Measuring the Rate Change of a Non-Static Book of Property and Casualty Insurance Business

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Abstract

Motivation. Calculated rate change factors can substantially affect loss ratio forecasts and thus are critical parameters for enterprise risk management (ERM). However, current methods are not well suited to a changing book of business.

Method. The analysis first explores the conceptual underpinnings of rate change and then applies the conclusions of this analysis to several practical problems.

Results. The proposed approach shows improved accuracy as compared to current methods, with particular significance for a non-static book of business.

Conclusions. I conclude that "rate change" measures the change in premium *relative to loss potential*. One can then apply this conceptual formulation in order to solve several problems that one confronts in practice: how to adjust for shifts in limits and deductibles, how to blend together changes in exposures when the portfolio uses several different exposure bases, and how to properly weight together granular measures of rate change (e.g., for each policy, subline, etc.) into an overall rate change for the entire portfolio.

1. Introduction

Several questions arise when calculating rate change factors and adjusting premium to current level. Some of these questions are:

1. "I have measured rate changes for several different sublines or multiple individual policies; how do I weight them together to obtain one blended rate change factor for the overall portfolio?"
2. "I am measuring rate change for excess casualty policies, which cover auto liability and also general liability claims; how do I combine rate changes for these two sublines, which have different exposure bases? More generally, how do I combine any two sublines that have different exposure bases? Is it possible to obtain one overall number for 'exposure change' when the sublines have different exposure bases?"
3. "How do I account for changes to a policy's limit and deductible when measuring the renewal policy's rate change?"
4. "When we implement rate increases and rate decreases for various classes of business, volume tends to grow in those classes for which we have decreased rates and volume tends to decline in those classes of business for which we have increased rates. Thus, rate changes tend to generate additional shifts in the mix of business in our portfolio; how do I properly reflect this shift when calculating rate change for the total book of business?"

2. The Theory and Purpose of Rate Change Factors

In order to answer these detailed questions, we need to first examine the fundamental principles underlying the theory of rate change.

This paper asserts that the general theory of rate change factors is not well defined. How should one calculate a company's rate change factors? The answer to this question depends upon the answer to the following question: For what purpose will we use these rate change factors?

In theory, rate change factors can be used for several different purposes. For example, one potential use of rate change factors is strategic: to enable management to better run the company. Under this approach, rate change factors indicate how the company is performing: they tell management where performance is improving and where it is slipping, thus allowing for better steering of the business and better implementation of strategy. If in fact this is the purpose of the rate change factors, then consider the dynamic situation in which policies currently issued by the company have higher deductibles than policies issued in the past. As the deductibles increase, the stable volume of losses in the deductible layer disappears and the company covers policies that have more variability, lower premium volume, and (because of fixed costs) higher expense ratios. Therefore, if the goal of the company is to understand the true nature of its performance, traditional rate change factors, which ignore shifts in required risk load and shifts in expense ratios, will fall short of the desired goal. Rather, the company must implement an approach whereby each policy in the portfolio, accounting for risk load and fixed expenses, is priced to a target premium; then, the company can evaluate how its actual premium compares to the target premium and how this ratio of "actual to target" changes over time. In a dynamic environment with changing policy provisions, only such an approach can give complete information to management about the performance and direction of the company's rate adequacy.

Given that most rate change factors do not typically account for all the aspects of shifts in required risk load and of shifts in expense ratios, the question persists: what good are rate change factors, for what purpose can we use them, and how does this affect how we calculate them?

Traditional rate change factors therefore appear to be much more relevant to a second purpose: formulating a loss ratio projection for a book of business. Such a projection is often helpful for operational needs, such as estimating initial loss reserves, or for transactional purposes, such as effecting reinsurance treaties. In order to forecast the projected loss ratio, the actuary often begins by looking at historical experience data; in order to make the data relevant to the projected period, the losses and premium are adjusted to current level.

Therefore, in order to understand the role of rate change factors, we must investigate the nature of the traditional loss ratio projection and articulate its assumptions.

3. Projecting Loss Ratio Using Adjusted Historical Data

What is the nature of the loss ratio projection framework? Losses (in aggregate for any given historical year) are simply adjusted to current cost level; they are typically not adjusted in any way to incorporate changes in mix of business or changes in policy provisions such as deductibles and limits. Premium is adjusted to what it "would be" had the historical policies been written today (or, more precisely, during the projected period). Just as with losses, there seem to be no adjustments for shifts in the mix of business or in policy features. Thus traditional methods appear to be relevant only for the limited situation of a static book of business (or one that changes only glacially).

How can traditional loss ratio projection be appropriate then for many books of business, which sustain significant changes in policies, classes of business, exposures, limits, and deductibles?

One answer to this challenge is simply to concede: yes, using historical data to project the future only makes sense when the portfolio is reasonably static, but not when it undergoes significant changes. This surrender appears especially applicable to the "extended exposures" method for adjusting premium to current level. After all, the extended exposures approach takes historical policies and simply re-rates the policies at today's rates; but if the types of policies in the portfolio have changed, the mix of business has shifted, and the limits and deductibles are different, what is the relevance of restating the policies of the historical portfolio?

Nevertheless, I believe that one can defend the use of historical data and adjusting for rate change by advancing the following reasoning. The goal of analyzing adjusted historical data is not to measure the **amount** of losses and premium that would occur from the historical portfolio, adjusted to today's dollars; rather, the goal is to measure premium and losses with respect to each other, i.e., the **interrelationship** of premiums to losses, and to measure what this relationship from the historical period would be in today's environment. Thus, even when the insurer's portfolio of policies undergoes significant change, traditional loss ratio projection can be highly relevant, but only to the extent that the analysis focuses on measuring the relationship between premium and losses; specifically, the focus should be on what this relationship will be in the projected period. This understanding of the purpose of using adjusted historical premium and losses, in turn, has ramifications for our understanding of what rate change factors should do and how we should calculate them.

4. Algebraic Representation

Let:

- Premium(portfolio(t), rates(t)) = premium for historical period t
- Loss(portfolio(t), cost(t)) = losses for historical period t
- LP(portfolio(t)) = loss potential for portfolio for historical period t; reflects limits, deductibles, and exposure base units
- LP(portfolio(t+1)) = loss potential for portfolio for projected period t+1; reflects limits, deductibles, and exposure base units
- LP(portfolio(t+1))/ LP(portfolio(t)) = multiplier to adjust loss potential for portfolio at time t to loss potential for portfolio at time t+1
- Trend(t, t+1) = cost(t+1) / cost(t)

Let's assume that there are changes in the book of business relating to exposures, limits, and deductibles.

We want to adjust losses and premium to the basis of the current book, so we must measure:

Fully Adjusted Losses = Loss(portfolio(t+1), cost(t+1)) =

$$Loss(portfolio(t), cost(t)) * \frac{LP(portfolio(t+1))}{LP(portfolio(t))} * Trend(t, t+1) \qquad (4.1)$$

And

Fully Adjusted Premium = Premium(portfolio(t+1), rates(t+1))

Multiplying and dividing by equal quantities, we derive:

Fully Adjusted Premium =

$$Premium(portfolio(t),rates(t)) * \frac{LP(portfolio(t+1))}{LP(portfolio(t))} *$$

$$\frac{Premium(portfolio(t+1),rates(t+1))}{Premium(portfolio(t),rates(t)) * \frac{LP(portfolio(t+1))}{LP(portfolio(t))}} \qquad (4.2)$$

As stated above, and as implied by equation (4.1), in theory the losses should be adjusted to reflect all changes in loss potential, whether from changes in exposures, mix of business, limits, deductibles, etc. Nevertheless, if we focus on the interrelationship of losses and premium, we note that the shift in loss potential appears both in equation (4.1) and in equation (4.2). Thus, if we look at the loss ratio and divide losses by premium, we divide equation (4.1) by equation (4.2) and cancel the factor for shift in loss potential. Then:

Adjusted Loss Ratio(portfolio(t+1), rates(t+1), cost(t+1)) =

$$\frac{Fully\ Adjusted\ Losses}{Fully\ Adjusted\ Premium} = \frac{Adjusted\ Losses}{Adjusted\ Premium} \qquad (4.3)$$

Where

Adjusted Losses =

$$Loss(portfolio(t),cost(t)) * Trend(t,t+1) \qquad (4.4)$$

And

Adjusted Premium =

$$Premium(portfolio(t),rates(t)) *$$

$$\frac{Premium(portfolio(t+1),rates(t+1))}{Premium(portfolio(t),rates(t)) * \frac{LP(portfolio(t+1))}{LP(portfolio(t))}} \qquad (4.5)$$

Note that equation (4.4) for adjusted losses is similar to equation (4.1) but no longer has any factor for changes in loss potential from exposures, limits, and deductibles. Therefore, the practice of not adjusting losses for these shifts in loss potential is sustainable, but only if one simultaneously defines adjusted premium properly.

Now, let us define the Rate Change Factor as the multiplier which converts historical premium to adjusted premium. Then:

Adjusted Premium =

$$Premium(portfolio(t), rates(t)) * Rate\ Change\ Factor \tag{4.6}$$

Then combining formulas (4.5) and (4.6), we derive

Rate Change Factor =

$$\frac{Premium(portfolio(t+1), rates(t+1))}{Premium(portfolio(t), rates(t)) * \frac{LP(portfolio(t+1))}{LP(portfolio(t))}} \tag{4.7}$$

Or, equivalently,

$$Rate\ Change\ Factor = \frac{Premium(t+1)}{Premium(t) * Shift\ in\ Loss\ Potential} \tag{4.8}$$

Equation (4.8) demonstrates that one must calculate the rate change factor using the ratio of two quantities:

1. Actual premium in period (t+1)
2. Actual premium in period (t) transformed for all shifts in loss potential, including exposures, limits, deductibles, etc.

To summarize, we have demonstrated three points:

1. To obtain an Adjusted Loss Ratio, the losses in the numerator do not need to be adjusted for changes in loss potential, thus somewhat exonerating current practice.
2. The premium must be adjusted by one factor, which we define as the rate change factor.
3. The rate change factor is thus defined by equation (4.8), which shows that premium from the prior period must be restated for changes in loss potential before measuring the change in rate level.

5. Applications

We will now apply the conclusions of the discussion above to solve the problems raised at the beginning of this paper.

Exhibit 1A
Change in Exposures

Expiring Policies

	Premium	Exposures	Premium per Exposure
Red Trucks	12,000,000	600	20,000
Green Trucks	4,000,000	400	10,000
Total	16,000,000	1,000	16,000

Renewing Policies

	Premium	Exposures	Premium per Exposure
Red Trucks	8,640,000	360	24,000
Green Trucks	4,480,000	560	8,000
Total	13,120,000	920	14,261

Exhibit 1B
Traditional Rate Change Calculations

Method 1: Average Rate per Exposure Unit

[1]	[2]	[3]	[4] = [3] / [2] -1
	Expiring Premium Per Exposure	Renewing Premium Per Exposure	Change
Red Trucks	20,000	24,000	20.00%
Green Trucks	10,000	8,000	-20.00%
Total	16,000	14,261	-10.87%

Methods 2 and 3: Weighted Average of Rate Changes

[1]	[2]	[3]	[4]
	Change	Expiring Premium Weight	Renewing Premium Weight
Red Trucks	20.00%	75.00%	65.85%
Green Trucks	-20.00%	25.00%	34.15%
Weighted Average		10.00%	6.34%

In this example, we show three traditional methods of measuring rate change:

1. Calculate the weighted average premium per exposure; measure this quantity for the renewal portfolio relative to the expiring portfolio for the rate change.
2. Measure the rate change of each class or policy in the portfolio; blend these rate changes together using a weighted average; use expiring premium as the weights.
3. Measure the rate change of each class or policy in the portfolio; blend these rate changes together using a weighted average; use renewing premium as the weights.

Note that all of the traditional methods produce different answers, none of which is correct.

The workbook below shows the proposed new approach.

Exhibit 1C
Proposed New Approach to Calculating Rate Change

[1]	[2] Expiring Premium	[3] Renewing Exposures / Expiring Exposures	[4] = [3] * [2] Expiring Premium Restated For Change in Exposure	[5] Renewing Premiums	[6] = [5] / [4] -1 Rate Change
Red Trucks	12,000,000	0.60	7,200,000	8,640,000	20.00%
Green Trucks	4,000,000	1.40	5,600,000	4,480,000	-20.00%
Total	16,000,000		12,800,000	13,120,000	2.50%

Exhibit 1D
Comparison Exhibit

Method	Description	Calculated Rate Change
1	Ratio of Average Rate per Exposure Unit	-10.87%
2	Expiring Premium Weighted Average of Rate Changes	10.00%
3	Renewing Premium Weighted Average of Rate Changes	6.34%
Proposed	Adjust Expiring Premium for Change in Loss Potential	2.50%

The proposed approach builds upon the prior conceptual understanding and equation (4.8); thus, expiring premium must be adjusted or “restated” for all shifts in loss potential before measuring rate change. In Exhibit 1D, we see that the proposed approach can generate significantly different rate change factors than other methods.

The proposed framework for measuring rate change also allows us to solve the problem of how to deal with a portfolio with multiple, dissimilar exposure bases.

The exhibits below demonstrate the proposed approach.

Exhibit 2A
Dissimilar Exposure Bases

Expiring

	Premium	Exposure Base	Exposures	Premium per Exposure
Jane's Contracting	12,000,000	sales (000s)	600	20,000
Jill's Stores	4,000,000	square feet (000s)	400	10,000
Total	16,000,000	undefined	undefined	undefined

Renewing

	Premium	Exposure Base	Exposures	Premium per Exposure
Jane's Contracting	8,640,000	sales (000s)	360	24,000
Jill's Stores	4,480,000	square feet (000s)	560	8,000
Total	13,120,000	undefined	undefined	undefined

Exhibit 2B
Measuring "Change in Premium from Changes in Exposure Base Units"

Proposed Approach to Measuring Rate Change

[1]	[2]	[3]	[4]	[5] = [4] * [2]	[6]	[7] = [6] / [5] -1
	Expiring Premium	Exposure Base	Renewing Exposures / Expiring Exposures	Expiring Premium Restated For Change in Exposure	Renewing Premiums	Rate Change
Jane's Contracting	12,000,000	sales (000s)	0.600	7,200,000	8,640,000	20.00%
Jill's Stores	4,000,000	square feet (000s)	1.400	5,600,000	4,480,000	-20.00%
Total	16,000,000	loss potential	0.800	12,800,000	13,120,000	2.50%

Measuring Exposure Change for Total Book

[1]	[2]	[3]	[4] = [3] / [2]	[5] = [3] / [2] -1
	Expiring Premium	Expiring Premium Restated For Change in Exposure	Ratio	Change in Premium from Changes in Exposure Base Units
Total	16,000,000	12,800,000	0.800	-20.00%

Initially, the disparate exposure bases of the classes of business prevent us from measuring the exposure base change for the total book. However, by restating the expiring premium for shifts in exposure bases, we create a new way to measure total exposure base change; we simply measure the total change in premium arising from changes in exposure bases. Thus, the proposed procedure of restating expiring premium for shifts in loss potential provides a framework for measuring the total exposure base change for a portfolio that has multiple, incongruous exposure bases.

Next, we look at shifts in deductibles.

Exhibit 3A
Change in Deductibles

Expiring

	Premium	Square Feet (000s)	Limit	Deductible	ILF Index = ILF(Limit) - ILF(Deductible)	Premium per Exposure
Joe's Stores	13,500,000	900	1,000,000	-	1.00	15,000
Bill's Stores	9,000,000	900	1,000,000	250,000	0.50	10,000
Total	22,500,000	1,800				12,500

Renewing

	Premium	Square Feet (000s)	Limit	Deductible	ILF Index = ILF(Limit) - ILF(Deductible)	Premium per Exposure
Joe's Stores	8,977,500	900	1,000,000	250,000	0.50	9,975
Bill's Stores	14,400,000	900	1,000,000	-	1.00	16,000
Total	23,377,500	1,800				12,988

Exhibit 3B
Traditional Rate Change Calculations

Class	Change	Expiring Premium Weight	Renewing Premium Weight
Joe's Stores	33.00%	60.00%	38.40%
Bill's Stores	-20.00%	40.00%	61.60%
Weighted Average		11.80%	0.35%

Exhibit 3C
Proposed New Approach to Calculating Rate Change

[1]	[2] Expiring Premium Restated For Change in Exposure	[3] Renewing ILF Index / Expiring ILF Index	[4] = [3] * [2] Expiring Premium Restated For Change in Exposure **and** Change in Limit, Deductible	[5] Renewing Premiums	[6] = [5] / [4] -1 Rate Change
Joe's Stores	13,500,000	0.50	6,750,000	8,977,500	33.00%
Bill's Stores	9,000,000	2.00	18,000,000	14,400,000	-20.00%
Total	22,500,000		24,750,000	23,377,500	-5.55%

Exhibit 3D
Comparison Exhibit

Method	Description	Calculated Rate Change
1	Expiring Premium Weighted Average of Rate Changes	11.80%
2	Renewing Premium Weighted Average of Rate Changes	0.35%
Proposed	Adjust Expiring Premium for Change in Loss Potential	-5.55%

Again, we see the importance of measuring rate change only after restating expiring premium for changes in loss potential.

In the numerical example above (Exhibits 3A through 3D), we use ILFs (increased limits factors) to measure the change in loss potential from changing limits and deductibles. However, there is more than one type of ILF. "Loss ILFs" measure the relationship of loss costs of different limits and deductibles; they derive from measures of Limited Expected Value (LEV, aka LAS or Limited Average Severity). "Premium ILFs," however, measure the relationship of premium the company charges for different limits and deductibles; they incorporate LEVs, risk load, and expenses. So when measuring rate change and restating premium for changes to limits and deductibles, which ILFs should one use?

Equation (4.8) demonstrates that when measuring rate change one must restate expiring premium for changes in **loss potential**. Therefore, when restating expiring premium for changes to limits and deductibles, it is more accurate to use Loss ILFs than Premium ILFs; afterwards, one can then measure the rate change factor as the ratio of renewing premium to restated expiring premium.

6. Ramifications for Enterprise Risk Management

We have seen that the proposed methodology can generate rate change factors that differ substantially from factors calculated by current methods. Moreover, the methodology one chooses can even affect the direction of the rate change, converting a measured rate increase into a measured rate decrease. Thus, proper measurement can lead to a more accurate estimate of both the direction and the magnitude of rate change, and can lead to a more accurate prediction of the expected loss ratio. However, although a more precise prediction of a firm's expected loss ratio is quite important for minimizing the risk of inaccurate pricing, does it have broader ramifications for enterprise risk management?

The following list delineates some areas in which improved measurement of rate change and loss ratio can have direct ramifications upon enterprise risk management (ERM).

6.1 Estimating the Cost and Benefit of Hedging

A firm's estimate of mean loss ratio affects in two ways the estimated probability that losses may be large enough to trigger a reinsurance hedge. First, it affects the measurement of how far the reinsurance attaches from the mean loss (i.e., reinsurance attachment point minus mean loss). If the estimate of the expected loss ratio is too low, this difference will be high, leading the firm to underestimate the true probability of recovering losses from reinsurance hedging. A second factor that affects the measured probability of triggering the reinsurance hedge is volatility. The volatility estimate is often quantified in terms of the coefficient of variation (CV); thus, standard deviation will equal the CV multiplied by the estimated mean loss ratio. As a result, an underestimated loss ratio can translate into an underestimated standard deviation and an underestimated probability of triggering reinsurance coverage.

In turn, this underestimated probability of triggering reinsurance affects two important metrics. First, the firm will underestimate the expected value of reinsurance recoveries; because the "cost" of reinsurance equals price minus recoveries, the firm will therefore overestimate the cost of reinsurance hedging. Second, the firm will underestimate the likelihood that the reinsurance hedge will provide mitigation of downside losses and thus will underestimate the "benefit" of the reinsurance hedge.

6.2 Reputational Risk

As mentioned in Section 2, a firm can use an estimated expected loss ratio to calculate initial loss reserves. In particular, longer tail excess liability lines often have minimal loss emergence as of 12 months; as a result, the firm often sets loss reserves by multiplying the year's premium by the estimated expected loss ratio. Ultimately, if the firm underestimates the expected loss ratio, there will tend to be an upward drift over time of booked ultimate losses, as higher actual losses replace lower initial loss estimates. On the one hand, the underestimation of reserves does not actually generate any losses or "cause" losses to be higher—the paid losses are merely a manifestation and consequence of writing the insurance policies in the first place. However, if the firm consistently underestimates its reserves, both investors and regulators may lose faith in the accuracy of the firm's published financial statements. Therefore, accurately calculating rate change factors and estimating the expected loss ratio can reduce the operational risk of underestimating liabilities and can reduce the reputational risk of losing credibility with both regulators and investors.

6.3 Strategic Risk

A firm often needs to make decisions about its pricing strategy. Sometimes the firm wants to increase its market share, and thus desires to reduce rates in order to attract more business. Other times, the firm wants to enhance profitability, and thus desires to expand margins via rate increases. Ultimately, the firm's ability to execute its pricing strategy and to manage strategic risk depends upon its ability to accurately measure the rate change of its book of business.

7. Summary

Quantitative analysis that projects an expected loss ratio often makes use of historical experience data and rate change factors. The appropriate application of such an analysis and the accurate calculation of rate change factors require a clear understanding of the conceptual foundations that underpin these methods. Having explored these foundational concepts, we conclude that the key goal of analyzing historical data is to forecast the interrelationship of losses and premiums for the projected book of business. Thus, one must measure rate change factors by first adjusting expiring premium for changes in all sources of loss potential, whether for changes in exposure base units, shifts in limits and deductibles, or changes in other sources of loss potential (e.g., the company's percentage share of a policy, the time duration of the policy, etc.). As a result, one can take the theory of measuring rate change factors and can apply it towards solving problems of measuring rate change in practice.

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