An Investigation of Select Birth Cohorts*

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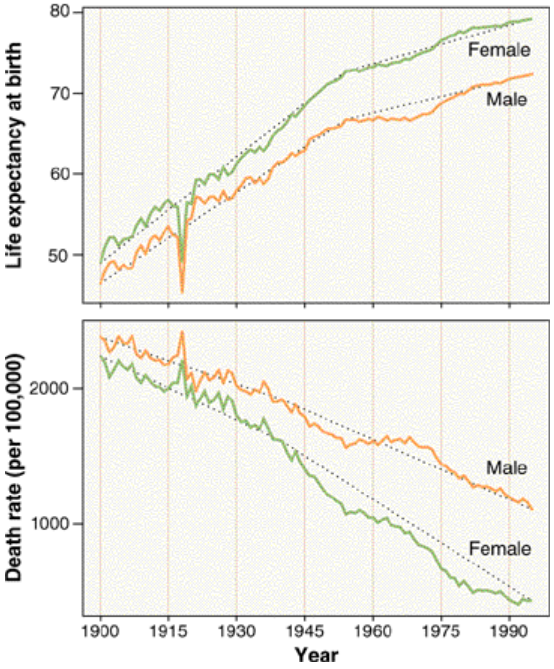
Abstract

The 20th century has witnessed unprecedented improvement in longevity in the United States, as well as worldwide, commonly termed (among actuaries) "mortality improvement." While this is quite an achievement for public health and public policy in general, it is also a source of risk for governments and corporations providing retirement benefits. In this work we ask whether *select birth cohorts* of unusually high improvements in longevity exist, where they exist and what their relationships may be in various countries. We begin by discussing criteria for defining such cohorts, and then we identify them from data. We then study the effect of these cohorts on the cost of retirement annuities and compare this effect to interest rates sensitivity.

Introduction

There has been an unprecedented improvement in longevity in the United States in the 20th century. The figure below shows both the increase in the life expectancy at birth and the reduction in the death rate over the past century in the United States.[1] Similar improvements occurred worldwide, with particularly large improvements in England and Wales and in Japan. Actuaries commonly call this process "mortality improvement."[2] While mortality improvements represent a significant achievement for public health and public policy, the improvements are also a source of fiscal risk for the governments and corporations providing retirement benefits.



The life expectancy at birth is a summary statistic, but it is not well-suited to provide insight into the fiscal risk faced by governments or the financial risk faced by insurers selling life annuities. The mortality rates for birth cohorts, however, are important in valuing life annuity contracts, and the aggregation of those mortality rates for the retired population is important in determining government burdens. Hence, the focus of this study will be the estimation and comparison of mortality rates for birth cohorts and the determinations of immediate annuity values and government burdens.

The first reorganization of time series data on age-specific mortality rates in a manner that allowed one to distinguish rates pertaining to persons born in the same year was probably due to Derrick (Derrick, 1927). Based on a graphical examination of the data, Derrick effectively argued that cohort rates provided a more consistent basis for projecting mortality than period rates. Subsequently, Kermack, McKendrick, and McKinlay (Kermack, McKendrick et al., 1934) provided a convincing demonstration of the power of the cohort method. "They . . . noted 'a general tendency for numbers of approximately the same magnitude to be arranged diagonally in the Tables . . . it is now to be noted that a diagonal line in the diagram represents the course of a group of people all born in a particular year'" (Hobcraft, Menken et al., 1982, p. 16).

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[1] The figure is reproduced from Wilmoth 1998.

[2] This phrase is used despite the inherent contradiction of this term and the actual nature of the process, which, we believe, should be termed "longevity improvement."

Despite the early work by Derrick and then by Kermack, et. al., mortality data has historically been examined in the context of an age-period-cohort model, and some controversy exists concerning the existence of the cohort effect. In spite of any controversy, the notion of cohort is certainly conceptually important if one believes that mortality-improving advances that occur at one point in the time continuum have different impacts on different age groups and that those impacts may persist and evolve in different ways. The transformation of the social, genetic, etc. world will impact people of different ages in different ways, and the resulting transformations may persist. Therefore, a cohort meaning is introduced in an age-time specification. Ryder captures this notion nicely by saying:

> ". . . transformations of the social world modify people of different ages in different ways; the effects of these transformations are persistent. In this way a cohort meaning is implanted in the age-time specification. Two broad orientations for theory and research flow from this position: first, the study of intra-cohort temporal development throughout the life cycle; second, the study of comparative cohort careers, i.e., inter-cohort temporal differentiation in the various parameters that may be used to characterize these aggregate histories." (Ryder, 1965, p. 861)

Hobcraft, et. al., also motivates cohorts by saying:

> "Cohort effects occur whenever the past history of individuals exerts an influence on their current behaviour in a way that is not fully captured by an age variable. If only events that occur prior to the initial observation influence cohort behaviour, then the linear model is appropriate. However, cohorts are continuously exposed to influences that affect their biological susceptibilities and social propensities. Obvious examples are wars and epidemics that may break out in the middle of a cohort's life and leave an imprint on all subsequent behaviour. If these disturbances affect all cohorts then alive in similar fashion, they can best be treated in the form of lagged period effects. But if, as seems more likely, their imprint is differentiated by age and becomes embodied in cohorts differentially, then a more complex form of cohort analysis is required." (Hobcraft, Menken et al., 1982, pp. 10-11)

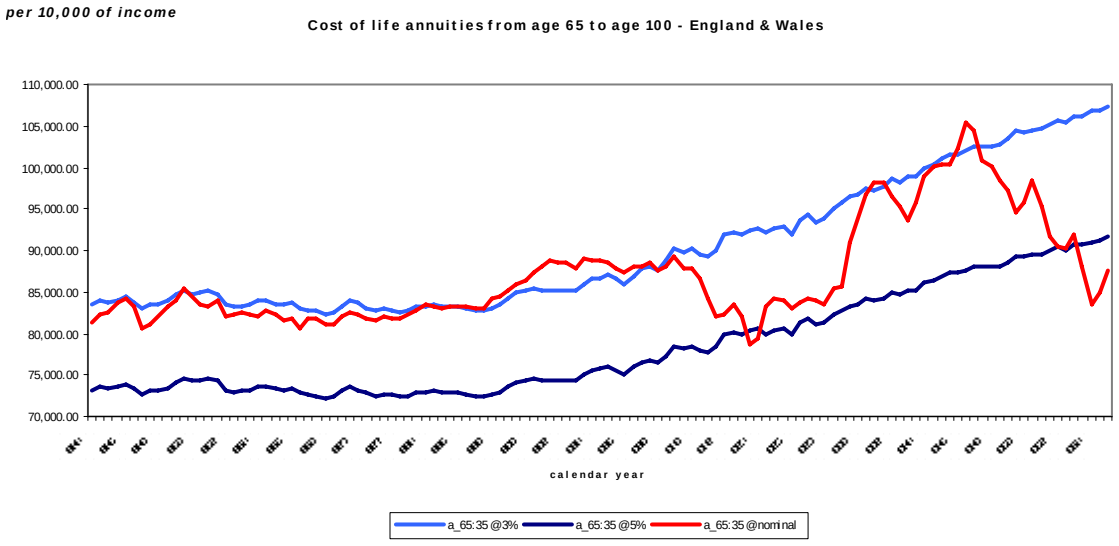
Since the generation life tables show the experience of birth cohorts, those tables may be important for an historical perspective of population growth, but they also provide a different means of studying cohort mortality and cohort projections. It may

also therefore be claimed that the generation life tables are crucial in pricing life contracts.

Willets (Willets; 1999, 2004) has noted that the actual historical longevity improvements do not occur in a smooth, upward fashion, but rather exhibit certain patterns, with some generations having better improvements than others. The generations experiencing relatively high improvements are often termed "select birth cohorts." However, the actual structure of such cohorts, the correlations among them in various countries and their impact on the prices of retirement instruments remain unexplored. In this project, we propose to investigate these issues. Let us note that there does not seem to be a universally accepted measure of longevity improvement as, for example, just measuring improvement in life expectancy at birth does not capture the full nature of the process. We will work on developing such measures.

To illustrate the phenomenon that we study here, consider Figure 1. The figure shows the cost of life annuities, i.e., present value of payments, from age 65 by birth year for England and Wales at 3 percent (top curve), 5 percent (bottom curve) and nominal historical interest rate when the given cohort turned 65. The mortality data was obtained from the Human Mortality Database (2004). The dates on the horizontal axis indicate the date of issue of the annuity.

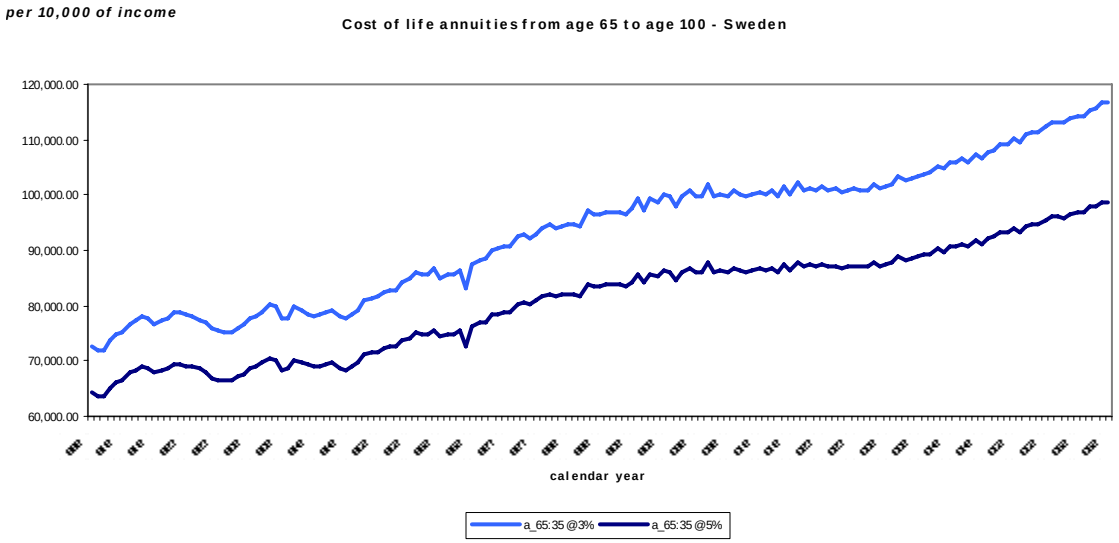
Figure 1



We see that, for example, at 3 percent, the cost of life annuities from age 65 has been generally increasing, but the curve showing it has many local maxima and minima in the period illustrated. These oscillations of the cost of retirement increase the risk of funding it. Figure 2 shows the same phenomena for Sweden, also based on Human Mortality Database (2004). In Figure 2, the bottom graph shows the value of a life

annuity from age 65 at 5 percent, and the top one gives the same information at 3 percent.

Figure 2



Again, we see oscillations in the value of the retirement annuity. Of course, both graphs also illustrate a pronounced increase in the cost of a retirement annuity due to improvements in longevity.

This improvement of longevity is of concern for public policymakers and for private insurers. Most governments worldwide are, to a greater or lesser degree, involved in provision of retirement benefits for their citizens. The cost of those benefits can be greatly affected, and in fact already have been, by increasing longevity. Insurance companies offering pensions and annuities respond to such a challenge by conservative pricing, and some go as far as not offering some products. Democratic governments generally do not have the option of not offering retirement support of any kind or of drastically reducing its levels, thus it is to their benefit if estimates of the cost of pension provision can be made accurately and if long-term problems can be diagnosed early. If longevity increases at an uneven pace, certain cohorts of retirees will prove themselves to be a greater burden on the public pension system than expected, while other cohorts may create an illusion of a lower cost. Since public pension systems seek stable and predictable funding, those surprises are undesirable.

Some insurance companies in the business of pension provision view longevity improvement as a significant and largely unpredictable source of risk in pricing lifetime annuities. But, of course, other important market participants believe that this risk also represents a great business opportunity. We believe that better research in this area can be of great value to both of these groups of pension providers.

1. Defining a Select Birth Cohort: Mortality Improvement

A *select birth cohort* with respect to *mortality improvement* is defined here as an age cohort or generation characterized by greater rates of mortality improvement than previous and subsequent generations. It is generally expected, at least in the last 100 years, that subsequent generations live longer than the previous ones. But among the birth cohorts, or subsequent generations, there are some whose improvement in mortality, or longevity, is especially high. Willets (1999) calls this the "cohort effect," describing it as a "wave of rapid improvements, rippling upwards through mortality rates in the United Kingdom." For example, in his estimation, for the past four decades, people born between 1925 and 1945 have benefited from faster mortality improvements than those born in adjacent generations. The implication is that when a select birth cohort reaches retirement, this cohort's lower mortality will be quite a shock to the retirement system. Willets (1999) documents such a phenomenon for the generation born in Great Britain in the 1930s, with emphasis on 1931. This generation is now in their 70s, and if they continue experiencing the same mortality improvements that they did in younger ages, they will be a strong example of the nature of this problem.

How should we define mortality improvement? Let us analyze this issue.

Suppose that we have mortality rates for ages 0, 1, 2 and 3 for cohorts born years $z, z+1, z+2, z+3$, and we want to compare mortality improvements for them. Let the mortality rates be (each column represents a point in time, the left superscript is a notation we introduce for the year of birth of the cohort studied):

$$\begin{bmatrix} {}^{z}q_0 & {}^{z}q_1 & {}^{z}q_2 & {}^{z}q_3 & & & \\ & {}^{z+1}q_0 & {}^{z+1}q_1 & {}^{z+1}q_2 & {}^{z+1}q_3 & & \\ & & {}^{z+2}q_0 & {}^{z+2}q_1 & {}^{z+2}q_2 & {}^{z+2}q_3 & \\ & & & {}^{z+3}q_0 & {}^{z+3}q_1 & {}^{z+3}q_2 & {}^{z+3}q_3 \end{bmatrix} \qquad (1)$$

Then the improvements (lowering) in mortality between these cohorts are:

$$\begin{bmatrix} {}^{z}q_0 - {}^{z+1}q_0 & {}^{z}q_1 - {}^{z+1}q_1 & {}^{z}q_2 - {}^{z+1}q_2 & {}^{z}q_3 - {}^{z+1}q_3 & & \\ & {}^{z+1}q_0 - {}^{z+2}q_0 & {}^{z+1}q_1 - {}^{z+2}q_1 & {}^{z+1}q_2 - {}^{z+2}q_2 & {}^{z+1}q_3 - {}^{z+2}q_3 & \\ & & {}^{z+2}q_0 - {}^{z+3}q_0 & {}^{z+2}q_1 - {}^{z+3}q_1 & {}^{z+2}q_2 - {}^{z+3}q_2 & {}^{z+2}q_3 - {}^{z+3}q_3 \end{bmatrix} \qquad (2)$$

It would be natural to ask which of the three mortality improvements was the largest, in order to compare them. The rates stated in (2) are absolute mortality improvements, expressed as vectors of changes in mortality rates. The standard way of

measuring mortality improvement is not through these absolute rates but rather via the relative rates:

$$-\left(\frac{{}^{z}q_x - {}^{z-1}q_x}{{}^{z-1}q_x}\right) = 1 - \frac{{}^{z}q_x}{{}^{z-1}q_x} \tag{3}$$

However, for the practical comparisons, e.g., Willets, 2004, it is the central death rate that is used, instead of the mortality rate, so that the mortality improvement between two birth cohorts, as measured for the cohort born in year *z*, is given by:

$${}^{z}i_x = -\left(\frac{{}^{z}m_x - {}^{z-1}m_x}{{}^{z-1}m_x}\right) = 1 - \frac{{}^{z}m_x}{{}^{z-1}m_x} \tag{4}$$

This still does not tell us how exactly a birth cohort can distinguish itself among others in its mortality improvement.

Willets (2004) proposes use of smoothed central death rates for the calculation of mortality improvement. Let us explain this methodology. One of the most commonly used parametric models of mortality, commonly termed the "Gompertz Law" (Bowers et al., 1997), assumes that the natural logarithm of the force of mortality is a linear function of age, i.e.,

$$\ln \mu(x) = x \ln c + \ln B, \tag{5}$$

so that

$$\mu(x) = Bc^x. \tag{6}$$

It is common to use the central death rate calculation from empirical data as an estimator of the force of mortality (Bowers et al., 1997). In that case, one could also assume that the central death rate is a linear function of age. Willets (2004) uses such an assumption for a smoothing procedure for his investigation of the cohort effect. The smoothing procedure is a method of aggregation of populations across birth years, so that the combined effect of unusually high mortality improvement over a group of birth cohorts can be investigated. Additionally, Willets (2004) compares mortality improvements among smoothed groups of birth cohorts in a calendar year, not across birth years. We will present his procedure in more detail, as it is a starting point for our analysis.

In his analysis, Willets (1999, 2004) creates a smoothed central death rate for a person aged *x* in calendar year *z*, obtained by a log-linear regression on the raw central death rates of nine birth cohorts, centered around the birth cohort considered. The

regression is applied to the central death rates of the cohorts with birth years $z-x-4$, $z-x-3$, $z-x-2$, $z-x-1$, $z-x$, $z-x+1$, $z-x+2$, $z-x+3$, $z-x+4$, at age x each, with the assumed relationship of the form:

$$\ln({}^{z-x}m_x) = \alpha(x, z-x) + \beta(x, z-x)\cdot(z-x) + \varepsilon\left(z-x\right), \tag{7}$$

where $\varepsilon\left(z-x\right)$ is the residual. Using the parameters $\alpha(x, z-x)$ and $\beta(x, z-x)$ derived in this regression procedure, the smoothed central death rate in the calendar year z at age x is

$$\ln({}^{z-x}\tilde{m}_x) = \alpha(x, z-x) + \beta(x, z-x)\cdot(z-x). \tag{8}$$

To assess the relative mortality improvement rate for those aged x in year z, Willets compares this value to the corresponding value for the cohort born in $z-x-1$ to obtain a smoothed mortality improvement measure:

$${}^{z-x}\Delta\tilde{m}_x = \frac{{}^{z-x-1}\tilde{m}_x - {}^{z-x}\tilde{m}_x}{{}^{z-x-1}\tilde{m}_x} = 1 - \frac{{}^{z-x}\tilde{m}_x}{{}^{z-x-1}\tilde{m}_x}. \tag{9}$$

It would seem natural to repeat the procedure described above and obtain the value ${}^{z-x-1}\tilde{m}_x$ from a log-linear regression on the "original" values of the cohorts $z-x-5$, $z-x-4$, $z-x-3$, $z-x-2$, $z-x-1$, $z-x$, $z-x+1$, $z-x+2$, $z-x+3$ at age x, and as a result obtain the following: ${}^{z-x}\tilde{m}_x = e^{\alpha(x,z-x)+\beta(x,z-x)\cdot(z-x)}$, ${}^{z-x-1}\tilde{m}_x = e^{\alpha(x,z-x-1)+\beta(x,z-x-1)\cdot(z-x-1)}$, and

$${}^{z-x}\Delta\tilde{m}_x = 1 - \frac{e^{\alpha(x,z-x)+\beta(x,z-x)\cdot(z-x)}}{e^{\alpha(x,z-x-1)+\beta(x,z-x-1)\cdot(z-x-1)}}. \tag{10}$$

But Willets instead assumes that the parameters of the regression done before still hold and applies them directly, resulting in a much simpler formula, ${}^{z-x}\tilde{m}_x = e^{\alpha(x,z-x)+\beta(x,z-x)\cdot(z-x)}$, ${}^{z-x-1}\tilde{m}_x = e^{\alpha(x,z-x)+\beta(x,z-x)\cdot(z-x-1)}$, and

$$\begin{aligned} {}^{z-x}\Delta\tilde{m}_x &= 1 - \frac{e^{\alpha(x,z-x)+\beta(x,z-x)\cdot(z-x)}}{e^{\alpha(x,z-x)+\beta(x,z-x)\cdot(z-x-1)}} \\ &= 1 - \frac{e^{\alpha(x,z-x)}e^{\beta(x,z-x)\cdot(z-x)}}{e^{\alpha(x,z-x)}e^{\beta(x,z-x)\cdot(z-x-1)}} \\ &= 1 - e^{\beta(x,z-x)}. \end{aligned} \tag{11}$$

Willets shows relative mortality improvement rates for ages 30-84 in calendar years 1965-1997 for England and Wales and for ages 40-100 in calendar years 1950-1999 for Japan. Subsequently, the values in excess of a certain percentage are shown for each calendar year, but accounting only for ages 30-84 and 40-100, respectively. This is how Willets derives the idea of the cohort effect for those born in the 1930s in England and Wales. As an alternative to Willets' criterion for showing values in the tables, which was based on the maximum for each calendar year, one could consider a comparison of values at a specific age, i.e., taking the maximum of the values for each specific year and showing only values in excess of a given percentage. This approach also compares different cohorts, but at a fixed age, which is more closely related to the definition of relative mortality improvement (recall that it compares mortality rates at a specific age with the corresponding value for the cohort before). When applying this alternative criterion to the same "smoothed" data used by Willets, the result is quite different from the one obtained by him: there is no significant evidence of select birth cohorts.

Another idea would be to "narrow" the smoothing period, i.e., perform a log-linear regression not to periods of ± 4 years, but instead narrow the periods to ± 2 years and ±1 year. Further narrowing would result in taking the original mortality rates and not smoothing at all. In our research, we determined that by gradually narrowing the smoothing period, the cohort patterns in Willets' diagrams are becoming less obvious and weaker, and eventually they disappear.

The approach used by Willets, aggregating the data, may be appropriate for public policymakers, who often deal with the population as a whole, and may be able to smooth out some intergenerational effect through issuance of public debt. For the purpose of risk management by the private sector and through possible market instruments trading mortality risk, we believe it is necessary to investigate mortality improvement more directly, cohort by cohort.

We propose that a *select birth cohort* be defined by a criterion that points out its mortality improvement exceeding two neighboring birth cohorts. The first such criterion that we propose is based on individual age comparisons, and it defines a select birth cohort as follows:

Definition 1: A *select birth cohort* with respect to *mortality improvement* is a birth cohort whose mortality improvement exceeds that of the birth cohort just before it and just after it at the majority of individual ages.

We do admit that this is not an ideal criterion, but previous works of Willets (1999, 2004) on this subject have not provided any specific definition and relied exclusively on graphs in identifying select birth cohorts. We want to establish analytical criteria instead. We will also attempt to provide an integrated all-age criterion in what follows.

Let us note that under a uniform distribution of deaths (UDD) (Bowers et al., 1997) in the year of death assumption, we have the following relationship between the mortality rate q_x, force of mortality, cohort size and the central death rate m_x:

$$m_x \underset{\text{UDD}}{=} \mu\left(x+\frac{1}{2}\right) \underset{\text{UDD}}{=} \frac{q_x}{1-\frac{1}{2}q_x} = \frac{2d_x}{l_x + l_{x+1}}. \tag{12}$$

Note that the central death rate may exceed 1, and under the UDD assumption, its maximum value is 2 when $q_x = 1$. In general, the central death rate is unbounded from above, unlike the mortality rate. This is simply illustrated by considering a population of size 1 and the one member of the population dying instantly at the very beginning of the year. In this example, the number of deaths, i.e., the numerator in the definition of the central death rate, is 1, while the exposure to death, i.e., the denominator in the definition of the central death rate, is zero, as for the whole year nobody else is available to die in the population.

Under UDD, using central death rates for mortality improvement definition overstates that mortality improvement as compared to the one described by mortality rate q_x, as long as mortality declines with age, because under UDD,

$$1-\frac{{}^{z+1}m_x}{{}^{z}m_x} \underset{\text{UDD}}{=} 1-\frac{\dfrac{{}^{z+1}q_x}{1-\frac{1}{2}\,{}^{z+1}q_x}}{\dfrac{{}^{z}q_x}{1-\frac{1}{2}\,{}^{z}q_x}} = 1-\frac{{}^{z+1}q_x}{{}^{z}q_x}\frac{2-{}^{z}q_x}{2-{}^{z+1}q_x} > 1-\frac{{}^{z+1}q_x}{{}^{z}q_x}. \tag{13}$$

In general, i.e., without the UDD assumption, however, the central death rate does consider the timing of deaths during the year of death, and this makes it a better measure for our purpose than the mortality rate, as the length of life during the year of death does contribute to longevity.

Let us illustrate the proposed criterion for measurement of mortality improvement with a simple mortality table of hypothetical population of guinea pigs.

Consider five generations of five guinea pigs born in each of the years 1999, 1998, 1997, 1996 and 1995, with the following central death rates at ages 1 through 5 (we assume that 5 is the limiting age for these guinea pigs):

$$\begin{bmatrix} {}^{1995}m_0 & {}^{1996}m_0 & {}^{1997}m_0 & {}^{1998}m_0 & {}^{1999}m_0 \\ {}^{1995}m_1 & {}^{1996}m_1 & {}^{1997}m_1 & {}^{1998}m_1 & {}^{1999}m_1 \\ {}^{1995}m_2 & {}^{1996}m_2 & {}^{1997}m_2 & {}^{1998}m_2 & {}^{1999}m_2 \\ {}^{1995}m_3 & {}^{1996}m_3 & {}^{1997}m_3 & {}^{1998}m_3 & {}^{1999}m_3 \\ {}^{1995}m_4 & {}^{1996}m_4 & {}^{1997}m_4 & {}^{1998}m_4 & {}^{1999}m_4 \end{bmatrix} = \begin{bmatrix} \frac{4}{17} & \frac{4}{17} & \frac{4}{19} & \frac{4}{20} & \frac{4}{20} \\ \frac{4}{13} & \frac{4}{13} & \frac{4}{14} & \frac{4}{15} & \frac{4}{15} \\ \frac{4}{9} & \frac{4}{9} & \frac{4}{11} & \frac{4}{11} & \frac{4}{12} \\ \frac{4}{5} & \frac{4}{5} & \frac{4}{7} & \frac{4}{8} & \frac{4}{8} \\ 2 & 2 & \frac{4}{3} & 1 & \frac{4}{3} \end{bmatrix}. \quad (14)$$

The above data correspond to the following ages (in years) of death for the five cohorts:

Birth year 1995	0.25	1.25	2.25	3.25	4.50
Birth year 1996	0.25	1.25	2.25	3.25	4.50
Birth year 1997	0.75	1.50	2.75	3.75	4.75
Birth year 1998	1.00	1.75	2.75	4.00	5.00
Birth year 1999	1.00	1.75	3.00	4.00	4.75

If we were to consider mortality rates, all five cohorts follow the simple De Moivre's Law (Bowers et al., 1997) with limiting age of 5, and the mortality rates do not improve among them at all. But by using central death rates, we obtain the following mortality improvement measures for the 1996, 1997, 1998, and 1999 cohorts:

$$\begin{bmatrix} {}^{1996}i_0 & {}^{1997}i_0 & {}^{1998}i_0 & {}^{1999}i_0 \\ {}^{1996}i_1 & {}^{1997}i_1 & {}^{1998}i_1 & {}^{1999}i_1 \\ {}^{1996}i_2 & {}^{1997}i_2 & {}^{1998}i_2 & {}^{1999}i_2 \\ {}^{1996}i_3 & {}^{1997}i_3 & {}^{1998}i_3 & {}^{1999}i_3 \\ {}^{1996}i_4 & {}^{1997}i_4 & {}^{1998}i_4 & {}^{1999}i_4 \end{bmatrix} = \left[\begin{array}{c|c|c|c} 0 & \frac{2}{19} & \frac{1}{20} & 0 \\ 0 & \frac{1}{14} & \frac{1}{15} & 0 \\ 0 & \frac{2}{11} & 0 & \frac{1}{12} \\ 0 & \frac{2}{7} & \frac{1}{8} & 0 \\ 0 & \frac{1}{3} & \frac{1}{4} & -\frac{1}{3} \end{array}\right] \quad (15)$$

Here are the comparisons of mortality improvement rates for these cohorts:

1997 versus 1996:	higher improvement at every age
1998 versus 1997:	lower improvement at every age
1999 versus 1998:	higher improvement at age 2, lower improvements otherwise

Clearly, using the definition we gave, the birth cohort of 1997 is a select birth cohort (it is marked in (15)). But it is actually an extreme case of such a situation, where this cohort has a higher mortality improvement at *every age* than the cohort preceding it and the cohort following it. We do not require such a strong version of the criterion in our definition.

2. Defining a Select Birth Cohort: Longevity Improvement

In order to introduce the alternative definition of a select birth cohort that we want to propose in this work, let us calculate complete life expectancies (Bowers et al., 1997) for every age for the five hypothetical cohorts of guinea pigs described above:

$$\begin{bmatrix} {}^{1995}\mathring{e}_0 & {}^{1996}\mathring{e}_0 & {}^{1997}\mathring{e}_0 & {}^{1998}\mathring{e}_0 & {}^{1999}\mathring{e}_0 \\ {}^{1995}\mathring{e}_1 & {}^{1996}\mathring{e}_1 & {}^{1997}\mathring{e}_1 & {}^{1998}\mathring{e}_1 & {}^{1999}\mathring{e}_1 \\ {}^{1995}\mathring{e}_2 & {}^{1996}\mathring{e}_2 & {}^{1997}\mathring{e}_2 & {}^{1998}\mathring{e}_2 & {}^{1999}\mathring{e}_2 \\ {}^{1995}\mathring{e}_3 & {}^{1996}\mathring{e}_3 & {}^{1997}\mathring{e}_3 & {}^{1998}\mathring{e}_3 & {}^{1999}\mathring{e}_3 \\ {}^{1995}\mathring{e}_4 & {}^{1996}\mathring{e}_4 & {}^{1997}\mathring{e}_4 & {}^{1998}\mathring{e}_4 & {}^{1999}\mathring{e}_4 \end{bmatrix} = \begin{bmatrix} \frac{23}{10} & \frac{23}{10} & \frac{27}{10} & \frac{29}{10} & \frac{29}{10} \\ \frac{29}{16} & \frac{29}{16} & \frac{35}{16} & \frac{19}{8} & \frac{19}{8} \\ \frac{4}{3} & \frac{4}{3} & \frac{7}{4} & \frac{23}{12} & \frac{23}{12} \\ \frac{7}{8} & \frac{7}{8} & \frac{5}{4} & \frac{3}{2} & \frac{11}{8} \\ \frac{1}{2} & \frac{1}{2} & \frac{3}{4} & 1 & \frac{3}{4} \end{bmatrix} \qquad (16)$$

Now let us calculate the *absolute improvement in life expectancy* of each cohort versus the previous cohort, defined as ${}^{z}ie_k = {}^{z}\mathring{e}_k - {}^{z-1}\mathring{e}_k$. We have:

$$\begin{bmatrix} {}^{1996}ie_0 & {}^{1997}ie_0 & {}^{1998}ie_0 & {}^{1999}ie_0 \\ {}^{1996}ie_1 & {}^{1997}ie_1 & {}^{1998}ie_1 & {}^{1999}ie_1 \\ {}^{1996}ie_2 & {}^{1997}ie_2 & {}^{1998}ie_2 & {}^{1999}ie_2 \\ {}^{1996}ie_3 & {}^{1997}ie_3 & {}^{1998}ie_3 & {}^{1999}ie_3 \\ {}^{1996}ie_4 & {}^{1997}ie_4 & {}^{1998}ie_4 & {}^{1999}ie_4 \end{bmatrix} = \begin{bmatrix} 0 & \frac{4}{10} & \frac{2}{10} & 0 \\ 0 & \frac{6}{16} & \frac{3}{16} & 0 \\ 0 & \frac{5}{12} & \frac{2}{12} & 0 \\ 0 & \frac{3}{8} & \frac{1}{4} & -\frac{1}{8} \\ 0 & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \end{bmatrix} \tag{17}$$

In this case, the cohort born in 1997 again shows the greatest absolute improvement in life expectancy at every age but age 4, at which it has the same absolute improvement as the 1998 cohort. Now consider the *relative improvement in life expectancy*:

$${}^{z}rie_k = \frac{{}^{z}\mathring{e}_k - {}^{z-1}\mathring{e}_k}{{}^{z-1}\mathring{e}_k}.$$

Using this measure, we get:

$$\begin{bmatrix} {}^{1996}rie_0 & {}^{1997}rie_0 & {}^{1998}rie_0 & {}^{1999}rie_0 \\ {}^{1996}rie_1 & {}^{1997}rie_1 & {}^{1998}rie_1 & {}^{1999}rie_1 \\ {}^{1996}rie_2 & {}^{1997}rie_2 & {}^{1998}rie_2 & {}^{1999}rie_2 \\ {}^{1996}rie_3 & {}^{1997}rie_3 & {}^{1998}rie_3 & {}^{1999}rie_3 \\ {}^{1996}rie_4 & {}^{1997}rie_4 & {}^{1998}rie_4 & {}^{1999}rie_4 \end{bmatrix} = \begin{bmatrix} 0 & \frac{4}{23} & \frac{2}{27} & 0 \\ 0 & \frac{6}{29} & \frac{3}{35} & 0 \\ 0 & \frac{5}{16} & \frac{2}{21} & 0 \\ 0 & \frac{3}{7} & \frac{1}{5} & -\frac{1}{12} \\ 0 & \frac{1}{2} & \frac{1}{3} & -\frac{1}{4} \end{bmatrix} \tag{18}$$

We define a "longevity improvement select birth cohort" as follows:

Definition 2: A *select birth cohort* with respect to *longevity improvement* is a birth cohort whose relative longevity improvement exceeds that of the birth cohort just before it and just after it at the majority of individual ages.

We see that when using this longevity improvement criterion, the 1997 cohort of the hypothetical guinea pigs is a select birth cohort, improving at every age more than either of its neighboring cohorts.

The two criteria we propose are related but not always equivalent. Consider now a simpler hypothetical population of guinea pigs of shorter lifespan, born in 2000, 2001, and 2002, dying at exact ages (in years):

2000 birth cohort	0.50	1.00	2.50
2001 birth cohort	0.25	2.00	2.25
2002 birth cohort	0.50	1.00	2.50

Then their mortality and life expectancy parameters are:

$$\begin{bmatrix} {}^{2000}m_0 & {}^{2001}m_0 & {}^{2002}m_0 \\ {}^{2000}m_1 & {}^{2001}m_1 & {}^{2002}m_1 \\ {}^{2000}m_2 & {}^{2001}m_2 & {}^{2002}m_2 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{4}{9} & \frac{2}{5} \\ 1 & \frac{1}{2} & 1 \\ 2 & 4 & 2 \end{bmatrix}$$

$$\begin{bmatrix} {}^{2000}\overset{\circ}{e}_0 & {}^{2001}\overset{\circ}{e}_0 & {}^{2002}\overset{\circ}{e}_0 \\ {}^{2000}\overset{\circ}{e}_1 & {}^{2001}\overset{\circ}{e}_1 & {}^{2002}\overset{\circ}{e}_1 \\ {}^{2000}\overset{\circ}{e}_2 & {}^{2001}\overset{\circ}{e}_2 & {}^{2002}\overset{\circ}{e}_0 \end{bmatrix} = \begin{bmatrix} 1 & \frac{3}{2} & 1 \\ \frac{3}{4} & \frac{9}{8} & \frac{3}{4} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{2} \end{bmatrix}$$

The mortality and longevity improvement parameters are:

$$\begin{bmatrix} {}^{2001}i_0 & {}^{2002}i_0 \\ {}^{2001}i_1 & {}^{2002}i_1 \\ {}^{2001}i_2 & {}^{2002}i_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{9} & \frac{1}{10} \\ \frac{1}{2} & -1 \\ -1 & \frac{1}{2} \end{bmatrix}, \quad \begin{bmatrix} {}^{2001}ie_0 & {}^{2002}ie_0 \\ {}^{2001}ie_1 & {}^{2002}ie_1 \\ {}^{2001}ie_2 & {}^{2002}ie_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{3}{8} & -\frac{3}{8} \\ -\frac{1}{4} & \frac{1}{4} \end{bmatrix}$$

$$\begin{bmatrix} {}^{2001}rie_0 & {}^{2002}rie_0 \\ {}^{2001}rie_1 & {}^{2002}rie_1 \\ {}^{2001}rie_2 & {}^{2002}rie_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{3} \\ \frac{1}{2} & -\frac{1}{3} \\ -\frac{1}{2} & 1 \end{bmatrix}$$

We see that the 2001 cohort improves less than the 2002 cohort under the mortality improvement criterion, but it improves more than the 2002 cohort under the relative life expectancy improvement criterion.

A measure using complete life expectancy may, however, not always be practical. Data available empirically, in the form of mortality tables, is discrete and annual. In that case, a *curtate life expectancy* (Bowers et al., 1997) must be used. However, the online Human Mortality Database (2004) provides estimates of complete life expectancy using an approximation methodology specified in the Human Mortality Database's methods protocol, and we use that form of data for our empirical estimates.

We will now apply these quantitative definitions of select birth cohorts to data from selected countries worldwide, using data from the online Human Mortality Database (2004).

4. Select Birth Cohorts in Empirical Data

Human Mortality Database (2004), further referred to as "HMD," is an online collection of data assembled jointly by the University of California at Berkeley and by the Max Planck Institute for Demographic Research in Rostock, Germany. This project collects continuously increasing set of data concerning human mortality, separated by gender and combined, for a wide variety of countries worldwide. The longest data series are available for Sweden, and England and Wales (combined), but a wide variety of countries are represented.

We have used the data available from HMD to seek select birth cohorts with respect to mortality improvement and with respect to relative longevity improvement in various countries.

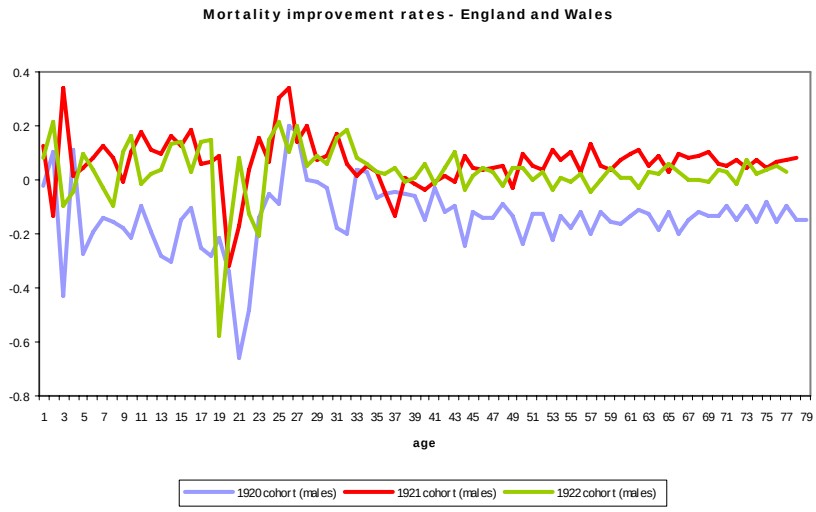
Table 1 presents our findings of select birth cohorts in England and Wales data. The first column identifies years of birth of select birth cohorts for males, the second one for females and the third one for the combined population (both). This compares to select birth cohorts as identified by the relative longevity improvement in Table 2.

TABLE 1.
Mortality Improvement Select Birth Cohorts in England and Wales

M	F	B
1742	1741	1741
1744	1744	1743
1747	1746	1745
1749	1747	1746
1750	1751	1750
1752	1758	1752
1899	1916	1919
1919	1919	1921
1921	1921	1925
1946	1946	1941
1948	1948	1946
1963	1981	1948
1970		1970

The following two figures show the improvement rates for two select birth cohorts relative to the birth cohorts on either side. Observe that the 1921 cohort dominated the 1920 and 1922 rather consistently from 1943 forward, but not by much.

FIGURE 3
1921 Select Birth Cohort Relative to 1920 and 1922



The next figure charts the 1946 select birth cohort against the cohorts on either side. Here the effect is more dramatic.

FIGURE 4
1946 Select Birth Cohort Relative to 1945 and 1947

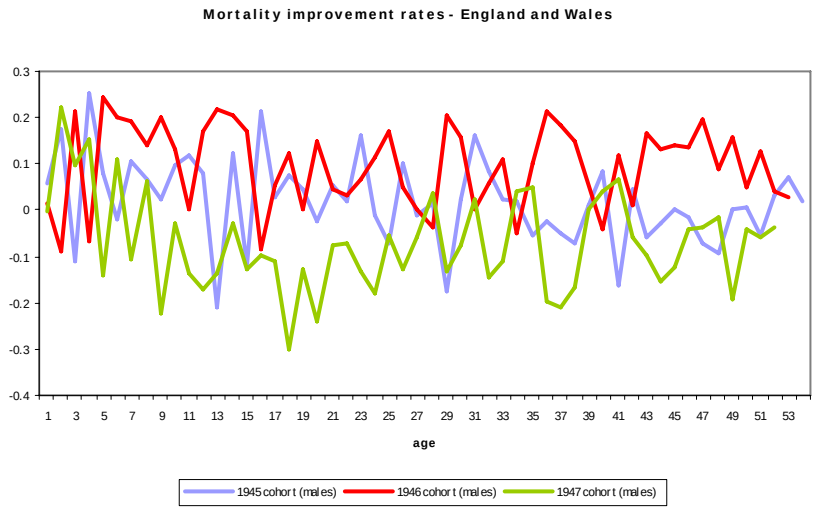


TABLE 2
Relative Longevity Improvement Select Birth Cohorts in England and Wales

M	F	B
1737	1734	1734
1739	1735	1735
1741	1736	1736
1743	1737	1737
	1739	1739
	1741	1741

While the comparison of relative longevity improvement cohorts is not unexpected, it is rather striking that there are so few relative longevity improvement cohorts and that there are none in the modern times. We have also looked at mortality improvement measured in terms of five-year central death rates: ${}_5m_x = {}_5d_x / {}_5L_x$. When using these five-year central death rates, we obtain the five-year select birth cohorts for England and Wales given in Table 3. This illustrates the pattern we had indicated with respect to the Willets analysis: narrowing the band of aggregated data lessens the chance of identifying a select birth cohort.

TABLE 3
Five-Year Mortality Improvement Select Birth Cohorts for England and Wales

M	F	B
1797	1797	1797
1807	1798	1807
1808	1799	1808
1809	1800	1809
1810	1807	1810
1817	1808	1817
1818	1809	1818
1819	1810	1819
1820	1817	1820
1826	1818	1826
1827	1819	1827
1828	1820	1828
1829	1826	1829
1830	1827	1851
1851	1828	1852
1861	1829	1861
1862	1837	1862
1871	1851	1871
1872	1861	1872
1906	1862	1881
1919	1871	1882
1934	1872	1905
1945	1881	1906
1946	1882	1919
1967	1902	1936
1976	1905	1946
1977	1906	1976
1978	1919	1977
	1936	1978
	1942	1979
	1959	
	1962	
	1976	
	1979	

This data identifies the generation born in years 1936-40 (in addition to many others, however!) as a select birth cohort, although for men the years 1934-1938 are stronger, while the stronger years are 1936-1940 for women. We have also performed our analysis for other countries. Japan produced the following birth cohorts (Table 4).

TABLE 4
Five-Year Mortality Improvement Select Birth Cohorts for Japan

M	F	B
1851	1846	1849
1853	1847	1853
1859	1849	1855
1888	1857	1888
1897	1865	1891
1899	1888	1897
1906	1897	1899
1908	1899	1906
1913	1901	1908
1919	1904	1913
1921	1906	1915
1923	1908	1919
1930	1913	1921
1932	1919	1925
1934	1921	1930
1937	1925	1932
1939	1930	1934
1943	1932	1937
1946	1934	1939
1948	1937	1943
1966	1939	1946
1970	1943	1948
1981	1946	1966
1986	1948	1979
1996	1960	1981
	1964	1996
	1966	
	1990	

But using a relative longevity improvement gave no male select birth cohorts in Japan; 1844, 1845 and 1846 for females; and only 1845 for the overall population. Switzerland data gave us select birth cohorts under mortality improvement stated in Table 5.

However, under longevity improvement criterion, very little is revealed, and, in particular, for the entire population, only recent birth years 1997, 1999 and 2001, for which only several years' worth of data are available, show any select cohort characteristics.

TABLE 5
Five-Year Mortality Improvement Select Birth Cohorts for Switzerland

M	F	B
1780	1781	1779
1807	1783	1780
1813	1818	1782
1818	1848	1787
1821	1855	1799
1825	1921	1807
1877	1964	1818
1921	1986	1855
1928	1995	1921
1981	1997	1984
1993	2001	1991
1995		1993
1997		1995
1999		1999

One more criterion for identification of select birth cohorts that we develop here is based on a graphical method of presenting data by Willets (1999, 2004). We will call it a "percentile criterion" for identification of select birth cohorts. For a given birth cohort, mark its mortality improvement at age x if at that age that mortality improvement exceeds 80 percent of mortality improvements at the same age in the data set under consideration. If more than 50 percent of the ages are so marked, we will term the birth cohort as the "percentile select birth cohort.".

Applying the percentile criterion, we were able to identify 1919 as a select cohort in England and Wales for both males and females as well as for the overall population. Furthermore, the 1946 male-only cohort is select. In Canada, those born in 1902 belong to a select cohort, and the French cohorts of 1916, 1919, 1920, 1921 and 1941 are select regardless of the gender.

For Italy, people born in 1919 and 1921 are identified as select cohorts as well as females born in 1818 and males born in 1945. For Japan, 1939 and 1946 are the only select cohorts under this criterion when regarding the male, the female and the aggregate population.

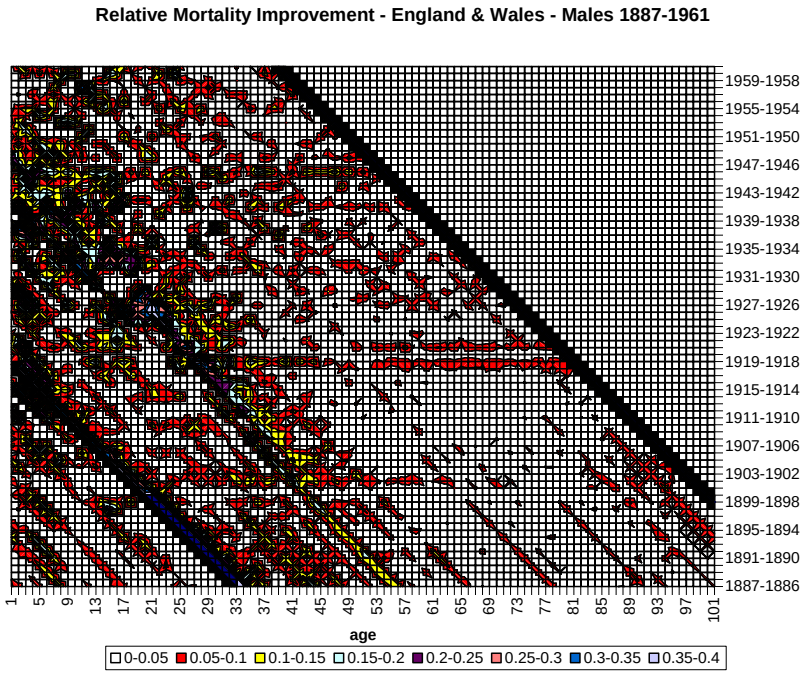
For the United States, 1902 and 1946 are select cohorts for both sexes, while 1919 and 1929 are identified as select cohorts among males and 1934 among females.

However, there are countries for which the percentile criterion does not identify any (the Netherlands, Sweden) or almost no (Norway: only 1816, Switzerland: only 1818) select cohorts.

Finally, a very common method of display of the cohort effect is a graphical representation of rates of mortality improvement. Figure 5 shows, in different shades of grey, the rates of mortality improvement in England and Wales. Note that ages are on the horizontal axis and birth years are on the vertical axis. Hence, a calendar year is read on the diagonal, e.g., we can see the 1918 Spanish flu epidemic in the lower left quadrant of the figure as a diagonal line. The select cohorts are seen as horizontal lines; those lines that are primarily red at 1919, 1921 and 1946 identify three of the select birth cohorts identified in Table 1.

The conclusion of our analysis is that while select cohorts do appear rather regularly, they do not show in a uniform manner with respect to the methodology of analysis, and thus their effect on the cost of retirement does not appear as profound as the overall pattern of improvement.

FIGURE 5
Male Rates of Mortality Improvement in England and Wales



5. Cost of Life Annuities and Select Birth Cohorts

In the early part of this paper, we already observed the oscillations in the value of life annuities from age 65 superimposed on the general pattern of increasing cost of such annuities in England and Wales, as well as in Sweden. A similar pattern can be observed in Italy and France. In Figures 6 and 7, in both cases the top curves represent the cost of life annuities at age 65 paid at 3 percent, while the bottom curves represent the actuarial present value of a life annuity but at a 5 percent discount rate.

In both cases we see that the graphs are not continuous in their upward movement but rather we see the same kind of oscillations we saw in England and Wales, and Sweden. We did not, however, see such oscillations in the data in the United States and Japan (although there was much less data available for these two countries, making it necessary to consider future analysis of the issue).

FIGURE 6
Cost of Life Annuities in Italy

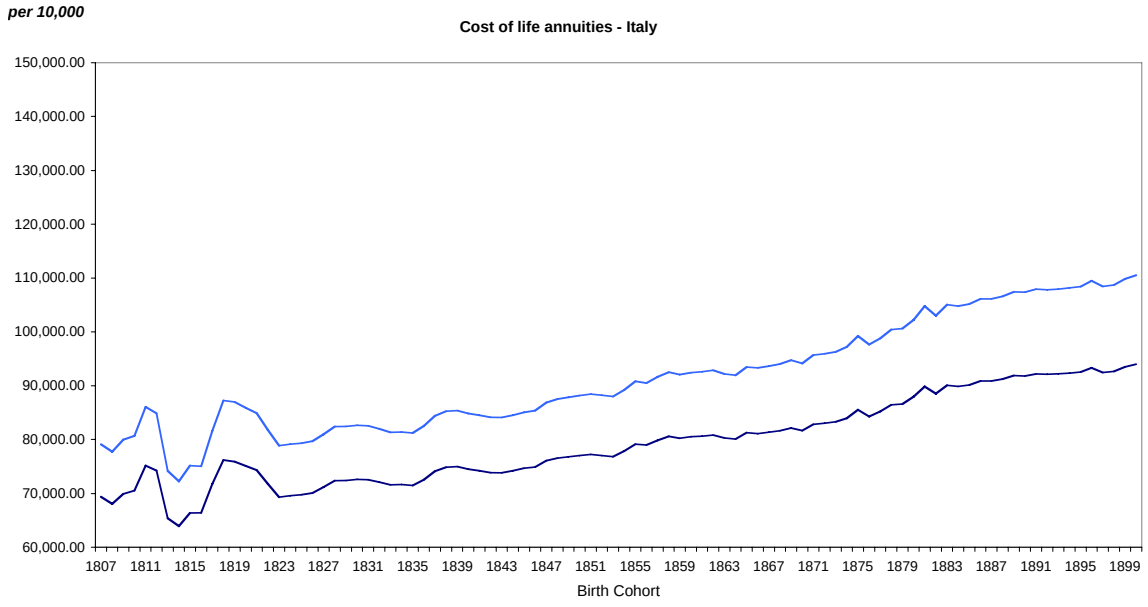
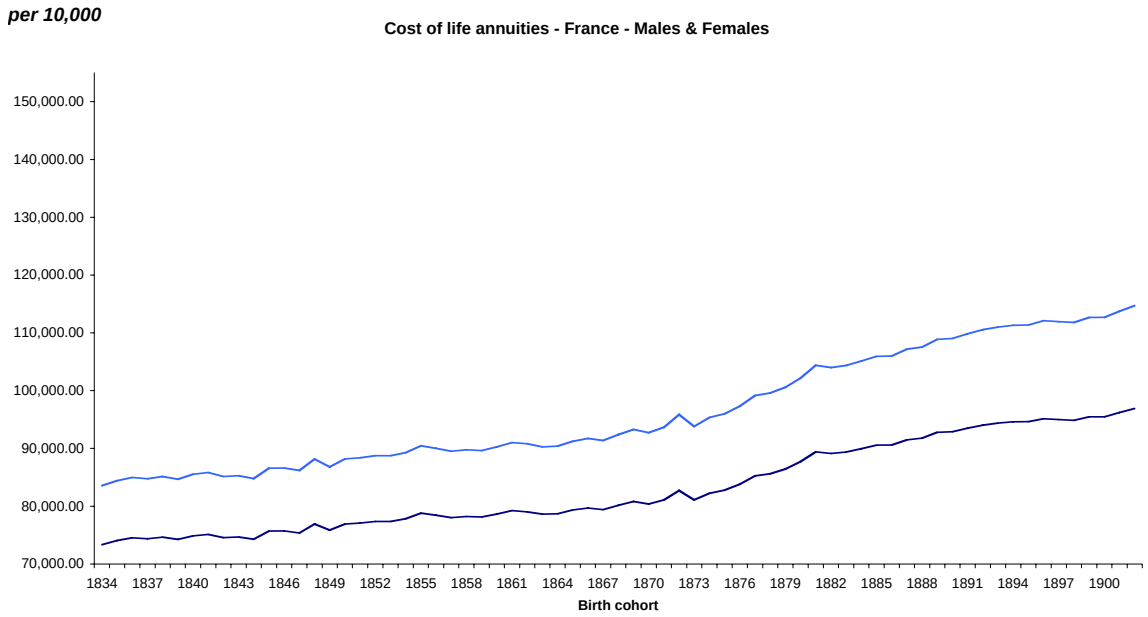


FIGURE 7
Cost of Life Annuities in France



The oscillations observed make it necessary to ask whether they are large enough to be considered in the pricing of life annuities by private providers of life annuities, i.e., private insurance companies and pension plans. Governments issuing pensions do not generally price life annuities individually and thus are more interested in the overall burden of retirement benefits, which we will endeavor to illustrate later in this work.

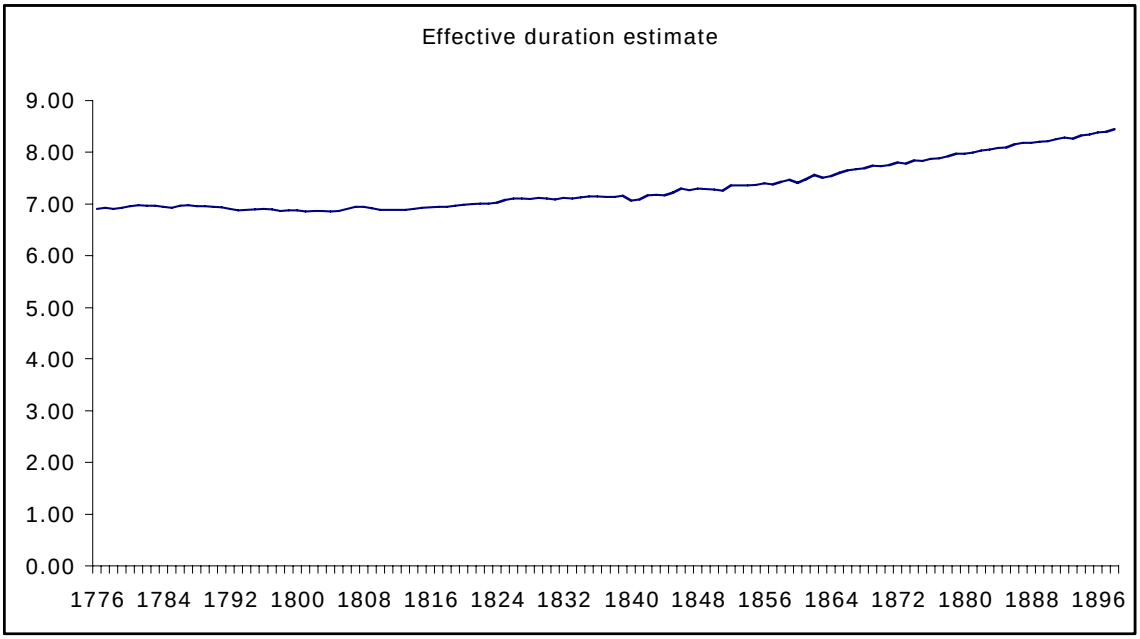
We have analyzed the variability in the cost of life annuities from age 65 for England and Wales and found that the maximum upward oscillations occurred for the following birth cohorts, with the level of increase also given in Table 6.

TABLE 6
Maximum Upward Oscillations

1852	2.15%
1861	1.83%
1846	1.66%
1865	1.34%
1845	1.27%
1841	1.26%
1876	1.11%

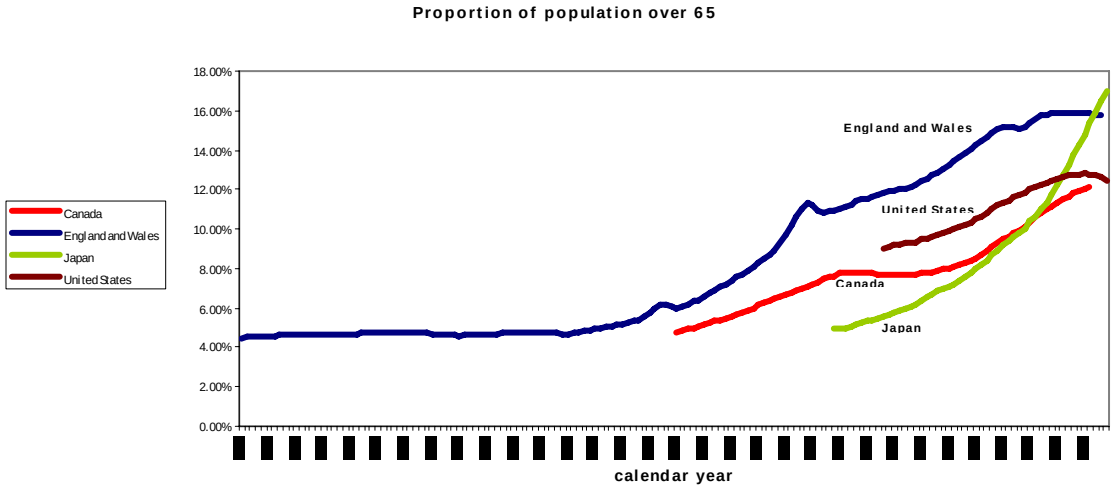
In order to compare this effect with the effect of interest rates, we estimated the effective duration (see, for example, Gajek, Ostaszewski and Zwiesler, 2005) of the life annuity from age 65 to age 100 for England and Wales for the birth cohorts under consideration and obtained Figure 8.

FIGURE 8
Effective Duration of a Life Annuity From age 65,
Estimated Near 3% Discount Rate



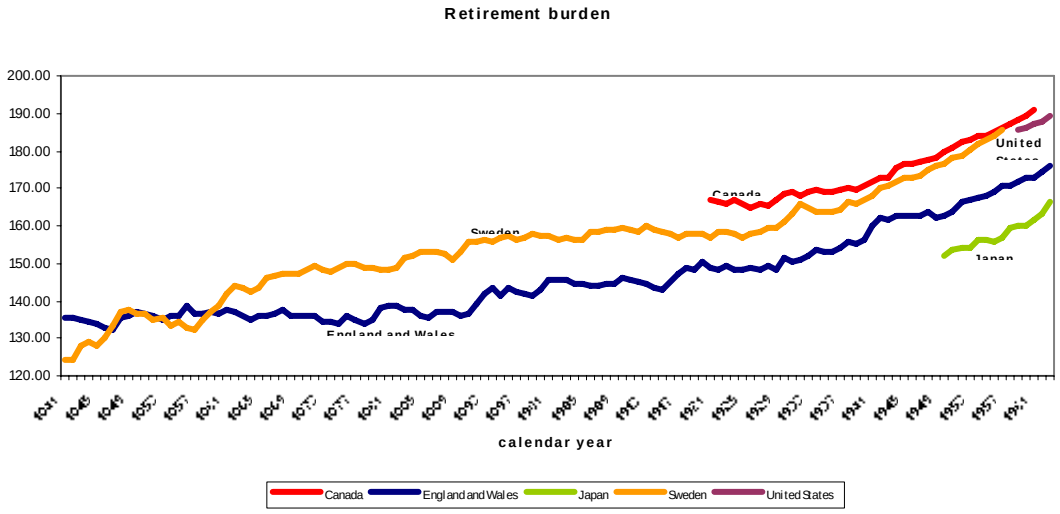
In comparing these estimates of duration for other countries (France, Sweden, Italy and United States), we generally found that the duration varied between six and nine. Given these estimates, we see that oscillations of the value of retirement annuities appear bounded from above by oscillations produced by roughly 40 basis points variability in interest rates. While this may not appear to be a very large value, it probably should be considered by insurance firms planning for the cost of providing retirement benefits. On the other hand, governmental decision-makers may be more concerned about the cost of paying retirement benefits in aggregate to the entire population age 65 and older. The magnitude of this problem can be indicated by observing the percentage of population age 65 and older in several countries, presented below.

FIGURE 9
Percentage of Population Over 65 in Canada, England, Japan and the United States



Another possible measure of the burden of retirement burden on public finances is the total actuarial present values of life annuities to all aged 65 and above calculated at a given point in time, at 3 percent (notably, it is not weighted by the population size, so that it is a simplified measure of burden). We will term this value the "retirement burden." This value is shown for several countries in Figure 10.

FIGURE 10
Retirement Burden in Various Countries



6. Conclusions

Our analysis indicates that while select birth cohorts do exist in various countries, they do not appear to be very common, especially with respect to relative longevity improvement criterion, and they do not appear correlated across countries. Their effect on the cost of life annuities varies, and seems to be somewhere in the range of up to a maximum of an effect equivalent to about a 40-basis-point change in interest rates. The retirement burden, i.e., the total cost of all life annuities issued instantly to all age 65 and above, shows some cohort pattern in the United Kingdom and Sweden, with less of a cohort pattern in other countries analyzed.

We believe that these issues should be subject to further study, especially as more detailed data on mortality becomes available.

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