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Some Financing Options for Social Security

by

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ABSTRACT

This is the fourth paper on n-year roll forward reserve financing of public benefit systems, especially Old-Age, Survivors and disability Insurance (OASDI), prepared by University of Michigan faculty and students. As the papers have developed, some ideas have been simplified, while others have become more complex.

The concept of n-year roll forward reserve financing is reviewed. Comparisons of data between the 1993 and 1994 annual Trustees Reports of the Office of the Actuary, Social Security Administration are presented.

The contribution of this paper is the discussion of various means of replacing the required annual incomes on a new money basis with an actuarial equivalent series for use in n-year roll forward reserve financing. The first method applied a level percentage rate to the projected increasing taxable payrolls over a fixed term of m-years. Illustrations for m = 4, 8, 12, and 16 with n = 1 and 2 are provided. The second method uses a moving term of m-years that advances one year at the end of each year. This second method receives some mention in the Trustees Reports. Advantages and disadvantages of the fixed term and the moving term approaches are discussed. The two methods have been analyzed with the use of a theoretical model assuming a constant force of interest and a constant rate of growth of the payroll base, as well as a more realistic model based on the year-by-year requirements of OASDI.

It is concluded that the theoretical model requires further attention. This research can be extended to expand our knowledge of OASDI financing and the applicability of n-year roll forward reserve financing.

Keywords: Social Security financing, n-year roll-forward, reserve financing, pension funding

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As in previous papers, a main concept is that of n-year roll-forward reserve financing. The concept applies readily to the financing of Social Security programs. The paper was prepared under the National Science Foundation program of Research Experience for Undergraduates, in co-ordination with the Actuarial Program at the University of Michigan.

Reports of the boards of Trustees of Old-Age, Survivors and Disability Insurance (OASDI), of Hospital Insurance, (HI), and of Supplementary Medical Insurance (SMI), and extensive data, provided by Richard Foster, Deputy Chief Actuary, Social Security Administration, were most helpful to us. In return, we hope the information developed here, from these Reports and data, will be of assistance in the financial management of Social Security. While our studies relate to programs in the United States of America, the ideas may be of use in other countries, with mature or developing economies.

I. THE CONCEPT OF N-YEAR ROLL FORWARD RESERVE FINANCING

1.1 Introduction. This concept is not new. It has been developing over many years in regard to OASDI, and may find application in other large public benefit systems. In recent years, faculty and students at the University of Michigan have explored the concept (see [5], [12], [13], [14]). The last reference, in particular, provides an overview of the concept and what it would mean in the financing of Social Security.

In this paragraph, we will explore the concept in general terms, and in the subsequent paragraph will provide a more precise, mathematical description. Prerequisites for such a financing method are a year-by-year projection of the outgo for benefits and administration, a year-by-year projection of the income source upon which taxes or contributions are levied, and a projection of interest rates to be earned by reserve funds. Also, it is assumed that there is on hand a reserve equivalent to the outgoes for the next n years. This reserve will be used to provide such outgoes, and in each year the reserve will be replenished by an amount of new money equivalent to the projected outgo for the year that is n years ahead. Such amount for calendar year k is denoted by ${}_nI_k$ and, in theory, assures that, at the end of the current year, there will be a reserve equivalent to the outgoes for the subsequent n years. In essence, n-year roll-forward reserve financing is pay-as-you-go financing (or assessment financing), projected n years.

To be more precise, and to relate to formula (3.1.1) of [14], we assume that interest is at the constant force δ, that during the calendar year k, the outgo is O_k, and the required annual new money income to replenish the reserve is ${}_nI_k$. For simplicity, each of O_k, and ${}_nI_k$ is assumed to occur at mid-year. Then at 12/31/k, there is on hand a reserve, $A_{12/31/k}$, equal to the present value of the outgoes of the next n years, thus

$$ {}_{n}A_{12/31/k} = O_{k+1}e^{-\delta/2} + O_{k+2}\,e^{-3\delta/2} + \cdots + O_{k+n}\,e^{-(2n-1)\delta/2} \qquad (1.1.1) $$

The growth of the reserve over the year $k + 1$ is given by

$$ {}_{n}A_{12/31/k}\,e^{\delta} + \left({}_{n}I_{k+1} - O_{k+1}\right)e^{\delta/2} = {}_{n}A_{12/31/k+1} \qquad (1.1.2) $$

where

$$ {}_{n}I_{k+1} = O_{k+1+n}\,e^{-n\delta} \qquad (1.1.3) $$

At the end of year $k + 1$, the theoretical amount on hand is

$$ {}_{n}A_{12/31/k+1} = O_{k+2}\,e^{-\delta/2} + \cdots + O_{k+1+n}\,e^{-(2n-1)\ \delta/2} \qquad (1.1.4) $$

that is, the reserve has rolled forward to the present value of outgoes for the n-years subsequent to 12/31/k+1 . This process continues in each year $k + j$, in accordance with (1.1.2), with $k + 1$ replaced by $k + j$, $j = 2, 3, \cdots$.

We have assumed a constant force of interest, but one can see that the roll-forward reserve process will also occur if specified varying forces of interest are utilized. That process is simply more complicated to express. The calculations in this paper have used the varying forces of interest obtained through data supplied by the Office of the Actuary.

1.2 Comparison of n-Year Roll-Forward Reserve Financing and Classical Pension Funding. This has been done already in Section 1.1 of [14] but some different emphases will be used here. As an alternative to classical pension funding, n-year roll-forward reserve funding can be employed for large public pension systems. We have seen that this would require projection, for a considerable term of years, of the annual outgoes of the pension system, the required annual incomes and the required year-end reserves sufficient to pay the outgoes for the subsequent n-years. This provides a modern, actuarial discipline for financing large public pension systems on a quite different basis than that of classical pension funding. What are the essential differences?

One basic difference is that in making actuarial valuations for a pension fund, the theory considers the closed group of participants on the valuation date, and traces their survival to the end of their lives. Under n-year roll-forward reserve financing, the theory considers the open group of present and future participants, and seeks to maintain year-end reserves equivalent to the outgoes for the subsequent n-years. For classical pension funding, one considers the participation of members on the valuation date during their remaining survival which may extend over many years. For n-year roll-forward reserve funding, one considers the open group of present and future participants only through a limited number of n-year terms. Both methods of financing are theoretical models, and may be adjusted regularly for gains and losses experienced.

Under classical pension funding, there may be the semblance of more equity for individuals, or groups of individuals, since contributions on their behalf may be accumulated for many years before being paid out in benefits. With n-year roll-forward reserve financing, if n is relatively small, contributions to the system may be paid out in benefits soon after receipt. In such cases, equity depends on the laws and regulations covering the pension system, and on the service records and collective bargaining agreements, rather than on the accumulation of individual amounts in the pension fund.

Note the phrase, "semblance of more equity," as a characteristic of classical pension funding. By spreading accrued liabilities over a fixed or moving term of years, and adjusting benefits periodically to keep up with inflation, the individual equity of classical pension funding may be more apparent than real. On the other hand, since the required annual incomes under n-year roll-forward reserve financing may increase (as percents of payroll), from year to year, a smoothing to a level percent of increasing payrolls may be required. This can be accomplished by linking a number of n-year terms together to form a longer term of m years, where m is a multiple of n. Such a process will be discussed in Part III of this paper.

Under classical pension funding, there is a regular (often yearly) adaptation to experience, and the initial closed group is adjusted to the participants on the new valuation date. For n-year roll-forward reserve financing, there may be yearly adjustments to experience, and possibly the m-years of level percent funding may be adjusted as the experience develops. Thus, classical pension funding, and n-year roll-forward reserve financing, have a number of common ideas, but generally classical pension funding will result in a much larger fund than would n-year roll-forward reserve funding (see [3]).

The fund accumulated under classical pension funding may be needed to permit settlement of accrued benefits in case the pension system is to be terminated. Although a considerable fund (and, in case of OASDI, a massive fund) develops under n-year roll-forward reserve financing, nevertheless it may fall far short of providing adequate settlements in case of system termination. Thus, n-year roll-forward reserve financing needs some assurance of indefinite continuation of the pension system. This may be forthcoming in large public systems with benefits that do not exceed financial capacity.

Our last paragraph has touched on the scope of application of n-year roll-forward reserve financing. The inference is that such financing is inappropriate unless the benefit system is assured of indefinite continuation. However, a very large national system, such as OASDI is not well served by classical pension funding because of the huge funds that would develop from long-term accumulations, [3]. A semi-private system, such as Teachers Insurance and Annuity System - College Retirement Equities Fund, devoted to benefits for higher education staff, may approach the borderline between n-year roll-forward reserve financing, and classical pension funding. At present, it is mainly under the latter.

One difficulty with a literal application of n-year roll-forward reserve financing, is that the required annual incomes, ${}_nI_k$, may increase from year to year as percents of the increasing payrolls. To meet this problem, we have developed in Part III, equivalent sets of payments by means of a level percent of the payrolls $W_k, W_{k+1}, \cdots, W_{k+m-1}$ for a fixed term of m years. This has been done for $m = 4, 8, 12, 16$; for $n = 1, 2$; and for various starting years k. Thus, a number of financing options for fixed terms are presented.

We also considered spreading the present value requirement over moving terms of m years. We then found our guiding theoretical model to be inadequate as it does not consider the growth of the present value requirement in future years. A next major step will be to utilize a more dynamic model involving growth of the required incomes, taxable payrolls and interest earnings. This should shed new information on the "summarized cost rates" appearing in the Reports of the OASDI Trustees.

II. DATA FROM 1993 AND 1994 PROJECTIONS SOLUTIONS FOR "ROLLER-COASTER" FINANCING

2.1 Basic Data Derived from 1993 and 1994 Projections. In Section 1.1 we have introduced the notations; k , for calendar year; W_k , for effective taxable payroll in year k ; O_k , for outgo in year k ; ${}_nI_k$, for the required new money income in year k ; and ${}_nA_{12/31/k}$, for the required reserve at the end of year k . The latter two symbols are in regard to n-year roll-forward reserve financing. Especially when we consider the financing of OASDI we shall need the concept of effective taxable payroll, W_k, in calendar year k (See Taxable Payroll in [2], Glossary).

Through the very helpful co-operation of the Office of the Actuary, we received in 1993 and 1994, year-by-year projected values of W_k and O_k . Also, we were furnished values of the projected annual interest rates, which we interpreted as effective annual rates, from which we calculated the corresponding forces of interest, δ_k . These data are summarized in TABLE 2.1.1.

From the data furnished by the Office of the Actuary, we computed values of ${}_nI_k$, $100\left({}_nI_k/W_k\right)$, and ${}_nA_{12/31/k}$, for $n = 1$ and $n = 2$. The computations were based on formulas (1.1.1) and (1.1.3), modified when necessary to take account of varying forces of interest. The results are summarized in TABLE 2.1.2.

It is to be noted from TABLE 2.1.1, that the 1993 projected effective taxable payroll rises from 2,817 billions in 1994 to 36,369 billions in 2042. In the 1994 projections, the rise is from 2,790 billions in 1994 to 33,963 billions in 2142. The 1994 projected taxable payrolls are lower than those projected in 1993, and this has considerable impact on the inferences drawn from the projections (See [2] and [17]).

Table 2.1.1

Summary Comparison of δ_k, W_k, O_k for 1993, 1994 Projections, Alternative II
(Dollar Figures in Billions)

Year k	Force of Interest δ_k		Effective Taxable Payroll W_k		Outgo O_k	
			1993	1994	1993	1994
1994	7.60%	7.6048%	2,817	2,790	326	325
1998	6.62%	6.7006%	3,488	3,418	404	401
2002	6.37%	6.3810%	4,375	4,247	508	503
2006	6.23%	6.2580%	5,519	5,336	639	635
2010	6.20%	6.1960%	6,914	6,667	815	818
2014	6.20%	6.1960%	8,585	8,251	1,069	1,084
2018	6.20%	6.1960%	10,581	10,121	1,425	1,452
2022	6.20%	6.1960%	12,978	12,356	1,893	1,922
2026	6.20%	6.1960%	15,897	15,072	2,478	2,500
2030	6.20%	6.1960%	19,535	18,446	3,174	3,177
2034	6.20%	6.1960%	24,066	22,644	3,569	3,963
2038	6.20%	6.1960%	29,614	27,768	4,924	4,850
2042	6.20%	6.1960%	36,369	33,963	6,036	5,911

Table 2.1.2

Summary Comparisons of $_nI_k$, $100(_nI_k/W_k)$, $_nA_{12/31/k}$ for n=1,2 and for 1993, 1994 Projections
Alternative II
(Dollar Figures in Billions)

Year k	Required Annual Incomes $_1I_k$ 1993	1994	Percent of Taxable Payroll $100(_1I_k/W_k)$ 1993	1994	Required Reserve at 12/31/k $_1A_{12/31/k}$ 1993	1994
1994	319	318	11.31	11.39	331	330
1998	400	396	11.46	11.60	413	410
2002	505	500	11.55	11.77	522	516
2006	636	633	11.52	11.86	656	653
2010	818	823	11.83	12.34	843	849
2014	1,079	1,096	12.57	13.28	1,113	1,130
2018	1,440	1,466	13.61	14.49	1,485	1,512
2022	1,906	1,933	14.69	15.64	1,966	1,994
2026	2,484	2,501	15.62	16.59	2,562	2,580
2030	3,165	3,162	16.20	17.14	3,265	3,261
2034	3,960	3,922	16.46	17.32	4,085	4,045
2038	4,871	4,789	16.45	17.25	5,024	4,940
2042	5,971	5,841	16.42	17.20	6,159	6,025

Year k	Required Annual Incomes $_2I_k$ 1993	1994	Percent of Taxable Payroll $100(_2I_k/W_k)$ 1993	1994	Required Reserve at 12/31/k $_2A_{12/31/k}$ 1993	1994
1994	313	312	11.11	11.18	657	654
1998	397	392	11.38	11.48	824	816
2002	502	498	11.48	11.73	1,040	1,030
2006	634	633	11.49	11.86	1,310	1,306
2010	822	830	11.89	12.44	1,691	1,705
2014	1,089	1,108	12.69	13.43	2,237	2,273
2018	1,454	1,479	13.74	14.61	2,985	3,038
2022	1,917	1,941	14.77	15.71	3,944	3,996
2026	2,485	2,497	15.63	16.57	5,125	5,156
2030	3,153	3,143	16.14	17.04	6,517	6,503
2034	3,924	3,877	16.30	17.12	8,133	8,045
2038	4,816	4,728	16.26	17.03	9,991	9,817
2042	5,909	5,773	16.25	17.00	12,253	11,979

The outgo values in TABLE 2.1.1 are closely tracked by the required annual incomes, ${}_1I_k$ and ${}_2I_k$ shown in TABLE 2.1.2. These incomes intertwine in a rather complicated way. More insight on this intertwining may be gained from ([14], Figure 2.1.5).

The required percents of taxable payroll clearly indicate the impact of retirements from the "baby-boom" generation; such retirements beginning about year 2010. This shows the need for future increases of OASDI income to provide an appropriate balance between OASDI outgo and income.

Another comment is that the current OASDI Trust Funds (combined) now exceed the required year-end reserve, ${}_1A_{12/31/1994}$, of 1-year roll-forward reserve financing. It will not be until the year 2001 that the same will hold approximately for ${}_2A_{12/31/2001}$. The values of ${}_2A_{12/31/k}$ are approximately double the values for ${}_1A_{12/31/k}$. This is to be expected since ${}_2A_{12/31/k}$ is equivalent to the outgoes for two years, rather than the one year goal of ${}_1A_{12/31/k}$.

2.2 Solutions for "Roller-Coaster" Financing. Section 5.1 of [14] discusses how roll-forward reserve financing would solve the problem in current OASDI financing that, in 1993 estimates, a massive fund of almost five trillion dollars would develop in OASDI by year 2024, but then be exhausted rapidly by year 2036. The solution of this problem by n-year roll-forward reserve financing with $n = 1, 2, 3$ and 4 was considered. Since the combined OASDI Trust Funds now exceed, ${}_1A_{12/31/1993}$, 1-year roll-forward reserve financing could begin immediately. (See Figure 5.1 in [14]).
It was estimated that it could be continued possibly to year 2031 under present legislation. Then to maintain 1-year roll-forward reserve financing, the required (new money) contribution rate was estimated to be 16.30 percent of taxable payroll, while current legislation provides for

12.40 percent. An increase in the contribution rate would then be required, or other adjustments made.

Last year's Research Experience for Undergraduate students favored a solution which aims at providing 2-year roll-forward reserve financing. One practical method to reach this goal would be to wait until December 31, 2001, at which time ${}_{2}A_{12/31/2001}$ was estimated to be fully available from the fund developing under current legislation. This situation would hold until about year 2025, by which time ${}_{2}I_{k}$ would require more than the current 12.4 percent of effective taxable payroll. It was thought that a 2-year roll-forward reserve would instill more confidence in OASDI's continued ability to pay benefits than would a 1-year roll-forward reserve, and would provide Congress more time to make any adjustments agreed to be necessary.

What situations are estimated from 1994 data? In Figure 2.1, there appears a comparison of ${}_{1}A_{12/31/k}$ with the fund arising from current legislation. As of 12/31/1994, the OASDI combined Trust Funds are estimated to reach 431 billions, which is well in excess of ${}_{1}A_{12/31/1994}$. This favorable situation is estimated to continue until year 2024 (7 years earlier than estimated by the 1993 data). By 2024, the required percent of effective taxable payroll to maintain 1-year roll-forward reserve financing is estimated at 16.2 percent. Congressional action would then be necessary.

Figure 2.2 compares ${}_{2}A_{12/31/k}$ with the fund arising from current legislation for OASDI. The estimates are based on 1994 data. It is now estimated that ${}_{2}A_{12/31/2001}$ will be 19.4 billions in excess of the fund provided by current law. Some additional financing will be needed to close this gap, but this appears feasible (see Part III). Then 2-year roll-forward reserve financing could be continued from 2002 until 2017. Appropriate adjustments would then be necessary.

Figure 2.1

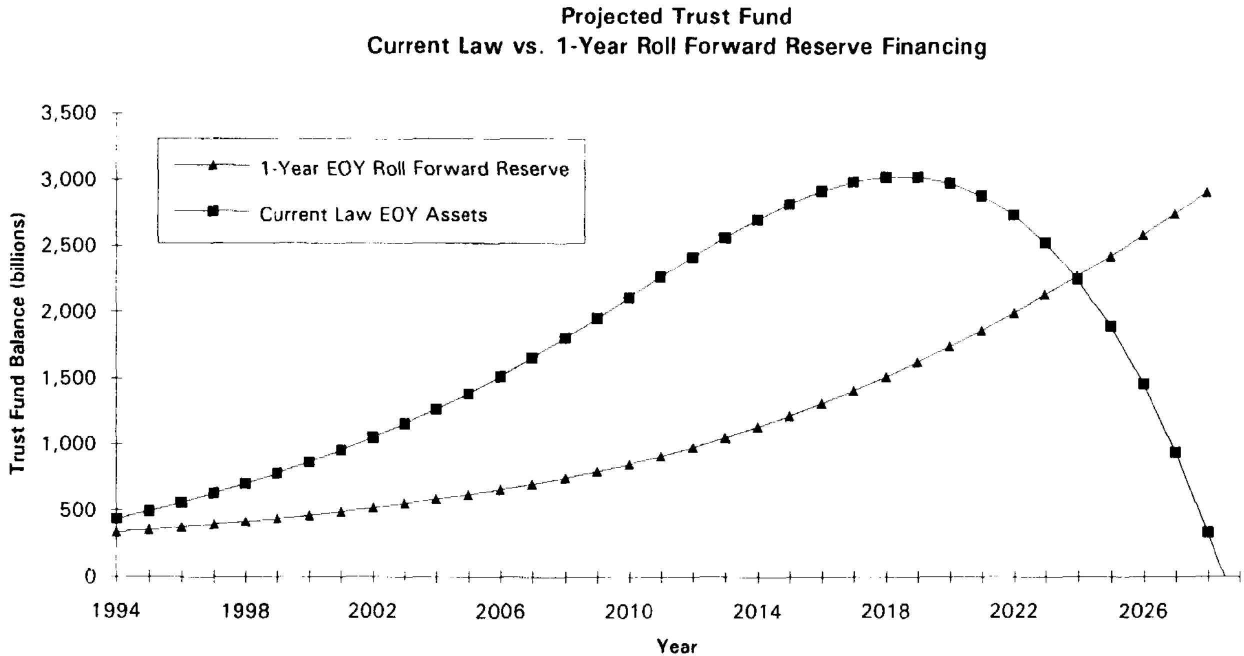
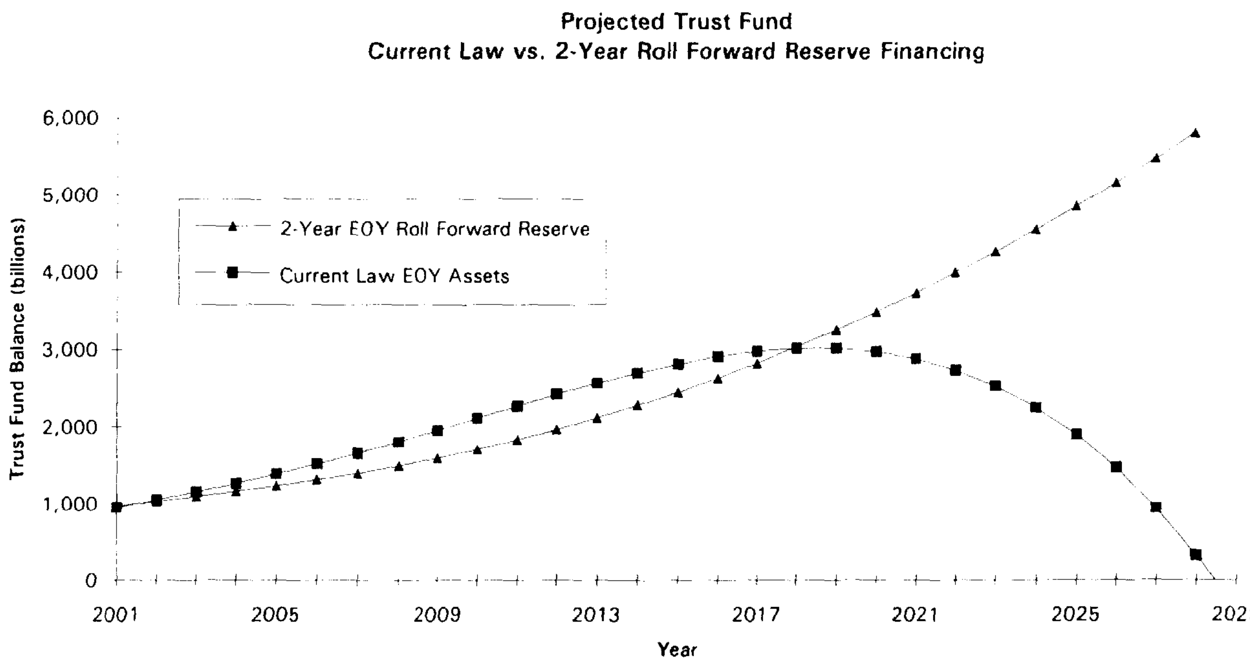

Projected Trust Fund
Current Law vs. 1-Year Roll Forward Reserve Financing
Trust Fund Balance (billions)
3,500
3,000
2,500
2,000
1,500
1,000
500
0
1-Year EOY Roll Forward Reserve
Current Law EOY Assets
1994
1998
2002
2006
2010
2014
2018
2022
2026
Year

Figure 2.2


Projected Trust Fund
Current Law vs. 2-Year Roll Forward Reserve Financing
6,000
5,000
4,000
3,000
2,000
1,000
0
Trust Fund Balance (billions)
2-Year EOY Roll Forward Reserve
Current Law EOY Assets
2001
2005
2009
2013
2017
2021
2025
2029
Year

III. APPLICATION OF n-YEAR ROLL-FORWARD RESERVE FINANCING TO OASDI

3.1 Introduction. Near the end of Section 1.2, it was indicated that a literal application of n-year roll-forward reserve financing to OASDI would yield required contribution rates, $100\left({}_nI_k/W_k\right)$, which would change from year to year. This could be unacceptable for the administration of OASDI, particularly in regard to handling the payroll taxes. To eliminate this difficulty, we have sought level percent (of taxable payrolls) rates for various m-year terms. An immediate question is whether the computation of such rates shall be for <u>fixed</u> terms of m-years, or should be recalculated at the beginning of each year, that is for <u>moving</u> terms which advance one year at the end of each year.

For the initial year, the computations, on a fixed term or moving term basis, are the same. But for large m, the level percent contribution applied to the first year's payroll, may provide for less than interest on the obligation. That obligation we have taken to be the present value at the beginning of the m-year term of the required annual incomes, ${}_nI_k$, during the term. As a consequence of less than interest being paid on the obligation, that obligation (in current year dollars) may increase. This would be manageable perhaps, on a fixed term basis, with larger incomes in the later years offsetting the interest deficits in the early years. On a moving term basis, with m large, an interest deficit is repeated each year, and the obligation (in current year dollars) increases indefinitely. For that reason, we have chosen to do most of our computations on a fixed term basis, and will return to the implications of a moving term later.

We then encounter a new problem. If level contribution rates are determined for successive m-year terms, then at the junction point between such terms there may be a considerable difference in the required rates.

To consider this problem, we have computed the level percent rates required for fixed terms of 4, 8, 12 and 16 years, and have computed what differences occur at the junction points. This has been done for 1993 and 1994 data, and for $n = 1$ and 2 .

3.2 Level Percent Contribution Rates for the $n = 1$ Case. We start with the present values at the beginning of each block of m-years. These present values are denoted by

$_1I(k, k+m-1)$ = Present value at 1/1/k of the required annual incomes $_1I_{k+j}$, $j = 0, 1, \ldots, m - 1$ for the m-year term consisting of calendar years $k, k + 1, \ldots, k + m - 1$.

$W(k, k+m-1)$ = Present value at 1/1/k of the effective taxable payrolls W_{k+j}, $j = 0, 1, \ldots, m - 1$ for the m-year term consisting of the calendar years $k, k + 1, \ldots, k + m - 1$.

From these, we calculate the level percent contribution rates, namely

$$_1r(k, k + m - 1) = 100\ [\,_1I(k, k + m - 1)/W\ (k, k + m - 1)] \qquad (3.2.1)$$

Results are tabulated in TABLE 3.2.1 for 1994 data.

It should be noted that as of 1/1/94, the combined OASDI Trust Funds stood at 378 billions, which is considerably in excess of $_1A_{1/1/1994} = 314$ billions, as computed in ([14], TABLE OASDI-D-1). Thus 1-year roll-forward reserve financing could begin immediately. In the early years of an m-year level contribution rate term, the income provided thereby will exceed the required annual income $_1I_{k+j}$, and in the later years, will be less. The reserve at $1/1/k + m$ aims to be sufficient to provide O_{k+m}.

Note also the differences in rates at the junction points. These differences will be an important consideration in selecting what fixed term of m-years should be utilized.

Table 3.2.1

Computation of m-Year Level Contribution Rates
To Provide the Required Annual Incomes for 1-Year Roll Forward Reserve Financing of OASDI
Based on the Alternative II Projections for the 1994 Report of the OASDI Trustees
(In Billions)

Valuation Date 1/1/k	m=4 $_1I$ (k, k+3)	m=4 W (k, k+3)	Level Percent 100 ($_1I$/W)	Difference in rates at Junction Points
1994	1,193	10,392	11.48	
1998	1,514	12,984	11.66	0.19
2002	1,924	16,291	11.81	0.15
2006	2,461	20,497	12.01	0.19
2010	3,234	25,544	12.66	0.65
2014	4,321	31,506	13.72	1.05
2018	5,755	38,557	14.92	1.21
2022	7,537	47,042	16.02	1.10
2026	9,664	57,440	16.83	0.80
2030	12,136	70,428	17.23	0.41
2034	14,946	86,370	17.31	0.07
2038	18,227	105,874	17.22	-0.09

Valuation Date 1/1/k	m=8 $_1I$ (k, k+7)	m=8 W (k, k+7)	Level Percent 100 ($_1I$/W)	Difference in rates at Junction Points
1994	2,328	20,128	11.57	
2002	3,835	32,201	11.91	0.34
2010	6,607	50,134	13.18	1.27
2018	11,637	75,272	15.46	2.28
2026	19,136	112,407	17.02	1.56
2034	29,172	169,002	17.26	0.24

Valuation Date 1/1/k	m=12 $_1I$ (k, k+11)	m=12 W (k, k+11)	Level Percent 100 ($_1I$/W)	Difference in rates at Junction Points
1994	3,439	29,528	11.64	
2006	7,613	59,589	12.78	1.13
2018	17,524	110,262	15.89	3.12
2030	34,904	202,331	17.25	1.36

Valuation Date 1/1/k	m=16 $_1I$ (k, k+15)	m=16 W (k, k+15)	Level Percent 100 ($_1I$/W)	Difference in rates at Junction Points
1994	4,541	38,709	11.73	
2010	13,696	95,986	14.27	2.54
2026	36,906	215,356	17.14	2.87

Table 3.2.2

Level m-Year Contribution Rates
To Provide the Required Annual Incomes for 1-Year Roll Forward Reserves Financing of OASDI
Based on Alternative II Projections for the 1993 and 1994 Reports of the OASDI Trustees

	m=4		m=8		m=12		m=16	
Calendar Year	**1993 Data**	**1994 Data**	**1993 Data**	**1994 Data**	**1993 Data**	**1994 Data**	**1993 Data**	**1994 Data**
1994	11.35	11.48	11.43	11.57	11.47	11.64	11.50	11.73
1995	│	│	│	│	│	│	│	│
1996	│	│	│	│	│	│	│	│
1997	↓	↓	│	│	│	│	│	│
1998	11.52	11.66	│	│	│	│	│	│
1999	│	│	│	│	│	│	│	│
2000	│	│	│	│	│	│	│	│
2001	↓	↓	↓	↓	│	│	│	│
2002	11.54	11.81	11.57	11.91	│	│	│	│
2003	│	│	│	│	│	│	│	│
2004	│	│	│	│	│	│	│	│
2005	↓	↓	│	│	↓	↓	│	│
2006	11.60	12.01	│	│	11.88	12.78	│	│
2007	│	│	│	│	│	│	│	│
2008	│	│	│	│	│	│	│	│
2009	↓	↓	↓	↓	│	│	↓	↓
2010	12.07	12.66	12.50	13.18	│	│	13.47	14.27
2011	│	│	│	│	│	│	│	│
2012	│	│	│	│	│	│	│	│
2013	↓	↓	│	│	│	│	│	│
2014	12.94	13.72	│	│	│	│	│	│
2015	│	│	│	│	│	│	│	│
2016	│	│	│	│	│	│	│	│
2017	↓	↓	↓	↓	↓	↓	↓	↓

In TABLE 3.2.2, a comparison is given for 24 years of the level contribution rates for $m = 4, 8, 12$ and 16, and $n = 1$, based on 1993 data and 1994 data. This TABLE 3.2.2 may provide further insight into how to select the fixed term m.

3.3 Level Percent Contribution Rates for the $n = 2$ Case. Again, we start with the present values at the beginning of each block of m-years. These present values are denoted by

$_2I(k, k + m - 1)$ = Present value at 1/1/k of the required annual incomes $_2I_{k+j}$, $j = 0, 1, \ldots, m - 1$ for the m-year term consisting of the calendar years $k, k + 1, \ldots, k + m - 1$.

$W(k, k + m - 1)$ has been defined already in Section 3.2 and is unchanged as we go from $n = 1$ to $n = 2$.

From these present values, we calculate the level percent contribution rate, namely

$$_2r(k, k + m - 1) = 100\ [_2I(k, k + m - 1)/W(k, k + m - 1)] . \qquad (3.3.1)$$

Results are tabulated in TABLE 3.3.1.

Note that it is not until 1/1/2002 that the assets under current law approximate the required reserve $_2A_{1/1/k}$. In fact, the fund projected to be on hand, 951,806 (millions), is 19,390 (millions) less than $_2A_{1/1/2002}$ = 971,196 (millions.) To cover this gap between the reserve required to be on hand and the actual estimated fund, we add the 19,390 million to the present value $_2I$ (2002, 2002 + m - 1) to be spread over the payrolls of the first m-years. The adjustment, 19,390, is relatively small in comparison to $_2I(2002, 2002 + m - 1)$. The question arises why not start 2-year roll forward financing earlier than year 2002! The answer is you could, by adding the appropriate adjustment to the initial required present

Table 3.3.1

Computation of m-Year Level Contribution Rates
To Provide the Required Annual Incomes for 2-Year Roll Forward Reserve Financing of OASDI
With a One-Time Adjustment to $_2I$ (k, k+(m-1)) in 2002
Based on the Alternative II Projections for the 1994 Report of the OASDI Trustees
(In Billions)

Valuation Date 1/1/k	m=4 $_2I$ (k, k+3)	m=4 W (k, k+3)	Level Percent 100 ($_2I$/W)	Difference in rates at Junction Points
2002	1,936	16,291	11.88	
2006	2,470	20,497	12.05	0.17
2010	3,266	25,544	12.78	0.73
2014	4,369	31,506	13.87	1.08
2018	5,796	38,557	15.03	1.17
2022	7,556	47,042	16.06	1.03
2026	9,632	57,440	16.77	0.71
2030	12,038	70,428	17.09	0.32
2034	14,768	86,370	17.10	0.01
2038	18,003	105,874	17.00	-0.09
2042	22,024	129,409	17.02	0.01
2045	27,102	157,808	17.17	0.16

Valuation Date 1/1/k	m=8 $_2I$ (k, k+7)	m=8 W (k, k+7)	Level Percent 100 ($_2I$/W)	Difference in rates at Junction Points
2002	3,853	32,201	11.96	
2010	6,675	50,134	13.31	1.35
2018	11,693	75,272	15.53	2.22
2026	19,027	112,407	16.93	1.39
2034	28,820	169,002	17.05	0.13
2042	43,176	252,575	17.09	0.04

Valuation Date 1/1/k	m=12 $_2I$ (k, k+11)	m=12 W (k, k+11)	Level Percent 100 ($_2I$/W)	Difference in rates at Junction Points
2002	5,829	47,663	12.23	
2014	13,495	90,255	14.95	2.72
2026	28,023	165,020	16.98	2.03
2038	51,702	303,005	17.06	0.08

Valuation Date 1/1/k	m=16 $_2I$ (k, k+15)	m=16 W (k, k+15)	Level Percent 100 ($_2I$/W)	Difference in rates at Junction Points
2002	7,893	62,547	12.62	
2018	23,283	143,746	16.20	3.58
2034	55,121	322,860	17.07	0.88

Table 3.3.2

Level m-Year Contribution Rates
To Provide the Required Annual Incomes for 2-Year Roll Forward Reserves Financing of OASDI
Based on Alternative II Projections for the 1993 and 1994 Reports of the OASDI Trustees

	m=4		m=8		m=12		m=16	
Calendar Year	**1993 Data**	**1994 Data**	**1993 Data**	**1994 Data**	**1993 Data**	**1994 Data**	**1993 Data**	**1994 Data**
2002	11.47	11.88	11.54	11.96	11.74	12.23	12.06	12.62
2003								
2004								
2005	↓	↓						
2006	11.61	12.05						
2007								
2008								
2009	↓	↓	↓	↓				
2010	12.16	12.78	12.61	13.31				
2011								
2012								
2013	↓	↓			↓	↓		
2014	13.07	13.87			14.08	14.95		
2015								
2016								
2017	↓	↓	↓	↓			↓	↓
2018	14.13	15.03	14.61	15.53			15.29	16.20
2019								
2020								
2021	↓	↓						
2022	15.12	16.06						
2023								
2024								
2025	↓	↓	↓	↓	↓	↓	↓	↓

value, but the impact on ${}_2r(k, k + 3)$, for example, would be considerable if k is much less than 2002.

The differences in the rates at the junction points, as tabulated in TABLE 3.3.1, should be considered in the selection of the term m , if 2-year roll-forward reserve financing of OASDI is undertaken.

In TABLE 3.3.2, a comparison is given for 24 years of the level contribution rates for terms of 4, 8, 12 and 16 years, based on 1993 data and 1994 data, for the 2-year roll-forward reserve financing of OASDI.

3.4 Operation of a Level Percent Contribution Rate. In Section 3.2, we set up a process by (3.2.1) to provide, by a level percent contribution rate, ${}_1r(k, k + m - 1)$ applied to the payrolls W_{k+j}, $j = 0, 1, 2, \ldots, m - 1$, for the required annual incomes, ${}_1I_{k+j}$, in those m years if 1-year roll-forward reserve financing is to operate. We illustrate this process for the years 2010, 2011, ..., 2025, using 1994 data, by the computations in TABLE 3.4.1. From TABLE 3.2.1 we have ${}_1r(2010, 2025) = 14.27\%$. Since year-by-year required contribution rates for the years 2010, 2011, ..., 2025 are an increasing series, a level percent contribution rate for the m years, will produce more than the required annual incomes, ${}_2I_{2010+j}$, in the early years, and less than the required annual incomes in the later years. The accumulated excess funds in the early years are gathered into a supplementary fund which will be depleted by the deficits in the later years. If F_k denotes the supplementary fund at the end of year k , we have

$$F_k = F_{k-1}\, e^{\delta} + \left[{}_1r(2010,\ 2025)\ W_k - {}_1I_k\right] e^{\delta/2}. \qquad (3.4.1)$$

This can be checked, except for rounding differences, in TABLE 3.4.1 . One notes, as it should, that F_k depletes to 0 by the end of year 2025. However, the Total Reserve is aimed to equal ${}_1A_{12/31/2025}$, and is planned to

Table 3.4.1

Illustration of Operation of Level Percent Contribution Rate
n=1, m=16, δ=.0619602, ${}_1r_{(2010, 2025)}$=14.27% with 1994 data

(1) Calendar Year k	(2) W_k	(3) ${}_1r_{(2010, 2025)} * W_k$	(4) ${}_1I_k$	(5) $[(3) - (4)] * e^{\delta/2}$	(6) Supplementary Fund $F_k = F_{k-1} * e^{\delta} + (5)$	(7) Required Reserve ${}_1A_{12/31/k}$	(8) Total Reserve (6) + (7)
2010	6,667	951	823	132	132	849	981
2011	7,042	1,005	883	126	267	910	1,177
2012	7,427	1,060	948	115	399	978	1,377
2013	7,829	1,117	1,019	101	526	1,051	1,577
2014	8,251	1,177	1,096	84	644	1,130	1,774
2015	8,689	1,240	1,179	63	748	1,216	1,964
2016	9,153	1,306	1,269	39	834	1,309	2,143
2017	9,626	1,373	1,364	9	897	1,407	2,304
2018	10,121	1,444	1,466	-23	931	1,512	2,444
2019	10,642	1,518	1,573	-57	934	1,623	2,557
2020	11,188	1,596	1,687	-94	900	1,740	2,641
2021	11,757	1,678	1,807	-133	825	1,864	2,688
2022	12,356	1,763	1,933	-175	702	1,994	2,696
2023	12,984	1,853	2,066	-220	528	2,131	2,658
2024	13,644	1,947	2,205	-267	295	2,275	2,569
2025	14,340	2,046	2,350	-314	0	2,424	2,424

provide a smooth start for 1-year roll-forward reserve financing for the following term of 16 years, namely (2026, 2041).

3.5 Level Percent Contribution Rates Computed on a Moving Term Basis. For some years, we have been concerned about the implication of discharging present value obligations by a level percent contribution rate levied against increasing payrolls. One immediate question is whether the level percent rate is levied for a fixed term of m years, or for a moving term of m years which advances one year at the end of each year. Another is whether interest is considered in terms of the current year's dollars, or in terms of abstract "real dollars" which adjust to inflation in some way. We are indebted to Alan Sonnanstine, A.S.A., for sharing illustrations with us of the moving term case. These have assisted in clarifying our thinking on the problem, and will continue to do so.

The issues are quite mathematical, and will be pursued in more depth in the Appendix. Here we will say that if m is much beyond 15, a level percent contribution rate on increasing payrolls may produce less than required interest (in current year's dollars). If a moving term is used, the obligation (in current year's dollars) may increase indefinitely. On the other hand, if the process yields at least interest in constant dollars on the obligation, there may be some decrease in the obligation in terms of such dollars.

A disadvantage of using a fixed term is that at junction points of successive terms there may be a considerable difference in the contribution rates. An advantage is that it may be more easily understood. A disadvantage of moving terms for OASDI is that it produces a year-by-year series of varying rates to which the benefit system must accommodate.

Table 3.5.1

Moving Term Contribution Rates
To Provide 1-Year Roll Forward Financing of OASDI
For a 16-Year Term
Based on Alternative II Projections for the 1994 Report of the OASDI Trustees
(In Billions)

Calendar Year (k)	$_{1}I$ (k, k+15)	W (k , k+15)	$_{1}r$ (k , k+15)
2010	13,696	95,986	14.27
2011	14,680	101,014	14.53
2012	15,725	106,272	14.80
2013	16,831	111,782	15.06
2014	17,999	117,563	15.31
2015	19,229	123,628	15.55
2016	20,522	130,002	15.79
2017	21,877	136,699	16.00
2018	23,294	143,746	16.20
2019	24,772	151,162	16.39
2020	26,311	158,963	16.55
2021	27,913	167,180	16.70
2022	29,578	175,837	16.82
2023	31,308	184,959	16.93
2024	33,105	194,576	17.01
2025	34,969	204,698	17.08
2026	36,906	215,356	17.14

This latter point can be tested for OASDI financing by computation of the moving term, m-year level percentages. An example is displayed in TABLE 3.5.1.

There, values of ${}_1I(k, k+15)$, $W(k, k+15)$ and ${}_1r(k, k+15)$ are shown for moving 16-year terms, beginning with the year 2010. Thus, for example, for the 16-year term (2015, 2030), the contribution rate is 15.55%; for the term (2016, 2031), it is 15.79% ; and for the term (2017, 2032), it is 16.00% .

The case illustrated in TABLE 3.5.1 is a relatively favorable one, with $m = 16$, so that interest (in current year dollars) is paid on the obligation, ${}_1I(k, k+15)$. The process does <u>not</u> provide level percent contribution rates from year to year.

IV. FURTHER STUDIES TO BE MADE

The financing of Social Security and public employee retirement systems is an inexhaustible subject that should have much further study. We hope that the University of Michigan faculty and actuarial students will undertake such studies on a long-term basis. Six projects come immediately to mind:

1. A theoretical model, that we and others have been using as a guide considers a single present value liability to be discharged by level percent contributions of increasing payrolls over an extended term. The model incorporates a constant rate of interest and a constant rate of growth of payrolls. Somewhat predictably, if the level percent contributions are determined for m-years at the beginning of each year, on the basis of the outstanding original liability, the contribution rates will tend to decrease, even when m is large. The model is inadequate as it does not allow for the growth of the liability but considers only the initial single liability. A rate of growth of the liabilities stream should be included in the model, in addition to the assumed constant rates of interest and of payroll growth. Thereby, the theoretical model becomes more complicated and will permit much exploration, including that of conditions leading to stabilization of the whole system. Fortunately, OASDI projects streams of outgo and of taxable payroll which can be studied readily alongside such a theoretical model for OASDI.

2. In 1994, our studies concentrated on the intermediate Alternative II set of assumptions. There may be reason to pursue such studies on the optimistic Alternative I assumptions, and on the pessimistic Alternative III assumptions. At some time in the future, the possible variance from Alternative II assumptions may be examined by more sophisticated means.

3. The time is ripening for widespread application of n-year roll-forward reserve financing of large public benefit systems in well-developed economies. In particular, such financing may find a role in health insurance programs. We believe it may also have a role for pension benefits. However, there are deeply entrenched concepts of classical pension funding which will take at least a generation-length of education, research and selling efforts to overcome. There is much simplicity, flexibility, and power in the concept of n-year roll-forward financing, and that concept may eventually prevail for large public systems.

4. The role to be played by n-year roll-forward reserve financing in developing and immature economies, can be another fertile field for actuarial study.

5. The adaptation of n-year roll-forward reserve financing to assist the stabilization of disability insurance systems is a challenge for actuaries interested in such an application of the concept.

6. Finally, we hope that the University of Michigan faculty and students, and others, will consider thoroughly the key role played by the "summarized cost rate" in Reports of the Board of Trustees of OASDI. What are suitable limits for the concept, and what can n-year roll-forward reserve financing ideas do to assist in defining such limits?

A future study, not yet called for, would indicate what could happen to pension funding and Social Security during and after a hyper-inflation. A 1977 paper by J. R. Trowbridge [16] on Assessmentism, (a funding method then used in France), should be consulted for insights on these matters.

Some background for these projects is given in the Appendix.

APPENDIX

In this Appendix, which is addressed to actuaries, we shall use the term "amortization". The usual actuarial sense of the term is that a series of payments are involved, which in some sense are level, and such that each payment is used initially to provide interest on the outstanding principal, with the balance of the payment providing for some repayment of principal. If the payment does not suffice to pay all required interest, it is assumed that the balance can be borrowed at the valuation rate, and the outstanding principal is increased by the amount borrowed.

A.1 Amortization Over a Fixed Term. We consider that there is a present value, A_k, at time 1/1/k, to be discharged by a level percent of payrolls W_k, W_{k+1}, ... W_{k+m-1} over the fixed term of calendar years k, k + 1, ..., k + m - 1 . The required payments are assumed to be made at the middle of the year (both for income and for outgo). It is further assumed that interest is constant at force δ , and payrolls are growing at rate τ , so that $W_{k+j} = W_k e^{\tau j}$. Our initial equation is

$$A_k = {}_n r(k, k + m - 1) e^{-\delta/2} W_k \ddot{a}_{\overline{m}|\delta-\tau}, \qquad \text{(A1)}$$

where r(k, k + m - 1) denotes the level percent rate, and the payment in calendar year k + j is

$${}_n r(k, k + m - 1) W_{k+j},$$

and has value at time 1/1/k equal to

$${}_n r(k, k + m - 1) e^{-\delta/2} W_k e^{-(\delta-\tau)j}.$$

By summing this relation over the years k + j, j = 0, 1, 2, ..., m - 1 ; one obtains the amortization equation (A1).

The amortization payment at the middle of year k (when j = 0) is then

$${}_n r(k, k + m - 1) W_k = \frac{A_k e^{\delta/2}}{\ddot{a}_{\overline{m}|\ \delta-\tau}} \qquad \text{(A2)}$$

or equivalently

$$\frac{A_k e^{\delta}}{\ddot{a}_{\overline{m}|\,\delta-\tau}} \tag{A3}$$

at the end of year k.

From now on, we shall assume that annuity-values are calculated at force of interest $\delta - \tau$, and shall omit $\delta - \tau$ from the annuity-symbol.

The outstanding principal, A_{k+1}, at time 1/1/(k + 1) is given retrospectively by

$$A_{k+1} = A_k e^{\delta} - {}_n r(k, k + m - 1)\, W_k\, e^{\delta/2}$$

$$= {}_n r(k, k + m - 1)\, e^{-\delta/2}\, W_k\, \ddot{a}_{\overline{m}|}\, e^{\delta} - {}_n r(k, k + m - 1)\, W_k\, e^{\delta/2}$$

$$= {}_n r(k, k + m - 1)\, e^{\delta/2}\, W_k\, [\ddot{a}_{\overline{m}|} - 1]. \tag{A4}$$

Then from (A1) and (A4)

$$\frac{A_{k+1}}{A_k} = e^{\delta}\left[1 - \frac{1}{\ddot{a}_{\overline{m}|}}\right]. \tag{A5}$$

This is a key formula for fixed term amortization. It may be shown that

$$e^{\delta}\left[1 - \frac{1}{\ddot{a}_{\overline{m}|}}\right] \lesseqgtr 1 \tag{A6}$$

according as

$$d = (1 - e^{-\delta}) \lesseqgtr \frac{1}{\ddot{a}_{\overline{m}|}} \tag{A7}$$

This relation (A7) requires that the amortization factor $1/\ddot{a}_{\overline{m}|}$ (which is calculated at force $\delta - \tau$) at least equals the interest-in-advance rate $d = 1 - e^{-\delta}$ if A_{k+1} is to be less than A_k. This may limit the size of m. Under current assumptions, m must be less than 20 if $\frac{1}{\ddot{a}_{\overline{m}|}}$ is to equal or exceed d. Keep in mind that $\ddot{a}_{\overline{m}|}$ is based on the force $\delta - \tau$, not δ.

The foregoing is in the context of a theoretical model with constant force of interest, δ ; and payrolls, $W_{k+j} = W_k e^{\tau j}$, increasing at constant rate, τ. When applied to OASDI, we seek to provide the required annual incomes, ${}_nI_k$, for a term of m-years by means of a level percent applied to the increasing payrolls, W_{k+j} , $j = 0, 1, 2, \ldots, m - 1$. The initial present value is then ${}_nI(k, k + m - 1)$ to be amortized by the level percent

$${}_nr(k, k + m - 1) = 100 \, [{}_nI(k, k + m - 1)/W(k, k + m - 1)] \, ,$$

applied to the payrolls in the m-year term. A question arises as to how to define and compute the outstanding principal in this complex amortization, and what value would be obtained from such a computation. The analysis illustrated in Section 3.4 and TABLE 3.4.1 is informative, but does not directly involve the outstanding principal of the amortization. At any rate, we do not need, and do not express formulas for the outstanding principal in our fixed m-year term amortization, for calendar years $k + j$ with $j > 1$.

A.2 Amortization Over Moving Terms. A moving term amortization would involve a recomputation of the requirement at time 1/1/(k + 1). Such requirement will be denoted by $\tilde{A}_{k+1}$. The next step would be spreading of $\tilde{A}_{k+1}$ by a level percent of the payrolls W_{k+1} , W_{k+2}, ..., W_{k+m} .

We here meet a limitation of our general theoretical model — it provides little information on how $\tilde{A}_{k+1}$ relates to A_{k+1} where the latter is the outstanding principal at 1/1/(k + 1) under the original amortization of A_k .

As a first attempt to estimate the new amortization rate required, we substitute A_{k+1} for the unknown $\tilde{A}_{k+1}$, and study

$$A_{k+1} = {}_nr(k + 1, k + m) \, e^{-\delta/2} \, W_{k+1} \, \ddot{a}_{\overline{m}|} \tag{A8}$$

Comparing (A8) with (A1), we have

$$\frac{A_{k+1}}{A_k} = \frac{{}_n r(k+1,\ k+m)\ W_{k+1}}{{}_n r(k,\ k+m-1)\ W_k},$$

and from this and (A5) obtain

$$\frac{{}_n r(k+1,\ k+m)}{{}_n r(k,\ k+m-1)} = \frac{A_{k+1}}{A_k}\ \frac{W_k}{W_{k+1}}$$

$$= e^{\delta}\left(1 - \frac{1}{\ddot{a}_{\overline{m}|}}\right)\frac{1}{e^{\tau}} \qquad \text{(A9)}$$

But

$$e^{\delta-\tau}\left(1 - \frac{1}{\ddot{a}_{\overline{m}|}}\right)$$

$$= e^{\delta-\tau}\left(\frac{\ddot{a}_{\overline{m}|} - 1}{\ddot{a}_{\overline{m}|}}\right)$$

$$= e^{\delta-\tau}\frac{a_{\overline{m-1}|}}{\ddot{a}_{\overline{m}|}}$$

$$= \frac{\ddot{a}_{\overline{m-1}|}}{\ddot{a}_{\overline{m}|}} \qquad \text{(A10)}$$

when we remember all annuity-values are based on force $\delta-\tau$.

Thus, for the amortization, with A_{k+1} in place of $\tilde{A}_{k+1}$, we have

$$\frac{{}_n r(k+1,\ k+m)}{{}_n r(k,\ k+m-1)} = \frac{\ddot{a}_{\overline{m-1}|}}{\ddot{a}_{\overline{m}|}} \qquad \text{(A11)}$$

so that ${}_n r(k+1,\ k+m) < {}_n r(k,\ k+m-1)$ for finite values of m.

What happens in moving amortization operations based on current year dollars? Here, we define the outstanding principal, A_{k+j}, at the beginning of calendar year k+j, by the recursion relation

$$A_{k+j+1} = A_{k+j}e^{\delta} - \frac{A_{k+j}e^{\delta}}{\ddot{a}_{\overline{m}|}}$$

$$= A_{k+j}e^{\delta}\left(1 - \frac{1}{\ddot{a}_{\overline{m}|}}\right). \qquad \text{(A12)}$$

In (A12), we are thinking of the equivalent amortization payment $\frac{A_{k+j}e^{\delta}}{\ddot{a}_{\overline{m}|}}$ at the end of year k+j [see (A3)]. However, we have stated that $e^{\delta}\left(1 - \frac{1}{\ddot{a}_{\overline{m}|}}\right) \gtreqless 1$ according as $d = 1 - e^{-\delta} \gtreqless \frac{1}{\ddot{a}_{\overline{m}|}}$. Hence, from (A12) $A_{k+j+1} > A_{k+j}$ if the interest rate in advance, d, exceeds the amortization factor $\frac{1}{\ddot{a}_{\overline{m}|}}$. Under this assumption, the outstanding principals, A_{k+j}, $j = 0,1,2,3....$ form an increasing series in current dollars.

Our strategy here is to define first the outstanding principal (in current year dollars) for this moving term amortization by means of the recurrence relation (A12). This provides a familiar foundation, for proceeding to constant dollars.

What happens in operations based on constant dollars? Here, the rate of interest-in-advance, equivalent to the force $\delta-\tau$, is $1-e^{-(\delta-\tau)}$, and A_{k+j}, measured in constant dollars, becomes $A_{k+j}/e^{\tau j}$. We now have from (A12)

$$A_{k+j+1}/e^{\tau(j+1)} = (A_{k+j}/e^{\tau j})\ e^{\delta-\tau}\ (1 - \frac{1}{\ddot{a}_{\overline{m}|}}) \qquad \text{(A13)}$$

$$= (A_{k+j}/e^{\tau j})\ \frac{\ddot{a}_{\overline{m-1}|}}{\ddot{a}_{\overline{m}|}},\ \text{by (A10).}$$

The equation (A13) indicates that the outstanding principal at 1/1/(k+j+1), in constant dollars, equals the outstanding principal at

$1/I/(k+j)$ in constant dollars, reduced by the factor $e^{\delta-\tau}\left(1 - \frac{1}{\ddot{a}_{\overline{m}|}}\right) = \frac{\ddot{a}_{\overline{m-1}|}}{\ddot{a}_{\overline{m}|}}$.

Thus, here the $A_{k+j}/e^{\tau j}$ form a decreasing series. An interesting result is obtained by multiplying through (A13) by $[1 - e^{-(\delta-\tau)}]$, if $\delta>\tau$. This yields

$$[1 - e^{-(\delta-\tau)}](A_{k+j+1}/e^{\tau(j+1)}) = \frac{(A_{k+j}/e^{\tau j})}{\ddot{a}_{\overline{m}|}}[1 - e^{-(\delta-\tau)(m-1)}]$$

$$< \frac{(A_{k+j}/e^{\tau j})}{\ddot{a}_{\overline{m}|}}. \qquad \text{(A14)}$$

This shows that interest-in-advance on the outstanding principal (in constant dollars) at the beginning of year k+j+1 (end of year k+j) is less than the amortization payment in year k+j. Thus, some repayment of outstanding principal in terms of constant dollars actually occurs, even for large values of the amortization term of m years. In current year dollars, however, if m>20 the outstanding principal may increase indefinitely, which is a questionable situation.

The foregoing is on the assumption that $\delta>\tau$. If $\delta<\tau$, then

$$\ddot{a}_{\overline{m}|}=[1-e^{-(\delta-\tau)m}]/[1-e^{-(\delta-\tau)}]=[e^{(\tau-\delta)m}-1]/[e^{\tau-\delta}-1]=\ddot{s}_{\overline{m}|\tau-\delta} \qquad \text{(A15)}$$

so that, by (A13), $A_{k+j}/e^{\tau j}$, j=0,1,2,3... is a decreasing series. We have not pursued this matter further because of the limited nature of this special amortization process.

This relates to a problem with moving amortizations of the type described. The foregoing relations have considered moving amortization of a <u>single</u> present value, evaluated at the beginning of the initial calendar year. All subsequent present values depend on this initial one, and as one keeps spreading this obligation by the moving amortization, the outstanding principal in constant dollars will decrease. This process does not consider the changing stream of obligations that emerge in a dynamic system over a term of years. To test what may happen, we return to OASDI financing.

A.3 Moving Term Amortization Applied to OASDI Financing. The mechanics of applying moving term amortization to OASDI has been presented in Section 3.5, and an illustration thereof is given in TABLE 3.5.1. We shall not repeat the material of Section 3.5, but will add some comments on TABLE 3.5.1.

It is to be noted that the moving amortization rates increase from year-to-year, from 14.27 percent of taxable payroll in 2010 to 17.16 percent in year 2026. These are substantially in excess of the 12.40 percent now allotted to OASDI financing, and adjustments would be required. The percents form an increasing series, unlike what we have seen in (A11). This confirms the need for a more dynamic model than what we have been using for studying OASDI financing (see Part IV, project 1.)

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