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CREDIBILITY WITH INCOMPLETE INFORMATION IN GROUP INSURANCE

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ABSTRACT

In experience rating, insurers set rates, for a particular policy or risk, based on a blend of the claim experience of that risk during a prior period and a mean rate. The extent that the risk's own experience is used is called the credibility. Very often, the exact amount of incurred claims during the prior period, which is called the experience period, is not yet known when the rate needs to be set. This is due to the fact that many claims, particularly for non life coverages, can take many months or years to be fully settled. In addition, some of the claims will not be reported until well after the date of incurral. Insurers will generally set claim reserves for these estimated liabilities. The claim reserve for unknown claims is called Incurred But Not Reported (IBNR). Most credibility models published assume that claims incurred during the prior period are known. This paper deals with how the credibility levels change based on this lack of knowledge. Group insurance models are used, but some of the techniques could apply to any insurance risk.

0. Introduction

Let X and Y be claim random variables. Throughout this paper the approximation $Y \approx zX + c$ will be solved by least squares. That is, the quantity $E\left[(Y - zX - c)^2\right]$ will be minimized with respect to the constants z and c. The solution is $z = \dfrac{C(X,Y)}{V(X)}$. $E(X) = \mu_X$ is the expectation of X, $V(X) = E\left[(X-\mu_X)^2\right]$ is the variance of X, and $C(X,Y) = E\left[(X-\mu_X)(Y-\mu_Y)\right]$ is the covariance of X and Y. The constant z is called the credibility.

1. Unbiased Estimated Claim Amounts for a Single Risk

Claim Amounts in year i: X_i

Claims estimated for year i: $\hat{X}_i$

Assume that $C(\hat{X}_1, X_2 | X_1) = 0$ and $E(\hat{X}_i | X_i) = X_i$

Approximate: $X_2 \approx z_e \hat{X}_1 + c$

$$\text{Then } z_e = \frac{C(\hat{X}_1, X_2)}{V(\hat{X}_1)} = \frac{C[E(\hat{X}_1|X_1), E(X_2|X_1)] + E[C(\hat{X}_1, X_2|X_1)]}{V(\hat{X}_1)}$$

$$= \frac{C[X_1, E(X_2|X_1)]}{V(\hat{X}_1)} = \frac{C(X_1,X_2)}{V(\hat{X}_1)} = z\frac{V(X_1)}{V(\hat{X}_1)} = z\frac{V(X_1)}{V[E(\hat{X}_1|X_1)] + E[V(\hat{X}_1|X_1)]}$$

$$= z\frac{V(X_1)}{V(X_1) + E[V(\hat{X}_1|X_1)]} = \frac{z}{1 + \dfrac{E[V(\hat{X}_1|X_1)]}{V(X_1)}}$$

Where: $z = \dfrac{C(X_2, X_1)}{V(X_1)}$ resulting from $X_2 \approx zX_1 + c$.

Thus the credibility of the estimated claims will be lower than if the actual claims had been used.

2. Estimation Based on Observed Claims

Of course, often $E(\hat{X}_1 | X_t) \neq X_1$.

In this model (group health or casualty) let CX be observed, where $0 \leq C \leq 1$ and C is independent of the X_ts. C is the random ratio of observed to total claims that is assumed *independent of the actual amount of claims.*

Let $\hat{X}_1 = rCX_1$ for constant $r \geq 1$. Here r is an estimation or reserving factor set by the insurer. The insurer will usually estimate $r = E\left(\frac{1}{C}\right)$. Note that it is not generally true that $r = \frac{1}{E(C)}$.

In fact $r = E\left(\frac{1}{C}\right) > \frac{1}{E(C)}$.

$$z_i = \frac{C(X_i, X_j)}{r\mu_c\sigma_i^2 + \frac{r}{\mu_c}\sigma_c^2(\mu_i^2 + \sigma_i^2)} \leq z.$$

Where $\sigma_i^2 = V(X_i)$, $\mu_i = E(X_i)$, $\mu_c = E(C)$, and $\sigma_c^2 = V(C)$.

3. Group Model - Fixed Benefit (e.g. Life Insurance) -- Claims Known

Claims in year t: $X_t = \sum_{i=1}^{n_t} YI_{it}b_i$, where Y is an unknown risk parameter random variable independent of the I's. I_{it} are indicator random ($I_{it} \in \{0,1\}$) variables that a claim has occurred for risk i in year t and are mutually independent.

Define: $P\{I_{it} = 1\} = p_i$, $e = \sum_{i=1}^{n_t} p_i$, $\bar{b} = \frac{\sum_{i=1}^{n_t} p_i b_i}{e}$, $s_b^2 = \frac{\sum_{i=1}^{n_t} p_i(b_i - \bar{b})^2}{e} = \frac{\sum_{i=1}^{n_t} p_i b_i^2}{e} - \bar{b}^2$,

and $\bar{X} = \frac{1}{n}\sum_{t=1}^{n} X_t$.

For the approximation $X_{n+1} \approx z\bar{X} + c$

$z = \frac{nef}{nef + k}$, where $k = \frac{\mu_Y^2 + \sigma_Y^2}{\sigma_Y^2} \approx \frac{\mu_Y^2}{\sigma_Y^2}$, $\mu_Y = E(Y)$, $\sigma_Y^2 = V(Y)$, and

$f = \frac{\bar{b}^2}{\bar{b}^2 + s_b^2}$.

Note that the alternative formulation: $P\{I_{it} = 1\} = Yp_i$ yields the same result except that $k = \frac{\mu_Y}{\sigma_Y^2}$. If we take $\mu_Y = 1$ then the formulas are identical.

4. Group Life Insurance with IBNR (incurred but not reported)

Same as section 3, except only $r < 1$ of the claims have been reported in year n.

So $X_n = \sum_{i=1}^{n_t} b_i[YrI_{in} + (1-r)p_i]$ or observed claims plus a IBNR reserve equal to manual claims.

Then $z = \dfrac{(n-1+r)ef}{(n-1+r)ef+k}$. Note that this will also be less than the credibility in section 3.

5. Group Non Life Insurance - Claims Known

Each claim has a random severity. A common example of this might is group disability income.

Definitions same as in section 3, except: $X_t = \sum_{i=1}^{m} YI_{it} B_{it}$ Then we still have $z = \dfrac{nef}{nef+k}$ but

now $f = \dfrac{\bar{b}^2}{\bar{b}^2 + s_b^2 + \bar{\sigma}_B^2}$, where $\bar{\sigma}_B^2 = \dfrac{\sum_{i=1}^{m} p_i V(B_{it})}{e}$.

6. Group Non Life Insurance - Partial Information on Severity - Right Censoring

This model would be applicable to group long term disability income.

The research for this section has not been completed.