43%	2.37%
35	2.65%	2.40%	1.80%	2.22%	3.05%	2.19%	2.32%	2.37%
40	2.22%	1.97%	1.48%	1.73%	2.74%	2.19%	2.21%	2.37%
45	2.18%	1.93%	1.45%	1.69%	2.28%	2.19%	2.08%	2.37%
50	2.31%	2.06%	1.55%	1.81%	2.10%	2.19%	1.96%	2.37%
55	2.42%	2.17%	1.63%	2.01%	2.09%	2.46%	1.83%	2.37%
60	2.40%	2.15%	1.61%	1.95%	2.09%	2.75%	1.69%	2.37%
65	2.46%	2.21%	1.66%	2.05%	2.04%	3.00%	1.55%	1.74%
70	2.69%	2.44%	1.83%	2.24%	1.97%	3.17%	1.40%	1.32%
75	2.82%	2.57%	1.93%	2.39%	1.82%	3.21%	1.25%	1.02%
80	2.67%	2.42%	1.82%	2.24%	1.51%	3.04%	1.09%	0.78%
85	2.24%	1.99%	1.49%	1.77%	1.22%	2.57%	0.92%	0.59%
90	1.69%	1.44%	1.08%		0.86%	1.85%	0.75%	0.43%
95	1.34%	1.09%	0.81%		0.59%	1.15%	0.57%	0.30%
100	1.25%	1.00%	0.75%		0.45%	0.68%	0.39%	0.18%
Arithmetic mean								
60-	2.50%	2.25%	1.69%	2.05%	1.70%	2.91%	1.25%	1.16%

4.4 Comparison of Mortality Rates

The following two figures compare the mortality rates for a person aged 65 in 2005 projected by DAV 2004 R with the corresponding mortality rates projected by the previous German standard table DAV 1994 R, by the Swiss table ER 2000, by the Austrian table AVÖ 1996R and by the U.K. table IA 92 mc.

As can be seen, there are huge differences in the projected rates. The Swiss mortality rates are by far the most conservative. The kinks at ages 61 and 66 are due to the selection factors in DAV 2004 R.

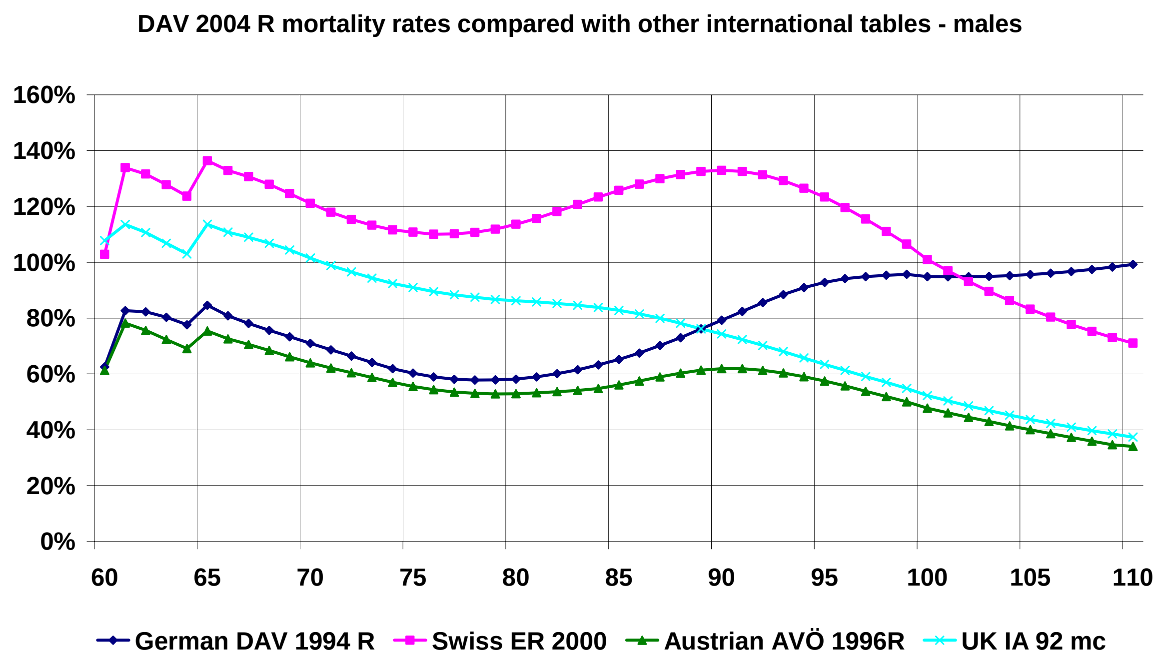


Figure 11. Comparison of DAV 2004 R mortality rates with international mortality rates for a male person aged 65 in 2005

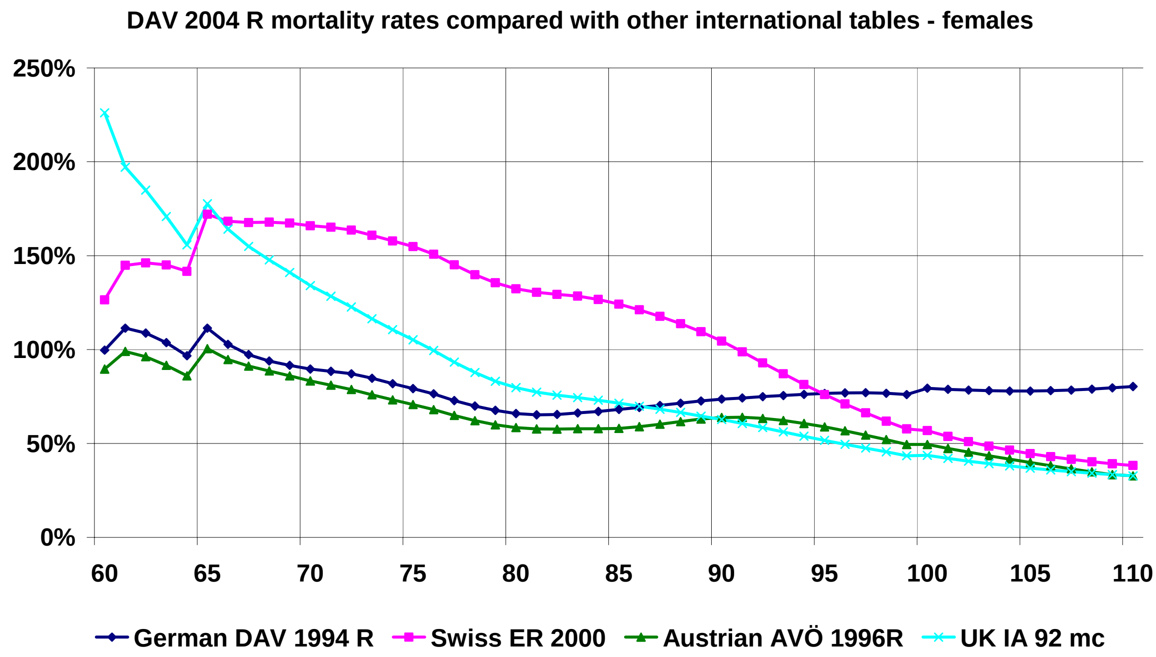


Figure 12. Comparison of DAV 2004 R mortality rates with international mortality rates for a female person aged 65 in 2005

4.5 Comparison of Net Premiums

The following tables contain comparisons of net single premiums[2] and net annual premiums[3] for the following mortality tables:

- German table DAV 2004 R,
- Swiss table ER 2000,
- U.K. table IA 92 mc, and
- Austrian table AVÖ 1996R.

The purpose is not a price comparison, but a compressed comparison of the different mortality tables.

Net premiums for males are lower for DAV 2004 R than for the Swiss mortality table. For females, net premiums for DAV 2004 R and the Swiss mortality table are similar. Net premiums for DAV 2004 R are higher than for the U.K. and Austrian mortality tables.

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[2] Net single premiums for immediate annuities are given by $\ddot{a}_x$. Net single premiums for deferred annuities are given by ${}_{n|}\ddot{a}_x$.

[3] Net annual premiums for deferred annuities are given by $\dfrac{{}_{n|}\ddot{a}_x}{\ddot{a}_{x,n}}$.

Begin in 2005

Interest rate 2.75%

Net premium in € for annual annuity payment of 1 € in advance

Males

		Absolute				Relative		
		1	2	3	4	2 in % of 1	3 in % of 1	4 in % of 1
		Germany	Switzerland	UK	Austria	Switzerland	UK	Austria
		DAV 2004 R	ER 2000	IA 92 mc	AVÖ 1996R			
Immediate annuity								
Age at issue 60								
Birth year 1945	Net single premium	19.608	20.397	19.015	17.383	104.0%	97.0%	88.7%
Deferred annuity								
Payout phase starting at age 60								
Deferment period 20 years								
Birth year 1965								
Age at issue 40								
	Net single premium	11.982	12.540	11.177	9.835	104.7%	93.3%	82.1%
	Net annual premium	0.776	0.810	0.720	0.641	104.4%	92.8%	82.6%
Deferment period 30 years								
Birth year 1975								
Age at issue 30								
	Net single premium	9.488	9.835	8.634	7.601	103.7%	91.0%	80.1%
	Net annual premium	0.461	0.479	0.418	0.371	104.0%	90.6%	80.6%
Payout phase starting at age 65								
Deferment period 20 years								
Birth year 1960								
Age at issue 45								
	Net single premium	10.522	11.074	9.655	8.282	105.3%	91.8%	78.7%
	Net annual premium	0.685	0.717	0.625	0.545	104.7%	91.3%	79.5%
Deferment period 30 years								
Birth year 1970								
Age at issue 35								
	Net single premium	8.394	8.787	7.492	6.404	104.7%	89.3%	76.3%
	Net annual premium	0.409	0.428	0.364	0.315	104.6%	88.8%	77.0%

Begin in 2005

Interest rate 2.75%

Net premium in € for annual annuity payment of 1 € in advance

Females

		Absolute				Relative		
		1	**2**	**3**	**4**	**2 in % of 1**	**3 in % of 1**	**4 in % of 1**
		Germany	**Switzerland**	**UK**	**Austria**	**Switzerland**	**UK**	**Austria**
		DAV 2004 R	ER 2000	IA 92 mc	AVÖ 1996R			
Immediate annuity								
Age at issue 60								
Birth year 1945	Net single premium	21.409	21.761	20.615	19.836	101.6%	96.3%	92.7%
Deferred annuity								
Payout phase starting at age 60								
Deferment period 20 years								
Birth year 1965								
Age at issue 40								
	Net single premium	13.048	13.059	12.130	11.652	100.1%	93.0%	89.3%
	Net annual premium	0.840	0.841	0.777	0.751	100.1%	92.5%	89.4%
Deferment period 30 years								
Birth year 1975								
Age at issue 30								
	Net single premium	10.252	10.156	9.300	9.037	99.1%	90.7%	88.2%
	Net annual premium	0.496	0.491	0.448	0.437	99.1%	90.4%	88.2%
Payout phase starting at age 65								
Deferment period 20 years								
Birth year 1960								
Age at issue 45								
	Net single premium	11.691	11.726	10.719	10.188	100.3%	91.7%	87.1%
	Net annual premium	0.756	0.757	0.688	0.660	100.2%	91.0%	87.3%
Deferment period 30 years								
Birth year 1970								
Age at issue 35								
	Net single premium	9.242	9.162	8.241	7.923	99.1%	89.2%	85.7%
	Net annual premium	0.448	0.444	0.397	0.385	99.1%	88.7%	85.8%

Acknowledgements

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Appendix Table Values of DAV 2004 R

Part A Selection Factors and Base Tables

Selection Factors	Males	Females
f^1	0,670538	0,712823
f^{2-5}	0,876209	0,798230

	Base Table 1st Order				Base Table 2nd Order			
	Aggregate Table		Selection Table Benefit Payment Period		Aggregate Table		Selection Table Benefit Payment Period	
	Males	Females	Males	Females	Males	Females	Males	Females
Age	q_x	q_y	q_x^6	q_y^6	q_x	q_y	q_x^6	q_y^6
0	0,003439	0,002694	0,002546	0,002549	0,004076	0,003226	0,003018	0,003053
1	0,000317	0,000280	0,000234	0,000265	0,000375	0,000335	0,000278	0,000317
2	0,000214	0,000160	0,000158	0,000152	0,000253	0,000192	0,000187	0,000182
3	0,000158	0,000124	0,000117	0,000117	0,000187	0,000148	0,000139	0,000140
4	0,000122	0,000101	0,000090	0,000095	0,000145	0,000120	0,000107	0,000114
5	0,000108	0,000078	0,000080	0,000074	0,000128	0,000094	0,000095	0,000089
6	0,000102	0,000081	0,000076	0,000076	0,000121	0,000097	0,000090	0,000091
7	0,000087	0,000080	0,000065	0,000076	0,000104	0,000096	0,000077	0,000091
8	0,000099	0,000069	0,000073	0,000065	0,000117	0,000082	0,000087	0,000078
9	0,000084	0,000068	0,000062	0,000065	0,000100	0,000082	0,000074	0,000077
10	0,000083	0,000066	0,000061	0,000062	0,000098	0,000079	0,000073	0,000075
11	0,000098	0,000071	0,000073	0,000067	0,000117	0,000085	0,000086	0,000081
12	0,000104	0,000075	0,000077	0,000071	0,000123	0,000090	0,000091	0,000085
13	0,000114	0,000079	0,000084	0,000075	0,000135	0,000094	0,000100	0,000089
14	0,000140	0,000092	0,000103	0,000087	0,000165	0,000110	0,000122	0,000104
15	0,000192	0,000120	0,000142	0,000114	0,000228	0,000144	0,000169	0,000136
16	0,000276	0,000144	0,000205	0,000137	0,000328	0,000173	0,000243	0,000164
17	0,000364	0,000166	0,000270	0,000157	0,000432	0,000199	0,000320	0,000188
18	0,000596	0,000235	0,000442	0,000223	0,000707	0,000282	0,000523	0,000267
19	0,000630	0,000238	0,000467	0,000225	0,000747	0,000285	0,000553	0,000269
20	0,000627	0,000230	0,000464	0,000218	0,000743	0,000275	0,000550	0,000260
21	0,000636	0,000211	0,000471	0,000199	0,000754	0,000252	0,000558	0,000239
22	0,000625	0,000215	0,000463	0,000203	0,000741	0,000257	0,000548	0,000243
23	0,000642	0,000201	0,000475	0,000190	0,000761	0,000240	0,000563	0,000227
24	0,000622	0,000222	0,000460	0,000210	0,000737	0,000266	0,000546	0,000251
25	0,000617	0,000225	0,000457	0,000213	0,000731	0,000270	0,000541	0,000255
26	0,000616	0,000225	0,000456	0,000213	0,000730	0,000270	0,000540	0,000255
27	0,000627	0,000235	0,000471	0,000222	0,000743	0,000281	0,000558	0,000266
28	0,000613	0,000258	0,000485	0,000244	0,000726	0,000309	0,000574	0,000293
29	0,000603	0,000280	0,000502	0,000265	0,000715	0,000335	0,000595	0,000317
30	0,000598	0,000291	0,000515	0,000275	0,000709	0,000348	0,000610	0,000329
31	0,000605	0,000302	0,000546	0,000292	0,000717	0,000361	0,000647	0,000350
32	0,000626	0,000318	0,000568	0,000329	0,000742	0,000381	0,000674	0,000394
33	0,000663	0,000344	0,000601	0,000357	0,000786	0,000413	0,000712	0,000427
34	0,000713	0,000385	0,000653	0,000401	0,000845	0,000461	0,000774	0,000480
35	0,000775	0,000434	0,000697	0,000445	0,000918	0,000519	0,000826	0,000533

	Base Table 1st Order				Base Table 2nd Order			
	Aggregate Table		Selection Table Benefit Payment Period		Aggregate Table		Selection Table Benefit Payment Period	
	Males	Females	Males	Females	Males	Females	Males	Females
Age	q_x	q_y	q_x^6	q_y^6	q_x	q_y	q_x^6	q_y^6
36	0,000850	0,000488	0,000751	0,000498	0,001008	0,000585	0,000890	0,000596
37	0,000944	0,000547	0,000821	0,000561	0,001119	0,000656	0,000973	0,000671
38	0,001047	0,000605	0,000878	0,000606	0,001242	0,000725	0,001041	0,000725
39	0,001153	0,000666	0,000968	0,000656	0,001367	0,000798	0,001148	0,000785
40	0,001261	0,000735	0,001083	0,000743	0,001495	0,000881	0,001284	0,000890
41	0,001372	0,000809	0,001169	0,000823	0,001626	0,000968	0,001386	0,000986
42	0,001483	0,000885	0,001288	0,000903	0,001758	0,001059	0,001527	0,001082
43	0,001603	0,000959	0,001403	0,001000	0,001900	0,001149	0,001663	0,001198
44	0,001732	0,001033	0,001532	0,001079	0,002053	0,001237	0,001815	0,001292
45	0,001871	0,001113	0,001719	0,001200	0,002217	0,001332	0,002038	0,001437
46	0,002025	0,001203	0,001872	0,001291	0,002400	0,001440	0,002219	0,001546
47	0,002194	0,001301	0,002074	0,001395	0,002601	0,001558	0,002458	0,001671
48	0,002373	0,001406	0,002268	0,001553	0,002813	0,001683	0,002688	0,001860
49	0,002563	0,001512	0,002526	0,001650	0,003038	0,001811	0,002994	0,001976
50	0,002762	0,001616	0,002838	0,001823	0,003274	0,001935	0,003364	0,002183
51	0,002981	0,001720	0,003029	0,001965	0,003534	0,002060	0,003591	0,002353
52	0,003212	0,001822	0,003358	0,002089	0,003807	0,002182	0,003980	0,002502
53	0,003449	0,001931	0,003684	0,002262	0,004088	0,002312	0,004366	0,002709
54	0,003684	0,002052	0,004054	0,002470	0,004367	0,002458	0,004805	0,002957
55	0,003911	0,002186	0,004419	0,002623	0,004636	0,002618	0,005238	0,003141
56	0,004134	0,002340	0,004872	0,002877	0,004901	0,002803	0,005775	0,003445
57	0,004370	0,002516	0,005388	0,003106	0,005179	0,003013	0,006387	0,003720
58	0,004627	0,002706	0,005888	0,003391	0,005485	0,003240	0,006980	0,004061
59	0,004932	0,002914	0,006541	0,003731	0,005846	0,003490	0,007753	0,004468
60	0,005299	0,003145	0,007226	0,004121	0,006281	0,003766	0,008565	0,004935
61	0,005777	0,003402	0,007922	0,004492	0,006848	0,004074	0,009390	0,005379
62	0,006383	0,003692	0,008590	0,004862	0,007566	0,004421	0,010182	0,005822
63	0,007119	0,004021	0,009229	0,005195	0,008438	0,004815	0,010939	0,006221
64	0,007963	0,004384	0,009933	0,005504	0,009439	0,005250	0,011774	0,006591
65	0,008886	0,004830	0,010714	0,005827	0,010533	0,005783	0,012699	0,006977
66	0,009938	0,005278	0,011662	0,006266	0,011779	0,006321	0,013823	0,007504
67	0,011253	0,005905	0,012834	0,006904	0,013339	0,007071	0,015212	0,008268
68	0,012687	0,006674	0,014099	0,007701	0,015038	0,007992	0,016712	0,009222
69	0,014231	0,007548	0,015456	0,008612	0,016869	0,009039	0,018321	0,010313
70	0,015887	0,008525	0,016920	0,009637	0,018832	0,010209	0,020056	0,011540
71	0,017663	0,009679	0,018547	0,010869	0,020937	0,011591	0,021984	0,013015
72	0,019598	0,010965	0,020408	0,012266	0,023230	0,013131	0,024190	0,014689
73	0,021698	0,012341	0,022511	0,013782	0,025719	0,014778	0,026683	0,016504
74	0,023990	0,013909	0,024873	0,015522	0,028436	0,016656	0,029483	0,018588
75	0,026610	0,015706	0,027614	0,017516	0,031542	0,018808	0,032731	0,020976
76	0,029533	0,017672	0,030689	0,019686	0,035006	0,021163	0,036376	0,023574
77	0,032873	0,019722	0,034200	0,021922	0,038965	0,023618	0,040539	0,026251
78	0,036696	0,022102	0,038203	0,024477	0,043496	0,026468	0,045283	0,029312
79	0,041106	0,024975	0,042787	0,027510	0,048724	0,029908	0,050717	0,032944
80	0,046239	0,028535	0,048081	0,031211	0,054808	0,034171	0,056992	0,037376
81	0,052094	0,032947	0,054068	0,035743	0,061748	0,039454	0,064088	0,042803
82	0,058742	0,038340	0,060821	0,041240	0,069628	0,045913	0,072092	0,049385
83	0,066209	0,044665	0,068363	0,047641	0,078479	0,053487	0,081033	0,057051

	Base Table 1st Order				Base Table 2nd Order			
	Aggregate Table		Selection Table Benefit Payment Period		Aggregate Table		Selection Table Benefit Payment Period	
	Males	Females	Males	Females	Males	Females	Males	Females
Age	q_x	q_y	q_x^6	q_y^6	q_x	q_y	q_x^6	q_y^6
84	0,074583	0,051737	0,076782	0,054741	0,088405	0,061956	0,091012	0,065553
85	0,083899	0,059541	0,086113	0,062514	0,099447	0,071302	0,102071	0,074862
86	0,094103	0,068187	0,096295	0,071076	0,111543	0,081656	0,114141	0,085115
87	0,105171	0,077684	0,107306	0,080444	0,124661	0,093028	0,127192	0,096333
88	0,116929	0,087911	0,118973	0,090508	0,138599	0,105275	0,141021	0,108385
89	0,129206	0,098662	0,131124	0,101071	0,153150	0,118149	0,155425	0,121035
90	0,141850	0,109614	0,143616	0,111814	0,168138	0,131265	0,170231	0,133899
91	0,154860	0,120510	0,156454	0,122478	0,183559	0,144313	0,185449	0,146670
92	0,168157	0,131383	0,169564	0,133104	0,199321	0,157333	0,200989	0,159395
93	0,181737	0,142265	0,182946	0,143725	0,215417	0,170365	0,216850	0,172113
94	0,195567	0,153185	0,196568	0,154369	0,231810	0,183442	0,232997	0,184860
95	0,209614	0,164128	0,210397	0,165023	0,248460	0,196546	0,249388	0,197618
96	0,223854	0,175065	0,224411	0,175662	0,265339	0,209643	0,265999	0,210358
97	0,238280	0,185958	0,238604	0,186250	0,282439	0,222688	0,282823	0,223038
98	0,252858	0,196824	0,252947	0,196808	0,299718	0,235701	0,299824	0,235681
99	0,267526	0,207667	0,267377	0,207342	0,317104	0,248685	0,316929	0,248296
100	0,278816	0,229739	0,278816	0,229739	0,330487	0,275117	0,330487	0,275117
101	0,293701	0,243350	0,293701	0,243350	0,348131	0,291416	0,348131	0,291416
102	0,308850	0,257319	0,308850	0,257319	0,366086	0,308144	0,366086	0,308144
103	0,324261	0,271655	0,324261	0,271655	0,384354	0,325311	0,384354	0,325311
104	0,339936	0,286368	0,339936	0,286368	0,402934	0,342930	0,402934	0,342930
105	0,355873	0,301467	0,355873	0,301467	0,421824	0,361012	0,421824	0,361012
106	0,372069	0,316962	0,372069	0,316962	0,441023	0,379567	0,441023	0,379567
107	0,388523	0,332860	0,388523	0,332860	0,460525	0,398606	0,460525	0,398606
108	0,405229	0,349169	0,405229	0,349169	0,480327	0,418136	0,480327	0,418136
109	0,422180	0,365896	0,422180	0,365896	0,500419	0,438167	0,500419	0,438167
110	0,439368	0,383046	0,439368	0,383046	0,520793	0,458705	0,520793	0,458705
111	0,456782	0,400622	0,456782	0,400622	0,541435	0,479752	0,541435	0,479752
112	0,474411	0,418626	0,474411	0,418626	0,562330	0,501312	0,562330	0,501312
113	0,492237	0,437055	0,492237	0,437055	0,583459	0,523382	0,583459	0,523382
114	0,510241	0,455906	0,510241	0,455906	0,604801	0,545956	0,604801	0,545956
115	0,528401	0,475170	0,528401	0,475170	0,626326	0,569024	0,626326	0,569024
116	0,546689	0,494832	0,546689	0,494832	0,648003	0,592570	0,648003	0,592570
117	0,565074	0,514872	0,565074	0,514872	0,669795	0,616569	0,669795	0,616569
118	0,583517	0,535264	0,583517	0,535264	0,691657	0,640988	0,691657	0,640988
119	0,601976	0,555969	0,601976	0,555969	0,713536	0,665783	0,713536	0,665783
120	0,620400	0,576942	0,620400	0,576942	0,735375	0,690898	0,735375	0,690898
121	1,000000	1,000000	1,000000	1,000000	1,000000	1,000000	1,000000	1,000000

Part B. Trend Functions

	Initial Trend 2nd Order		Target Trend 2nd Order		Trend 1st Order	
	Males	Females	Males	Females	Males	Females
Age	F(x)	F(y)	F(x)	F(y)	F(x)	F(y)
0	0,03045921	0,03045921	0,02275699	0,02275699	0,03303985	0,03303985
1	0,03045921	0,03045921	0,02275699	0,02275699	0,03303985	0,03303985
2	0,03045921	0,03045921	0,02275699	0,02275699	0,03303985	0,03303985
3	0,03045921	0,03045921	0,02275699	0,02275699	0,03303985	0,03303985
4	0,03045921	0,03045921	0,02275699	0,02275699	0,03303985	0,03303985
5	0,03045921	0,03045921	0,02275699	0,02275699	0,03303985	0,03303985
6	0,03045921	0,03045921	0,02275699	0,02275699	0,03303985	0,03303985
7	0,03045921	0,03045921	0,02275699	0,02275699	0,03303985	0,03303985
8	0,03045921	0,03045921	0,02275699	0,02275699	0,03303985	0,03303985
9	0,03045921	0,03045921	0,02275699	0,02275699	0,03303985	0,03303985
10	0,03045921	0,03045921	0,02275699	0,02275699	0,03303985	0,03303985
11	0,03045921	0,03045921	0,02275699	0,02275699	0,03303985	0,03303985
12	0,03045921	0,03045921	0,02275699	0,02275699	0,03303985	0,03303985
13	0,03045921	0,03045921	0,02275699	0,02275699	0,03303985	0,03303985
14	0,03045921	0,03045921	0,02275699	0,02275699	0,03303985	0,03303985
15	0,03045921	0,03045921	0,02275699	0,02275699	0,03303985	0,03303985
16	0,03045921	0,03045921	0,02275699	0,02275699	0,03303985	0,03303985
17	0,03045921	0,03045921	0,02275699	0,02275699	0,03303985	0,03303985
18	0,03045921	0,03045921	0,02275699	0,02275699	0,03303985	0,03303985
19	0,03045921	0,03045921	0,02275699	0,02275699	0,03303985	0,03303985
20	0,03045921	0,03045921	0,02275699	0,02275699	0,03303985	0,03303985
21	0,03045921	0,03045921	0,02275699	0,02275699	0,03303985	0,03303985
22	0,03045921	0,03045921	0,02187596	0,02275699	0,03303985	0,03303985
23	0,03043323	0,03040765	0,02043855	0,02271861	0,03301381	0,03298816
24	0,02881264	0,03029140	0,01923494	0,02263210	0,03138903	0,03287161
25	0,02752183	0,03020168	0,01827591	0,02256532	0,03009490	0,03278166
26	0,02653992	0,03010539	0,01754616	0,02249365	0,02911046	0,03268512
27	0,02583092	0,02994760	0,01701912	0,02237620	0,02839963	0,03252693
28	0,02536650	0,02969009	0,01667385	0,02218451	0,02793402	0,03226875
29	0,02508959	0,02935894	0,01646795	0,02193800	0,02765640	0,03193675
30	0,02493426	0,02894826	0,01635245	0,02163226	0,02750067	0,03152501
31	0,02483416	0,02836840	0,01627801	0,02120050	0,02740031	0,03094365
32	0,02474212	0,02757348	0,01620958	0,02060850	0,02730804	0,03014668
33	0,02462828	0,02657932	0,01612492	0,01986797	0,02719391	0,02914996
34	0,02449507	0,02544282	0,01602585	0,01902117	0,02706035	0,02801054
35	0,02439063	0,02426505	0,01594819	0,01814336	0,02695565	0,02682974
36	0,02434739	0,02311820	0,01591603	0,01728835	0,02691229	0,02567995
37	0,02434472	0,02208468	0,01591404	0,01651761	0,02690961	0,02464377
38	0,02434439	0,02121613	0,01591380	0,01586975	0,02690928	0,02377300
39	0,02432191	0,02050145	0,01589708	0,01533655	0,02688675	0,02305650
40	0,02428500	0,01992856	0,01586963	0,01490906	0,02684974	0,02248214
41	0,02426450	0,01952555	0,01585438	0,01460830	0,02682919	0,02207810
42	0,02424813	0,01930199	0,01584221	0,01444145	0,02681278	0,02185396
43	0,02423142	0,01923720	0,01582978	0,01439310	0,02679602	0,02178902
44	0,02418724	0,01930711	0,01579692	0,01444528	0,02675173	0,02185910
45	0,02410346	0,01946812	0,01573461	0,01456544	0,02666774	0,02202052
46	0,02398254	0,01970017	0,01564468	0,01473862	0,02654651	0,02225316
47	0,02382519	0,01996800	0,01552764	0,01493850	0,02638875	0,02252168
48	0,02364638	0,02025161	0,01539464	0,01515013	0,02620949	0,02280602
49	0,02346664	0,02055358	0,01526094	0,01537544	0,02602928	0,02310876

	Initial Trend 2nd Order		Target Trend 2nd Order		Trend 1st Order	
	Males	**Females**	**Males**	**Females**	**Males**	**Females**
Age	**F(x)**	**F(y)**	**F(x)**	**F(y)**	**F(x)**	**F(y)**
50	0,02331582	0,02085973	0,01514874	0,01560386	0,02587807	0,02341569
51	0,02320900	0,02117083	0,01506928	0,01583595	0,02577098	0,02372758
52	0,02311376	0,02146215	0,01499843	0,01605328	0,02567549	0,02401966
53	0,02302553	0,02169881	0,01493279	0,01622981	0,02558704	0,02425692
54	0,02293885	0,02186952	0,01486830	0,01635714	0,02550013	0,02442806
55	0,02284194	0,02197616	0,01479620	0,01643668	0,02540298	0,02453498
56	0,02272244	0,02201318	0,01470729	0,01646429	0,02528317	0,02457210
57	0,02258219	0,02199130	0,01460295	0,01644797	0,02514256	0,02455016
58	0,02243636	0,02192024	0,01449444	0,01639497	0,02499636	0,02447892
59	0,02230882	0,02180639	0,01439954	0,01631005	0,02486849	0,02436478
60	0,02223622	0,02169000	0,01434552	0,01622323	0,02479571	0,02424808
61	0,02224764	0,02161569	0,01435402	0,01616781	0,02480715	0,02417359
62	0,02236662	0,02162912	0,01444255	0,01617782	0,02492644	0,02418705
63	0,02259603	0,02176735	0,01461324	0,01628093	0,02515644	0,02432563
64	0,02292896	0,02203222	0,01486095	0,01647849	0,02549022	0,02459119
65	0,02335122	0,02238686	0,01517508	0,01674299	0,02591357	0,02494674
66	0,02383259	0,02279537	0,01553315	0,01704763	0,02639617	0,02535629
67	0,02434107	0,02323941	0,01591133	0,01737873	0,02690595	0,02580147
68	0,02484958	0,02370775	0,01628948	0,01772791	0,02741577	0,02627101
69	0,02531438	0,02419153	0,01663510	0,01808856	0,02788177	0,02675603
70	0,02569230	0,02466260	0,01691607	0,01843969	0,02826066	0,02722831
71	0,02594418	0,02509284	0,01710332	0,01876036	0,02851318	0,02765966
72	0,02605113	0,02545692	0,01718283	0,01903168	0,02862041	0,02802468
73	0,02599425	0,02574882	0,01714055	0,01924919	0,02856339	0,02831732
74	0,02576921	0,02594249	0,01697325	0,01939350	0,02833777	0,02851150
75	0,02538270	0,02602289	0,01668589	0,01945340	0,02795026	0,02859209
76	0,02485163	0,02597743	0,01629101	0,01941953	0,02741783	0,02854652
77	0,02419360	0,02580295	0,01580165	0,01928952	0,02675811	0,02837159
78	0,02343096	0,02549572	0,01523440	0,01906059	0,02599351	0,02806358
79	0,02258309	0,02506486	0,01460361	0,01873950	0,02514346	0,02763160
80	0,02167445	0,02450425	0,01392748	0,01832166	0,02423250	0,02706955
81	0,02071541	0,02381520	0,01321366	0,01780802	0,02327100	0,02637873
82	0,01972490	0,02300732	0,01247624	0,01720567	0,02227796	0,02556878
83	0,01874172	0,02209869	0,01174408	0,01652807	0,02129226	0,02465783
84	0,01779590	0,02111098	0,01103958	0,01579131	0,02034403	0,02366759
85	0,01688653	0,02005839	0,01036206	0,01500595	0,01943234	0,02261230
86	0,01601311	0,01896550	0,00971119	0,01419030	0,01855670	0,02151662
87	0,01515958	0,01784943	0,00907499	0,01335712	0,01770100	0,02039770
88	0,01430337	0,01672797	0,00843666	0,01251967	0,01684261	0,01927338
89	0,01343795	0,01560419	0,00779132	0,01168026	0,01597498	0,01814674
90	0,01262835	0,01453824	0,00752827	0,01088382	0,01516333	0,01707808
91	0,01192950	0,01358644	0,00752827	0,01017249	0,01446271	0,01612386
92	0,01134117	0,01274847	0,00752827	0,00954608	0,01387288	0,01528376
93	0,01086315	0,01202404	0,00752827	0,00900445	0,01339365	0,01455749
94	0,01049529	0,01141290	0,00752827	0,00854744	0,01302486	0,01394480
95	0,01023747	0,01091485	0,00752827	0,00817495	0,01276639	0,01344549
96	0,01008961	0,01052972	0,00752827	0,00788688	0,01261815	0,01305938
97	0,01005034	0,01025737	0,00752827	0,00768315	0,01257878	0,01278634
98	0,01005034	0,01009773	0,00752827	0,00756372	0,01257878	0,01262629
99	0,01005034	0,01005034	0,00752827	0,00752827	0,01257878	0,01257878
100	0,01005034	0,01005034	0,00752827	0,00752827	0,01257878	0,01257878
101	0,01005034	0,01005034	0,00752827	0,00752827	0,01257878	0,01257878
102	0,01005034	0,01005034	0,00752827	0,00752827	0,01257878	0,01257878

	Initial Trend 2nd Order		Target Trend 2nd Order		Trend 1st Order	
	Males	Females	Males	Females	Males	Females
Age	F(x)	F(y)	F(x)	F(y)	F(x)	F(y)
103	0,01005034	0,01005034	0,00752827	0,00752827	0,01257878	0,01257878
104	0,01005034	0,01005034	0,00752827	0,00752827	0,01257878	0,01257878
105	0,01005034	0,01005034	0,00752827	0,00752827	0,01257878	0,01257878
106	0,01005034	0,01005034	0,00752827	0,00752827	0,01257878	0,01257878
107	0,01005034	0,01005034	0,00752827	0,00752827	0,01257878	0,01257878
108	0,01005034	0,01005034	0,00752827	0,00752827	0,01257878	0,01257878
109	0,01005034	0,01005034	0,00752827	0,00752827	0,01257878	0,01257878
110	0,01005034	0,01005034	0,00752827	0,00752827	0,01257878	0,01257878
111	0,01005034	0,01005034	0,00752827	0,00752827	0,01257878	0,01257878
112	0,01005034	0,01005034	0,00752827	0,00752827	0,01257878	0,01257878
113	0,01005034	0,01005034	0,00752827	0,00752827	0,01257878	0,01257878
114	0,01005034	0,01005034	0,00752827	0,00752827	0,01257878	0,01257878
115	0,01005034	0,01005034	0,00752827	0,00752827	0,01257878	0,01257878
116	0,01005034	0,01005034	0,00752827	0,00752827	0,01257878	0,01257878
117	0,01005034	0,01005034	0,00752827	0,00752827	0,01257878	0,01257878
118	0,01005034	0,01005034	0,00752827	0,00752827	0,01257878	0,01257878
119	0,01005034	0,01005034	0,00752827	0,00752827	0,01257878	0,01257878
120	0,01005034	0,01005034	0,00752827	0,00752827	0,01257878	0,01257878
121	0,00000000	0,00000000	0,00000000	0,00000000	0,00000000	0,00000000