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**SIMPLIFIED CASH FLOW TESTING OF  
TRADITIONAL PARTICIPATING WHOLE LIFE INSURANCE**

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**Simplified Cash Flow Testing  
of  
Traditional Participating Whole Life Insurance**

**Abstract:**

The process of cash flow testing is viewed as arduous and time consuming by those delegated the responsibility of ensuring asset adequacy for reserve liabilities. The process is further compounded by complex and inflexible mainframe computing systems that were not designed to conduct cash flow testing on specialized lines of business. Most insurance company mainframe computing systems were designed to support accounting and asset segmentation functions that allow for a myriad of analyses at a business unit, subsidiary, or total company level. Cash flow testing is done at the product level, requiring the expertise of pricing actuaries and accountants resident in product development areas. Reliance upon investment officers is necessary for projecting cash flows from assets backing reserve liabilities, but not necessarily for projecting contractual cash flows (i.e., premiums, benefits, and expenses). Diverse product features and embedded options compound the process even more, requiring special skills on the part of the actuary to model and interpret projected and actual asset and liability cash flows. Such a wide range of activities must commence and terminate, for some, within a short window of time around fiscal year-end, thus making it clear that cash flow testing activities can consume a great deal of manpower and computing resources.

Cash flow testing participating insurance does not have to be arduous or time consuming. This paper will develop a model for cash flow testing traditional participating insurance that utilizes pricing tools and assumptions to project reserve liabilities. The model makes use of finance and accounting tools to monitor the reasonableness of ending and intermediate surplus results that emerge under the various interest rate scenarios imposed. The model explicitly incorporates the interest rate scenarios defined in the NAIC's Standard Valuation Law and New York Regulation 126 (NY126). By varying dividend levels and lapse rate assumptions in tandem with the regulatory interest rate scenarios, the model becomes a handy tool for anticipating how a policyholder might behave in a given economic climate; and therefore, in effect, the model recognizes some of the more option-like features of participating whole life insurance.

## **I. Introduction**

### **A. An Overview of the Problem**

Reserve tests for participating whole life insurance products have rarely been the object of intense scrutiny by insurance regulators, financial and investment professionals, or actuaries. These products accumulate premiums at low guaranteed rates of interest over the whole life of the insured to pay an eventual death benefit determined at policy issue. As long as the policy remains in-force, the insured can take advantage of product features such as policy loan and rider options and enjoy experience refunds that are paid in the form of dividends. The safety of participating whole life products to insureds and insurers was never in question until the early 1990s.

The declining interest rate environment of the early 1990's raised fears that insurers might not be able to meet the minimum guarantees on participating whole life insurance products. Solvency was the concern that pushed cash flow testing center stage to measure the ability of reserves to provide for contractual obligations and to indicate, if necessary, a remedial course of action. The lack of simplified computer and cash flow testing techniques increased the difficulty and cost of analyzing asset adequacy in step with changing interest rates, leaving many insurers to rely on costly seriatim based valuation systems for results.

The need for technology that is easy to implement becomes even more paramount when one considers the emerging prominence of the role of the Appointed Actuary. The amended version of the 1980 Standard Valuation Model Law not only requires appointed actuaries to certify that "reserves and designated surplus make good and sufficient provisions for all future obligations ..., (134). p. 8)" it also requires appointed actuaries to report directly to their Boards of Directors any and all events that may endanger company solvency. Unfortunately, the financial and actuarial literature offers little in the way of techniques for cash flow testing participating insurance. Computer searches of the "Business Periodicals Ondisc," "Infotrac," and "NEXIS" databases, and the indexes of publications for Insurance: Mathematics and Economics, and the Society of Actuaries had little to reveal on the subject of cash flow testing participating insurance. This writer found only a few discussions of assumptions and techniques directed at cash flow testing participating life insurance and the necessity of even engaging in such an activity. It is this writer's hope the ensuing discussion will help lessen the paucity of literature on this subject.

This paper will discuss a methodology for measuring the cash flow sensitivity of participating whole life insurance under various interest rate scenarios using techniques that are easy to develop without the power and expense of mainframe computing systems. New business asset shares once considered only a pricing tool are introduced in this paper as a viable tool for projecting participating whole life insurance liabilities in cash flow testing models. Through the use of simplifying assumptions and retro-fitting adjustments the asset share model office

projection will simulate the liabilities for the block of participating whole life insurance under study. The asset shares are not a good fit coming into the model because of an excessively high new money rate assumption used to model dividends, gross premium margins higher than necessary, and liability flows out of line with actual experience. These problems are addressed by re-calibrating the asset share dividends and liabilities to actual experience using dividend and gains adjustment factors, avoiding the cost of modeling these flows from scratch.

To be viable this technique must reproduce operating conditions prevailing at the start of the projection period and produce run-off liability patterns that are plausible and consistent with each interest rate and concomitant policy loan and lapse rate scenario imposed. As a natural consequence, the responsiveness of dividends is modeled under each scenario in line with the philosophy of this insurer. Dividends have long been viewed as a "safety net" for gain from operations on participating business and the main reason insurers have unanimously agreed that it is hard to lose money on traditional participating whole life business. It is also the reason many believe cash flow testing this business is either unnecessary or that it need not be extensive. To do otherwise, an insurer would incur a great expense without much benefit. This view is changing, albeit slowly. As will be demonstrated, the asset share model office projection technique can alleviate the concerns of the early 1990s without the expense of mainframe valuation systems.

The model office projection is used only to model product liabilities and cash flows under the regulatory interest rate scenarios. It is not used to model cash flows from the assets backing those liabilities. Reliance upon the abilities of investment specialists to model asset cash flows under the regulatory interest rate scenarios is permissible under current valuation law and should be assumed by the reader for this discussion. The investment specialists of this insurer certified that cash flows for existing assets were modeled taking into account "call" and "non-call" probabilities appropriate for each of the regulatory interest rate scenarios. While a discussion of the investment model used to derive those flows is beyond the scope of this paper, it is essential

to state that it is assumed that reinvestment of future net cash flows in the liability model presented herein is restricted to non-callable bonds as a simplifying assumption.

## **B. A Review of the Literature**

The wealth of literature on the subject of cash flow testing targets Single Premium Deferred Annuities (SPDAs). SPDAs are admittedly highly sensitive to fluctuations in interest rates and one of the main reasons for the development of New York Regulation 126 [13]. Michael E. Mateja {[42], [43]} discussed how to cash flow test SPDAs extensively at the 1985 Symposium for the Valuation Actuary (SVA) in Chicago, Illinois, and again at the 1988 Symposium for the Valuation Actuary in Toronto, Ontario. Mr. Mateja outlines three stages of cash flow testing: cash flow development, processing, and interpretation. Developing needed product and investment cash flows, according to Mateja, requires a thorough knowledge of product features and underlying assets. “Product cash flows are defined in the contract and, if positive, will generate investment cash flows.”(43, p. 10-3) ” But product cash flows can be more difficult to predict than investment cash flows, since insurers are ultimately dependent on policyholders to honor and comply with the terms for premium payment defined in their contracts, whereas investment cash flows generally arise from “safe” investments with known and, often, fixed payment schedules. Modeling product and investment cash flows under varying investment environments requires some knowledge of option-pricing theory, which Mateja says is a new skill actuaries must acquire if they are to understand the effects of those options on surplus projections and he lists the principal features of investment cash flows requiring such skills. Mateja outlines concisely the dependencies among an insurer’s cash flows: investment cash flows are dependent upon product cash flows and product cash flows are dependent upon policyholder behavior which, in turn, is dependent upon a host of influences external to the insurance operating environment and the external investment marketplace. Cash flow testing becomes art when it captures all such influences.

Processing cash flows requires a model capable of quantifying the interplay among operational variables such as interest rates, lapse rates, mortality rates, premiums, asset prepayment rates, etc. Mateja states a few fundamental principles underlying this interplay:

*“With increasing interest rates, lapse rates will increase, and investment prepayments will decrease. Just the opposite happens in decreasing interest rate scenarios. ((43), p. 10-9)”*

*“Other cash flows, which are dependent on the major product and investment cash flows, such as expenses, FIT, and future surrender benefits, will also be dependent on the interest scenarios. ((43), p. 10-10)”*

*“Once call protection has expired, you can assume that the more economically advantageous it is to refinance, the more likely it is that borrowers will do so. ((43), p. 10-11)”*

*“If net cash flow is positive, it must be reinvested; if it is negative, it is necessary to borrow or sell assets. Assumptions in this regard can materially influence results. ((43), p. 10-11)”*

*“The typical accounting system in an insurance company doesn’t understand cash flows, so you will scramble a bit((43), p. 10-13)” ... “The fundamental problem is that the information required for statutory accounting purposes is not the same as needed for cash flow analysis...sinking fund terms, refinancing dates, call information, and call premiums. ((43), p. 10-13)”*

*“Another real-world problem is control, i.e., making sure you’re looking at the same assets as liabilities. New business, trading, and commitments are some areas that should be of particular concern as you address the problem of control. ((43), p. 10-13)”*

Mateja’s principles are insightful and should be used as guiding principles in developing a cash flow testing model. He then talks about his third stage of cash flow testing, interpreting cash flows, which is done by example.

Philip K. Polkinghorn [52] discussed the sensitivity of SPDAs to "key" cash flow assumptions at another SVA as recently as 1992. Polkinghorn pointed out very early in his presentation that

- *“many of the assumptions made in cash flow testing are judgmental,*
- *results can be sensitive to modest changes in assumptions,*
- *the valuation actuary has as part of his/her responsibility to perform sensitivity analysis, ((52), p.1)”*

before discussing his cash flow testing results for an SPDA product.

The current Standard Valuation Law [48] and the Actuarial Standards Board, Actuarial Standard of Practice (ASP) Numbers 14 and 22 {[1], [3]}, implicitly support Polkinghorn's assertion of the judgmental nature of cash flow testing, leaving the decision to conduct cash flow testing up to valuation actuaries. However, all three standards require annual actuarial certification of reserves by appointed actuaries be delivered to their Insurance Commissioners. Should an appointed actuary conclude cash flow testing is not needed on a particular line of business, justification must be given in the opinion.

James Milholland [46] contends exercising judgment in cash flow testing is vitally important in determining when cash flow testing is needed and the degree to which it should be carried out. He supports the development of a company policy on cash flow testing which "would include company guidelines about the nature and extent of testing as well as the circumstances under which testing might not be required....such a policy would serve to document conformity with standards and also serve as internal guidance. (46), p. 3)" Milholland's remarks reflect his concern that ASP Number 14, and possibly Number 22, will be viewed more as additional work by actuaries than as a useful tool.

Gregory Jacobs [34] discussed how to integrate valuation concepts with cash flow testing at the 1988 Symposium for the Valuation Actuary in Dallas, Texas. He began by talking about the three classic forms of risk every insurer should be monitoring: C-1, C-2, and C-3. These three risks are defined as follows:

***C-1 Risk: Credit or Default Risk.*** *An insurance company can suffer a loss as a result of an asset that incurs a significant decrease in market value or becomes worthless purely as a result of events other than movements in interest rates.*

***C-2 Risk: Pricing Risk.*** *Simply stated, an insurance company can suffer a loss due to unforeseen changes in experience levels with respect to mortality, morbidity, expenses, etc.*



*C-3 Risk: Interest Rate Duration Risk. Simply stated, an insurance company can be selected against when interest rates move. This antiselection may cause a block of business to be unprofitable, meaning that the assets backing the reserves are not sufficient to cover the risk. ([34], pgs. 1 - 4)''*

Understanding the effects of these risks is the responsibility of the Valuation Actuary, according to Jacobs. Other responsibilities include: "understanding and obeying the standards of practice established by the profession and the regulations regarding the measurement of risk; relying on key individuals in the company, such as the Chief Executive Officer (CEO) and the Chief Investment Officer (CIO), for continuation of stated policy and/or good business judgment in managing company affairs; and rendering an opinion as to the adequacy of reserves. ([34], p. 6)'' Jacobs was followed by Arnold A. Dicke [34], who discussed various approaches to developing interest rate scenarios for cash flow testing. Dicke is critical of the yield curve scenarios defined in New York Regulation 126. He says the drawbacks to using such a static set of yield curves are:

- "1. The number of possible scenarios is unlimited, so no single scenario is likely to occur in real life.*
- 2. There is no unique way to make a probability statement about the outcome of a test. ([34], p. 14)''*

Dicke offers as alternatives the Transition Probability Approach and the Successive Ratio Model. Surprisingly, neither of these two methods produced results significantly different from those resulting from the regulatory scenarios. The last presenter, Douglas C. Doll [34], discussed how to set various cash flow testing assumptions for SPDAs and Universal Life products.

Cash flow testing SPDAs has enjoyed a great deal of attention from actuaries, financial and investment analysts, and regulators. Not only is this product line sensitive to changes in interest rates, it is also structurally simpler to model. SPDAs often involve only a single large premium which is subject to an interest rate term structure that evolves with time at the discretion of management. Management decides crediting interest rates beyond any initial interest rate guarantee period after consideration of marketplace variables. Reserving for SPDAs can also be

simplistic in the absence of multiple premium payments. Participating insurance is not as simplistic. Multiple premium payments can be expected and reserves exist on a variety of valuation bases, especially reserves for older blocks of participating insurance which continue to be a rich source of disintermediation risk. More recent issues of participating insurance contain variable policy loan interest rate provisions which lessen an insurer's exposure to the risk of disintermediation more than under fixed policy loan interest rate provisions. The model presented in this paper will utilize a simple means for measuring exposure under both types of provisions.

Dr. Allen Brender [12] is one of the few to make a contribution to the literature on the subject of modeling and cash flow testing participating insurance. He shares some of the same concerns regarding cash flow testing participating insurance that the above researchers have with regard to cash flow testing SPDAs. Dr. Brender discussed the "what" and the "how" of modeling participating whole life insurance issued by the Solvent Stock Life Insurance Company at the 1988 Symposium for the Valuation Actuary in Toronto, Ontario. Solvent Stock had been issuing whole life insurance for twenty-five years when Dr. Brender presented his model at the 1988 SVA. His model ignores riders and employs the "three-factor formula" for his dividend scale with tax reserve adjusted interest and mortality components, while the expense component is treated as a balancing item. The policy loan rate assumed is fixed at 6% for policies issued prior to 1969, 9% for the period 1969 through 1974, and variable thereafter. These assumptions are in line with tradition in the development of dividend scales and in line with how policy loan rate provisions have changed over time from a fixed rate provision to a provision which allows for the direct recognition of policy loan activity in dividend payouts to a provision of variable policy loan rates which recognizes interest rate movements in the financial marketplace.

Dr. Brender is also concerned with the assets underlying the block of participating insurance that he modeled. Call and prepayment patterns on bonds and mortgages are modeled in line with Solvent Stock Life's internal investment policy for these assets. A "buy and hold" strategy is

adopted and no trading is permitted as a simplifying assumption. Another simplifying assumption he makes is that “policy years and calendar years coincide; in effect, the division only operates one day a year, January 1. (112), p. 229)” He admits that this is not a very plausible assumption, but he still found his results under various scenarios to be quite reasonable and realistic. Dr. Brender rationalized the use of simplified computing systems in his modeling of participating life insurance as follows:

*“When you consider that a single scenario requires at least five runs of the valuation system, that at least twenty to thirty scenarios will have to run, and that most companies’ valuation systems require many hours to run on a mainframe for a single run, the cost in computer time and dollars of using a seriatim based valuation system seems prohibitive. (112), p. 231)”*

Dr. Brender adopted the Canadian Life and Health Insurance Association (CLHIA) prescribed scenarios in his work and only one resulted in negative net income. None of the prescribed scenarios resulted in surplus approaching zero. Non-CLHIA prescribed scenarios were also tested though not extensively discussed, but Dr. Brender suggests they involved varying interest rate scenarios and that all but one produced satisfactory results by CLHIA standards.

It is clear that cash flow testing is a major responsibility for actuaries in the United States and Canada. And although much of the research analyzes SPDAs, much can be learned from this research to help actuaries develop global approaches to analyzing participating and other types of insurance products for cash flow testing. For example, cash flow testing should begin with a thorough study of product features, identifying in particular those most likely to induce antiselection in response to changes in the financial marketplace. The next section of this paper examines the premium payment patterns and typical policy options common to many participating whole life products. This discussion is followed by the technical core of the Participating Cash Flow Testing Model (PCFTM) using new business asset shares, results under regulatory and non-regulatory interest rate scenarios, and their interpretation. And, lastly, criticisms and directions for further research are discussed.

## **II. The Product and its Characteristics**

### **A. Premium Payment Patterns**

The premiums on whole life policies can be structured to meet the specific needs of individual policyholders. They can be: level over the whole life of the insured, set at one level for an initial period of years and increased to another level over the remaining life of the insured, vanish altogether after a specified number of years with dividends making the payments if insurer experience is good, or be on a strictly limited pay basis independent of dividend experience. The block of participating business cash flow tested by the model presented in this paper consists of:

1. Level Premium Whole Life (LPWL), and
2. Modified Premium Whole Life (MPWL).

Policyholders may attach an Additional Insurance Protection (AIP) rider or a Paid-Up Insurance (PUI) rider (but not both) to the base policy under certain conditions and policy restrictions. An AIP rider is a combination of one-year term insurance and paid-up additions and may only be attached at issue. A PUI rider may be attached after issue but only to policies with initial face amounts in excess of \$25,000 for LPWL and \$50,000 for MPWL. Both riders require additional premiums and have the effect of accelerating policy cash values and reducing annual premiums (or shortening premium payment periods), in addition to increasing insurance protection in-force. While in effect, each type of rider entitles the policyholder to additional dividends because the additional insurance protection acquired is itself participating. Rider dividends are payable in the same manner as dividends on the base policy. They may be taken in cash, left on deposit, used to buy additional paid-up or term insurance, or used to reduce policy premiums. Using rider dividends to reduce policy premiums is the most prevalent election on this block of business and partly structures the “Vanishing Premiums” feature. The policyholder can apply base dividends, rider dividends, and the cash values of insurance additions (or some combination) to sharply decrease “out-of-pocket” annual premiums or to shorten the policy’s premium payment period. Even the election of LPWL over MPWL can materially effect the

vanish point of base policy premiums because LPWL premiums are generally greater than MPWL premiums.

### **Level Premium Whole Life**

Level Premium WL is designed to be a limited-pay life insurance policy, requiring premiums to the later of age 85 or 15 years. The premiums on LPWL are normally higher than premiums on a comparable modified premium whole life policy for an insured of the same age and underwriting profile, but dividends are payable starting in the second policy year. Higher premiums mean higher cash values in the early policy years and early dividends mean the total annual premium outlay can be reduced starting in the second policy year. The addition of an AIP or PUI rider can further reduce the policyholder's net annual outlay and accelerate the growth of the policy's cash value and even more so if purchased with dividends. Policyholders may also pay additional premiums over and above those defined for the base policy but only to the extent the additional premiums do not cause the policy to be viewed as an investment rather than as a purchase for death protection by Internal Revenue standards.

### **Modified Premium Whole Life**

Modified premium whole life supports a 2-step level premium payable over the life of the policy. The premiums are low in the first five policy years and increase to a higher level in the sixth policy year and remain at that level. On average the sixth year premium step increase is 25%. Low going-in premiums make insurance protection more affordable for young families, single parent families as well as retired couples and individuals in the 25 to 85 age group with at least \$30,000 of annual income. Base policy dividends become payable in the sixth policy year and can be used to offset the premium increase thereafter, maintaining the "out-of-pocket" cost to the policyholder. The addition of an AIP or PUI rider further reduces the cost to policyholders while simultaneously increasing policy cash values.

## B. Common Policyholder Options

Table 1 lists some of the features common to the block of participating whole life insurance under study in this paper. And while the specifics may differ among insurers, the effect these options can have on the profitability of participating whole life in various interest rate environments is a common concern. Therefore, brief discussions of these options are included for completeness and they are specific to the block of business under study.

**Table 1:**

**Policy Features Common to Participating Whole Life Insurance**

**Premium Payment Patterns**

Level or Modified at Issue

**Surrender Provision**

Guaranteed Value Available at any Time

**Crediting Interest Rates**

3.5% Guaranteed on Settlement Options  
and Deferred Surrender Benefit Payments

**Dividend Payment Policy**

Payable, except if in-force as Extended Term Insurance  
Dividends can also be used to purchase paid-up additions

**Minimum Dividend Rate**

Earn  $\geq$  3.5% if left on deposit

**Policy Loan Rate**

$\leq$  Max{Moody's Corporate Bond Yield lagged 2 months, 5.5%} on Post-1980 Issues  
Fixed Rates of 5%, 6%, and 8% on Pre-1980 Issues

**Premium Payment Patterns**

Policyholders can use level and modified premium whole life insurance to maximize cash value and insurance protection. Initial premiums can be reduced in the early years with premium increases offset by dividends in the later years and vanish altogether in the presence of policy riders. Alternatively, premiums can be set at one level for the entire premium paying period of the policy, producing larger cash values sooner and an earlier premium vanish point. Both

policy types guarantee that the level of premiums on the date of issue will meet objectives for death protection and savings. The major advantage of modified premium whole life is its low going in cost for the policyholder. This feature makes it more affordable for young, retired, and single parent families than level premium whole life. Since its introduction in the late 1980s, reserves on modified premium have grown steadily, now accounting for more than 20% of total reserves for this block as of 12/31/94. The upward trend is expected to continue.

### **Surrender Provision**

The policyholder may surrender at any time after issue but only for the guaranteed value of the policy. This amount equals the guideline policy cash value plus dividends and applicable interest due under the policy and attached riders, less any amounts of indebtedness. Policy values are computed in accordance with 80CSO, a 5% continuous interest basis, as reflected in the Tables of Computations defined in the contract and are specific to underwriting issue status. While policyholders are free to surrender at any time after issue, they have “paid a high initial cost to purchase the contract, and will not recover this cost unless the contract is held for a long period of time.”<sup>(21, p. 3)</sup> Historical surrender data for this block of business suggests policyholders are reluctant to surrender but will in extreme economic conditions. This block of business experienced its highest surrender rates during the periods 1926 to 1945, which includes the years of the Great Depression (1929-1939), and 1977 to 1990, a period marked by great inflation. During the first period, surrender rates climbed to nearly 7% before falling to just under 4% as a percentage of reserves in the years following. Surrender rates surged again in the second period, rising from 3.80% in 1977 to 8.67% in 1981 before a slow decent to 4.02% in 1990. This insurer responded by reducing dividends during both periods and in fact reduced dividends to zero for a number of years during the Great Depression at which time this block of business was about 35 years old. Using dividends to save operating gain is not a new or foreign concept to insurers. In fact, some insurers and researchers alike opine the focus of cash flow testing participating products should be to determine the capacity of dividends to sustain

operating gain in extreme interest rate environments, and not the adequacy of assets to do the same.

### **Credited Interest Rates**

Credited interest rates on participating products are not transparent to policyholders. Excess interest is paid as a function of insurer experience and “bundled” with gains from mortality and expenses in the annual dividends paid. This is a major way in which traditional participating products differ from their universal life counterparts. Policyholders understand this difference and many prefer the “unbundled” feature of universal life products. The “unbundled” feature of universal life products can be viewed as operationally equivalent to the dividend option of traditional whole life products. And yet, the unbundled feature has raised more concerns with regard to asset adequacy analysis than the dividend option.

### **Dividend Payment Policy & Minimum Dividend Rate**

As with all participating products dividends are payable at the discretion of the insurer. They are that portion of divisible surplus which results from better than expected insurer gain from operation. Most insurers, as does this one, utilize the three-factor contribution method to apportion surplus for distribution to policyholders. Under this method dividends are explicitly computed as the sum of excess interest earnings and mortality and expense savings. Policyholders are free to take these distributions in cash, leave them on deposit, apply them to any indebtedness, or fund PUI or AIP riders as previously discussed. With the exception of the cash option, each of these options is supported by reserves on the books of this insurer. Dividends left on deposit to accumulate with interest account for more than 50% of aggregate reserves held by this insurer for dividend options. Accumulated dividend reserves should be examined for asset adequacy since once left on deposit they are subject only to variations in interest rates, not variations in mortality or expenses. A minimum interest rate of 3.5% is guaranteed on these funds and a cash flow testing model could also attempt to measure this insurer’s ability to achieve this minimum.



### **Policy Loan Rate**

Policy loans are policyholder options and a huge source of cash outflow antiselection under fixed policy loan interest rate provisions, but less so under variable policy loan interest rate provisions or direct recognition provisions. Policies issued on this block prior to 1980 were issued with fixed interest rates of 5%, 6%, and 8%. This insurer experienced high disintermediation rates during the late 70s and early 80s when interest rates were high and like other insurers adopted the variable loan interest rate provision prescribed by the NAIC. This new provision dramatically slowed withdrawal rates on new issues but was ineffective on in-force policies since it could not be applied retrospectively. During that time this insurer took additional steps to control disintermediation by introducing a dividend enhancement plan. Under this plan, policyholders were allowed to exchange their old fixed rate policies for new variable rate policies with a potential for greater dividends. But as we shall see later this plan was not as successful as hoped. Most policyholders held onto their fixed rate policies. And today the total outstanding loan balance on this block of business is in excess of \$1 billion, a major opportunity loss of yield from possibly richer assets available in the marketplace. This insurer can only reclaim loaned funds if the policyholder repays the loan (which he/she is not obligated to do) or if the policyholder dies or surrenders the policy. Upon death or surrender, any indebtedness may be collected from the then available death benefit or cash value, respectively.

### **C. Dividend Options**

As Table 2 depicts, this insurer holds reserves for dividend options just as it does for the base policy. The reserves for paid-up additions and 1-year term insurance are on an actuarial basis and included in cash flow testing as a matter of course. The accumulated dividend reserves are not on an actuarial basis. This reserve resembles a savings account where dividend deposits are credited periodically with interest. The actual credited interest rate varies at the discretion of the insurer but it is guaranteed to meet or exceed the minimum guaranteed rate stipulated in the policy. The actual rate credited varies as a function of interest rates in the marketplace and this insurer will reduce the credited rate when investment yields are depressed. In declining interest

rate environments this insurer becomes concerned about not being able to provide the minimum interest credits on the dividend accumulation fund, fearing a “run-on-the-bank” reaction from policyholders. Unfortunately, the interest rate risk exposure for this fund is not covered in valuation law and many insurers may be ignoring it. The basic model presented in this paper can be modified to include a cash flow testing module for the dividend accumulation reserve and was for this insurer. The details of that module, however, are beyond the scope of this paper.

**Table 2**  
**Aggregate Dividend Reserves**  
**for**  
**Traditional Participating Whole Life Insurance**  
**(\$000)**

<b>Dividend Reserves:</b>	<b>1992</b>	<b>1993</b>	<b>1994</b>
<b>Accumulated Dividends*</b> (% of Grand Total)	<b>1,235,695</b> (60.0%)	<b>1,251,042</b> (56.9%)	<b>1,265,699</b> (56.1%)
<b>Paid-Up Additions</b> (% of Grand Total)	<b>814,251</b> (39.6%)	<b>939,672</b> (42.7%)	<b>976,691</b> (43.4%)
<b>1-Year Term Insurance</b> (% of Grand Total)	<b>8,679</b> (0.4%)	<b>9,158</b> (0.4%)	<b>10,219</b> (0.5%)
<b>Grand Total:</b> % Change:	<b>2,058,625</b>	<b>2,199,872</b> 6.86%	<b>2,252,610</b> 2.40%

\*This is not an actuarial present value simply dividend deposits accumulated with interest.

The AIP rider was created specifically for modified premium whole life when this product was introduced in 1988. Since this is a relatively new product, the reserves for the AIP rider are quite small as evidenced by Table 2. The rider is initially composed of one year term insurance only and as the table shows the reserves held for one year term account for less than 1% of total aggregate reserves. As the AIP rider matures, the term component is replaced with paid-up additions and those reserves account for a more significant portion of the aggregate total. What is not clear is how the interest rate sensitivity of reserves for one year term insurance and paid-up additions differs from that for base policy reserves. At this point in time, this is a more important consideration for paid-up additions than one year term mainly because of the size of

reserves held for paid-up additions. Reserves for paid-up additions are included in Exhibit 8 reserves but they can be cash flow tested in the same manner as either cash value reserves or an accumulation account. Treatment as cash value reserves implies paid-up additions are purchased primarily for death protection and subject to the same interest rate sensitivity as the base policy. On the other hand, if treated as an accumulation account, then it may be more appropriate to treat them in like manner to dividend accumulations to model their interest rate sensitivity in cash flow testing models. Under this treatment, the paid-up additions are merely another way for policyholders to treat traditional participating whole life insurance as an investment. For now, the PCFTM uses the former approach assuming paid-up additions have the same interest rate sensitivity as the base policy. Sufficient data is not yet available to help better model the interest rate sensitivity of the AIP rider, as one year term reserves are not significant enough at this time to make a difference.

### **III. The Objective of Cash Flow Testing**

#### **A. Researchers For & Against Cash Flow Testing Participating Whole Life**

Armand dePalo and James P. Rieskytl [21] discussed “when” to conduct cash flow testing on participating insurance at the 1987 Symposium for the Valuation Actuary in Dallas, Texas. The work they presented was prepared for inclusion in a Handbook for the Valuation Actuary. It is intended to help valuation actuaries distinguish between participating products that require extensive cash flow testing and participating products requiring less to measure exposure to C-3 risk. This distinction is predicated on the ability of participating products to be “self-supporting” under all but the most adverse conditions and whose dividends reflect the current earnings of the company (and have the ability to recover past losses). ((21), p. 1)” The more probable a participating product is likely to be self-supporting, the less extensive cash flow testing needs to be, according to dePalo and Rieskytl. Presumably, dePalo and Rieskytl are also using the term “self-supporting” to mean dividend scales and premium levels are more than sufficient to cover any unexpected future contingencies as well. They say:

*“The true participating whole life contract is a contract that, via dividends, is intended to give the policyholder insurance coverage at approximate cost, reflecting the actual experience of the insurer for the policy class. To accomplish this goal, the insurer gives only minimal guarantees and the policyowner assumes the risk of future variations in actual experience within limits set by the guarantees. As a result, classical asset/liability matching is inappropriate for true participating insurance. Rather, the objective sought by asset/liability matching is tied principally to future changes in dividends, not to future cash or loan values or interest rates assumed at issue (or various renewal points). The objective of participating ‘matching’ is to maximize the company’s dividend paying capability - to maximize the ability to match the dividends to changing interest rates (and other experience factors) as they emerge so as to provide ‘insurance at cost.’ This responsiveness varies between new money and portfolio based dividends but each system expects to reflect change as it emerges. ((21), p. 2)”*

Classical asset/liability matching (ALM) on participating business is not only inappropriate, but also an almost impossible exercise to complete. Macaulay and modified duration measures are inappropriate to ALM manage this business. As discussed by Cody [14], Macaulay and modified duration are useful tools only when cash flows are fixed. The cash flows of participating whole life insurance do not satisfy this requirement because insurers can and will alter the timing and amount of these cash flows through the dividend option. Participating liabilities are long in duration and insurers must roll over investments as prescribed by their investment policy to match liability cash flows. At present the only tool available to monitor that matching adequately is cash flow testing. Bad results may imply the insurer needs to lengthen assets while good results may allow the insurer to shorten the duration of the portfolio of assets backing participating liabilities.

Providing insurance at cost is the only constraint that must be satisfied according to dePalo and Rieskytl. They imply this constraint is implicitly satisfied if the gross premiums supporting participating insurance contracts are based on conservative assumptions, and, thus, extensive cash flow testing need not be conducted. These two researchers contend that participating contracts “can qualify for an exemption from extensive cash flow testing only if current and future dividends are meaningful in magnitude and can be substantially reduced to adjust for prior losses and future adverse variations in experience. ((21), p. 2)” Unfortunately, dePalo’s and Rieskytl’s exemption criteria are problematic because the phrases “extensive,” “meaningful in

magnitude,” and “substantially reduced” need to be quantified. Cash flow testing provides a means of quantifying the sufficiency of current and future dividends as well as the impact other risk conditions may have on participating insurance lines. Valuation actuaries are encouraged to utilize management’s philosophy with respect to reducing dividends in their cash flow testing models. It is important to know if management would impose a floor below which dividends cannot be reduced and incorporate that information into a cash flow testing model. Once dividends reach that floor, the model would know to start reducing operating gain, and then surplus, to absorb adverse experience. Valuation actuaries must also pay attention to portfolio yields implied by cash flow testing results. If insufficient, an insurer’s ability to provide minimum contractual guarantees to policyholders is compromised. This relationship is impossible to measure without conducting cash flow testing.

Richard Lambert [40] discusses some of the “how” and a great deal of the “when” to do cash flow testing on participating insurance, questioning the need to even do it at all. He recognizes very early in his discussion that most of the raw data needed to cash flow test products other than traditional participating insurance is of the same nature as the raw data needed to cash flow test participating insurance. The one exception he notes regards dividends. Dividends are integral only to participating product lines and Lambert discusses the importance of incorporating current company dividend policy in cash flow testing models. In particular, models need to know “what is the time lag between when interest rates drop and when the company reduces its dividends to reflect those lower rates. ([40], p. 1)” He also discusses adjusting dividend scales for interest rates and catastrophic illnesses like Acquired Immune Deficiency Syndrome, the mechanics of the contribution principle in the distribution of divisible surplus, and the need to recognize components of dividend scales unrelated to participating blocks of insurance. Lambert’s discussion of “when” to conduct cash flow testing on traditional participating product lines is grounded in the reality of trying to appease regulators without straining company capital and manpower resources. He cash flow tested a block of participating insurance under the New York

Regulation 126 scenarios including an inverted yield curve scenario and found twentieth year surplus projections under each of these scenarios to be about the same.

Several simple linear regression analyses he conducted further supported his position on minimizing capital and manpower to cash flow test participating insurance. His dependent variables consisted of actual historical data on the following cash flow items: premiums, claims, expenses, dividends, surrenders, policy loans, and cash flows from insurance operations. Each of these dependent variables was separately regressed against each of the following independent variables: calendar year, short-term interest rates, and long-term interest rates. There were a total of twenty-one distinct regressions, each using actual historical data on dependent and independent variables for the calendar years 1976 through 1992. Four regressions yielded significant results (i.e., statistical “t-scores” that have a low probability of occurring by chance alone): claims regressed against long term interest rates, surrenders regressed against short-term interest rates, surrenders regressed against long-term interest rates, and policy loans regressed against short-term interest rates. None of these results is particularly surprising. It is well documented that in rising interest rate environments good risks surrender leaving poor risks behind to carry the block of business. An immediate consequence of an increase in surrenders is an increase in claims experience. Millette [47] found not only this result for the block of business under study but he also concluded that short-term interest rates was the driver behind the high policy loan withdrawal rates on this block from 1977 to 1990. Other insurers could probably come to similar conclusions as Lambert and Millette after a study of their business. On the other hand, Lambert was surprised that dividends regressed against short-term and long-term interest rates did not yield significant “t-scores.” He explains this finding as follows:

*“It can be attributed to two things:*

- 1. Dividend interest rates are related more to portfolio rates than to new money rates.*
- 2. The normal annual growth in aggregate dividends due to the slope of the dividend scale dominates the changes in the dividend interest rate. ((40), p. 18) ”*

Lambert subsequently draws another conclusion from his regression analyses which supports his argument for not conducting cash flow testing on participating insurance on an annual basis. He states:

*“While certain cash-flow components are interest sensitive, the overall insurance cash flows are not. This is because the total insurance cash flows are dominated by cash flows that are not particularly interest sensitive: premiums, claims, expenses, and dividends. (140), p. 22)”*

Claims bear a direct relationship to an insurer’s mortality experience and that experience is known to be influenced by lapse rates, which, in turn, are directly influenced by interest rates. According to Douglas C. Doll [34], “it is generally accepted that, if there are extra lapses on life insurance policies, that’s going to increase mortality in the future, because some of those excess lapses are going to reflect select lives. The people who are impaired are more likely to keep a life insurance policy since they’re unable to get an equivalent policy elsewhere. (134), p. 42)” The point here is that claims (or paid death benefits which is presumably meant by Lambert’s reference) may not be directly influenced by movements in interest rates, but if only the policyholders in good health are lapsing, then mortality increases and that affects an insurer’s financial statements. Doll also highlights the effects of expense inflation on maintenance expenses and the secondary effect it has on reserves. He believes that under high lapse scenarios maintenance and overhead expenses may not be getting proper weighting in cash flow testing models and calls for more diligence in this area.

Researchers {[16], [37], [58]} are looking into the effect of life insurance policy options, like the withdrawal option, on the price (or value) sensitivity of life insurance liabilities and this interest extends to the liabilities of participating whole life contracts. At present mathematical tools exist to value options on the asset side of the balance sheet, but those tools have not proven to be useful for valuing options on the liability side of the balance sheet. So while Lambert’s observations may hold for the liabilities he analyzed, it is safe to forego generalizing his remarks to the liabilities of other insurers until more research is done.

Lambert and dePalo and Rieskytl are all of the opinion that participating insurance need not be subjected to extensive cash flow testing. Lambert recommends cash flow testing participating insurance every three years if there is little change in pricing assumptions, dividend scales, and investment policy but would prefer the Actuarial Standards Board to recommend simplified alternative asset adequacy measures. The team of dePalo and Reiskytl agrees with Lambert's defense of less extensive cash flow testing techniques, but they do not offer alternatives. They leave it up to valuation actuaries to develop simplified models for quantifying the sufficiency of reserves backing participating products. One such model is discussed in this paper.



## **B. What Should Be Measured?**

### **1. Pricing Considerations**

The basic question cash flow testing seeks to address is the ability of the existing book value of assets (set equal to the book value of existing reserves) to support future product liability obligations. Pricing inadequacy is not the focus. The product development process attempts to set premiums to achieve a desired profit goal, compensate agents competitively, and maintain a competitive share of the product's market. Participating products have an additional objective of returning some portion of earnings, those not needed, to policyholders in the form of dividends. Premium levels on participating products are set in expectation of returning earnings to policyholders by basing those earnings on conservative estimates of future mortality rates, interest rates, taxes, expenses, and contingencies for future catastrophic events. Harsh surrender penalties in the early policy years help to discourage policyholders from lapsing, allowing insurers to recover expenses and grow accumulated funds. Cash flow testing utilizes current premium levels and other funds, at the time of valuation, to analyze the match between asset flows and liability flows under assumptions for mortality rates, interest rates, taxes, expenses, and contingencies for future catastrophic events that match current-day conditions. Cash flow testing does not measure the appropriateness of pricing assumptions, although an insurer may use "the results of cash flow testing to determine the desirability of certain product features, investment strategies, or interest-crediting strategies (159), p. 126)" in future product development efforts.

### **2. Policyholder Dividends**

Dividends are an important component of future liability flows, since by design they are an integral component of participating products, functioning to provide insurance at cost. Policyholders are typically allowed to dispose of their share of divisible surplus in a variety of

ways, as evidenced in Exhibit 7 of the NAIC Annual Statement Blank, **Dividends and Coupons to Policyholders**. Some common options are to route dividend monies to pay renewal premiums, shorten endowment or premium-paying periods, buy additional paid-up insurance amounts or annuities, take dividend amounts in cash, or leave the money on deposit with insurers to accumulate at least at minimum contractually specified rates of interest. The most prevalent means by which dividends are distributed to policyholders is the **Contribution Principle** which requires that aggregate divisible surplus be distributed among policies in the same proportion as the policies are considered to have contributed to divisible surplus. The participating block of business under study utilizes the **Three-Factor Contribution Method** to rebate surplus to policyholders, and although other methods for distributing surplus are utilized by some insurers, only the three-factor method will be discussed in this paper. The three factor method says that excess earnings returned as dividends result from three sources: interest, mortality, and expense gains, formulated as:

$${}_kD = {}_kI + {}_kM + {}_kE \quad (8), p. 609$$

where

- ${}_kD$  = dividend per \$1,000 payable at the end of policy year k
- ${}_kI$  = excess interest factor for policy year k
- ${}_kM$  = mortality savings factor for policy year k
- ${}_kE$  = expense savings factor for policy year k

Bowers, et al.,[11] derive an explicit formulation using the notion of a fund share,  ${}_kF$ , which when combined with investment income and future premiums will mature the block of business with a high probability. A fund share, akin to an asset share, represents a target per policy share of the portfolio of assets backing the reserve liabilities for the entire block of policies. In its simplest three-term form, Bowers, et al., derive divisible surplus as follows:

$$\begin{aligned}
 {}_{k+1}D &= ({}_kF + G) (i'_{k+1} - i) && + && \text{Interest Gain} \\
 &+ (1 - {}_{k+1}F)(q^{(1)}_{x+k} - q^{(1)}_{x+k}) && + && \text{Mortality Gain}
 \end{aligned}$$

$$[E_k (1 + i) - E'_k (1 + i'_{k+1})]$$

**Expense Gain**

where  $x$  = age of issue

${}_kF$  = Beginning of year fund target

$G$  = Gross premiums for policy year  $k$

$E'_k = G c'_k + e'_k$  actual expense experience for policy year  $k$

$E_k = G c_k + e_k$  assumed expense experience for policy year  $k$

$c_k$  = contingency and profit loading for policy year  $k$

$e_k$  = expenses for policy year  $k$

$i'_{k+1}$  = actual interest experience for policy year  $k$

$i$  = assumed expense experience for policy year  $k$

$q^{(1)}_{x+k}$  = actual mortality experience for policy year  $k$

$q^{(1)}_{x+k}$  = assumed mortality experience for policy year  $k$

Black and Skipper discuss at least six different approaches insurers utilize to determine dividend interest. One insurer bases dividend scale interest on pre-tax net income inclusive of realized capital gains and exclusive of net income from policy loans. The asset base was not disclosed for this insurer, but Black and Skipper point out that one could legitimately use ledger assets, admitted assets, or invested funds with the above pre-tax net income. Other insurers are found to set this interest factor to a rate they judge will best reflect their investment earnings for several years and assume this rate as the portfolio rate, or use an average portfolio rate. Insurers could also turn to the investment generation method to set dividends. Under this method, dividend interest is based on the investment performance of assets that belong to the same generation as the base policy. This method is criticized for the erratic pattern of dividends it produces, an undesirable result from a marketing point of view.

Asset share studies base dividend interest rates on interest-bearing liabilities. "Interest-bearing liabilities include policy reserves, funds held under settlement agreements, dividend accumulations, and advanced premiums. Further, if certain items, such as dividend accumulations, have a minimum guarantee, some insurers credit these items with the guaranteed rate only and increase the net effective rate for regular policy dividend purposes. (18), p. 610)" This

means that the dividend interest rate used to credit paid dividends will likely differ from the interest rate implicit in asset share dividends.

Black and Skipper point out a key difference between the asset share mortality assumption and the dividend mortality assumption. Their point in this regard is that the dividend “mortality savings factor is usually expressed as a percentage of the assumed cost of insurance...depending only on the attained age of the insured and the insurer’s experience among all insureds at that age and duration. ([8], p. 612)” The cost of insurance factor is usually based on ultimate mortality for the calculation of dividend mortality and select mortality in asset share studies. Select lives will experience a greater increase in dividend mortality savings in their early years than in their ultimate lives.

### 3. Gains From Operations

Gains from operations directly influence the surplus position of an insurer. The **Summary of Operations** exhibit of the NAIC Annual Statement blank details the relationship between statutory gains from operations and statutory surplus. The upper half of the exhibit provides four measures of operational performance for a given year:

1. *Net Gain from Operations Before Dividends to Policyholders and Before Federal Income Taxes,*
2. *Net Gain from Operations After Dividends to Policyholders and Before Federal Income Taxes,*
3. *Net Gain from Operations After Dividends to Policyholders and After Federal Income Taxes and Before Realized Capital Gains or Losses, and*
4. *Net Income.*

Net gain from operations is the difference between operational inflows and operational outflows. Some examples of operational inflows include: premiums, annuity considerations, and deposit funds collected during the year; net investment income for the year; commissions, expenses, and

reserve adjustments on reinsurance ceded; amortization of the interest maintenance reserve; and other miscellaneous income. Some examples of operational outflows include: death, endowment, annuity, disability, and surrender benefits paid out during the year; increases in policy reserves and deposit funds; commissions, expenses, insurance taxes and fees; dividends; and federal income taxes paid during the year. Each of the gain from operations lines listed above includes all or some portion of an insurer's operational inflows and outflows. Net income is unique in that it combines the total gain for the year resulting from operational inflows and outflows with results from realized capital gains or losses on the sale or maturity of underlying assets.

#### **4. Surplus Projections**

Cash flow testing efforts should produce measures that allow valuation and appointed actuaries to determine whether or not "reserves and designated surplus make good and sufficient provisions for all future obligations." Two such measures are statutory surplus and its present value at points of interest in the projection period. Both measures are the result of efforts to model operational inflows and outflows, and include the effects of reinvestments. The duration(s) at which measurements are taken should support run-off patterns of liabilities. For some lines of business, the point at which most liabilities will be paid up might be of interest. For other lines, profit objectives set during the product development process may influence the valuation actuary's choice of duration at which to take measurements. Management may also have some interest in the determination of when measurements should be taken. Projected surplus and its present value at the twentieth duration was of particular interest in valuing this block. This duration allows comparisons of the performance of this block of business against its profit objective. Positive results would indicate a satisfactory response from this block of business to fluctuations in the interest, lapse, and mortality rates imposed on the model.

Negative results would suggest the need to hold additional reserves for this block of business, a response the Actuarial Standards Board would support. To do otherwise, appointed actuaries must justify this action in their actuarial opinions. Negative results may also imply the need to lengthen the duration of the portfolio of assets backing the liabilities to thwart the risk of asset inadequacy. The PCFTM did produce negative results for the block of business under study, but only under interest rate scenarios more severe than those prescribed by regulation. The decision to hold additional reserves under such severe scenarios is the responsibility of valuation and appointed actuaries, possibly assisted by investment and finance specialists to help evaluate the likelihood of such scenarios materializing in the marketplace. Performance results for the block of business under study are discussed in more detail in a later section of this paper.

#### **IV. Modeling Product Liability Outflows -- The Simplifying Technique**

##### **A. Using Asset Shares to Model the Block of Business**

Shane Chalke's Profit Testing System (PTS) is a widely used PC-based pricing tool. This system allows pricing actuaries to project experience on a planned product using experience units as small as individuals. The traditional approach is to band individuals on characteristics such as age, gender, underwriting classification, average policy size, sales distribution channels, etc., to form homogeneous groups of policyholders with "predictable" experience. The resulting distinct intersections represent the issue units or cells the PTS requires for modeling experience on new business. Assumptions for lapse rates, mortality, expenses, and portfolio yields, combined with standard assumptions for future interest and inflation rates, taxes, and premium loadings are imposed on each issue unit. The PTS allows insurers to develop premium constraints for each issue class, underwriting classification status, dividends, and gender, taking into account TAMRA 7-pay maximums. Dividends can also be modeled in line with an insurer's philosophy.

This insurer relies on its own experience for assumptions underlying issue units and requires dividends to increase with issue age except near age 100. The model office projection for this insurer is given in Table 3. It initially contained only 30 years worth of data which was not far enough into the future to reflect the “run-off” behavior of the business, a requirement of cash flow testing regulations. From that point on, premium income, reserves, surrender benefits, pre-tax gain, dividends, and surplus was modeled using accepted actuarial recursion formulas, insurer specific assumptions, and basic statutory accounting formulas to relate these elements. The projected liability flows of Table 3 were derived as follows:

**Liability Flows**  
 = (Investment Income - Pre-Tax Gain) - Policyholder Dividends - ΔTerminal Reserves

This relationship provides an initial level of net product liability flows. By substituting the standard expression for pre-tax gain the above expression becomes:

**Pre-Tax Gain**

= {(Investment Income - (Premiums + Investment Income - Death & Surrender Benefits - ΔTerminal Reserves - Expenses - Policyholder Dividends) }  
 - Policyholder Dividends - ΔTerminal Reserves

**or, simply**

**= Death & Surrender Benefits + Expenses - Premiums**

The asset shares are on a calendar year basis with a January 1 issue date. The initial liability flows are lagged a half year to convert policies to the standard actuarial mid-year issue assumption. This was achieved by prospectively averaging liability flows at adjacent durations. That is,

**Table 3**  
**ASSET SHARE MODEL OFFICE PROJECTION**  
**1993 NEW ISSUES**  
 (Note: Asset shares are on a calendar year basis.)  
 (\$Millions)

	(a) Gross Premium Income	(b) Increase in Reserves	(c) Policyholder Dividends	(d) Surrender Benefit
<b>Cumulative Sums</b>	<b>\$ 736.6</b>	<b>\$ 0.0</b>	<b>\$ 291.2</b>	<b>\$ 287.0</b>
<i>Durations From Issue</i>				
0	71.9	0.0	0.0	0.0
1	61.8	30.6	0.0	0.1
2	54.4	24.4	0.1	1.3
3	48.5	19.2	1.0	4.0
4	43.2	14.6	12.3	6.3
5	45.2	17.6	12.5	8.0
6	40.7	13.7	12.9	9.8
7	36.7	10.4	13.2	11.2
8	33.0	7.4	13.3	12.3
9	29.7	4.8	13.2	13.0
10	26.7	5.0	12.5	11.3
11	24.4	3.3	12.1	11.6
12	22.3	1.7	11.6	11.9
13	20.3	0.3	11.1	12.0
14	18.5	-0.8	10.6	12.0
15	16.7	-2.4	10.1	11.8
16	15.1	-3.1	9.8	11.6
17	13.7	-3.8	9.4	11.3
18	12.3	-4.3	9.0	11.0
19	11.1	-4.7	8.7	10.7
20	9.9	-1.9	8.0	7.0
21	9.1	-2.3	7.8	6.9
22	8.4	-2.7	7.6	6.8
23	7.8	-2.9	7.3	6.6
24	7.2	-3.3	7.1	6.6
25	6.6	-3.3	6.8	6.3
26	6.1	-3.5	6.6	6.1
27	5.6	-3.7	6.3	5.9
28	5.1	-3.8	6.0	5.7
29	4.6	-4.0	5.6	5.5
30	4.0	-4.5	5.3	5.3
31	3.5	-5.1	5.0	5.1
32	2.9	-6.0	4.6	4.8
33	2.4	-6.9	4.2	4.5
34	1.9	-7.8	3.8	4.2
35	1.5	-8.6	3.3	3.8
36	1.1	-9.1	2.9	3.3
37	0.8	-9.3	2.4	2.8
38	0.6	-9.0	2.0	2.4
39	0.4	-8.4	1.6	1.9
40	0.3	-7.5	1.2	1.4
41	0.2	-6.3	0.9	1.1
42	0.1	-4.9	0.6	0.7
43	0.1	-3.6	0.4	0.5
44	0.0	-2.5	0.2	0.3
45	0.0	-1.5	0.1	0.2
46	0.0	-0.9	0.1	0.1
47	0.0	-0.4	0.0	0.0
48	0.0	-0.2	0.0	0.0
49	0.0	-0.1	0.0	0.0



**Table 3**

**ASSET SHARE MODEL OFFICE PROJECTION  
1993 NEW ISSUES**

{Note: Asset shares are on a calendar year basis.}  
(\$Millions)

	(a) Gross Premium Income	(e) Reserves	(f) Investment Income	(g) Pre-Tax Gain From Operations (Including Interest on Surplus)
<b>Cumulative Sums</b>	<b>\$ 736.6</b>	<b>\$ 4,252.0</b>	<b>\$ 436.5</b>	<b>\$ 60.4</b>
<i>Durations From Issue</i>				
0	71.9	0.0	-5.1	-72.0
1	61.8	30.6	-0.6	10.7
2	54.4	55.0	2.1	13.1
3	48.5	74.2	4.4	12.3
4	43.2	88.8	6.7	2.8
5	45.2	106.4	8.6	5.2
6	40.7	120.1	10.1	4.1
7	36.7	130.5	11.4	3.3
8	33.0	137.9	12.4	2.7
9	29.7	142.7	13.1	2.1
10	26.7	147.7	13.5	1.8
11	24.4	151.0	13.9	2.0
12	22.3	152.7	14.3	2.0
13	20.3	153.0	14.4	1.9
14	18.5	152.2	14.5	1.9
15	16.7	149.8	14.4	1.1
16	15.1	146.7	14.2	1.2
17	13.7	142.9	13.9	1.3
18	12.3	138.6	13.6	1.3
19	11.1	133.9	13.2	1.4
20	9.9	132.0	12.8	1.6
21	9.1	129.7	12.7	1.7
22	8.4	127.0	12.5	1.7
23	7.8	124.1	12.3	1.7
24	7.2	120.8	12.1	1.7
25	6.6	117.5	11.9	1.7
26	6.1	114.0	11.6	1.7
27	5.6	110.3	11.3	1.8
28	5.1	106.5	11.1	1.8
29	4.6	102.5	11.2	1.9
30	4.0	98.1	10.9	1.9
31	3.5	92.9	10.5	1.8
32	2.9	86.9	10.1	1.8
33	2.4	80.0	9.6	1.8
34	1.9	72.2	9.0	1.8
35	1.5	63.7	8.3	1.8
36	1.1	54.6	7.5	1.8
37	0.8	45.3	6.7	1.9
38	0.6	36.3	5.9	1.9
39	0.4	27.9	5.2	2.0
40	0.3	20.4	4.4	2.0
41	0.2	14.1	3.8	2.1
42	0.1	9.2	3.3	2.2
43	0.1	5.6	3.0	2.3
44	0.0	3.1	2.7	2.4
45	0.0	1.6	2.6	2.5
46	0.0	0.7	2.5	2.6
47	0.0	0.3	2.6	2.7
48	0.0	0.1	2.6	2.8
49	0.0	0.0	2.7	2.9

**Table 3**

**ASSET SHARE MODEL OFFICE PROJECTION  
1993 NEW ISSUES**

*Interest Rate Assumed in Asset Shares: 9.30%*

*{Note: Asset shares are on a calendar year basis.}*

*(\$Millions)*

	(a) Gross Premium Income	(h) Ending Surplus	(f) - (b) - (c) - (g) Liability Flows (Un-Lagged)	Lagged Liability Flows (Mid-Year Assumption)
<b>Cumulative Sums</b>	<b>\$ 736.6</b>	<b>\$ 77.2</b>	<b>\$ 84.9</b>	<b>\$ 51.4</b>
<i>Durations From Issue</i>				
0	71.9	-48.6	66.9	12.5
1	61.8	-45.5	-41.9	-38.7
2	54.4	-40.0	-35.5	-31.8
3	48.5	-34.0	-28.1	-25.6
4	43.2	-33.1	-23.0	-24.9
5	45.2	-30.4	-26.7	-23.7
6	40.7	-26.9	-20.6	-18.1
7	36.7	-23.4	-15.5	-13.3
8	33.0	-20.3	-11.0	-9.0
9	29.7	-17.5	-7.0	-6.4
10	26.7	-15.2	-5.8	-4.7
11	24.4	-13.0	-3.5	-2.3
12	22.3	-10.9	-1.0	0.1
13	20.3	-8.9	1.1	2.0
14	18.5	-7.0	2.8	4.2
15	16.7	-5.7	5.6	6.0
16	15.1	-4.4	6.3	6.7
17	13.7	-3.1	7.0	7.3
18	12.3	-1.8	7.6	7.7
19	11.1	-0.6	7.8	6.5
20	9.9	0.6	5.1	5.3
21	9.1	1.9	5.5	5.7
22	8.4	3.1	5.9	6.1
23	7.8	4.2	6.2	6.4
24	7.2	5.4	6.6	6.7
25	6.6	6.4	6.7	6.8
26	6.1	7.5	6.8	6.9
27	5.6	8.5	6.9	7.0
28	5.1	9.5	7.1	7.4
29	4.6	10.5	7.7	7.9
30	4.0	11.5	8.2	8.5
31	3.5	12.5	8.8	9.2
32	2.9	13.4	9.6	10.0
33	2.4	14.4	10.4	10.8
34	1.9	15.2	11.2	11.4
35	1.5	16.1	11.7	11.8
36	1.1	17.0	11.9	11.8
37	0.8	17.9	11.7	11.4
38	0.6	18.8	11.1	10.6
39	0.4	19.7	10.1	9.4
40	0.3	20.6	8.7	7.9
41	0.2	21.5	7.1	6.3
42	0.1	22.5	5.5	4.7
43	0.1	23.5	3.9	3.2
44	0.0	24.6	2.6	2.0
45	0.0	25.7	1.5	1.1
46	0.0	26.9	0.8	0.5
47	0.0	28.1	0.3	0.1
48	0.0	29.3	0.0	-0.1
49	0.0	30.7	-0.1	-0.1

$$\text{Lagged Liability Flows}_t = (\text{Liability Flows}_t + \text{Liability Flows}_{t+1})/2$$

Table 4 depicts historical data on basic elements comprising the summary of operations for this block of business. The product inflows increased significantly in 1993 as did the liability outflows. The PCFTM uses this actual liability experience to adjust the pricing assumptions

**Table 4**  
**Selected Annual Statement Data**

<b>Results for Participating Whole Life</b>			
<b>Summary of Operations (\$Mil)</b>	<b>1991</b>	<b>1992</b>	<b>1993</b>
<b>Inflows:</b>			
Premiums (P)	901	994	1,038
Investment Income (II)	791	774	791
Other (O)	143	146	127
<b>Total Inflows:</b>	<b>1,835</b>	<b>1,864</b>	<b>1,956</b>
<b>Outflows:</b>			
Benefits (B)	864	846	806
Life Reserve Increase (V)	121	173	248
Expenses and Other Outflows (E)	356	344	364
PH Dividends (D)	321	302	296
<b>Total Outflows:</b>	<b>1,662</b>	<b>1,665</b>	<b>1,714</b>
<b>Pre-Tax Gain (G):</b>	<b>173</b>	<b>199</b>	<b>242</b>
<b>Other Operating Results (\$Mil):</b>			
<b>Ledger + Non-Ledger Assets (\$Mil):</b>	<b>10,280</b>	<b>10,600</b>	<b>10,865</b>
<b>Aggregate Life Reserves (\$Mil):</b>	<b>7,288</b>	<b>7,394</b>	<b>7,490</b>
<b>Surplus Account (\$Mil):</b>	<b>1,704</b>	<b>1,834</b>	<b>1,908</b>

embedded in the asset shares. This adjustment to the lagged liability flows is necessary for replicating current gain from operations at time zero in the projection. Pre-tax gain from operations including interest on surplus is shown in Table 3. The model will exclude interest on surplus to derive a target profitability ratio which is needed to adjust the asset share liability flows to current operating levels.

## **B. Conversion of Asset Shares into a Closed Block of Business**

A standard model office projection provides results from the present year to some future year. The projection period of the model office projection used by the PCFTM was 1993 to 2022. To understand how the mapping is done it is necessary to interpret each asset share model office projection as representative of a past issue's contribution to 1993 experience and this requires removing references to calendar years. This is done in Table 3 where each projection year is represented as a duration from issue. Taking 1993 as the valuation year of cash flow testing, the asset share values at duration zero are assumed to represent the 1993 open block experience for contracts zero durations from issue -- 1993 issues. The asset share values at duration one are assumed to represent the 1993 open block experience for contracts one duration from issue -- 1992 issues. The asset share values at duration two are assumed to represent the 1993 open block experience for contracts two durations from issue -- 1991 issues. Continuing in this manner, the asset share values at duration forty-nine are assumed to represent the 1993 open block experience for contracts forty-nine durations from issue -- 1944 issues. This mapping simulates the 1993 open block experience of contracts issued as far back as 1944 and every year thereafter. The row of cumulative values given at the top of Table 3 summarizes the simulated experience of these issues and represents the entire block's experience in 1993. It is noteworthy that the mapping reflects a premium growth rate of zero. As illustrated in the next section, sales levels on this block of business have been relatively flat since 1967. Therefore, a premium growth assumption is not warranted.

Closing the block requires restricting new entrants, although decrement through death or withdrawal remains permissible. Table 5 shows the 1993 closed block experience for this product line at duration zero, the valuation year of the cash flow testing. The mapping technique that produced 1993 closed block experience is applied at each duration beyond the valuation year

Table 5

**THE 1993 CLOSED BLOCK OF PARTICIPATING  
WHOLE LIFE BUSINESS**  
(Baseline Levels)

(\$Millions)

Assume a premium growth rate of zero percent  
Assume premiums are paid continuously

Dividend Adjustment Factor = -2.535%  
Gains Adjustment Factor = -2.100%  
Both Applied to Gross Premiums

Initial Asset Share  
Closed Block Dividends = 291.2  
Dividend to Mean Reserves Ratio = 3.95%  
Gains to Mean Reserves Ratio = 0.23%

Durations from Valuation	Closed Block Gross Premiums	Mean Reserves	Dividend Adjusted Policyholder Dividends	Surrender Benefits	Ratio of Surrender Benefits to Mean Reserves	Gain Adjusted Liability Flows
0	736.6	4,497.2	177.7	287.0	6.38%	0.0
1	700.6	4,473.9	178.6	287.0	6.42%	66.1
2	631.8	4,438.2	180.2	287.0	6.47%	52.2
3	575.7	4,377.4	182.2	286.9	6.56%	89.7
4	524.2	4,296.8	184.6	285.6	6.63%	120.4
5	478.4	4,199.8	186.4	281.6	6.71%	145.0
6	434.2	4,087.2	177.4	275.1	6.74%	168.9
7	391.2	3,960.4	168.3	267.1	6.75%	191.7
8	352.5	3,822.9	159.1	257.5	6.74%	208.9
9	317.7	3,677.7	149.7	246.3	6.70%	221.4
10	286.3	3,527.5	140.3	234.0	6.63%	229.8
11	258.1	3,373.4	131.1	221.0	6.55%	235.6
12	232.6	3,215.9	122.7	209.7	6.52%	239.7
13	209.2	3,056.6	114.6	198.1	6.48%	241.5
14	187.9	2,896.9	107.0	186.2	6.43%	241.0
15	168.5	2,738.2	99.9	174.2	6.36%	238.6
16	150.9	2,581.6	93.2	162.2	6.28%	234.0
17	135.0	2,428.3	86.9	150.4	6.20%	227.7
18	120.6	2,278.9	80.8	138.8	6.09%	220.8
19	107.6	2,134.1	74.9	127.5	5.98%	213.2
20	95.9	1,994.2	69.4	116.5	5.84%	205.3
21	85.4	1,857.9	64.1	105.8	5.70%	198.6
22	75.9	1,724.0	59.4	98.8	5.71%	193.1
23	67.2	1,592.8	54.8	91.9	5.77%	187.2
24	59.1	1,464.6	50.4	85.1	5.81%	181.0
25	51.6	1,339.8	46.2	78.5	5.86%	174.4
26	44.7	1,218.4	42.1	71.9	5.90%	167.6
27	38.3	1,100.6	38.1	65.6	5.96%	160.8
28	32.5	986.6	34.3	59.5	6.04%	153.8
29	27.1	876.5	30.7	53.6	6.12%	146.7
30	22.3	770.5	27.3	47.9	6.22%	139.2
31	18.0	668.9	24.1	42.4	6.34%	131.2
32	14.2	572.3	21.1	37.1	6.48%	122.6
33	11.0	481.4	18.3	32.0	6.64%	113.3
34	8.4	397.2	15.6	27.1	6.83%	103.2
35	6.2	320.3	13.2	22.6	7.06%	92.3
36	4.5	252.1	10.9	18.5	7.32%	80.9
37	3.2	192.6	8.8	14.7	7.64%	69.1
38	2.2	142.4	7.0	11.4	8.00%	57.3
39	1.5	101.4	5.4	8.6	8.44%	45.9
40	1.0	69.2	4.0	6.2	8.96%	35.3
41	0.6	44.9	2.8	4.3	9.59%	25.9
42	0.4	27.6	1.9	2.9	10.35%	18.0
43	0.2	16.0	1.3	1.8	11.28%	11.7
44	0.1	8.6	0.8	1.1	12.45%	7.0
45	0.1	4.2	0.4	0.6	13.94%	3.7
46	0.0	1.9	0.2	0.3	15.93%	1.7
47	0.0	0.7	0.1	0.1	18.81%	0.6
48	0.0	0.2	0.0	0.1	24.05%	0.0
49	.....	.....	.....	.....	.....	.....
50	.....	.....	.....	.....	.....	.....

to project run-off experience for this block. Mid-year assumptions are imposed at each duration to calculate closed block premiums and mean reserves. Surrender benefits at a given duration are summed prospectively over remaining durations. The asset share dividend and liability flows are adjusted by dividend and gain adjustment factors, respectively, to bring initial levels in line with 1993 actual dividend and gain experience. The dividend and gain adjustment factors are given in Table 5 along with the desired dividend-to-mean reserve and gain-to-mean reserve ratios that determine their values. The assumptions and methodology underlying these factors will be discussed in more detail in a later section of this paper.

### **C. Sales Levels**

Sales levels on this block of business have been fairly level since 1967. Table 6 gives sales levels on this block as measured by annualized issue premium. Annualized issue premiums increase at a mean rate of 2% per year, before and after accounting changes are introduced. The relatively flat sales level over this period allowed for a premium growth rate assumption of zero in the mapping of asset share premiums to prior issue years. A premium growth rate assumption can only be utilized before the block is closed. After the block is closed such an assumption becomes inappropriate.

**Table 6**  
**Participating Whole Life Annualized Issue Premium**  
**for**  
**Period: 1967 - 1992**

<u>Policy Year</u>	<u>Sales Level (\$Million)</u>	<u>Factors Affecting Sales Levels</u>
1967	78	
1968	89	
1969	88	
1970	93	
1971	92	
1972	91	
1973	90	
1974	89	
1975	94	
1976	100	
1977	107	
1978	106	
1979	119	
1980	125	
1981	128	<= Universal Life Insurance Introduced
1982	103	<= Variable Products Introduced
1983	95	
1984	89	
1985	80	<= Universal Life Insurance Re-Designed
1986	67	
1987	79	
1988	108	<= New Accounting Method Introduced & MPWL Product Introduced
1989	119	
1990	128	
1991	139	
1992	120	

**\*\*Note:** The new accounting method introduced in 1988 resulted in higher issue premiums and these new premiums are not completely comparable to issue premiums before that date.

#### **D. Resulting Closed Block Premiums**

The closed block premiums are not used directly to derive liability flows as previously discussed, but they are used in the PCFTM. Life insurance premiums are directly tied to the Deferred Acquisition Costs (DAC) tax provision, which allows insurers to amortize acquisition expenses at a rate of 7.7% of gross premiums per year for ten years as premium income is realized to cover them. This rate applies to life and noncancellable accident and health insurance premiums. The closed block premiums are factored-up by the ratio of 1993 actual reserves to asset share reserves at each duration prior to calculating amortization amounts. An initial unamortized amount is estimated and combined with amounts already subject to DAC tax. The run-off of DAC expenses parallels the run-off of gross premiums for the closed block. The closed block gross premiums are also used by the dividend and gains adjustment factors to adjust the asset share dividend and liability flows, respectively, to 1993 actual experience, as will be discussed in more detail in a later section. Finally, closed block premiums are used to develop assumptions for expense inflation over the projection period.

#### **E. Adjusting Policyholder Dividends to Match Current Experience**

By now it is well understood that the basic objective of participating whole life insurance is to provide policyholders with insurance at “cost” over the whole of their lives. Gross premiums on each policy are set to enable insurers to achieve this objective over the aggregate life of the policy. As the policy ages, contingencies originally provisioned for are measured against actual experience allowing the insurer to gauge what unused portion can be returned to policyholders. Unneeded earnings apportioned to policyholder dividends each year are reported in the Summary of Operations exhibit of the Annual Statement. Table 4 depicts the basic components of this exhibit for the participating whole life block under study and is reproduced in part in Table 7 below.



**Table 7**  
**Selected Annual Statement Data**

<b>Results for Participating Whole Life</b>			
<b>Summary of Operations (\$Mil)</b>	<b>1991</b>	<b>1992</b>	<b>1993</b>
Inflows:			
Total Inflows:	1,835	1,864	1,956
Outflows:			
Total Outflows:	1,341	1,361	1,418
Pre-Tax Gain Before Dividends:	494	504	538
PH Dividends (D):	321	302	296
PH Dividends as a % of Pre-Tax Gain:	65%	60%	55%
Pre-Tax Gain (G) After Dividends:	173	199	242
<b>PH Dividends as a % of Reserves:</b>			
	<b>4.4%</b>	<b>4.1%</b>	<b>4.0%</b>
Ledger + Non-Ledger Assets (\$Mil):	10,280	10,600	10,865
Aggregate Life Reserves (\$Mil):	7,288	7,394	7,490
Surplus Account (\$Mil):	1,704	1,834	1,908

Since dividends are provisioned for in gross premiums, the PCFTM uses gross premiums to bring the asset share dividends in line with actual experience. Table 5 depicts the projected experience for the 1993 closed block along with the dividend to mean reserves ratio, the dividend adjustment factor, and other measures relevant to this analysis. The initial asset share closed block dividends are also indicated for convenience and differ from the initial adjusted policyholder dividends by almost half. Initial closed block dividends are approximately 6.5% of the asset share mean reserves, much higher than actual experience. The asset share dividends are much higher than actual experience relative to reserves for two principal reasons:

1. *Asset share dividends are projected using new money rates and not portfolio rates which ultimately determine the actual dividends paid.*
2. *Gross premium margins assumed in the asset shares are much higher than actual experience.*

To retro-fit the asset share dividends to actual dividend experience at the end of 1993, the following relationship was applied:

$$= \left\{ \sum_{x=t}^{49} \text{Asset Share Policyholder Dividends}_x \right\} + \text{Dividend Adjustment Factor} * \text{Gross Closed Block Premiums}_{t+1}$$

*(The asset share dividends are taken from those given in Table 3.)*

Actual experience suggests initial asset share dividends should be adjusted to achieve a dividend to mean reserves ratio of 4.0%. Through trial and error, the value of the dividend adjustment factor is changed until the desired dividend to mean reserves ratio is achieved. This factor then becomes the percentage by which asset share dividends are adjusted using gross premiums to achieve this ratio at duration zero in the projection. Although a 4.0% target is indicated by actual results, a target ratio of slightly less (3.95%) was used in the PCFTM to reproduce actual 1993 dividends more precisely. When dividends under a 4.0% target are grossed-up by the ratio of 1993 actual reserves to asset share mean reserves, policyholder dividends are reproduced as \$300 million, slightly more than 1993 actual dividends of \$296 million paid. By decreasing the target ratio, grossed-up dividends reproduce 1993 actual dividends more exactly, giving a dividend adjustment factor of 2.54%. This action does not have a material effect on the results.

#### **F. Adjusting Liability Flows to Match Current Experience**

The model presented in this paper relies on an asset share model office projection for projected liability data for the block of business under study. As previously discussed, the unadjusted asset share results cannot be expected to reflect actual statutory operating gain as of the valuation date of the cash flow testing. They require a re-calibration to replicate current operating gain. This re-calibration is done by way of a **gains adjustment factor**. The gains adjustment factor re-adjusts the asset share liability flows so as to reproduce current profit margins at time zero in the projection period.

Table 4 gives historical summary of operations data for this block and is reproduced in part in Table 8 below. Table 8 displays a reduced set of items from the summary of operations page along with other annual statement items that will aid in developing an adjustment factor for the liability flows used in the PCFTM.

**Table 8**  
**Selected Annual Statement Data**

<b>Results for Participating Whole Life</b>			
<b>Summary of Operations (\$Mil)</b>	<b>1991</b>	<b>1992</b>	<b>1993</b>
<b>Inflows:</b>			
Total Inflows:	1,835	1,864	1,956
<b>Outflows:</b>			
Total Outflows:	1,341	1,361	1,418
Pre-Tax Gain Before Dividends:	494	504	538
PH Dividends (D):	321	302	296
<b>Pre-Tax Gain (G):</b>	<b>173</b>	<b>199</b>	<b>242</b>
<b>Pre-Tax Gain Net of Interest on Surplus:</b>	<b>29</b>	<b>58</b>	<b>98</b>
<b>Net Pre-Tax Gain* to Total Assets:</b>	<b>0.28%</b>	<b>0.55%</b>	<b>0.90%</b>
<b>3 Year Average Gains Target Ratio:</b>			<b>0.58%</b>
<b>Capital Losses at 35 BP:</b>			<b>-0.35%</b>
<b>1993 Net-Gains Target Ratio:</b>			<b>0.23%</b>
<b>Supplementary Data (\$Mil):</b>			
Ledger + Non-Ledger Assets:	10,280	10,600	10,865
Aggregate Life Reserves:	7,288	7,394	7,490
Surplus Account:	1,704	1,834	1,908

\* Net Pre-Tax Gain is the same as Pre-Tax Gain Net of Interest on Surplus

Pre-tax gain net of interest on surplus can vary dramatically from year to year and does for this insurer, although for the better. Net pre-tax gain as a percent of total assets also exhibits huge yearly increases over this three year period. This data seems to suggest good times are ahead for this insurer, but the PCFTM is going to resist using only 1993 results to gains-adjust liability flows. The PCFTM is concerned with the long-run earnings record of this insurer and will look

to average historical operating gain performance to gains-adjust projected liabilities. In view of the data, this would seem a rather conservative position. A three year average gains ratio of 0.58% indicates how this insurer has performed under recent conditions in the external marketplace and current social climate. More years could be justified if past conditions are felt to currently influence an insurer's operation in a significant way. Five year average results were about the same for this insurer.

Premiums and reserves for the closed block are taken directly from the model office. Investment income is adjusted downward to reflect an implied prevailing portfolio rate of 6.78%. Benefits and expenses are approximated by initial closed block premiums and initial closed block liability flows from Table 3. The adjusted policyholder dividends are those from Table 5. Interest on surplus is calculated for exclusion from pre-tax gain as previously indicated.

The gains adjustment amount (see Table 5) represents the amount of premium offset needed to achieve the desired gains ratio of 0.23% at duration zero in the projection period. This offset corrects the ill-fit of the mortality and expense assumptions inherent in the asset share premium scales by forcing the asset share liability flows to behave as if under current conditions of mortality and expenses, at least initially. After the initial duration, pre-tax gain is not constrained by this ratio but the liability flows do continue to evolve in the presence of the gains adjustment factor given in Table 5. The liability flows are projected at each duration  $t$  using the following relationship:

$$= \frac{1}{2} \cdot Liabilities_t + \left\{ \sum_{x=t+1}^{50} Liabilities_x \right\} - Gains Adjustment Factor \cdot Closed Block Gross Premiums_t$$

The first term reflects the mid-year assumption imposed on the model. The second term sums the remaining asset share liability flows to close the block of business and project its run-off pattern. The third term illustrates the use of gross premiums by way of the gains adjustment factor to bring initial liability flows in line with actual 1993 experience and adjusts remaining projected flows by the same percent of corresponding projected premiums.

Table 5 shows the gains adjustment factor, the gains adjusted liability flows, and the gain to reserves ratio. The gains adjustment amount in Table 5 is defined as the product of closed block premiums at duration zero and the gains adjustment factor. As previously mentioned, this amount reduces liabilities at duration zero to achieve a gains ratio of 0.23%. The gains ratio is the ratio of net pre-tax gain from operations to mean reserves.

Through trial and error, the value of the gains adjustment factor as represented in the above formula is changed until the initial liability flows converge to those that achieve the desired net pre-tax gain to mean reserves ratio. This factor then becomes the percentage by which subsequent liability flows are adjusted using gross premiums. Table 5 shows that a gains adjustment factor of -2.10% is needed to attain a net pre-tax gain to mean reserves ratio of 0.23%.

#### **G. Developing Sensitivity Measures**

Let's review. The model office projection provided the basic elements of a statutory income statement. A closed block of business was created by mapping projected income statement data back to previous issues so the run-off of 1993 closed block business can be observed. The asset share dividends have been adjusted to support this insurer's current estimation of funds not needed to mature the block and a rough estimate of initial liability flow projections have been

forced to conform to present-day conditions. The PCFTM needs to fold in a provision for future lapses. This will allow for observing the withdrawal sensitivity of the liability flows under different interest rate scenarios and the effect on projected surplus levels.

Table 9 illustrates the lapse function of the PCFTM under a level interest rate scenario. The durational lapse adjustment factors are defined for three subsets of the projection period and define anticipated lapse behavior. Level interest rates are not believed to alter the expected surrender behavior assumed in the asset shares, as they reflect current withdrawal assumptions used in pricing. Surrender patterns for this block are effectively modeled by the surrender benefits given in the model office projection. The mapping technique employed by the PCFTM to create a closed block of business utilizes the ratio of closed block surrender benefits to closed block reserves as a proxy for future baseline surrender behavior at each duration in the projection period. The lapse adjustment factors allow the PCFTM to tailor the baseline lapse behavior defined by the pricing to specific interest rate scenarios.

Under a level interest rate scenario, the lapse adjustment factor is defined as 1.0 for all durations. This means that the level of reserves, dividends, and liability flows do not require an adjustment to account for the level of surrenders implied by the asset share model office projection. As you will recall, the liability flows used in the model were derived as follows:

$$= (\text{Investment Income} - \text{Pre-Tax Gain}) - \text{Policyholder Dividends} - \Delta \text{Terminal Reserves.}$$

The surrender benefits from the model office were not used to directly derive liability flows. Instead, implied rates of surrender are derived as the ratio of surrender benefits to mean reserves using values for the closed block (see Table 5). These rates are assumed implicit in the level of reserves, dividends, and liability flows given in Table 5 and assumed to prevail in a level interest rate environment. Under varying interest rate scenarios, the level of reserves, dividends, and

Table 9

**LAPSE ADJUSTED LIABILITY FLOWS  
BY DURATION FROM VALUATION**

*Level Scenario*

(\$Millions)

Note:

*Survivorship Factor is defined  
as 1.00000 at duration 0  
and at duration 1.*

**Durational Lapse Adjustment Factors  
Under The Level Interest Rate Scenario**

<u>Durations</u>	<u>Factors</u>
1 through 5	1.0000
6 through 10	1.0000
11 and Beyond	1.0000

<u>Durations from Valuation</u>	<u>Survivorship Factors</u>	<u>Adjusted for Survivorship</u>			<u>Closed Block Premiums</u>
		<u>Mean Reserves</u>	<u>Policyholder Dividends</u>	<u>Gain Adjusted Liability Flows</u>	
0	1.00000	4,497.2	177.7	0.0	736.6
1	1.00000	4,473.9	178.6	66.1	700.6
2	1.00000	4,438.2	180.2	52.2	633.8
3	1.00000	4,377.4	182.2	89.7	575.7
4	1.00000	4,296.8	184.6	120.4	524.2
5	1.00000	4,199.8	186.4	145.0	478.4
6	1.00000	4,087.2	177.4	168.9	434.2
7	1.00000	3,960.4	168.3	191.7	391.2
8	1.00000	3,822.9	159.1	208.9	352.5
9	1.00000	3,677.7	149.7	221.4	317.7
10	1.00000	3,527.5	140.3	229.8	286.3
11	1.00000	3,373.4	131.1	235.6	258.1
12	1.00000	3,215.9	122.7	239.7	232.6
13	1.00000	3,056.6	114.6	241.5	209.2
14	1.00000	2,896.9	107.0	241.0	187.9
15	1.00000	2,738.2	99.9	238.6	168.5
16	1.00000	2,581.6	93.2	234.0	150.9
17	1.00000	2,428.3	86.9	227.7	135.0
18	1.00000	2,278.9	80.8	220.8	120.6
19	1.00000	2,134.1	74.9	213.2	107.6
20	1.00000	1,994.2	69.4	205.3	95.9
21	1.00000	1,857.9	64.1	198.6	85.4
22	1.00000	1,724.0	59.4	193.1	75.9
23	1.00000	1,592.8	54.8	187.2	67.2
24	1.00000	1,464.6	50.4	181.0	59.1
25	1.00000	1,339.8	46.2	174.4	51.6
26	1.00000	1,218.4	42.1	167.6	44.7
27	1.00000	1,100.6	38.1	160.8	38.3
28	1.00000	986.6	34.3	153.8	32.5
29	1.00000	876.5	30.7	146.7	27.1
30	1.00000	770.5	27.3	139.2	22.3
31	1.00000	668.9	24.1	131.2	18.0
32	1.00000	572.3	21.1	122.6	14.2
33	1.00000	481.4	18.3	113.3	11.0
34	1.00000	397.2	15.6	103.2	8.4
35	1.00000	320.5	13.2	92.3	6.2
36	1.00000	252.1	10.9	80.9	4.5
37	1.00000	192.6	8.8	69.1	3.2
38	1.00000	142.4	7.0	57.3	2.2
39	1.00000	101.4	5.4	45.9	1.5
40	1.00000	69.2	4.0	35.3	1.0

liability flows are modeled in the PCFTM as a function of the surrender rates implied by the closed block. For example, in a rising interest rate scenario surrender benefits are assumed to increase and the level of reserves, dividends, and liabilities should decrease in amount over time. The model requires the durational lapse adjustment factors to be 1.0 at duration zero for all scenarios to maintain initial operating conditions. Beyond duration zero, durational lapse adjustment factors can vary to suit the interest rate scenario imposed.

The survivorship factors compound the effect non-level interest rate scenarios have on the baseline reserves, dividends, and liabilities from time zero to a given duration. When interest rates remain level throughout the projection period, there is no effect on baseline levels as can be seen in the following formulation of the survivorship factors used in the PCFTM:

$$Survivorship_t = Survivorship_{t-1} * \{1 + Surrender\ Benefits\ to\ Re\ serves\ Ratio_t * (1 - Lapse\ Factor_t)\}$$

The purpose of the survivorship factors is to specify the level to which reserves, dividends, and liability flows should be reduced or increased under a given non-level interest rate scenario. A durational lapse adjustment factor of 1.0 appropriate for a level interest rate scenario will not change the baseline surrender behavior assumed in reserves, dividends, and liability flows. A durational lapse adjustment factor of 0.5 appropriate for a down interest rate scenario implies the baseline surrender behavior assumed in reserves, dividends, and liability flows should decrease, as policyholders tend to hold onto their policies. And a durational lapse adjustment factor of 2.0 appropriate for an up interest rate scenario implies the baseline surrender behavior assumed in reserves, dividends, and liability flows should increase, as policyholders tend to surrender their policies.



Factors of 1.0, 0.5, and 2.0 were chosen for the regulatory level, down, and up interest rate scenarios, respectively, for the PCFTM. As previously discussed, Millette [47] observed that withdrawals as a percentage of reserves on this block of business:

- exceeded 6% during the early years of the Great Depression (1930- 1934),
- declined to just over 3% in the later years,
- and fell nearly to zero during the years of World War II (1939 to 1945).

During one of the greatest inflationary times in the U.S., he observed surrender rates climb from 3.80% in 1977 to 8.76% in 1981 and then decline to 4.02% in 1990. His observations support the choice of durational lapse factors used under the regulatory interest rate scenarios. However, it is important to sensitivity test this block of business under non-regulatory interest rate scenarios as well. The durational lapse adjustment factors will take on values other than 0.5, and 2.0 under scenarios more extreme than the regulatory ones to assess the ability of the model to produce reasonable results.

The mean reserves are adjusted for lapses to account for the effect on persisting lives by the following relationship:

$$\text{Adjusted Mean Reserves}_t = \text{Survivorship}_t * \{ \text{Mean Reserves}_t \text{ Unadjusted for Survivorship} \}$$

PCFTM policyholder dividends and closed block premiums are similarly defined as illustrated in the following formulas:

$$\begin{aligned} & \text{Policyholder Dividends Adjusted for Survivorship}_t \\ & = \text{Survivorship}_t * \{ \text{Policyholder Dividends}_t \text{ Unadjusted for Survivorship} \} \end{aligned}$$

$$\begin{aligned} & \text{Closed Block Premiums Adjusted for Survivorship}_t \\ & = \text{Survivorship}_t * \{ \text{Closed Block Premiums}_t \text{ Unadjusted for Survivorship} \} \end{aligned}$$

The liability flows are further modeled to account for survivorship assumed under non-level interest rate scenarios. First, the liability flows net of surrender benefits for the current period are adjusted by the survivorship factor for the current period to reflect the change in liabilities associated with survivors from prior periods. Second, surrender benefits for the current period are adjusted for prior surrenders using the survivorship and durational lapse adjustment factors. The result is the amount of surrender benefits expected to be paid in the current period and it is combined with the survivorship adjusted liability flows to give a new level of liability outflows. The **Gain and Survivorship Adjusted Liability Flows** are defined as:

*Gain and Survivorship Adjusted Liability Flows*

$$\begin{aligned}
 &= \text{Gain Adjusted Liability Flows} \\
 &\quad \text{for } t < 2 \\
 &= (\text{Gain Adjusted Liability Flows}_t - \text{Surrender Benefits}_t) * \text{Survivorship}_t \\
 &\quad + \\
 &\quad (\text{Surrender Benefits}_t * \text{Survivorship}_{t-1} * \text{Durational Lapse Adjustment Factor}_t) \\
 &\quad \text{for } t \geq 2
 \end{aligned}$$

Table 9A shows the progression of lapse adjusted premiums, reserves, dividends, and liability flows under the pop-down interest rate scenario. Comparing corresponding values in Tables 9 and 9A, survivorship factors rapidly become accumulation factors under the pop-down scenario, which is indicative of a higher persistency rate than observed under the level scenario. Recall that the level scenario reflects the lapse behavior exhibited by the asset share surrender benefits and serves as a baseline level of experience. Under down scenarios, fewer lapses are expected, which means more premium income should be generated because the policies are persisting longer, and dividends should grow over time as more premium income is received. At the same time, reserves should grow to account for increasing mortality as persisting lives age, along with

Table 9A

**LAPSE ADJUSTED LIABILITY FLOWS  
BY DURATION FROM VALUATION**

**Pop-Down Scenario**

(\$Millions)

**Note:**

Survivorship Factor is defined as 1.00000 at duration 0 and at duration 1.

**Durational Lapse Adjustment Factors  
Under The Pop-Down Interest Rate Scenario**

<u>Durations</u>	<u>Factors</u>
1 through 5	0.5000
6 through 10	0.5000
11 and Beyond	0.5000

<u>Durations from Valuation</u>	<u>Survivorship Factors</u>	<u>Adjusted for Survivorship</u>			<u>Closed Block Premiums</u>
		<u>Mean Reserves</u>	<u>Policyholder Dividends</u>	<u>Gain Adjusted Liability Flows</u>	
0	1.00000	4,497.2	177.7	0.0	736.6
1	1.00000	4,473.9	178.6	66.1	700.6
2	1.03234	4,581.7	186.0	(98.9)	654.3
3	1.06617	4,667.1	194.3	(62.2)	613.8
4	1.10161	4,733.4	201.3	(29.7)	577.5
5	1.13855	4,781.7	212.3	(0.4)	544.7
6	1.17690	4,810.2	208.7	31.5	511.0
7	1.21662	4,818.3	204.8	65.3	476.0
8	1.25760	4,807.6	200.1	95.5	443.4
9	1.29972	4,780.0	194.6	122.5	412.9
10	1.34284	4,736.9	188.5	146.4	384.5
11	1.38683	4,678.3	181.9	168.6	358.0
12	1.43206	4,605.3	175.7	188.3	333.1
13	1.47847	4,519.0	169.5	205.9	309.4
14	1.52600	4,420.7	163.3	221.2	286.8
15	1.57455	4,311.4	157.3	234.3	265.4
16	1.62403	4,192.6	151.4	244.3	245.1
17	1.67433	4,065.8	145.5	251.6	226.1
18	1.72533	3,932.0	139.3	257.6	208.2
19	1.77689	3,792.1	133.1	262.3	191.3
20	1.82881	3,647.0	126.8	265.8	175.5
21	1.88090	3,494.5	120.5	271.3	160.7
22	1.93482	3,335.6	114.9	275.3	146.9
23	1.99066	3,170.7	109.1	278.6	133.8
24	2.04852	3,000.3	103.2	281.1	121.1
25	2.10856	2,825.0	97.4	282.7	108.8
26	2.17081	2,644.9	91.3	283.6	97.0
27	2.23555	2,460.4	85.3	283.9	85.7
28	2.30301	2,272.1	79.1	283.6	74.8
29	2.37348	2,080.3	72.9	282.6	64.4
30	2.44732	1,885.6	66.8	280.2	54.6
31	2.52489	1,688.8	60.9	276.0	45.4
32	2.60667	1,491.7	55.0	269.7	37.1
33	2.69323	1,296.6	49.2	260.6	29.7
34	2.78525	1,106.3	43.5	248.3	23.3
35	2.88355	924.1	38.0	232.5	18.0
36	2.98914	753.5	32.6	213.1	13.5
37	3.10325	597.6	27.4	190.7	10.0
38	3.22743	459.5	22.6	165.7	7.2
39	3.36363	340.9	18.1	139.3	5.0
40	3.51433	243.0	14.0	112.7	3.5

liabilities to account for higher dividend payouts and higher reserve increases each year. Table 9B shows how the product flows of Table 9 are altered under a pop-up interest rate scenario. Survivorship factors become discount factors indicating reductions in future liabilities due to withdrawals.

Though not shown in Table 9 or Table 9A, the liability flows are grossed-up once more by the ratio of actual 1993 reserves to lapse adjusted mean reserves as a final, but vitally necessary, step before they are combined with the asset flows. This last modification simply grosses the liability flows upward to 1993 levels.

#### **H. Policy Loans**

Policy loan activity was much more important in the early 80s and prior years when insurance contracts allowed policyholders to borrow against their cash values at modest interest rates, while at the same time interest rates in the financial services marketplace were significantly higher. Policyholders looking to arbitrage their investments in life insurance contracts borrowed heavily against them at low interest rates and invested those funds elsewhere at significantly higher interest rates. This arbitrage activity replicated the “run on the bank” environment of the U.S. Great Depression Era (1929-1939). Insurers responded in the 1980s and onward by integrating policy loan activity with dividend payout levels and moving away from fixed policy loan interest rates. Under the NAIC’s variable loan provision, insurers can tie policy loan interest rates to yields on Moody’s Bond Index with a lag. This practice dampens the effects of disintermediation on new issues but not on older business. Arbitrage opportunities still exist on older business and need to be modeled in cash flow testing. The model presented in this paper details some simple techniques for studying an insurer’s exposure to disintermediation risk on older blocks of business. A crosstabs analysis is introduced that can help insurers develop and monitor their exposure to disintermediation risk due to policy loans.

Table 9B

**LAPSE ADJUSTED LIABILITY FLOWS  
BY DURATION FROM VALUATION**

Pop-Up Scenario

(\$Millions)

**Note:**  
Survivorship Factor is defined  
as 1.00000 at duration 0  
and at duration 1.

**Durational Lapse Adjustment Factors  
Under The Pop-Up Interest Rate Scenario**

Durations	Factors
1 through 5	2.0000
6 through 10	2.0000
11 and Beyond	2.0000

Durations from Valuation	Survivorship Factors	Mean Reserves	Adjusted for Survivorship		
			Policyholder Dividends	Gain Adjusted Liability Flows	Closed Block Premiums
0	1.00000	4,497.2	177.7	0.0	736.6
1	1.00000	4,473.9	178.6	66.1	700.6
2	0.93533	4,151.2	168.5	354.4	592.8
3	0.87401	3,825.9	159.3	364.4	503.2
4	0.81591	3,505.8	150.6	364.5	427.7
5	0.76120	3,196.9	141.9	355.6	364.2
6	0.70992	2,901.5	125.9	343.6	308.2
7	0.66199	2,621.7	111.4	329.5	259.0
8	0.61740	2,360.2	98.2	311.0	217.7
9	0.57604	2,118.5	86.3	289.8	183.0
10	0.53782	1,897.2	75.5	267.3	154.0
11	0.50258	1,695.4	65.9	245.1	129.7
12	0.46980	1,510.8	57.6	224.9	109.3
13	0.43935	1,342.9	50.4	205.2	91.9
14	0.41110	1,190.9	44.0	186.1	77.3
15	0.38494	1,054.0	38.5	168.0	64.9
16	0.36075	931.3	33.6	150.8	54.5
17	0.33840	821.7	29.4	134.7	45.7
18	0.31779	724.2	25.7	120.0	38.3
19	0.29879	637.7	22.4	106.7	32.2
20	0.28133	561.0	19.5	94.6	27.0
21	0.26531	492.9	17.0	84.2	22.7
22	0.25010	431.2	14.9	76.0	19.0
23	0.23566	375.4	12.9	68.4	15.8
24	0.22196	325.1	11.2	61.4	13.1
25	0.20895	279.9	9.6	54.9	10.8
26	0.19661	239.6	8.3	48.9	8.8
27	0.18489	203.5	7.1	43.4	7.1
28	0.17373	171.4	6.0	38.4	5.6
29	0.16310	142.9	5.0	33.8	4.4
30	0.15295	117.8	4.2	29.6	3.4
31	0.14325	95.8	3.5	25.7	2.6
32	0.13397	76.7	2.8	22.1	1.9
33	0.12507	60.2	2.3	18.7	1.4
34	0.11653	46.3	1.8	15.6	1.0
35	0.10830	34.7	1.4	12.8	0.7
36	0.10037	25.3	1.1	10.3	0.5
37	0.09271	17.9	0.8	8.0	0.3
38	0.08529	12.1	0.6	6.0	0.2
39	0.07809	7.9	0.4	4.4	0.1
40	0.07109	4.9	0.3	3.0	0.1

An insurer's current policy loan experience is easily observable with a simple crosstabs analysis. A crosstabs is a matrix which classifies data by two of its attributes, with one attribute classifying the data by rows and the other attribute classifying the data by columns. For the PCFTM, outstanding policy loan balances were classified by policy issue year and by policy loan interest rate. Row and column sums aggregate outstanding balances by their respective attributes.

Table 10 displays outstanding policy loan balances for this block of business as of September 30, 1993. At a glance, it becomes immediately apparent the bulk (85%) of the \$1.6 billion balance is attributable to policies issued prior to 1981 with fixed policy loan rate provisions. Around this time, this insurer adopted the NAIC's variable policy loan provision and allowed inforce policyholders to convert their fixed loan rate policies to variable loan rate policies under a dividend enhancement plan which accounts for policy loan balances at variable rates on policies issued prior to 1981. The intent of the dividend enhancement plan was to curb exposure to disintermediation, but the crosstabs suggests very few policyholders converted their fixed rate loan provisioned policies to policies with the variable rate provision. As of 9/30/93, policies with variable loan rate provisions accounted for only 10% of the outstanding loan balance for this block of business. The top half of Table 10A summarizes the loan experience of the crosstabs analysis and the percentages given provide a foundation upon which to model future policy loan experience. The goal in modeling policy loans is to project outstanding balances at each duration that can be combined with the fixed income and real estate assets backing the reserve liabilities for this block. Policy loans are rightfully included with assets since policy cash values back policy loans ( i.e., cash values provide the collateral against which policy loans are made) and part of the policy loan interest is credited to policyholders.

CROSSTABS ANALYSIS OF OUTSTANDING POLICY LOAN BALANCES ON  
PARTICIPATING WHOLE LIFE INSURANCE AS OF 9/30/93

Table 10

(5000)

Policy Issue Year:	Policy Loan Rates:											Total \$Outstanding Balance	% of Total Outstanding Balance
	5.00% Fixed Rate	6.00% Fixed Rate	7.00%	7.50%	7.75%	8.00% Fixed Rate	8.00%	8.25%	8.50%	8.75%	9.25%		
1918		0										0	0%
1919		7										7	0%
1920		4										4	0%
1921		10										10	0%
1922		31						1				32	0%
1923		23			1							24	0%
1924		23										23	0%
1925		46	1		1			3				50	0%
1926		64	0			0	0					65	0%
1927		93										93	0%
1928		140			0			4				144	0%
1929		145		4				1				150	0%
1930		156						0				156	0%
1931		252				2	0	5				260	0%
1932		221		1		2	0	4				228	0%
1933		267			0			3				270	0%
1934		307			4	1	0					312	0%
1935		489			3			10				502	0%
1936		623			3	2	0	5				633	0%
1937	1	791	2	1	0	0	0	2		0		797	0%
1938		984	1		4	2	0	6				997	0%
1939	1,310		2	6	8	2	0	3		0		1,331	0%
1940	2,204		2	7	14	3	0	22				2,253	0%
1941	2,954		6	8	4	13	1	32				3,018	0%
1942	2,440		9	3	1	5	0	13		0		2,471	0%
1943	2,550				4	2	0	18		0		2,575	0%
1944	3,262		4		40	2	0	47				3,356	0%
1945	4,052		15		64	9	1	36				4,178	0%
1946	7,326		29	99	26	21	1	79		1		7,585	0%
1947	6,231		4		57	31	2	38	1		1	6,364	0%

CROSSTABS ANALYSIS OF OUTSTANDING POLICY LOAN BALANCES ON  
PARTICIPATING WHOLE LIFE INSURANCE AS OF 9/30/93

Table 10

(\$000)

Policy Issue Year:	Policy Loan Rates:										Total \$Outstanding Balance	% of Total Outstanding Balance	
	5.00% Fixed Rate	6.00% Fixed Rate	7.00%	7.50%	7.75%	8.00% Fixed Rate	8.00%	8.25%	8.50%	8.75%			9.25%
1948	6,504		7	3	54	14	1	47	4	5		6,639	0%
1949	7,518		6	16	21	22	1	60		5	1	7,649	0%
1950	10,244		12	18	96	46	3	151	21	2	0	10,593	1%
1951	11,815		50	5	44	34	2	158	3		2	12,113	1%
1952	13,072		19	19	82	35	2	145	0		0	13,376	1%
1953	15,679	1	29	31	63	78	5	190	4	2	4	16,086	1%
1954	17,403		17	13	78	42	3	130	5	10	9	17,710	1%
1955	20,140		34	43	71	74	5	311	18	0	1	20,698	1%
1956	22,344		26	49	121	68	5	270	4	2	2	22,888	1%
1957	30,096		51	228	153	133	9	464	17	9	3	31,162	2%
1958	35,860		57	100	287	158	11	453	25	4	5	36,959	2%
1959	39,405		72	161	257	155	11	511	24	11	2	40,608	3%
1960	34,162		56	83	303	202	14	512	18	1	3	35,354	2%
1961	37,850		71	91	288	203	14	519	18	1	4	39,060	2%
1962	42,114		90	90	284	134	9	593	20	7	6	43,348	3%
1963	51,043	1	126	133	456	164	11	728	17	27	4	52,709	3%
1964	52,500		84	161	415	226	15	787	46	8		54,242	3%
1965	58,567		174	252	423	218	15	760	38	4	4	60,455	4%
1966	57,688		90	118	422	271	19	910	16	4	4	59,542	4%
1967	60,915		100	210	464	318	22	1,129	22	5	15	63,200	4%
1968	66,487	1	221	216	573	351	24	963	17	5	6	68,864	4%
1969	61,536		108	138	1,063	279	19	937	51	9	6	64,146	4%
1970	32,904	24,975	162	105	439	289	20	1,054	37	5	5	59,994	4%
1971	10,547	47,413	192	131	461	430	29	1,624	46	5	2	60,879	4%
1972	9,455	45,183	201	197	428	244	17	889	81	17	3	56,716	4%
1973	7,683	38,447	76	139	383	277	19	833	29	12	9	47,907	3%
1974	7,585	35,920	55	121	365	214	15	764	32	8	6	45,085	3%
1975	16,955	31,952	63	150	336	405	28	705	25	2	16	50,636	3%
1976	11,940	35,282	94	115	396	300	21	778	13	10	3	48,952	3%
1977	11,077	36,311	107	115	327	449	31	891	48	5	2	49,362	3%



CROSSTABS ANALYSIS OF OUTSTANDING POLICY LOAN BALANCES ON  
PARTICIPATING WHOLE LIFE INSURANCE AS OF 9/30/93

Table 10

(\$000)

Policy Issue Year:	Policy Loan Rates:											Total \$Outstanding Balance	% of Total Outstanding Balance
	5.00% Fixed Rate	6.00% Fixed Rate	7.00%	7.50%	7.75%	8.00% Fixed Rate	8.00%	8.25%	8.50%	8.75%	9.25%		
1978	3,452	13,577	5	35	92	25,615	1,751	250	8	8		44,794	3%
1979	2	18,727	270	20	162	31,015	2,121	117	5	2	2	52,444	3%
1980	1	3,244			78	38,990	2,666	86	1	1		45,068	3%
1981	1					39,216	2,682					41,899	3%
1982		2	1	4		27,150	1,856	55				29,068	2%
1983	1		1,070	1,282	3,751	15,077	1,031	9,227	457	28	21	31,944	2%
1984			1,143	1,668	5,935	7,319	500	9,740	603	49	52	27,009	2%
1985			1,302	2,272	5,391	4,917	336	8,678	703	51	26	23,676	1%
1986			699	648	4,279	3,653	250	5,859	333	104	39	15,865	1%
1987			643	878	2,778	2,421	166	4,714	399	20	21	12,038	1%
1988			652	815	2,915	2,508	172	5,156	297	47	16	12,578	1%
1989			301	525	2,157	1,389	95	3,806	411	38	14	8,735	1%
1990			230	290	1,236	654	45	2,100	111	25	1	4,692	0%
1991			78	263	551	380	26	898	76	31		2,302	0%
1992			4	3	114	139	9	258				527	0%
1993								1				1	0%
<b>Total</b>													
<b>Outstanding</b>	<b>\$ 896,875</b>	<b>\$ 335,714</b>	<b>\$ 8,925</b>	<b>\$ 12,083</b>	<b>\$ 38,824</b>	<b>\$ 206,370</b>	<b>\$ 14,111</b>	<b>\$ 69,543</b>	<b>\$ 4,106</b>	<b>\$ 593</b>	<b>\$ 317</b>	<b>\$ 1,587,460</b>	<b>( 100 ) %</b>
<b>Balance</b>	<b>( 56 ) %</b>	<b>( 21 ) %</b>	<b>( 1 ) %</b>	<b>( 1 ) %</b>	<b>( 2 ) %</b>	<b>( 13 ) %</b>	<b>( 1 ) %</b>	<b>( 4 ) %</b>	<b>( 0 ) %</b>	<b>( 0 ) %</b>	<b>( 0 ) %</b>	<b>( 100 ) %</b>	
<b>(% of Total)</b>													

**Table 10A**  
**Outstanding Policy Loan Balances**  
**as of**  
**9/30/93**

Policy Loan Rate	% of Total Balance		
Fixed	5%		56%
	6%		21%
	8%		13%
Variable	Myriad		10%

<b>1994 Policy Loan Data</b>
------------------------------

Annual Expenses%	≈1.0%
Outstanding Balance as of 12/31/94 ⇒	\$1,733
Policy Loan Asset Base	\$10,868
Balance as a Percent of Asset Base	15.95%

Table 10B details the methodology behind the modeling of policy loans under a level interest rate scenario for illustrative purposes. It should be noted at the outset that this methodology is applied in the PCFTM to all the tested interest rate scenarios by varying new money rates and lapse adjustment factors consistently. The modeling starts with the reserve growth factor. It is used to keep policy loan balances in line with reserves and defined as follows:

$Reserve\ Growth\ Factor_t = Mean\ Reserve_t / Mean\ Reserve_{t-1}$
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Reserve adjusted policy loans on variable rate provisioned contracts are expected to increase at future durations at a rate of 1% per duration as follows:

$Pre - Adjusted\ Variable\ Balance = 1,733 * (13\% + 0.01 * t) * Reserve\ Growth\ Factor_t$
---

A similar relationship is applied to policy loans on fixed rate provisioned contracts. Fixed rate provisioned contracts are expected to experience a 1% decline in policy loans at future durations.

Table 10B

MODELING FUTURE POLICY LOAN EXPERIENCE  
UNDER THE LEVEL INTEREST RATE SCENARIO

(Millions)

Durations from Valuation	Reserve Growth Factor	Outstanding Balances on Variable Loans	Pre-Adjusted Outstanding Balances on Fixed Loans	New Money Rates	Adjusted Outstanding Balances at 5% Loan Rate	Adjusted Outstanding Balances at 6% Loan Rate	Adjusted Outstanding Balances at 8% Loan Rate	Annualized Interest	Pre-Tax Mean Interest	Total Outstanding Balance	After-Tax Cash Flow	Unlensed Amount at 5%	Unlensed Amount at 6%	Unlensed Amount at 8%	Change in Loans%
0	1.0000	173	1,560	5.93%	970	164	225	81	0	1,733	0	5,116	1,918	1,188	0.00%
1	0.9948	190	1,534	5.93%	849	299	161	70	75	1,499	283	5,138	1,946	1,228	3.06%
2	0.9869	205	1,505	5.93%	726	234	96	58	64	1,261	279	5,146	1,969	1,267	3.02%
3	0.9734	219	1,468	5.93%	602	167	29	45	51	1,018	277	5,125	1,980	1,300	2.95%
4	0.9554	232	1,424	5.93%	478	102	0	36	40	811	233	5,079	1,982	1,290	2.90%
5	0.9339	243	1,376	5.93%	356	37	0	28	32	636	195	5,011	1,976	1,246	2.88%
6	0.9088	252	1,323	5.93%	239	0	0	22	25	491	162	4,923	1,936	1,198	2.91%
7	0.8806	259	1,267	5.93%	127	0	0	18	20	386	117	4,816	1,854	1,147	2.97%
8	0.8500	265	1,208	5.93%	21	0	0	14	16	286	110	4,692	1,768	1,094	3.09%
9	0.8178	269	1,148	5.93%	0	0	0	13	14	269	26	4,479	1,680	1,040	3.21%
10	0.7844	272	1,087	5.93%	0	0	0	13	13	272	6	4,243	1,591	985	3.23%
11	0.7501	273	1,027	5.93%	0	0	0	13	13	273	8	4,067	1,503	930	3.23%
12	0.7151	273	967	5.93%	0	0	0	13	13	273	9	3,772	1,414	876	3.23%
13	0.6796	272	907	5.93%	0	0	0	13	13	271	10	3,539	1,327	822	3.23%
14	0.6442	268	848	5.93%	0	0	0	13	13	268	12	3,311	1,241	769	3.23%
15	0.6089	264	791	5.93%	0	0	0	13	13	264	13	3,088	1,158	717	3.23%
16	0.5740	259	736	5.93%	0	0	0	13	13	259	13	2,873	1,077	667	3.23%
17	0.5400	253	683	5.93%	0	0	0	12	13	253	14	2,665	1,000	619	3.23%
18	0.5067	246	632	5.93%	0	0	0	12	12	246	15	2,467	925	573	3.23%
19	0.4745	238	584	5.93%	0	0	0	12	12	238	15	2,278	854	529	3.23%
20	0.4434	231	538	5.93%	0	0	0	11	12	231	15	2,099	787	487	3.23%
21	0.4131	222	494	5.93%	0	0	0	11	11	222	16	1,928	723	447	3.23%
22	0.3833	213	452	5.93%	0	0	0	10	11	213	16	1,763	661	409	3.23%
23	0.3542	203	411	5.93%	0	0	0	10	10	203	17	1,605	602	373	3.23%
24	0.3257	192	372	5.93%	0	0	0	9	10	192	17	1,454	545	337	3.23%
25	0.2979	181	336	5.93%	0	0	0	9	9	181	17	1,309	491	304	3.23%
26	0.2709	169	300	5.93%	0	0	0	8	9	169	17	1,173	440	272	3.23%
27	0.2447	157	267	5.93%	0	0	0	8	8	157	17	1,043	391	242	3.23%
28	0.2194	144	236	5.93%	0	0	0	7	7	144	17	920	345	214	3.23%
29	0.1949	132	206	5.93%	0	0	0	6	7	132	17	804	301	187	3.23%
30	0.1713	119	178	5.93%	0	0	0	6	6	119	17	695	261	161	3.23%
31	0.1487	106	152	5.93%	0	0	0	5	6	106	17	593	223	138	3.23%
32	0.1272	93	128	5.93%	0	0	0	5	5	93	16	499	187	116	3.23%
33	0.1070	80	106	5.93%	0	0	0	4	4	80	16	413	155	96	3.23%
34	0.0883	67	86	5.93%	0	0	0	3	4	67	15	334	125	78	3.23%
35	0.0713	56	68	5.93%	0	0	0	3	3	56	14	265	99	62	3.23%
36	0.0560	45	52	5.93%	0	0	0	2	2	45	12	205	77	48	3.23%
37	0.0428	35	39	5.93%	0	0	0	2	2	35	11	153	58	36	3.23%
38	0.0317	26	29	5.93%	0	0	0	1	2	26	10	111	42	26	3.23%
39	0.0225	19	20	5.93%	0	0	0	1	1	19	8	78	29	18	3.23%
40	0.0154	13	13	5.93%	0	0	0	1	1	13	6	52	19	12	3.23%

The following formula defines the total fixed loan amounts prior to new money interest rate adjustments:

$$Pre - Adjusted Fixed Balance_t = 1,733 * (56\% + 21\% + 10\% - 0.01 * t) * Reserve Growth Factor_t$$

The 1% differential reflects the fact that the variable rate business is newer and may be expected to grow in size relative to the fixed rate business.

To account for changes in new money rates, the following relationship adjusts policy loan balances when new money rates exceed fixed policy loan rates by more than 3%:

$$Adjusted Outstanding Balance at F\% = \max \left\{ \begin{array}{l} 0, F\% Loan_{t-1} * Pre - Adjusted Fixed Balance Ratio + \\ (New Money Rate - F\% - 3\%) * Unloaned Amount \end{array} \right\}$$

where

$$Pre-Adjusted Fixed Balance Ratio = Pre-Adjusted Fixed Balance Ratio_t / Pre-Adjusted Fixed Balance Ratio_{t-1}$$

The 3% hurdle was judged appropriate for this block of business. Other insurers may use a different assumption. The term “F% Loan<sub>t-1</sub>” refers to adjusted outstanding balances on fixed loans given in Table 10A and “F%” represents the fixed policy loan rates of 5%, 6%, and 8%. The new money rates are level for this illustration, the level scenario. They should be made to vary as prescribed by Standard Valuation Law in the other regulatory interest rate scenarios. Results under the pop-down scenario are given in Table 10C. Under this scenario policy loan balances gradually decline to zero and after-tax cash flow increases. Under the pop-up scenario loan balances and cash flows rapidly go to zero (See Table 10D).

Annualized interest is the sum of policy loan interest less expenses on the total outstanding balance. The total outstanding balance is the sum of adjusted fixed loan balances plus the

Table 10C

MODELING FUTURE POLICY LOAN EXPERIENCE  
UNDER A POP-DOWN INTEREST RATE SCENARIO

(\$Millions)

Derivatives from Valuation	Reserve Growth Factor	Outstanding Balances on Variable Loans	Pre-Adjusted Outstanding Balances on Fixed Loans	New Money Rates	Adjusted Outstanding Balances at 5% Loan Rate	Adjusted Outstanding Balances at 6% Loan Rate	Adjusted Outstanding Balances at 8% Loan Rate	Annualized Interest	Pre-Tax Mean Interest	Total Outstanding Balance	After-Tax Cash Flow	Unloaned Amount at 5%	Unloaned Amount at 6%	Unloaned Amount at 8%	Change in Loans%
0	1.0000	173	1,560	5.93%	970	364	225	81	0	1,733	0	3,224	1,209	748	
1	0.9948	190	1,534	3.91%	823	296	169	65	73	1,478	303	3,303	1,251	789	3.01%
2	1.0188	212	1,554	3.91%	698	217	115	54	60	1,261	255	3,480	1,330	855	2.86%
3	1.0378	234	1,565	3.91%	561	171	55	42	48	1,020	272	3,647	1,407	922	2.76%
4	1.0525	255	1,569	3.91%	413	99	0	29	35	768	275	3,805	1,483	979	2.60%
5	1.0632	276	1,566	3.91%	257	24	0	19	24	557	227	3,955	1,556	978	2.40%
6	1.0696	297	1,557	3.91%	93	0	0	12	16	390	177	4,094	1,570	972	2.21%
7	1.0714	316	1,541	3.91%	0	0	0	9	11	316	81	4,144	1,554	962	2.01%
8	1.0690	333	1,519	3.91%	0	0	0	10	9	333	(12)	4,085	1,532	940	1.91%
9	1.0659	350	1,492	3.91%	0	0	0	10	10	350	(10)	4,012	1,505	931	1.91%
10	1.0533	365	1,460	3.91%	0	0	0	11	11	375	(8)	3,957	1,473	913	1.91%
11	1.0403	379	1,424	3.91%	0	0	0	11	11	390	(8)	3,830	1,436	899	1.91%
12	1.0240	390	1,384	3.91%	0	0	0	11	11	390	(5)	3,722	1,396	864	1.91%
13	1.0048	401	1,341	3.91%	0	0	0	12	12	401	(3)	3,606	1,352	837	1.91%
14	0.9830	409	1,295	3.91%	0	0	0	12	12	409	(1)	3,482	1,306	808	1.91%
15	0.9587	415	1,246	3.91%	0	0	0	12	12	415	1	3,351	1,257	778	1.91%
16	0.9323	420	1,196	3.91%	0	0	0	12	12	420	3	3,215	1,206	746	1.91%
17	0.9041	423	1,144	3.91%	0	0	0	12	12	423	5	3,076	1,153	714	1.91%
18	0.8743	424	1,091	3.91%	0	0	0	12	12	424	7	2,934	1,100	681	1.91%
19	0.8432	424	1,037	3.91%	0	0	0	12	12	424	9	2,790	1,046	648	1.91%
20	0.8109	422	984	3.91%	0	0	0	12	12	422	10	2,646	992	614	1.91%
21	0.7770	417	929	3.91%	0	0	0	12	12	417	12	2,499	937	580	1.91%
22	0.7417	411	874	3.91%	0	0	0	12	12	411	14	2,351	881	546	1.91%
23	0.7050	403	819	3.91%	0	0	0	12	12	403	16	2,201	826	511	1.91%
24	0.6671	393	763	3.91%	0	0	0	11	12	393	18	2,053	778	476	1.91%
25	0.6282	381	708	3.91%	0	0	0	11	11	381	19	1,903	714	442	1.91%
26	0.5881	367	652	3.91%	0	0	0	11	11	367	21	1,754	658	407	1.91%
27	0.5471	351	597	3.91%	0	0	0	10	10	351	23	1,606	602	373	1.91%
28	0.5052	333	543	3.91%	0	0	0	10	10	333	25	1,460	547	339	1.91%
29	0.4626	313	489	3.91%	0	0	0	9	9	313	26	1,315	493	305	1.91%
30	0.4193	291	436	3.91%	0	0	0	8	9	291	28	1,172	440	272	1.91%
31	0.3755	267	384	3.91%	0	0	0	8	8	267	29	1,033	387	240	1.91%
32	0.3317	241	333	3.91%	0	0	0	7	7	241	30	897	336	208	1.91%
33	0.2883	215	285	3.91%	0	0	0	6	7	215	31	766	287	178	1.91%
34	0.2460	188	239	3.91%	0	0	0	5	6	188	31	642	241	149	1.91%
35	0.2055	160	196	3.91%	0	0	0	5	5	160	31	527	198	122	1.91%
36	0.1675	134	157	3.91%	0	0	0	4	4	134	29	422	158	98	1.91%
37	0.1329	108	122	3.91%	0	0	0	3	4	108	28	328	123	76	1.91%
38	0.1022	85	92	3.91%	0	0	0	2	3	85	25	248	93	57	1.91%
39	0.0758	64	67	3.91%	0	0	0	2	2	64	22	180	68	42	1.91%
40	0.0540	47	47	3.91%	0	0	0	1	2	47	19	126	47	29	1.91%

Table 10D

MODELING FUTURE POLICY LOAN EXPERIENCE  
UNDER THE POP-UP INTEREST RATE SCENARIO

(\$Millions)

Duration from Valuation	Reserve Growth Factor	Pre-Adjusted		New Money Rates	Adjusted	Adjusted	Adjusted	Annualized Interest	Pre-Tax Mean Interest	Total Outstanding Balance	After-Tax Cash Flow	Unloaned Amount at 5%	Unloaned Amount at 6%	Unloaned Amount at 8%	Change in Loans%
		Outstanding Balances on Variable Loans	Outstanding Balances on Fixed Loans		Outstanding Balances at 5% Loan Rate	Outstanding Balances at 6% Loan Rate	Outstanding Balances at 8% Loan Rate								
0	1.0000	173	1,560	5.93%	970	364	225	81	0	1,733	0	5,116	1,918	1,188	0.00%
1	0.9948	190	1,534	8.93%	1,002	357	197	87	84	1,746	42	4,985	1,889	1,193	3.17%
2	0.9230	192	1,408	8.93%	966	326	156	81	84	1,640	160	4,527	1,734	1,119	3.25%
3	0.8507	192	1,283	8.93%	922	296	119	75	78	1,529	162	4,083	1,581	1,043	3.23%
4	0.7795	189	1,162	8.93%	873	267	86	69	72	1,415	160	3,660	1,433	966	3.22%
5	0.7108	185	1,047	8.93%	821	239	58	64	66	1,303	155	3,265	1,293	891	3.20%
6	0.6452	179	939	8.93%	767	214	33	58	61	1,193	149	2,898	1,160	817	3.19%
7	0.5830	172	839	8.93%	712	190	13	52	55	1,086	142	2,500	1,037	747	3.17%
8	0.5248	164	746	8.93%	657	168	0	48	50	989	130	2,233	923	676	3.16%
9	0.4711	155	661	8.93%	603	149	0	44	46	907	111	1,977	819	599	3.16%
10	0.4219	146	585	8.93%	552	131	0	40	42	829	105	1,730	725	530	3.17%
11	0.3770	137	516	8.93%	503	115	0	37	38	755	99	1,511	640	468	3.18%
12	0.3359	128	454	8.93%	457	101	0	33	35	686	92	1,315	564	411	3.19%
13	0.2986	119	398	8.93%	413	88	0	30	32	620	86	1,142	495	361	3.20%
14	0.2648	110	349	8.93%	372	77	0	27	29	559	80	989	434	316	3.21%
15	0.2344	102	305	8.93%	334	67	0	25	26	502	73	854	379	276	3.22%
16	0.2071	93	266	8.93%	299	58	0	22	24	450	67	737	331	241	3.23%
17	0.1827	85	231	8.93%	267	50	0	20	21	403	61	635	288	209	3.24%
18	0.1610	78	201	8.93%	238	43	0	18	19	360	55	546	251	182	3.25%
19	0.1418	71	174	8.93%	212	37	0	16	17	321	50	469	218	158	3.27%
20	0.1247	65	151	8.93%	188	32	0	14	15	285	45	402	189	137	3.28%
21	0.1096	59	131	8.93%	167	28	0	13	14	254	41	345	164	119	3.29%
22	0.0959	53	113	8.93%	147	24	0	11	12	224	37	294	141	102	3.30%
23	0.0835	48	97	8.93%	129	20	0	10	11	197	34	249	121	88	3.31%
24	0.0723	43	83	8.93%	112	17	0	9	9	172	31	210	104	75	3.32%
25	0.0622	38	70	8.93%	97	15	0	8	8	150	28	176	88	64	3.33%
26	0.0533	33	59	8.93%	83	12	0	7	7	129	25	147	74	54	3.34%
27	0.0452	29	49	8.93%	71	10	0	6	6	110	23	122	62	45	3.36%
28	0.0381	25	41	8.93%	60	8	0	5	5	94	20	100	52	37	3.37%
29	0.0318	21	34	8.93%	50	7	0	4	4	79	18	81	42	30	3.38%
30	0.0262	18	27	8.93%	42	6	0	3	3	66	16	66	34	25	3.39%
31	0.0213	15	22	8.93%	34	4	0	3	3	53	14	51	27	20	3.40%
32	0.0170	12	17	8.93%	27	3	0	2	2	43	12	40	22	16	3.42%
33	0.0134	10	13	8.93%	21	3	0	2	2	34	10	30	17	12	3.43%
34	0.0103	8	10	8.93%	16	2	0	1	2	26	9	23	13	9	3.44%
35	0.0077	6	7	8.93%	12	1	0	1	1	20	7	16	9	7	3.45%
36	0.0056	4	5	8.93%	9	1	0	1	1	14	6	12	7	5	3.47%
37	0.0040	3	4	8.93%	6	1	0	1	1	10	5	8	5	3	3.48%
38	0.0027	2	2	8.93%	4	0	0	0	0	7	4	5	3	2	3.49%
39	0.0018	1	2	8.93%	3	0	0	0	0	5	3	3	2	1	3.50%
40	0.0011	1	1	8.93%	2	0	0	0	0	3	2	2	1	1	3.52%

variable loan balance. This is the amount included with other assets backing reserve liabilities. The after-tax cash flow is generated as the change in policy loans plus after-tax mean interest at an assumed tax rate of 35% and included with other after-tax cash flows. The unloaned amounts use the outstanding balance percentages given in Table 10A to segment total assets (\$10,868 million) by policy loan interest rate. Thus, 5% fixed rate loans are assumed to belong to an initial asset pool of \$6,086 million; 6% loans are assumed to belong to an initial asset pool of \$2,282 million; 8% to a pool of \$1,413 million; and variable rate loans to a pool of \$1,087 million, for a total pool of \$10,868 million. The pool dwindles at each duration as do outstanding loan balances. The Change in Loan% column is the NAIC yield formulation which reflects the change in the total outstanding loan balance and the after-tax policy loan cash flow.

#### **I. Inflation**

Most of the inflation assumptions are implicit in the PCFTM. The cash flows were modeled by the Investment and Real Estate Departments and incorporate assumptions for inflation. Inflation assumptions for general insurance expenses are modeled in the PCFTM and described in the next section.

#### **J. Expenses**

Investment expenses are conservatively incorporated in the PCFTM at 20 basis points on reserves and reinvestment book values. Non-investment expenses subject to inflation are defined as 10% of premiums multiplied by inflation that has occurred since the starting point. Expense inflation factors are assumed to be one-half the new money rates defined in each scenario. Table 11 shows the progression of inflation factors and the level of expense inflation modeled in the PCFTM under the level, pop-down, and pop-up interest rate scenarios.

#### **K. Taxes**

Federal income taxes are assumed at a rate of 35% on cash flows, change in book values, interest income, and surplus. Tax credits (at the rate of 35%) for liability flows, reserve increases, and

Table 11

**EXPENSE INFLATION UNDER THE LEVEL,  
POP-DOWN, AND POP-UP SCENARIOS**  
(\$Millions)

Expenses subject to inflation are assumed to equal 10% of Premiums. Inflation assumed at half the new money rate.

Level Scenario			Pop-Down Scenario			Pop-Up Scenario		
Durations from Valuation	Inflation Factor	Expenses Due to Inflation	Durations from Valuation	Inflation Factor	Expenses Due to Inflation	Durations from Valuation	Inflation Factor	Expenses Due to Inflation
0	1.00000		0	1.00000		0	1.00000	
1	1.02905	3.1	1	1.02715	1.4	1	1.04465	3.1
2	1.06018	5.8	2	1.05247	2.6	2	1.09129	5.4
3	1.09161	8.1	3	1.07578	3.7	3	1.14002	7.0
4	1.12398	10.0	4	1.09692	4.7	4	1.19092	8.2
5	1.15711	11.7	5	1.11837	5.5	5	1.24410	8.9
6	1.19162	13.0	6	1.14023	6.3	6	1.29965	9.2
7	1.22695	14.0	7	1.16252	6.9	7	1.35767	9.3
8	1.26333	14.7	8	1.18525	7.4	8	1.41829	9.1
9	1.30079	15.3	9	1.20842	7.9	9	1.48162	8.8
10	1.33936	15.7	10	1.23204	8.2	10	1.54778	8.4
11	1.37907	15.9	11	1.25613	8.5	11	1.61688	8.0
12	1.41996	16.0	12	1.28069	8.7	12	1.68908	7.5
13	1.46206	16.0	13	1.30573	8.9	13	1.76450	7.0
14	1.50541	15.8	14	1.33125	8.9	14	1.84328	6.5
15	1.55005	15.6	15	1.35728	8.9	15	1.92558	6.0
16	1.59600	15.3	16	1.38381	8.9	16	2.01156	5.5
17	1.64333	14.9	17	1.41087	8.8	17	2.10138	5.0
18	1.69205	14.4	18	1.43845	8.7	18	2.19520	4.6
19	1.74222	13.9	19	1.46657	8.5	19	2.29322	4.2
20	1.79388	13.4	20	1.49524	8.3	20	2.39561	3.8
21	1.84706	12.8	21	1.52447	8.1	21	2.50257	3.4
22	1.90181	12.3	22	1.55428	7.8	22	2.61431	3.1
23	1.95822	11.6	23	1.58466	7.5	23	2.73104	2.7
24	2.01628	11.0	24	1.61564	7.2	24	2.85298	2.4
25	2.07606	10.2	25	1.64723	6.8	25	2.98037	2.1
26	2.13762	9.4	26	1.67943	6.3	26	3.11344	1.9
27	2.20100	8.6	27	1.71227	5.9	27	3.25246	1.6
28	2.26626	7.8	28	1.74574	5.4	28	3.39768	1.4
29	2.33345	6.9	29	1.77987	4.9	29	3.54939	1.1
30	2.40264	6.0	30	1.81467	4.3	30	3.70787	0.9
31	2.47388	5.2	31	1.85014	3.7	31	3.87342	0.7
32	2.54723	4.3	32	1.88631	3.2	32	4.04637	0.6
33	2.62275	3.6	33	1.92319	2.7	33	4.22704	0.4
34	2.70052	2.9	34	1.96079	2.2	34	4.41578	0.3
35	2.78059	2.2	35	1.99912	1.7	35	4.61295	0.2
36	2.86303	1.7	36	2.03821	1.4	36	4.81891	0.2
37	2.94792	1.3	37	2.07805	1.0	37	5.03408	0.1
38	3.03533	0.9	38	2.11808	0.8	38	5.25885	0.1
39	3.12533	0.7	39	2.16010	0.6	39	5.49366	0.1
40	3.21799	0.5	40	2.20233	0.4	40	5.73893	0.0



investment expenses offset the total tax liability. The differential earnings rate is also modeled. As previously discussed, DAC taxes are calculated according to statute and amortization amounts are included in the modeling of reinvestments.

## V. Actual Results Under Two Regulatory Scenarios

The results are in and we have pricing actuaries, accountants, and investment specialists to thank. Unknowingly, pricing actuaries have developed tools that can be used outside of the product development function. Those tools, combined with basic accounting principles, can be used to model insurance liabilities for cash flow testing models. Projected liability flows for the block of traditional participating whole life under study were simulated using new business asset shares, a unique mapping technique, and simplifying assumptions and adjustments to replicate current gain from operations at time zero. The investment specialists of this insurer applied the tools of their trade to model the option behavior of the assets backing product liabilities under the regulatory interest rate scenarios. Asset cash flows were made to respond as prudent investors would under the influence of moderate to extreme changes in interest rates. The PCFTM allows valuation actuaries to measure the interaction between projected asset and liability flows to determine the need to set up additional reserves.

### A. The Regulatory Interest Rate Scenarios

The NAIC's Standard Valuation Law and New York Regulation 126 both describe identical scenarios under which to test asset adequacy. Those scenarios prescribe the behavior of the yield curve over the projection period when valuing both assets and liabilities. Starting from an initial treasury yield curve with yield points specified for 3 and 6 month maturities, and 1, 2, 3, 4, 5, 7, 10, 15, 20, and 30 year maturities, the regulations define positive and negative **parallel shifts\*** of yields for cash flow testing. The regulatory scenarios are as follows:

***Level:** No shifts of initial yield curve throughout projection period.*

***Up:** Positive parallel shifts of 12.5 basis points each quarter for the first 10 years and level thereafter.*

**Up-Down:** *Positive shifts of 25 basis points each quarter for the first 5 years followed by negative shifts of 25 basis points each quarter for the next five years and level thereafter.*

**Pop-Up:** *Positive shifts of 75 basis points each quarter for the first year only. Though not prescribed in the regulations, the yield curve was inverted (i.e., reflected) at the end of the first quarter about the horizontal through the 7 year maturity yield point and remained inverted for one year. At the end of the first year, the inversion is removed and the yield curve is equivalent to a 300 basis point positive shift of the initial yield curve.*

**Down:** *Negative parallel shifts of 12.5 basis points each quarter for the first 10 years and level thereafter, subject to regulatory yield minimums.*

**Down-Up:** *Negative shifts of 25 basis points each quarter for the first 5 years followed by positive shifts of 25 basis points each quarter for the next five years and level thereafter, subject to regulatory yield minimums.*

**Pop-Down:** *Negative shifts of 75 basis points each quarter for the first year and level thereafter, subject to regulatory yield minimums.*

*{\*Note: A parallel shift means all yield points change by the same amount.}*

The regulations define minimum yields for the down scenarios as 50% of treasury yields plus a quality spread along all points of the initial yield curve. Additional yield points can be determined by interpolation or extrapolation using linear or, preferably, higher ordered methods. The resulting scenario yield curves are converted to spot curves to discount projected cash flows back to the valuation date. The PCFTM tested the block under study under all the regulatory scenarios and the results are presented in a later section. Results under the level and pop-down scenarios are discussed in detail to further elaborate on the model.

## B. The Level Scenario

As previously mentioned, once the asset share dividends are adjusted by the dividend adjustment factor and the initial asset share liability flows are adjusted by the gains adjustment factor, they are adjusted for survivorship and factored up by the ratio of 1993 actual reserves to initial asset share mean reserves. This last adjustment will ensure that factored dividends and liabilities will reproduce target performance ratios at duration zero. Assets and their flows, on the other hand, are factored down to the level of liabilities by the ratio of 1993 actual reserves to 1993 actual assets to satisfy regulatory equality constraints which requires assets to equal reserves at time zero. Now that all the flows are on the same playing field, we can examine how they perform in the PCFTM.

Most will not be surprised that surplus consistently increases under a level interest rate scenario. The central results for this scenario are given in Table 12 and backup data is given in Table 12A and Table 12B. As Table 12 depicts, surplus grows from an initial amount of zero to \$1,060 million by the end of the projection period and to more than half that amount by duration 20 under level interest rates and the baseline surrender behavior of the asset shares. Some explanation is called for to explain how surplus is derived in the PCFTM. Basic accounting says: **Assets = Liabilities + Surplus**. Thus, by simply transposing this equation, surplus is the difference between assets and liabilities. Valuation regulations on cash flow testing require setting initial surplus to zero which means assets must equal reserves at the initial duration in a cash flow testing model. This required condition was not initially met by the assets and liabilities used in the PCFTM. The assets for the level scenario are given in Table 12A which shows an initial book value of assets of \$10,868 million and initial asset cash flows of \$1,727 million. Both amounts reflect policy loan activity. The initial book value of asset share liabilities is \$4,497 million and initial liability flows are \$178 million. This imbalance is corrected by “factoring-up” the mean reserves and gains adjusted liability flows by the ratio of

**Table 12**  
**CASH FLOW TESTING RESULTS**  
**UNDER THE LEVEL**  
**INTEREST RATE SCENARIO**

(\$Millions)

*Assumptions:*

*Assets Flows are Factored-Down by the ratio  
of Reserves to the Book Value of Assets (69%)  
Asset Share New Money Rate: 9.3%  
Reinvestment Occurs Every Seven Years  
Prevailing Interest Rates Assumed to Start at 6.78%  
Before Default Charges and Investment Expenses  
NAIC Minimum Yield: 3.91%  
Default Charges Assumed at 3%  
Investment Expenses Assumed at 0.2%  
Assumed Tax Rate: 35%*

**Factored-Down Book Values + Book Value of Reinvestments**

<b>ASSETS</b>				
<i>Durations from Valuation</i>	<i>Un-Factored Book Values</i>	<i>Factored-Down Book Values</i>	<i>Book Value of Reinvestments</i>	<i>Factored-Down Asset Flows</i>
0	10,868	7,490		
1	9,601	6,617	893	1,190
2	8,568	5,905	1,627	990
3	7,564	5,213	2,278	940
4	6,658	4,589	2,818	841
5	5,422	3,737	3,540	1,032
6	4,605	3,174	3,945	708
7	3,702	2,551	4,383	737
8	3,195	2,202	4,534	443
9	2,710	1,868	4,663	414
10	2,277	1,570	4,747	362
11	2,108	1,453	4,636	172
12	1,904	1,312	4,541	189
13	1,777	1,225	4,391	134
14	1,655	1,140	4,236	128
15	1,488	1,026	4,106	151
16	1,381	952	3,942	110
17	1,298	895	3,769	92
18	1,197	825	3,612	103
19	1,121	772	3,444	83
20	1,019	702	3,301	98
21	843	581	3,216	148
22	789	544	3,049	60
23	736	507	2,885	59
24	683	471	2,725	57
25	630	434	2,571	56
26	578	398	2,421	54
27	527	363	2,277	53
28	476	328	2,137	51
29	427	294	2,004	49
30	378	261	1,876	47
31	331	228	1,755	44
32	286	197	1,642	42
33	242	167	1,537	39
34	202	139	1,442	36
35	164	113	1,358	32
36	130	90	1,286	28
37	100	69	1,227	24
38	75	51	1,182	20
39	54	37	1,149	17
40	37	25	1,130	13

**Table 12**  
**CASH FLOW TESTING RESULTS**  
**UNDER THE LEVEL**  
**INTEREST RATE SCENARIO**

(\$Millions)

*Assumptions:*

*Assets Flows are Factored-Down by the ratio of Reserves to the Book Value of Assets (69%)*  
*Asset Share New Money Rate: 9.3%*  
*Reinvestment Occurs Every Seven Years*  
*Prevailing Interest Rates Assumed to Start at 6.78% Before Default Charges and Investment Expenses*  
*NAIC Minimum Yield: 3.91%*  
*Default Charges Assumed at 3%*  
*Investment Expenses Assumed at 0.2%*  
*Assumed Tax Rate: 35%*

**Factored-Up & Lapse Adjusted Mean Reserves**

Durations from Valuation	LIABILITIES				
	Un-Factored Lapse Adjusted Mean Reserves	Factored-Up Lapse Adjusted Mean Reserves	Factored-Up Liability Flows	Factored-Up Policyholder Dividends	Projected Policyholder Dividends
0	4,497	7,490	178	296	296
1	4,474	7,451	114	298	298
2	4,438	7,392	93	300	300
3	4,377	7,290	158	304	309
4	4,297	7,156	211	307	298
5	4,200	6,995	254	311	297
6	4,087	6,807	295	295	270
7	3,960	6,596	334	280	242
8	3,823	6,367	363	265	215
9	3,678	6,125	385	249	188
10	3,528	5,875	399	234	170
11	3,373	5,618	409	218	155
12	3,216	5,356	415	204	138
13	3,057	5,091	418	191	125
14	2,897	4,825	417	178	113
15	2,738	4,560	413	166	106
16	2,582	4,300	405	155	98
17	2,428	4,044	394	145	85
18	2,279	3,796	382	134	83
19	2,134	3,554	368	125	77
20	1,994	3,321	355	116	71
21	1,858	3,094	343	107	65
22	1,724	2,871	333	99	60
23	1,593	2,653	323	91	56
24	1,465	2,439	311	84	51
25	1,340	2,231	300	77	47
26	1,218	2,029	288	70	43
27	1,101	1,833	275	64	39
28	987	1,643	263	57	35
29	876	1,460	250	51	31
30	770	1,283	237	45	28
31	669	1,114	223	40	25
32	572	953	208	35	22
33	481	802	192	30	19
34	397	661	174	26	16
35	320	534	156	22	13
36	252	420	136	18	11
37	193	321	116	15	9
38	142	237	96	12	7
39	101	169	77	9	5
40	69	115	59	7	4

**Table 12**  
**CASH FLOW TESTING RESULTS**  
**UNDER THE LEVEL**  
**INTEREST RATE SCENARIO**

(\$Millions)

*Assumptions*

Assets Flows are Factored Down by the ratio of Reserves to the Book Value of Assets (69%)  
 Asset Share New Money Rate: 9.3%  
 Reinvestment Occurs Every Seven Years  
 Prevailing Interest Rates Assumed to Start at 6.78%  
 Before Default Charges and Investment Expenses  
 NAIC Minimum Yield: 3.91%  
 Default Charges Assumed at 1%  
 Investment Expenses Assumed at 0.2%  
 Assumed Tax Rate: .35%

Durational Lapse Adjustment Factors by Durations		Results Under the Level Interest Rate Scenario
1 through 5 =>	1.0000	Ending Surplus = 1,040
6 through 10 =>	1.0000	20th Year Surplus = 682
11 and Beyond =>	1.0000	Present Value = 320

**Surplus**

Durations from Valuation	New Money Rates After Defaults	Surplus
0	5.93%	0
1	5.93%	59
2	5.93%	140
3	5.93%	201
4	5.93%	251
5	5.93%	283
6	5.93%	312
7	5.93%	338
8	5.93%	369
9	5.93%	406
10	5.93%	442
11	5.93%	470
12	5.93%	497
13	5.93%	525
14	5.93%	551
15	5.93%	572
16	5.93%	594
17	5.93%	619
18	5.93%	641
19	5.93%	663
20	5.93%	682
21	5.93%	702
22	5.93%	721
23	5.93%	739
24	5.93%	757
25	5.93%	774
26	5.93%	790
27	5.93%	807
28	5.93%	823
29	5.93%	838
30	5.93%	854
31	5.93%	870
32	5.93%	886
33	5.93%	903
34	5.93%	920
35	5.93%	938
36	5.93%	956
37	5.93%	976
38	5.93%	996
39	5.93%	1,017
40	5.93%	1,040

Table 12A

**GRAND TOTAL ASSET FLOWS BEFORE REINVESTMENT  
UNDER THE LEVEL INTEREST RATE SCENARIO**

(\$Millions)

<i>Durations from Valuation</i>	<i>Book Values</i>	<i>After-Tax Cash Flow</i>	<i>Post-Tax Implied Yield</i>	<i>Pre-Tax Implied Yield</i>
0	10,868			
1	9,601	1,727	4.60%	7.13%
2	8,568	1,436	4.53%	7.03%
3	7,564	1,364	4.57%	7.09%
4	6,658	1,220	4.52%	7.01%
5	5,422	1,497	4.42%	6.86%
6	4,605	1,028	4.29%	6.64%
7	3,702	1,069	4.07%	6.31%
8	3,195	642	4.00%	6.21%
9	2,710	600	3.99%	6.18%
10	2,277	526	3.81%	5.91%
11	2,108	250	3.72%	5.77%
12	1,904	275	3.62%	5.62%
13	1,777	194	3.70%	5.74%
14	1,655	185	3.73%	5.78%
15	1,488	219	3.39%	5.26%
16	1,381	160	3.75%	5.81%
17	1,298	133	3.83%	5.94%
18	1,197	149	3.89%	6.03%
19	1,121	121	3.96%	6.14%
20	1,019	143	3.91%	6.06%
21	843	214	4.15%	6.44%
22	789	86	4.10%	6.36%
23	736	85	4.22%	6.55%
24	683	83	4.34%	6.73%
25	630	81	4.45%	6.90%
26	578	79	4.56%	7.07%
27	527	76	4.66%	7.23%
28	476	74	4.76%	7.37%
29	427	71	4.84%	7.50%
30	378	68	4.91%	7.61%
31	331	64	4.96%	7.69%
32	286	60	4.99%	7.74%
33	242	56	5.01%	7.77%
34	202	52	5.01%	7.76%
35	164	47	4.98%	7.72%
36	130	41	4.93%	7.65%
37	100	35	4.86%	7.53%
38	75	30	4.76%	7.38%
39	54	24	4.63%	7.18%
40	37	19	4.47%	6.93%

**Table 12B**  
**REINVESTMENT CALCULATIONS**  
**UNDER THE LEVEL**  
**INTEREST RATE SCENARIO**  
(\$Millions)

Notes on Calculations:

- Investment expenses are 0.2% of the sum of reinvested book values and factored down assets.
- Full year post-tax interest equals interest from the previous period plus 65% (1-tax rate) of the difference between interest on current reinvested flows at the current new money rate and the same quantity seven years prior.
- Surplus is taxed at the rate of 35% on 70% of surplus interest calculated at new money rates.
- The PCFTM uses 70% of the new money rate serves as a proxy for the differential earnings rate.
- Total tax liability before DAC tax equals tax on asset cash flows plus full year post tax interest and tax on surplus and less tax credits.
- The DAC tax equals the tax rate times 7.7% of the DAC base (factored up by the ratio of initial actual reserves to initial asset share reserves) less any applicable unamortized amounts. The DAC base equals lapse adjusted premiums less premiums still subject to amortization for the prior ten-year period.

Durations from Valuation	Reinvested Cash Flow Amounts	Investment Expenses Assumed at .2%	Full Year Post-Tax Interest	Book Value of Reinvestments	Portfolio Rate On Reinvestments	Tax Credits On: Liability Flows, Reserve Increases, & Investment Expenses
0						
1	876.3	15.0	33.8	893.2	3.85%	135.5
2	719.5	15.0	61.5	1626.6	3.85%	122.2
3	639.3	15.1	86.2	2278.2	3.85%	133.3
4	530.0	15.0	106.6	2818.4	3.85%	136.4
5	708.2	14.8	133.9	3540.2	3.85%	141.6
6	397.2	14.6	149.2	3945.1	3.85%	137.4
7	429.8	14.2	165.8	4383.1	3.85%	132.7
8	1024.3	13.9	171.5	4534.0	3.85%	127.0
9	846.2	13.5	176.3	4663.1	3.85%	120.4
10	721.8	13.1	179.5	4747.2	3.85%	116.0
11	420.5	12.6	175.3	4635.7	3.85%	111.8
12	615.2	12.2	171.7	4540.9	3.85%	106.2
13	249.7	11.7	166.0	4390.6	3.85%	101.4
14	277.8	11.2	160.2	4235.6	3.85%	96.5
15	897.3	10.8	155.3	4106.2	3.85%	92.7
16	684.8	10.3	149.1	3941.7	3.85%	88.2
17	552.4	9.8	142.5	3769.0	3.85%	81.8
18	266.6	9.3	136.6	3612.1	3.85%	78.8
19	450.7	8.9	130.3	3444.4	3.85%	74.5
20	109.1	8.4	124.8	3301.1	3.85%	70.3
21	194.0	8.0	121.6	3215.7	3.85%	66.2
22	733.4	7.6	115.3	3048.7	3.85%	62.0
23	523.8	7.2	109.1	2884.6	3.85%	58.6
24	396.3	6.8	103.1	2725.4	3.85%	54.4
25	114.9	6.4	97.2	2570.8	3.85%	50.7
26	303.9	6.0	91.6	2421.2	3.85%	47.0
27	-32.7	5.6	86.1	2276.6	3.85%	43.3
28	57.5	5.3	80.8	2137.5	3.85%	39.7
29	602.3	4.9	75.8	2003.8	3.85%	36.2
30	398.7	4.6	71.0	1876.3	3.85%	32.5
31	277.7	4.3	66.4	1755.4	3.85%	28.9
32	3.7	4.0	62.1	1642.1	3.85%	25.3
33	201.1	3.7	58.1	1537.3	3.85%	21.9
34	-125.9	3.4	54.5	1442.3	3.85%	18.6
35	-24.9	3.2	51.4	1358.3	3.85%	15.5
36	531.8	2.9	48.6	1286.4	3.85%	12.6
37	340.8	2.8	46.4	1227.4	3.85%	10.0
38	232.9	2.6	44.7	1181.8	3.85%	7.7
39	-28.1	2.5	43.5	1149.4	3.85%	5.8
40	181.7	2.4	42.7	1129.6	3.85%	4.1



**Table 12B**  
**REINVESTMENT CALCULATIONS**  
**UNDER THE LEVEL**  
**INTEREST RATE SCENARIO**  
(\$Millions)

Durations from Valuation	Tax On Surplus	Surplus	Total After-Tax Net Interest Earned	Asset Base: Factored-Up Asset Share Reserves + Surplus	Pre-Tax NAIC Implied Portfolio Rate	Post-Tax NAIC Implied Portfolio Rate
0		0.0		7490.0	6.72%	
1	0.0	59.1	324.1	7510.1	6.80%	4.42%
2	0.9	139.5	315.4	7531.2	6.59%	4.28%
3	2.0	200.7	312.5	7491.1	6.54%	4.25%
4	2.9	251.0	303.4	7407.1	6.40%	4.16%
5	3.6	282.6	290.8	7277.2	6.22%	4.04%
6	4.1	311.8	277.1	7118.8	6.04%	3.92%
7	4.5	338.3	262.5	6934.2	5.86%	3.81%
8	4.9	368.7	252.9	6735.6	5.80%	3.77%
9	5.4	405.6	244.7	6530.7	5.78%	3.76%
10	5.9	441.8	233.7	6316.8	5.70%	3.71%
11	6.4	469.9	224.4	6088.2	5.67%	3.68%
12	6.8	497.3	214.8	5853.2	5.64%	3.66%
13	7.2	524.6	207.4	5615.2	5.67%	3.68%
14	7.6	551.3	199.1	5376.0	5.68%	3.69%
15	8.0	571.6	186.9	5131.9	5.57%	3.62%
16	8.3	594.0	181.9	4893.5	5.68%	3.69%
17	8.6	619.4	174.1	4663.6	5.71%	3.71%
18	9.0	641.4	166.3	4436.9	5.73%	3.72%
19	9.3	662.6	158.7	4216.8	5.75%	3.74%
20	9.6	682.3	150.3	4003.5	5.73%	3.73%
21	9.9	702.4	144.1	3796.7	5.79%	3.77%
22	10.2	721.4	136.1	3592.7	5.77%	3.75%
23	10.5	739.0	129.2	3391.8	5.80%	3.77%
24	10.7	756.7	122.4	3196.0	5.83%	3.79%
25	11.0	773.8	115.7	3005.1	5.85%	3.80%
26	11.2	790.4	109.0	2819.6	5.87%	3.82%
27	11.5	806.7	102.5	2639.7	5.89%	3.83%
28	11.7	822.6	96.1	2465.7	5.90%	3.84%
29	12.0	838.3	89.8	2298.0	5.91%	3.84%
30	12.2	853.9	83.7	2137.1	5.92%	3.85%
31	12.4	869.8	77.7	1983.7	5.91%	3.84%
32	12.6	886.0	72.0	1839.1	5.91%	3.84%
33	12.9	902.6	66.6	1704.4	5.90%	3.83%
34	13.1	919.8	61.6	1581.3	5.88%	3.82%
35	13.4	937.7	57.0	1471.4	5.86%	3.81%
36	13.6	956.3	53.0	1376.1	5.83%	3.79%
37	13.9	975.7	49.5	1296.5	5.81%	3.77%
38	14.2	996.1	46.7	1233.2	5.78%	3.76%
39	14.5	1017.5	44.5	1186.3	5.76%	3.75%
40	14.8	1039.9	42.9	1155.0	5.74%	3.73%

**Table 12B**  
**REINVESTMENT CALCULATIONS**  
**UNDER THE LEVEL**  
**INTEREST RATE SCENARIO**  
(\$Millions)

Beginning Unamortized  
DAC Tax Amount ==> \$ 51 Million

Durations from Valuation	Estimated Tax Liability Excluding DAC	DAC Tax	DAC Tax Base
0		23.1	627.0
1	121.2	17.0	490.1
2	121.1	11.5	368.6
3	102.6	6.6	259.6
4	87.7	2.2	161.3
5	67.6	-1.9	69.3
6	56.2	-5.8	-17.1
7	45.9	-9.3	-94.9
8	41.6	-12.4	-165.0
9	40.7	-15.3	-228.2
10	35.9	-9.5	-211.3
11	33.3	-8.6	-192.6
12	32.5	-7.9	-175.9
13	32.8	-7.2	-160.5
14	32.7	-6.6	-146.3
15	28.5	-6.0	-132.9
16	30.5	-5.4	-120.5
17	32.5	-4.9	-109.3
18	31.0	-4.4	-99.1
19	30.9	-4.0	-89.8
20	30.1	-3.6	-81.2
21	30.5	-3.3	-73.5
22	29.7	-3.0	-66.5
23	29.3	-2.7	-60.4
24	29.6	-2.5	-55.1
25	29.6	-2.3	-50.3
26	29.6	-2.1	-46.0
27	29.5	-1.9	-42.2
28	29.5	-1.7	-38.7
29	29.4	-1.6	-35.5
30	29.5	-1.5	-32.4
31	29.7	-1.3	-29.5
32	30.0	-1.2	-26.5
33	30.3	-1.1	-23.5
34	30.7	-0.9	-20.6
35	31.2	-0.8	-17.8
36	31.7	-0.7	-15.1
37	32.4	-0.6	-12.5
38	33.2	-0.5	-10.2
39	34.0	-0.4	-8.2
40	34.9	0.0	0.0

1993 actual reserves (\$7,490 million) to initial mean reserves (\$4,497 million). The assets and their flows are “factored-down” by the ratio of 1993 actual reserves to the initial book value of assets (\$10,868 million) which brings assets down to the same level as liabilities at the initial duration and surplus to zero as required. Subsequent flows on both sides of the equation are also factored to maintain consistency.

The book value of reinvestments is calculated as:

$$= \left\{ \sum_{x=t-6}^t \text{Re invested Cash Flow Amounts}_x \right\} + \frac{I}{2} * (\text{Full Year Post - Tax Interest}_t)$$

The above reflects the investment policy for this block which prescribes reinvestment of assets on average every seven years. Cash flow amounts as given in Table 12B are defined as follows:

$$\begin{aligned} \text{Reinvested Cash Flow Amounts}_t &= \text{Factored-Down Asset Flows}_t \\ &- \text{Factored-Up Liability Flows}_t \\ &- \text{Investment Expenses}_t \\ &+ \text{Reinvested Cash Flow Amounts}_{t-7} \\ &+ \text{Full Year Post-Tax Interest}_{t-1} \\ &+ \text{Tax Credits on Liabilities}_t \\ &- \text{Tax on Surplus}_t \\ &- \text{DAC Tax}_t \end{aligned}$$

As already discussed, dividends paid in any year result from experience gains due to interest, mortality, and expenses. The literature has indicated, in particular, that insurers employ a variety of methods to determine the interest component of the dividends they pay in any given year. Many insurers, including this one, relate dividend interest to the excess of portfolio yields over yields assumed in pricing when it is time to distribute surplus to policyholders. The PCFTM does not know (and does not need to) the dividend interest rate assumed in pricing to continue projecting dividends. It knows the initial dividends and the current pre-tax portfolio yield on underlying assets and projects changes in dividend levels by measuring the difference between

initial and projected portfolio yields at each duration  $t$ . If projected portfolio yields consistently rise above initial yields, then, theoretically, so should dividends. Likewise, if yields consistently fall, then dividends should not be far behind notwithstanding a lag. Dividends are modeled in the PCFTM using the following relationship:

$$\begin{aligned}
 \text{Projected Policyholder Dividends}_t &= \text{Max} \{ 0, \text{Factored-Up Dividends}_t, \\
 &\quad + (\text{Pre-Tax NAIC Implied Portfolio Yield}_{t,2} \\
 &\quad - \text{Pre-Tax NAIC Implied Portfolio Yield}_0) \\
 &\quad * \text{Factored-Up Mean Reserves}_{t,1} \\
 &\quad - \text{Expenses}_{t,5} \}
 \end{aligned}$$

This relationship says the following regarding this insurer's dividend philosophy:

1. *The management of this insurer is willing to reduce dividends to zero if conditions warrant such actions.*
2. *The time lag in lowering or raising dividends is two years behind current portfolio yields and five years behind current expense inflation.*
3. *Dividends will be reduced if portfolio yields drop below initial levels, and increase otherwise as a function of mean reserves.*

Notice that mean reserves and not initial reserves (*reserve at end of prior period plus current premium*) are used to determine the amount by which dividends are increased or decreased in a given year. Initial reserves are prescribed by the contribution principle. This discrepancy in the PCFTM is not viewed as a serious violation.

Now that the flows are modeled, surplus is calculated at each duration as follows:

$$\begin{aligned}
 \text{Surplus}_t &= \text{Factored- Down Asset Book Values}_t \\
 &\quad + \text{Reinvested Book Values}_t \\
 &\quad - \text{Factored-Up Mean Reserves}_t
 \end{aligned}$$

For reference, the following relationships were used to model the backup data in Table 12B and Table 13B at each duration:

1. *Investment expenses are 0.2% of the sum of reinvested book values and factored down assets.*
2. *Full year post-tax interest equals interest from the previous period plus 65% (1-tax rate) of the difference between interest on current reinvested flows at the current new money rate and the same quantity seven years prior.*
3. *Surplus is taxed at the rate of 35% on 70% of surplus interest calculated at new money rates. The PCFTM uses 70% of the new money rate serves as a proxy for the differential earnings rate.*
4. *Total tax liability before DAC tax equals tax on asset cash flows plus full year post tax interest and tax on surplus and less tax credits.*
5. *The DAC tax equals the tax rate times 7.7% of the DAC base (factored up by the ratio of initial actual reserves to initial asset share reserves) less 1/10 of the beginning unamortized amount which amortizes over 10 years.*
6. *The DAC base equals lapse adjusted premiums less 1/10 of premiums for the prior ten-year period.*

These relationships influence the surplus results under the level and other interest rate scenarios, but not dynamically. Other relationships could be substituted for those not defined by statute to “best fit” individual insurer characteristics, but even those characteristics will not prove dynamic enough to explain the variation in surplus levels ultimately observed in cash flow testing. In the PCFTM, the valuation actuary can vary new money rates and lapse rates easily, quickly, and dynamically to determine if liabilities and other policy guarantees are in danger of not being satisfied. Of course, any interest rate assumptions imposed on the liabilities must be consistent with assumptions underlying the assets modeled by the investment professionals. The PCFTM takes these pre-modeled assets and their flows as givens and presumes they were modeled consistently with the regulatory interest rate scenarios. The remaining results to be presented must be reviewed with this understanding.

### **C. The Pop-Down Scenario**

The central results for the pop-down scenario are given in Table 13, and the backup data is given in Table 13A and Table 13B. The pop-down interest rate scenario requires interest rates to drop

**Table 13**  
**CASH FLOW TESTING RESULTS**  
**UNDER THE POP-DOWN**  
**INTEREST RATE SCENARIO**

(\$Millions)

*Assumptions:*

*Assets Flows are Factored-Down by the ratio  
of Reserves to the Book Value of Assets (69%)  
Asset Share New Money Rate: 9.3%  
Reinvestment Occurs Every Seven Years  
Prevailing Interest Rates Assumed to Start at 6.78%  
Before Default Charges and Investment Expenses  
NAIC Minimum Yield: 3.91%  
Default Charges Assumed at 3%  
Investment Expenses Assumed at 0.2%  
Assumed Tax Rate: 35%*

**Factored-Down Book Values + Book Value of Reinvestments**

ASSETS				
Durations from Valuation	Un-Factored Book Values	Factored-Down Book Values	Book Value of Reinvestments	Factored-Down Asset Flows
0	10,868	7,490		
1	9,383	6,467	1,030	1,330
2	8,259	5,692	2,057	1,050
3	7,184	4,951	2,992	984
4	6,263	4,316	3,770	845
5	5,193	3,579	4,600	908
6	4,601	3,171	5,067	549
7	3,870	2,667	5,596	615
8	3,536	2,437	5,829	328
9	3,130	2,157	6,092	364
10	2,748	1,894	6,309	334
11	2,626	1,810	6,315	146
12	2,460	1,695	6,328	169
13	2,371	1,634	6,268	116
14	2,278	1,570	6,190	115
15	2,149	1,481	6,115	139
16	2,066	1,424	5,991	105
17	2,003	1,380	5,837	90
18	1,916	1,320	5,689	105
19	1,850	1,275	5,516	90
20	1,756	1,210	5,352	108
21	1,586	1,093	5,228	160
22	1,527	1,053	5,011	78
23	1,465	1,010	4,783	81
24	1,399	964	4,549	83
25	1,329	916	4,308	85
26	1,255	865	4,060	86
27	1,178	812	3,806	88
28	1,097	756	3,545	89
29	1,013	698	3,278	89
30	926	638	3,006	89
31	836	576	2,731	89
32	745	513	2,457	88
33	653	450	2,187	85
34	562	387	1,924	82
35	473	326	1,674	77
36	389	268	1,440	72
37	311	214	1,227	65
38	241	166	1,040	57
39	180	124	882	48
40	129	89	754	39

**Table 13**  
**CASH FLOW TESTING RESULTS**  
**UNDER THE POP-DOWN**  
**INTEREST RATE SCENARIO**

(\$Millions)

*Assumptions:*

*Assets Flows are Factored-Down by the ratio of Reserves to the Book Value of Assets (69%)*  
*Asset Share New Money Rate: 9.3%*  
*Reinvestment Occurs Every Seven Years*  
*Prevailing Interest Rates Assumed to Start at 6.78%*  
*Before Default Charges and Investment Expenses*  
*NAIC Minimum Yield: 3.91%*  
*Default Charges Assumed at 3%*  
*Investment Expenses Assumed at 0.2%*  
*Assumed Tax Rate: 35%*

**Factored-Up & Lapse Adjusted Mean Reserves**

Durations from Valuation	LIABILITIES				
	Un-Factored Lapse Adjusted Mean Reserves	Factored-Up Lapse Adjusted Mean Reserves	Factored-Up Liability Flows	Factored-Up Policyholder Dividends	Projected Policyholder Dividends
0	4,497	7,490	178	296	296
1	4,474	7,451	112	298	298
2	4,582	7,631	(160)	310	310
3	4,667	7,773	(97)	324	311
4	4,733	7,883	(42)	339	307
5	4,782	7,964	8	354	296
6	4,810	8,011	63	348	262
7	4,818	8,025	120	341	220
8	4,808	8,007	171	333	187
9	4,780	7,961	217	324	149
10	4,737	7,889	257	314	133
11	4,678	7,792	295	303	112
12	4,605	7,670	328	293	92
13	4,519	7,526	358	282	79
14	4,421	7,362	383	272	66
15	4,311	7,180	405	262	63
16	4,193	6,983	422	252	56
17	4,066	6,771	434	242	53
18	3,932	6,549	444	232	46
19	3,792	6,316	451	222	42
20	3,647	6,074	457	211	39
21	3,495	5,820	465	201	36
22	3,336	5,555	472	191	34
23	3,171	5,281	477	182	35
24	3,000	4,997	480	172	29
25	2,825	4,705	482	162	27
26	2,645	4,405	483	152	26
27	2,460	4,098	483	142	25
28	2,272	3,784	481	132	25
29	2,080	3,465	479	121	23
30	1,886	3,140	474	111	23
31	1,689	2,813	466	101	22
32	1,492	2,484	454	92	20
33	1,297	2,159	438	82	18
34	1,106	1,842	417	72	17
35	924	1,539	390	63	16
36	753	1,255	357	54	15
37	598	995	319	46	14
38	459	765	277	38	12
39	341	568	233	30	9
40	243	405	188	23	7

**Table 13**  
**CASH FLOW TESTING RESULTS**  
**UNDER THE POP-DOWN**  
**INTEREST RATE SCENARIO**

(\$Millions)

*Assumptions*

Assets Flows are Factored-Down by the ratio of Reserves to the Book Value of Assets (69%)  
 Asset Share New Money Rate: 9.3%  
 Reinvestment Occurs Every Seven Years  
 Prevailing Interest Rates Assumed to Start at 6.78%  
 Before Default Charges and Investment Expenses  
 NAIC Minimum Yield: 3.91%  
 Default Charges Assumed at 3%  
 Investment Expenses Assumed at 0.2%  
 Assumed Tax Rate: 35%

<i>Durational Lapse Adjustment Factors by Durations</i>		<i>Results Under the Level Interest Rate Scenario</i>
1 through 5 =>	1.0000	Ending Surplus = 1,040
6 through 10 =>	1.0000	20Th Year Surplus = 682
11 and Beyond =>	1.0000	Present Value = 320

**Surplus**

<i>Durations from Valuation</i>	<i>New Money Rates After Defaults</i>	<i>Surplus</i>
0	5.93%	0
1	3.91%	46
2	3.91%	118
3	3.91%	171
4	3.91%	203
5	3.91%	216
6	3.91%	227
7	3.91%	239
8	3.91%	259
9	3.91%	288
10	3.91%	313
11	3.91%	333
12	3.91%	353
13	3.91%	376
14	3.91%	398
15	3.91%	416
16	3.91%	432
17	3.91%	446
18	3.91%	461
19	3.91%	475
20	3.91%	488
21	3.91%	501
22	3.91%	508
23	3.91%	512
24	3.91%	516
25	3.91%	519
26	3.91%	520
27	3.91%	520
28	3.91%	517
29	3.91%	512
30	3.91%	504
31	3.91%	495
32	3.91%	486
33	3.91%	478
34	3.91%	469
35	3.91%	461
36	3.91%	453
37	3.91%	446
38	3.91%	441
39	3.91%	438
40	3.91%	439



Table 13A

**GRAND TOTAL ASSET FLOWS BEFORE REINVESTMENT  
UNDER THE POP-DOWN INTEREST RATE SCENARIO**

(\$Millions)

<i>Durations from Valuation</i>	<i>Book Values</i>	<i>After-Tax Cash Flow</i>	<i>Post-Tax Implied Yield</i>	<i>Pre-Tax Implied Yield</i>
0	10,868			
1	9,383	1,930	4.50%	6.98%
2	8,259	1,524	4.64%	7.19%
3	7,184	1,428	4.68%	7.25%
4	6,263	1,226	4.63%	7.18%
5	5,193	1,318	4.43%	6.87%
6	4,601	797	4.28%	6.64%
7	3,870	892	3.88%	6.01%
8	3,336	476	3.90%	6.04%
9	3,130	529	3.77%	5.85%
10	2,748	485	3.54%	5.49%
11	2,626	212	3.41%	5.28%
12	2,460	245	3.16%	4.91%
13	2,371	168	3.33%	5.17%
14	2,278	167	3.22%	4.99%
15	2,149	202	3.33%	5.17%
16	2,066	152	3.31%	5.14%
17	2,003	131	3.17%	5.23%
18	1,916	153	3.43%	5.32%
19	1,850	130	3.49%	5.41%
20	1,756	157	3.56%	5.51%
21	1,586	232	3.78%	5.86%
22	1,527	114	3.62%	5.62%
23	1,465	117	3.74%	5.80%
24	1,399	120	3.86%	5.98%
25	1,329	123	3.97%	6.16%
26	1,255	125	4.09%	6.34%
27	1,178	127	4.20%	6.51%
28	1,097	129	4.30%	6.67%
29	1,013	129	4.39%	6.81%
30	926	129	4.48%	6.94%
31	836	129	4.55%	7.05%
32	745	127	4.60%	7.13%
33	653	124	4.64%	7.19%
34	562	119	4.66%	7.23%
35	473	112	4.66%	7.23%
36	389	104	4.64%	7.19%
37	311	94	4.59%	7.12%
38	241	82	4.52%	7.01%
39	180	70	4.42%	6.86%
40	129	57	4.29%	6.65%

**Table 13B**  
**REINVESTMENT CALCULATIONS**  
**UNDER THE POP-DOWN**  
**INTEREST RATE SCENARIO**  
(\$Millions)

Notes on Calculations:

- Investment expenses are 0.2% of the sum of reinvested book values and factored down assets.
- Full year post-tax interest equals interest from the previous period plus 65% (1-tax rate) of the difference between interest on current reinvested flows at the current new money rate and the same quantity seven years prior.
- Surplus is taxed at the rate of 35% on 70% of surplus interest calculated at new money rates.
- The PCFTM uses 70% of the new money rate serves as a proxy for the differential earnings rate.
- Total tax liability before DAC tax equals tax on asset cash flows plus full year post tax interest and tax on surplus and less tax credits.
- The DAC tax equals the tax rate times 7.7% of the DAC base (factored up by the ratio of initial actual reserves to initial asset share reserves) less any applicable unamortized amounts. The DAC base equals lapse adjusted premiums less premiums still subject to amortization for the prior ten-year period.

Durations from Valuation	Reinvested Cash Flow Amounts	Investment Expenses Assumed at .2%	Full Year Post-Tax Interest	Book Value of Reinvestments	Portfolio Rate On Reinvestments	Tax Credits On: Liability Flows, Reserve Increases, & Investment Expenses
0						
1	1017.3	15.0	25.9	1030.2	2.54%	135.1
2	1013.9	15.0	51.6	2057.0	2.54%	120.4
3	923.6	13.4	75.1	2992.3	2.54%	129.1
4	768.2	11.7	94.6	3770.3	2.54%	135.6
5	819.7	10.2	115.5	4600.4	2.54%	138.5
6	461.1	8.8	127.2	5067.4	2.54%	133.6
7	522.3	7.3	140.4	5596.4	2.54%	126.2
8	1246.9	6.4	146.3	5828.9	2.54%	121.5
9	1273.4	7.4	152.9	6091.7	2.54%	114.5
10	1137.7	6.9	158.3	6308.5	2.54%	113.8
11	775.1	6.1	158.5	6315.5	2.54%	110.5
12	831.9	5.2	158.8	6327.7	2.54%	106.4
13	401.7	5.1	157.3	6267.5	2.54%	104.4
14	445.6	4.1	155.3	6189.8	2.54%	101.3
15	1173.1	4.0	153.5	6115.0	2.54%	101.6
16	1150.5	5.3	150.3	5990.6	2.54%	99.7
17	986.4	5.1	146.5	5837.4	2.54%	98.1
18	628.8	4.7	142.8	5689.2	2.54%	94.8
19	660.6	3.9	138.4	5515.8	2.54%	92.3
20	239.6	3.9	134.3	5351.7	2.54%	90.1
21	323.4	2.9	131.2	5228.0	2.54%	87.5
22	958.9	2.8	125.8	5011.1	2.54%	85.3
23	925.1	4.0	120.0	4782.9	2.54%	84.2
24	755.5	3.9	114.2	4549.0	2.54%	80.2
25	391.0	3.4	108.1	4308.2	2.54%	77.2
26	416.0	2.6	101.9	4060.5	2.54%	74.1
27	-12.0	2.6	95.5	3805.6	2.54%	71.2
28	65.8	1.6	89.0	3544.8	2.54%	67.9
29	695.7	1.6	82.3	3278.2	2.54%	64.6
30	656.3	2.8	75.4	3006.0	2.54%	61.3
31	484.3	2.6	68.5	2731.3	2.54%	56.8
32	120.5	2.1	61.7	2457.4	2.54%	51.7
33	149.2	1.3	54.9	2187.1	2.54%	46.6
34	-271.7	1.2	48.3	1924.2	2.54%	41.6
35	-181.7	0.2	42.0	1673.6	2.54%	36.2
36	465.1	0.3	36.1	1440.0	2.54%	30.9
37	446.0	1.5	30.8	1227.0	2.54%	26.2
38	299.5	1.3	26.1	1040.0	2.54%	21.1
39	-35.4	0.9	22.1	882.1	2.54%	16.0
40	23.2	0.2	18.9	754.5	2.54%	11.4

**Table 13B**  
**REINVESTMENT CALCULATIONS**  
**UNDER THE POP-DOWN**  
**INTEREST RATE SCENARIO**  
*(\$Millions)*

<i>Durations from Valuation</i>	<i>Tax On Surplus</i>	<i>Surplus</i>	<i>Total After-Tax Net Interest Earned</i>	<i>Asset Base: Factored-Up Asset Share Reserves + Surplus</i>	<i>Pre-Tax NAIC Implied Portfolio Rate</i>	<i>Post-Tax NAIC Implied Portfolio Rate</i>
0		0.0		7490.0	6.72%	
1	0.0	46.1	310.3	7497.1	6.56%	4.23%
2	0.4	118.3	304.4	7749.0	6.32%	4.07%
3	1.1	170.8	297.8	7943.6	6.00%	3.87%
4	1.6	203.2	287.0	8086.4	5.65%	3.65%
5	1.9	215.5	269.5	8179.2	5.22%	3.37%
6	2.1	227.5	257.1	8238.6	4.93%	3.18%
7	2.2	239.0	240.1	8263.6	4.58%	2.95%
8	2.3	258.6	236.7	8265.5	4.51%	2.91%
9	2.5	288.2	229.8	8249.0	4.38%	2.82%
10	2.8	313.3	221.6	8202.4	4.23%	2.73%
11	3.0	333.4	216.5	8125.0	4.17%	2.69%
12	3.2	353.2	209.9	8023.1	4.08%	2.63%
13	3.4	375.5	209.3	7901.8	4.13%	2.66%
14	3.6	397.5	204.4	7760.0	4.10%	2.64%
15	3.8	415.7	201.8	7596.1	4.13%	2.66%
16	4.0	431.8	195.8	7414.4	4.10%	2.64%
17	4.1	446.1	191.6	7217.5	4.11%	2.65%
18	4.3	461.2	187.1	7009.7	4.13%	2.67%
19	4.4	475.4	182.6	6791.0	4.16%	2.68%
20	4.6	488.1	177.3	6562.0	4.17%	2.69%
21	4.7	500.7	173.6	6320.7	4.23%	2.73%
22	4.8	508.4	164.8	6063.7	4.18%	2.70%
23	4.9	511.8	158.1	5792.5	4.19%	2.70%
24	4.9	516.0	151.9	5513.0	4.22%	2.72%
25	4.9	519.0	145.5	5223.9	4.26%	2.75%
26	5.0	520.4	139.0	4925.4	4.30%	2.78%
27	5.0	519.5	131.5	4617.3	4.33%	2.79%
28	5.0	516.7	124.2	4300.8	4.38%	2.82%
29	4.9	511.7	115.8	3976.3	4.40%	2.84%
30	4.9	503.9	106.3	3644.2	4.39%	2.83%
31	4.8	495.1	97.1	3107.8	4.40%	2.84%
32	4.7	486.5	88.2	2970.8	4.42%	2.85%
33	4.7	477.8	79.3	2637.1	4.45%	2.87%
34	4.6	468.8	69.9	2311.3	4.44%	2.86%
35	4.5	460.5	61.2	1999.6	4.47%	2.88%
36	4.4	453.1	52.4	1708.0	4.44%	2.86%
37	4.3	446.0	43.3	1441.3	4.33%	2.79%
38	4.3	440.8	36.0	1206.0	4.27%	2.76%
39	4.2	438.5	29.8	1006.3	4.23%	2.73%
40	4.2	439.0	24.9	843.7	4.23%	2.73%

**Table 13B**  
**REINVESTMENT CALCULATIONS**  
**UNDER THE POP-DOWN**  
**INTEREST RATE SCENARIO**  
(\$Millions)

Beginning Unamortized  
DAC Tax Amount = > \$ 51 Million

Durations from Valuation	Estimated Tax Liability Excluding DAC	DAC Tax	DAC Tax Base
0			
1	111.8	23.1	627.0
2	116.1	17.9	510.6
3	96.1	13.1	404.6
4	75.6	8.7	307.0
5	53.7	4.7	216.4
6	44.4	0.7	128.3
7	34.8	-3.1	42.1
8	34.1	-6.7	-38.1
9	35.0	-10.1	-112.9
10	27.8	-13.2	-182.6
11	26.3	-7.8	-173.9
12	24.9	-7.4	-164.5
13	26.7	-7.0	-156.1
14	26.1	-6.7	-148.2
15	24.4	-6.3	-140.6
16	23.1	-6.0	-132.9
17	22.3	-5.6	-125.3
18	22.9	-5.3	-118.3
19	22.6	-5.0	-111.7
20	21.8	-4.7	-105.3
21	22.0	-4.5	-99.2
22	18.5	-4.2	-93.2
23	16.4	-3.9	-87.8
24	16.9	-3.7	-82.9
25	16.2	-3.5	-78.6
26	15.3	-3.4	-74.7
27	13.8	-3.2	-71.2
28	12.5	-3.1	-68.1
29	10.9	-2.9	-65.1
30	8.9	-2.8	-62.3
31	7.8	-2.7	-59.4
32	7.2	-2.5	-56.2
33	6.5	-2.4	-52.6
34	5.7	-2.2	-48.5
35	5.3	-2.0	-44.1
36	5.1	-1.8	-39.5
37	4.6	-1.6	-34.7
38	5.1	-1.3	-29.9
39	6.2	-1.1	-25.3
40	7.3	-0.9	-20.9

by 300 basis points in the first year and remain at that reduced level thereafter, as long as reduced rates do not fall below defined NAIC minimums. The minimum rates are defined using treasury yields plus a weighted average spread on public and private bonds of designated quality ratings. For the pop-down and the other down type scenarios, the NAIC minimum yield utilized by the PCFTM is 3.91%.

Surplus results under the pop-down scenario were also as expected: positive over the projection period. But ending surplus under the pop-down scenario differed from ending surplus under the level scenario by a whopping 58%. Twentieth year surplus differed by 28% from its counterpart in the level scenario. Interest rates declined in the first year only and then remained level at the NAIC minimum yield. The assets and their flows increase sharply in value due to this decline, while the liability flows become negative only for a short time. After this initial one-time shock, the flows exhibit patterns similar to those under the level interest rate scenario, although at a reduced level. This scenario appears to measure the responsiveness of surplus to an initial shock only by allowing interest rates to remain level thereafter. Historical interest rates, whether new money, portfolio, or bond yields, have never exhibited such patterns for prolonged periods of time, and it seems unreasonable to expect interest rates projected under the regulatory interest rate scenarios to exhibit such patterns.

The pop-down interest rate scenario did not have a dramatic effect on surplus over the level, but the effect on portfolio yields was notable. The ending pre-tax portfolio yield under the pop-down scenario was 4.23% and 4.17% at the 20th duration. *{Under the level interest rate scenario, the corresponding yields are 5.74% and 5.73%.}* With policy guarantees in the 2.5% to 5.0% range on participating policies, such declines in yield can significantly reduce profit margins and may force an insurer to depend on other lines of business or surplus for relief. Under scenarios where

interest rates decline more than under the prescribed pop-down, the portfolio yields become deeply depressed relative to policy guarantees and surplus becomes negative.

## **VI. Validating the Integrity of the Model**

### **A. Comparisons to Actual Experience**

Like any mathematical model, the PCFTM is only of actuarial value if the results it produces are consistent with assumptions and representative of the block of business under study in the judgment of the valuation actuary. We have already seen how this block behaves under variations in interest rates and lapse experience. Surrenders are expected to decrease in the presence of declining interest rates, while reserves, dividends, and other liability flows are expected to increase along with premium income. The converse is expected for a short time in the presence of increasing interest rates and a mix is expected when interest rates are made to vary by duration (see appended graphs). These are only a few ways to check the internal consistency of the model.

Since the block is closed, only the run-off experience is being projected. The rate at which reserves decline at each duration should appear reasonable, gradually grading downward by duration under some scenarios and more rapidly under others. The valuation actuary should rely on the understanding of product development actuaries to make sure the run-off is consistent with prior experience and the characteristics of the block. In the PCFTM, declines in reserves under the level interest rate scenario approached double digits towards the last quarter of the projection period. Prior to the last quarter, reserves experienced more gradual declines by duration. The same patterns of consistency should hold for scenario liability flows. Analyzing the asset flows is a bit more complex.

Valuation actuaries will need to know the type, quality, and quantity of assets backing reserve liabilities, understand how the options should respond to changes in assumptions, how often

assets are acquired and mature, and the manner in which reinvestments are handled to determine if asset flows are responding as expected. The appended graphs depict the behavior of assets under the regulatory interest rate scenarios. These assets include those modeled by the investment and real estate departments and the book value of reinvestments modeled in the PCFTM. Both should be inspected. If irregularities are observed, it is the responsibility of valuation actuaries to investigate possible causes with the aid of investment professionals.

## **B. Implied Portfolio Rates**

The literature and regulations on cash flow testing discuss only the effect of varying assumptions on surplus. The effect on portfolio yields is not discussed. By projecting portfolio yields in cash flow testing models, valuation actuaries can determine the degree to which minimum guarantees on contractual obligations will be satisfied. Projected portfolio yields under the level and pop-down interest rate scenarios and other results are given in Table 12B and Table 13B, respectively. The yields on a pre-tax basis under the level scenario exceeded 5.0% by more than 50 basis points at all durations. Results under the pop-down scenario were not as favorable. Portfolio yields under the pop-down scenario dropped by more the 100 basis points from initial levels to 4.23% by the end of the projection period but ending surplus finished “in the black” at more than \$400 million. This result is significant, since it gives us a benchmark against which to compare minimum guarantees and other obligations. Removing the NAIC minimum allows for investigating how far interest rates need to fall to produce negative surplus or severely depressed portfolio yields on this block of business. For the block under study, falling interest rates are a more significant risk factor than rising interest rates because of participating policy guarantees. In the next section, results from the PCFTM under more extreme non-regulatory interest rate scenarios will be presented to help illuminate those conditions that produce disastrous results for the block of participating whole life insurance under study.

## **VII. Interpreting the Results**

Maybe dePalo and Rieskytl were right when they said a “true participating product” is one that is “self-supporting under all but the most adverse conditions.” But maybe not. Their definition leaves valuation actuaries to their own devices to judge whether or not current conditions embody the kind of adversity they speak of to decide if a block of participating insurance is self-supporting. The regulatory interest rate scenarios may not be sufficiently diverse to enable valuation actuaries to measure the degree to which a block of participating business is self-supporting by the above definition. By conducting cash flow testing, more extreme scenarios can be imposed on cash flows to help valuation actuaries judge and evaluate the conditions necessary to disable an insurer’s ability to mature a block of participating business. The block of participating whole life under study fared very well under all the regulatory “up” scenarios but less so under the regulatory “down” and more adverse non-regulatory scenarios. The use of the phrase “up scenario(s)” should be taken to mean the initial movement of interest rates is upward and subsequent downward movements may or may not follow. The use of the phrase “down scenario(s)” is similarly defined.

### **A. Ending Surplus Under the Regulatory Interest Rate Scenarios**

Table 14 summarizes some key results under all the regulatory interest rate scenarios. In evaluating these results, valuation actuaries can establish criteria by which to judge their significance. The direction and magnitude of ending and intermediate surplus levels is certainly one set of criteria, and the strength of portfolio yields relative to minimum guarantees is another. The minimum guarantees for this block fall in the 2.5% to 5.0% range for dividends left on deposit, settlement options, cash and surrender values, and reserves. Earnings below this range would compromise this insurer’s ability to mature this block of business. None of the regulatory “up” scenarios suggest this insurer is in danger of not being able to keep its promises to policyholders, and, recall, under “up” scenarios interest rates are increasing and policyholders are expected to lapse or secure policy loans. Surplus increases under all regulatory down scenarios and remains positive throughout the projection period while portfolio yields slip under the upper



**Table 14**  
**PCFTM Results Under**  
**Regulatory Interest Rate Scenarios**  
(\$Millions)

<b>Interest Rate Scenario:</b>	<b>Negative Cash Flow Duration</b>	<b>20th Year Surplus</b>	<b>20th Year Portfolio Rate</b>	<b>Ending Surplus</b>	<b>Ending Portfolio Rate</b>
<b>Level:</b> {no change} <b>Asset Cash Flows:</b>	27	682	5.73%	1,040	5.74%
<b>Down:</b> {-50 bp for 10, level} <b>Asset Cash Flows:</b>	34	481	4.17%	429	4.23%
<b>Down-Up:</b> {-100 bp for 5, +100 bp for 5, level} <b>Asset Cash Flows:</b>	34	642	5.93%	677	6.09%
<b>Pop-Down:</b> {-300 bp for 1, level} <b>Asset Cash Flows:</b>	34	488	4.17%	439	4.23%
<b>Up:</b> {+50 bp for 10, level} <b>Asset Cash Flows:</b>	11	738	8.98%	1,604	10.70%
<b>Up-Down:</b> {+100 bp for 5, -100 bp for 5, level} <b>Asset Cash Flows:</b>	11	651	5.69%	1,055	5.72%
<b>Pop-Up:</b> {+300 bp for 1, level} <b>Asset Cash Flows:</b>	18	714	7.78%	1,391	8.71%

*(New Money Rates assumed to start at 5.93% for all scenarios. Lapse factors are half normal under down scenarios, twice normal under up scenarios, and normal (1.0) under level scenarios. Portfolio yields are on a pre-tax basis)*

*(NAIC Minimum Yield of 3.91% was applied in all Down Scenarios.)*

*("± X bp for Y" means the rates will change by the amount and direction indicated each year for Y years. For example, {+300 bp for 1, level} means rates increase by 300 basis points per year for 1 year and remain level thereafter)*

bound of minimum guarantees in two of the down scenarios. Liability flows move as expected but asset cash flows take on negative values and much earlier under "up" scenarios than under "down" scenarios. There are several reasons why this happens.

Under up scenarios, policyholders are withdrawing at a greater rate leaving fewer premium dollars to offset the liability outflows -- policyholder dividends, death and surrender benefits, and expenses. Recall, the portfolio yield implicit in dividends is lagged two years while expense inflation though lagged five years reflects half the new money rate. Over time, any dividend gain due to improvement in portfolio yield is more than offset by the inflation component of expenses. This means dividends are decreasing more rapidly under up scenarios than they would under down scenarios. Surrender benefit are higher because lapses are assumed to occur at twice the rate assumed in the level scenario. Reserves are also affected by this assumption and decline more rapidly as reflected in the survivorship factors. They move from 1.0 to 0.0 under up scenarios implying the number of persisting lives decreases throughout the projection period and presumably its the young, "good" risks that are surrendering their policies. This is an old block of business, the average policy size is small, and many policies are likely to be beyond their premium paying periods. The little premium income that remains is insufficient in combination with investment income to generate non-negative net cash flow in advancing periods. These negative cash flow amounts are then invested at high interest rates and then reinvested seven year later at even higher interest rates in some scenarios. This directly reduces surplus.

The asset cash flows become negative under regulatory up scenarios but not enough in magnitude or frequency to have a deleterious effect on surplus. To determine when this block of business becomes vulnerable to high interest rates, more extreme interest rate scenarios are imposed. The next section examines how this block of business performs under more extreme up and down scenarios. These extreme variations help establish the conditions under which this block fails to be "self-supporting" according to dePalo's and Reiskytl's criteria. Some of these variations will include changes in assumption regarding the durational lapse adjustment factors. The PCFTM produces results that are consistent with each interest rate scenarios and lapse assumptions imposed as we shall see.

## **B. Ending Surplus Under Some Non-Regulatory Interest Rate Scenarios**

It is not clear if the regulatory scenarios embody the kind of adversity dePalo and Rieskytl think necessary to determine whether or not this block of participating whole life insurance is “self-supporting.” Portfolio yields pierce the range of minimum guarantees, but surplus never becomes negative. However, as the following results will show, small variations in the regulatory scenarios can result in adverse results. Small changes to the regulatory down scenarios produce more adverse results than small changes to the regulatory up scenarios, suggesting declining interest rate climates pose a more immediate risk for participating whole life insurance than rising interest rate climates. In declining climates, policyholders are more likely to keep their policies inforce rather than surrender. Those with the means of continuing their premium payments will do so to the disadvantage of the insurer, exposing the insurer to reinvestment risk. The net effect will likely lengthen contract liabilities in an environment where portfolio yields are decreasing.

For this exercise it was important to determine the degree to which interest rates must move to have both a marginally and totally adverse effect on this block of business. Even though surplus becomes negative under some of the non-regulatory scenarios, it may not necessarily mean this insurer cannot thwart off insolvency should some of those scenarios become realized. Secondly, it was important to measure the sensitivity of the results to the lapse assumptions imposed under the various scenarios. Testing for these two conditions will also help measure the ability of the PCFTM to produce results that are in line with intuition, if the PCFTM is to eliminate the need for modeling liability cash flows using expensive mainframe valuation systems.

### **The Effect of More Extreme Interest Rate Scenarios**

Under more severe down scenarios with no minimums, the yields fall dangerously within the range of minimum policy guarantees and ending surplus becomes negative (see Table 14A). One huge initial shock of -330.8 basis points in year 1 was sufficient to produce a noticeable amount of negative surplus (-\$1 mil.) at the end of the projection period. Two initial down shocks, -300

basis points in year 1 and -50 basis points in year 2, depress yields below 3.0% halfway through the projection period and cause ending surplus to plummet well below its initial level. However, surplus remains positive until duration 31 and asset cash flows are positive until duration 27. The Pop-Down No Minimum scenario removes the NAIC floor of 3.91% and allows yields in this case to fall to 2.93%. Removing this constraint does not have nearly the effect on ending surplus as the 1 Huge Shock No Minimum or 2 Down Shocks No Minimum scenarios. The asset cash flows under all three turn negative at the same time (duration 27) but they are significantly more negative under the 2 Down Shocks No Minimum scenario. This results in reinvestment of huge negative cash flows which directly reduces surplus.

Under the Extreme Down-Down-Down, interest rates don't drop as quickly as they did under the regulatory Pop-Down; rather, they grade more gradually but over more periods. The regulatory scenarios confine interest rate variations to at most the first ten years in the projection period and only move interest rates in at most two directions before allowing them to level off. Allowing interest rates to vary an additional 5 years under the Extreme Down-Down-Down scenario was sufficient to produce disastrous results for this block. Rates drop 75 basis points per year for the first 5 years, 25 basis points per year for the second five year period, and 12.5 basis points for a third five year period before being allowed to level off. This pattern prevents new money rates from falling below zero. The result was -\$415 million of surplus at duration 20 and a 1.16% portfolio yield which falls well below the range of minimum guarantees this insurer promises to policyholders. At the end of the projection, the numbers are -\$1,590 and -0.18% respectively.

Under the Down-Down-Down and Down-Up-Down, the surplus results are positive at the 20th duration, but portfolio yields are still below minimum policy guarantees. The magnitude of the interest rate drops in the Down-Down-Down scenario account for the slightly better performance of the block than under the Down-Up-Down. During the up portion of the Down-Up-Down scenario, the block of business has an opportunity to recover but when hit with another down cycle surplus plunges to -\$884 million. This is a big difference between the non-regulatory

**Table 14A**  
**PCFTM Results Under Some**  
**Non-Regulatory Interest Rate Scenarios**  
*(\$Millions)*

<i>Interest Rate Scenario:</i>	<i>Durational Lapse Adj. Factor</i>	<i>20th Year Surplus</i>	<i>20th Year Portfolio%</i>	<i>Ending Surplus</i>	<i>Ending Portfolio%</i>
<b>Pop-Down No Minimum:</b> {-300 for 1, level}	0.5	399	3.33%	230	3.45%
<b>1 Huge Shock No Minimum:</b> {-330.8 for 1, level}	0.5	306	3.06%	-1	3.45%
<b>2 Down Shocks No Minimum:</b> {-300 for 1, -50 for 1, level}	0.5	234	2.90%	-161	3.70%
<b>Up-Up-Up:</b> {100 for 5, 100 for 5, 100 for 5}	2.0	662	12.28%	2,284	20.66%
<b>Extreme Up-Up:</b> {200 for 5, 100 for 5, level}	2.0	485	10.57%	1,591	20.63%
{300 for 5, 200 for 5, level}	2.0	140	8.14%	351	30.11%
{400 for 1, 300 for 4, 200 for 5, level}	2.0	55	6.54%	-226	32.75%
{400 for 5, 100 for 5, 50 for 5, level}	2.0	-130	1.96%	-6,629	33.23%
<b>Up-Down-Up:</b> {400 for 5, -50 for 5, +50 for 5, level}	2.0	184	8.33%	575	25.42%
<b>Down-Up-Down:</b> {-100 for 5, +50 for 5, -50 for 5, level}	0.5	93	1.68%	-884	-0.35%
<b>Down-Down-Down:</b> {-50 for 5, -25 for 5, -12.5 for 5, level}	0.5	58	2.17%	-695	-0.66%
<b>Extreme Down-Down-Down:</b> {-75 for 5, -25 for 5, -12.5 for 5, level}	0.5	-415	1.16%	-1,590	-0.18%
<b>Moderate Down - Moderate Up:</b> {-100 for 5, +40 for 5, level}	0.5	325	3.33%	138	3.47%
{-118 for 5, +60 for 5, level}	0.5	301	3.42%	145	3.58%

*(New Money Rates assumed to start at 5.93% for all scenarios. Portfolio yields are on a pre-tax basis.)*

*("1 X bp for Y" means the rates will change by the amount and direction indicated each year for Y years. For example, {+300 for 1, level} means rates increase by 300 basis points per year for 1 year and remain level thereafter)*

scenarios presented here and the regulatory scenarios. The regulatory scenarios have fewer periods of variation: years 1 through 5 and years 6 through 10. After year 10, rates are allowed to remain level during which time the block of business has time to recover and will if interest rates are sufficient. Observe what happens under the Moderate Down-Moderate Up scenarios. They are similar to the regulatory Down-Up scenario for the first five years, dropping rates by 100/118 basis points per year. In the second five year period rates move up 40/60 basis points per year and then remain level. Surplus remains positive although portfolio yields fall within the range of minimum guarantees. Portfolio yields under the regulatory Down-Up were well above the range of minimum guarantees, primarily because in the second five year period rates moved upward a significant 100 basis points.

The non-regulatory up scenarios illustrate quite vividly how deleterious extremely high interest rates can be for this block of business. The asset cash flows become negative almost immediately in all the Extreme Up-Up-Up scenarios, as early as duration 2. Under the regulatory up scenarios the earliest the asset cash flows turn negative is duration 11. It is significant to state that the asset cash flows can return to a positive level later on in the projection and do in all the up scenarios tested. But in the non-regulatory up scenarios they are more often negative and more sizably so than in the regulatory up scenarios. This means there are more instances in the non-regulatory up scenarios where negative asset cash flows are reinvested than in the regulatory up scenarios. The reasons up scenarios cause asset cash flows to turn negative have already been discussed. Reduced premium flow, high lapses, and escalating inflation become more severe under several of the Extreme Up-Up-Up scenarios. The first two Extreme Up-Up-Up scenarios cause asset cash flows to become negative but their magnitude and frequency do not prevent surplus from increasing with duration. Interest rates are not extreme enough nor subject to enough variation to impact surplus adversely. Even when the durational lapse assumption is increased, surplus is only marginally affected under these two interest rate scenarios. Under others, the change is significant.

### The Effect of the Durational Lapse Adjustment Factors

Table 14B gives the results for the same interest rate scenarios as Table 14A but assumes different assumptions for durational lapse adjustment factors. The results given in Table 14B assume durational lapse adjustment factors of 0.25 for down scenarios and 2.5 for all the up scenarios except for 1, the Up-Up-Up scenario which assumes a factor of 3.0. Recall, the durational lapse adjustment factors affect the rate at which policyholders lapse. This assumption directly affects the surrender benefits in the liability flows, and the survivorship factors. A factor of 1.0 implies surrenders will occur at the rate implicit in the asset shares and survivorship will be 1.0 at all durations. A factor of 2.5 implies surrender benefits are 2.5 times that implicit in the asset shares and survivorship is decreasing by a factor of 1.5. Recall the following formulas:

***Gain & Survivorship Adjusted Liability Flows<sub>t</sub>***

$$\begin{aligned}
 &= \text{Gain Adjusted Liability Flows}_{t-1} \\
 &\quad \text{for } t < 2 \\
 &= (\text{Gain Adjusted Liability Flows}_{t-1} - \text{Surrender Benefits}_{t-1}) * \text{Survivorship}_t \\
 &\quad + \\
 &\quad (\text{Surrender Benefits}_{t-1} * \text{Survivorship}_{t-1} * \text{Durational Lapse Adjustment Factor}_t) \\
 &\quad \text{for } t \geq 2
 \end{aligned}$$

***Survivorship<sub>t</sub>***

$$= \text{Survivorship}_{t-1} * \{1 + \text{Surrender Benefits to Reserves Ratio}_t * (1 - \text{Lapse Factor}_t)\}$$

The survivorship factor alters premiums, dividends, reserves, and liability flows to reflect persisting lives as well as withdrawals. For example, a factor of 0.25 means 75% of the asset share surrender rates contribute to survivorship, whereas a factor of 0.5 implies only 50% make a contribution. Respectively, 25% and 50% of the baseline surrender benefits augment liability flows. The effect of the durational lapse adjustment factors was measured by using a factor of 0.25 in down scenarios and 2.5 and 3.0 in the up scenarios. By comparing the results in tables

**Table 14B**  
**PCFTM Results Under Some**  
**Non-Regulatory Interest Rate Scenarios**  
**with**  
**Different Durational Lapse Adjustment Factors**  
*(\$Millions)*

<i>Interest Rate Scenario:</i>	<i>Durational Lapse Adj. Factor</i>	<i>20th Year Surplus</i>	<i>20th Year Portfolio%</i>	<i>Ending Surplus</i>	<i>Ending Portfolio%</i>
<b>Pop-Down No Minimum:</b> {-300 for 1, level}	0.25	348	3.27%	-147	3.84%
<b>1 Huge Shock No Minimum:</b> {-330.8 for 1, level}	0.25	236	3.01%	-440	4.22%
<b>2 Down Shocks No Minimum:</b> {-300 for 1, -50 for 1, level}	0.25	148	2.85%	-647	5.43%
<b>Up-Up-Up:</b> {100 for 5, 100 for 5, 100 for 5, level}	3.0	549	14.59%	2,330	20.71%
<b>Extreme Up-Up:</b> {200 for 5, 100 for 5, level}	2.5	448	11.72%	1,714	20.69%
{300 for 5, 200 for 5, level}	2.5	76	6.36%	338	30.54%
{400 for 1, 300 for 4, 200 for 5, level}	2.5	-45	0.99%	-1,112	31.75%
{400 for 5, 100 for 5, 50 for 5, level}	2.5	-451	-78.87%	-203,806	33.20%
<b>Up-Down-Up:</b> {400 for 5, -50 for 5, +50 for 5, level}	2.5	-10	3.22%	-135	26.04%
<b>Down-Up-Down:</b> {-100 for 5, +50 for 5, -50 for 5, level}	0.25	-42	1.63%	-1,934	-0.10%
<b>Down-Down-Down:</b> {-50 for 5, -25 for 5, -12.5 for 5, level}	0.25	-93	2.12%	-1,412	-0.11%
<b>Extreme Down-Down-Down:</b> {-75 for 5, -25 for 5, -12.5 for 5, level}	0.25	-672	1.10%	-3,176	-0.13%
<b>Moderate Down - Moderate Up:</b> {-100 for 5, +40 for 5, level}	0.25	268	3.28%	-246	3.93%
{-118 for 5, +60 for 5, level}	0.25	246	3.36%	-227	4.00%

*(New Money Rates assumed to start at 5.93% for all scenarios. Portfolio yields are on a pre-tax basis.)*

*(" ± X bp for Y" means the rates will change by the amount and direction indicated each year for Y years. For example, {+300 for 1, level} means rates increase by 300 basis points per year for 1 year and remain level thereafter)*



14A and 14B, the effect of a 0.25 factor in the down scenarios almost always impacts surplus levels significantly, especially ending surplus. The point at which asset cash flows turn negative is unchanged in all but three of the down scenarios (the Pop-Down No Minimum, 1 Huge Shock No Minimum, and 2 Down Shocks No Minimum, see Table 14C) where negative asset cash flows are observed 5 to 7 durations earlier. In these three scenarios, the change in durational lapse adjustment factor has a huge effect on survivorship. By the end of the projection period, survivorship factors are more than 3 times initial levels.

Surplus levels in the Up-Up-Up scenario are only moderately influenced by a change in durational lapse factor. The survivorship factors decrease more rapidly under a 3.0 factor than under a 2.0 factor, but interest rates do not increase enough to generate sizably negative cash flows. The first two Extreme Up-Up scenarios under a 2.5 factor have a similar effect on surplus. When interest rates are pushed very high, the results are even more disastrous than observed under a 2.0 factor. The change to a 2.5 factor just pushes results more in the same direction which is in line with intuition. The duration at which asset cash flows turn negative occurs 1 duration earlier under a 2.5 lapse adjustment factor while surplus turns negative at most 11 durations earlier. The change in durational lapse adjustment factors has a significant effect on surplus by causing the asset cash flows to become negative almost immediately and significantly in magnitude.

Under the Moderate Down-Moderate Up scenarios it again becomes obvious that the change in lapse assumption has a greater affect on the magnitude of reinvested cash flows rather than on the duration at which cash flows become negative. Asset cash flows turn negative at the same duration under a 0.25 assumption as they do under a 0.5 assumption, but surplus turns negative 7 durations earlier.

In summary, the results of tables 14A, 14B, and 14C illustrate how adverse interest rates and lapse rates need to be to adversely affect surplus for participating whole life insurance. Up scenarios have to push interest rates very high, very fast or for long periods of time to deplete surplus. In the down scenarios, a single down shock of 300 bp and no NAIC minimum impacts surplus and this scenario

**Table 14C**  
**PCFTM Results Under Some**  
**Non-Regulatory Interest Rate Scenarios**  
**Comparisons of Durations First Negative Cash Flows are Observed**  
**Under Different Assumptions for Durational Lapse Adjustment Factors**  
*(\$Millions)*

<i>Interest Rate Scenario:</i>	<i>Duration at Which Surplus Turns Negative Table 14A vs. 14B</i>	<i>Duration at Which Asset Cash Flows Turn Negative Table 14A vs. 14B</i>
<i>Pop-Down No Minimum: {-300 for 1, level}</i>	-- vs. 35	27 vs. 34
<i>1 Huge Shock No Minimum: {-330 8 for 1, level}</i>	40 vs. 28	27 vs. 34
<i>2 Down Shocks No Minimum: {-300 for 1, -50 for 1, level}</i>	31 vs. 25	27 vs. 32
<i>Up-Up-Up: {100 for 5, 100 for 5, 100 for 5, level}</i>	-- vs. --	6 vs. 2
<i>Extreme Up-Up: {200 for 5, 100 for 5, level}</i>	-- vs. --	3 vs. 2
<i>{300 for 5, 200 for 5, level}</i>	-- vs. --	3 vs. 2
<i>{400 for 1, 300 for 4, 200 for 5, level}</i>	29 vs. 18	2 vs. 2
<i>{400 for 5, 100 for 5, 50 for 5, level}</i>	15 vs. 6	2 vs. 2
<i>Up-Down-Up: {400 for 5, -50 for 5, +50 for 5, level}</i>	-- vs. 19	2 vs. 2
<i>Down-Up-Down: {-100 for 5, +50 for 5, -50 for 5, level}</i>	22 vs. 20	27 vs. 27
<i>Down-Down-Down: {-50 for 5, -25 for 5, -12.5 for 5, level}</i>	22 vs. 19	27 vs. 27
<i>Extreme Down-Down-Down: {-75 for 5, -25 for 5, -12.5 for 5, level}</i>	15 vs. 14	27 vs. 27
<i>Moderate Down - Moderate Up: {-100 for 5, +40 for 5, level}</i>	-- vs. 33	34 vs. 34
<i>{-118 for 5, +60 for 5, level}</i>	-- vs. 33	34 vs. 34

*Note:*

- A "--" duration designation means negative cash flows are not observed over projection period, although under the down scenarios cash flows are declining.
- New Money Rates assumed to start at 5.93% for all scenarios. Portfolio yields are on a pre-tax basis.
- "x X bp for Y" means the rates will change by the amount and direction indicated each year for Y years. For example, {+300 bp for 1, level} means rates increase by 300 basis points per year for 1 year and remain level thereafter.

may not be as remote a possibility as most of the other extreme scenarios examined. Modest movements in interest rates combined with huge withdrawal activity could have the same effect as the results of the PCFTM illustrate.

## **VIII. Criticisms and Directions for Further Research**

If there is an overwhelming need for cash flow testing traditional participating whole life insurance then that need is to determine whether it is “self-supporting under all but the most adverse scenarios.” Unfortunately, the line distinguishing adverse scenarios from non-adverse scenarios is not clear; what seems adverse for one product may not undermine the profitability of another. These distinctions become clearer by cash flow testing. The results of the PCFTM demonstrate that for this block of business the regulatory interest rate scenarios are not adverse enough to determine the conditions under which this block fails to be self-supporting. The regulatory scenarios do not move enough in any direction or for a long enough period of time to help establish the limits of vulnerability for this block. However, valuation actuaries are not limited to imposing just the regulatory scenarios in cash flow testing. If the regulatory scenarios prove insufficient in measuring the strength of reserves, then it is the responsibility of valuation actuaries to expand their cash flow testing activities. The problem of course is one of expense if only seriatim based valuation systems are available.

Lambert, dePalo and Rieskytl, and Dr. Brender understand how expensive mainframe valuation systems can limit cash flow testing activities. The PCFTM removes this limitation by minimizing the cost of modeling liabilities through the use of new business asset shares. They were not perfect coming into the PCFTM because of an excessively high new money rate assumption used to model dividends, gross premium margins higher than necessary, and liability flows out of line with actual experience. These problems were addressed by re-calibrating the asset share dividends and liabilities to actual experience with the dividend and gains adjustment factors, avoiding the cost of modeling these flows from scratch. Modeling reserve liabilities using mainframe valuation systems would require, at a minimum, reserve valuations for the

block at each duration in the projection period. Even if cost were not an issue, it does not seem possible to use valuation systems to prepare needed reserve valuations within the regulatory time constraints. When cost is factored in, the need for simplified cash flow testing methods is overwhelming.

The PCFTM addresses the need for simplified cash flow testing techniques and appears to produce results that are consistent with the lapse assumptions and interest rate scenarios imposed. One question that remains, however, is how the results of the PCFTM would compare with those of a mainframe valuation system. Unfortunately, this would be an expensive question to answer using mainframe systems and it cannot be answered by turning to the literature. The literature offers only Dr. Brender's model but it is based on Canadian actuarial standards and regulations and its results are not comparable to the results produced by the PCFTM, a model based on U.S. actuarial standards and regulations.

The lack of comparable models is not the only area where further research could help measure the reliability of the results produced by the PCFTM, however. Would a different set of new business asset shares materially change the level of projected liability flows? The mix of business assumptions would differ under a different model office projection, as would the pattern of liabilities. Intuitively, one might think such a change could be material. The gains adjustment factor should correct for some of the variation associated with a different model office projection, but possibly not enough. The gains adjustment factor re-calibrates the liability flows to reproduce current gain from operation. Its value is constant over the entire projection period and applied to premiums to further alter the level of projected liability flows. More research would help test the reliability of the gains adjustment factor after the start of the projection and may even suggest alternate methods of re-calibrating liability flows should be sought. This holds true for the dividend adjustment factor as well.

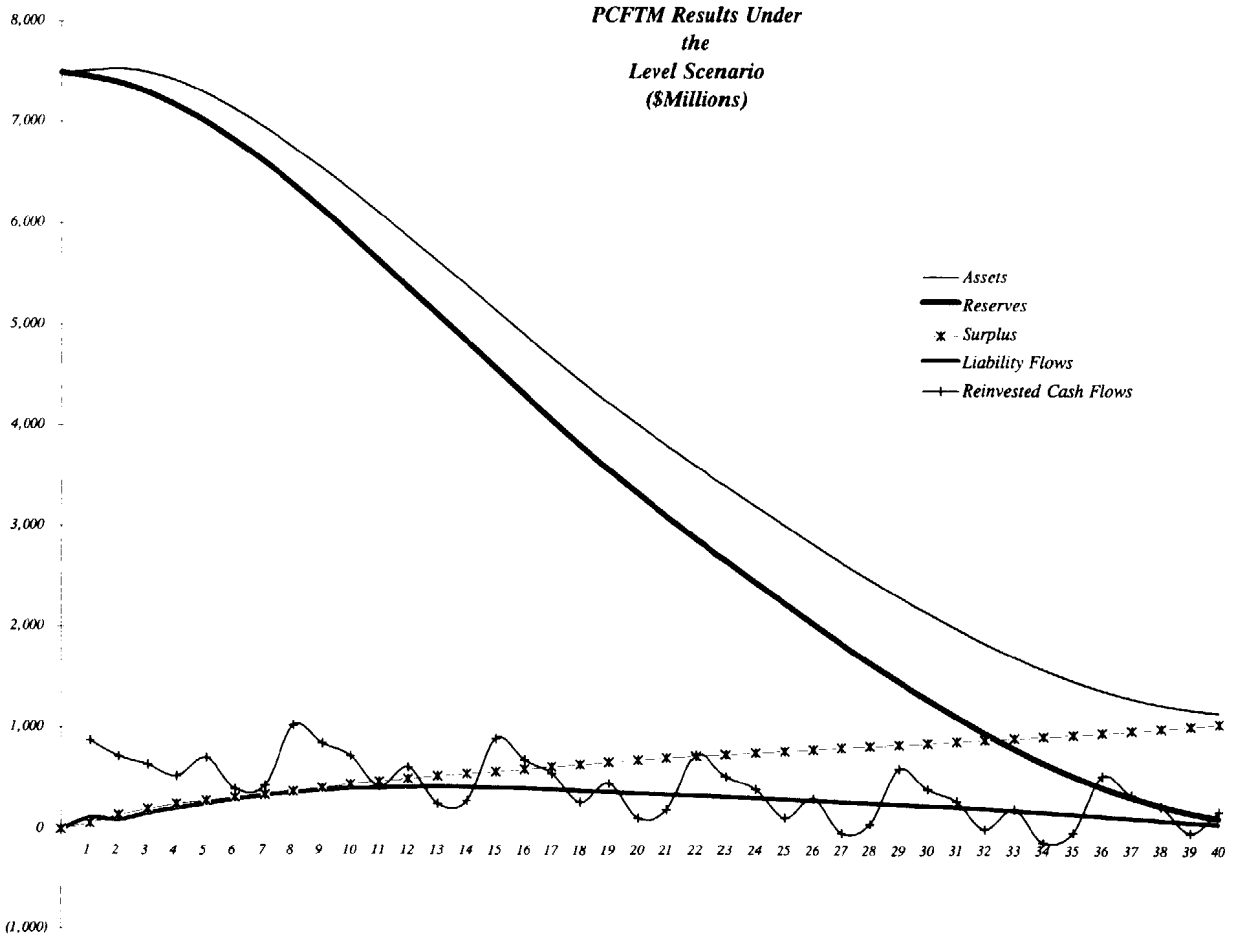
The durational lapse adjustment factors once set in a given scenario are held constant over the entire projection period. A factor of 2.0 in up scenarios and 0.5 in down scenarios are not unreasonable assumptions given the historical surrender behavior of this block under rising and declining interest rate climates. The sensitivity of these assumptions is measured by varying them to 2.5 and 3.0 in up scenarios and to 0.25 in down scenarios. But even in these additional cases assumptions remain fixed throughout the projection period. It is possible that it would have been better to grade the durational lapse adjustments from 1.0 more directly with movements in interest rates. This approach could have a significant effect on the duration at which negative cash flows are first observed. More research should focus on developing a formulaic relationship between the durational lapse adjustment factors and movements in interest rates.

Under rising interest rates, especially, it is observed how detrimentally the investment policy assumption for this block of business affects surplus. The investment policy requires the reinvestment of asset cash flows every seven years in non-callable bonds. The problem of course is that a negative cash flow at given duration will be reinvested seven years later with then possibly negative cash flows. This policy has a greater effect on surplus in up scenarios than down scenarios. In fact as the graphs in the appendix illustrate, this policy causes the cash flows to exhibit periodic behavior where the pattern is replicated every seven years. More research could reveal that reinvesting in instruments with a spread of maturities would eliminate this periodicity and produce a much smoother pattern of cash flows.

Finally, the interest rate sensitivity of various dividend options will become more significant for this block of business in the future. The AIP rider is a relatively new and fancy way policyholders can use one-year term insurance and paid-up additions to reduce premiums and accelerate the growth of policy cash values. One-year term reserves for this option were judged insignificant in size to warrant special consideration in the PCFTM and reserves for paid-up additions were assumed to have the same interest rate sensitivity as base policy reserves. This

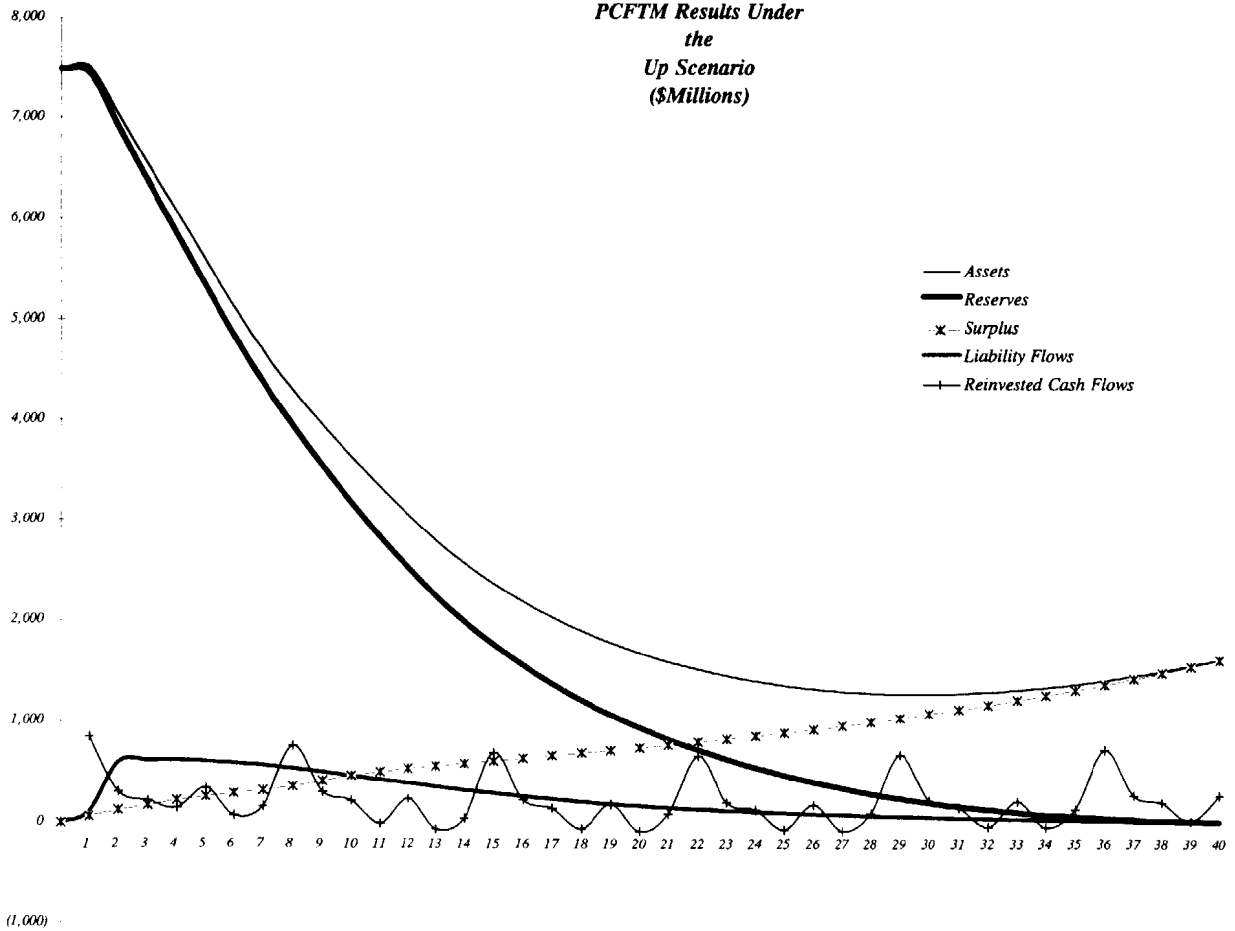
assumption may not hold for this block in the future as policyholders become more sophisticated in their understanding of the mechanics of this option. More research will determine the need for better handling of this option in cash flow testing models.

**IX. Appendix: Graphs of PCFTM Results**

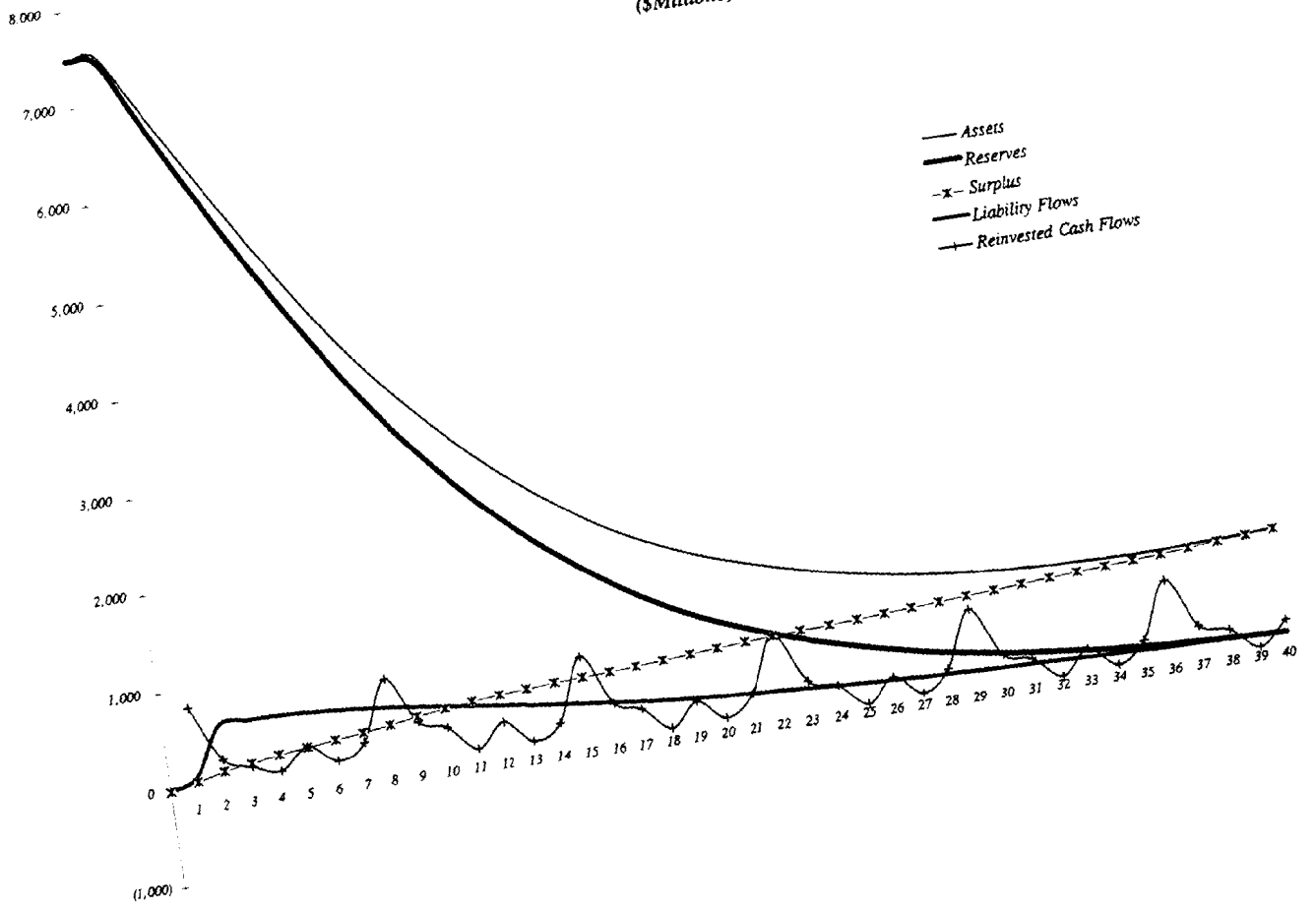


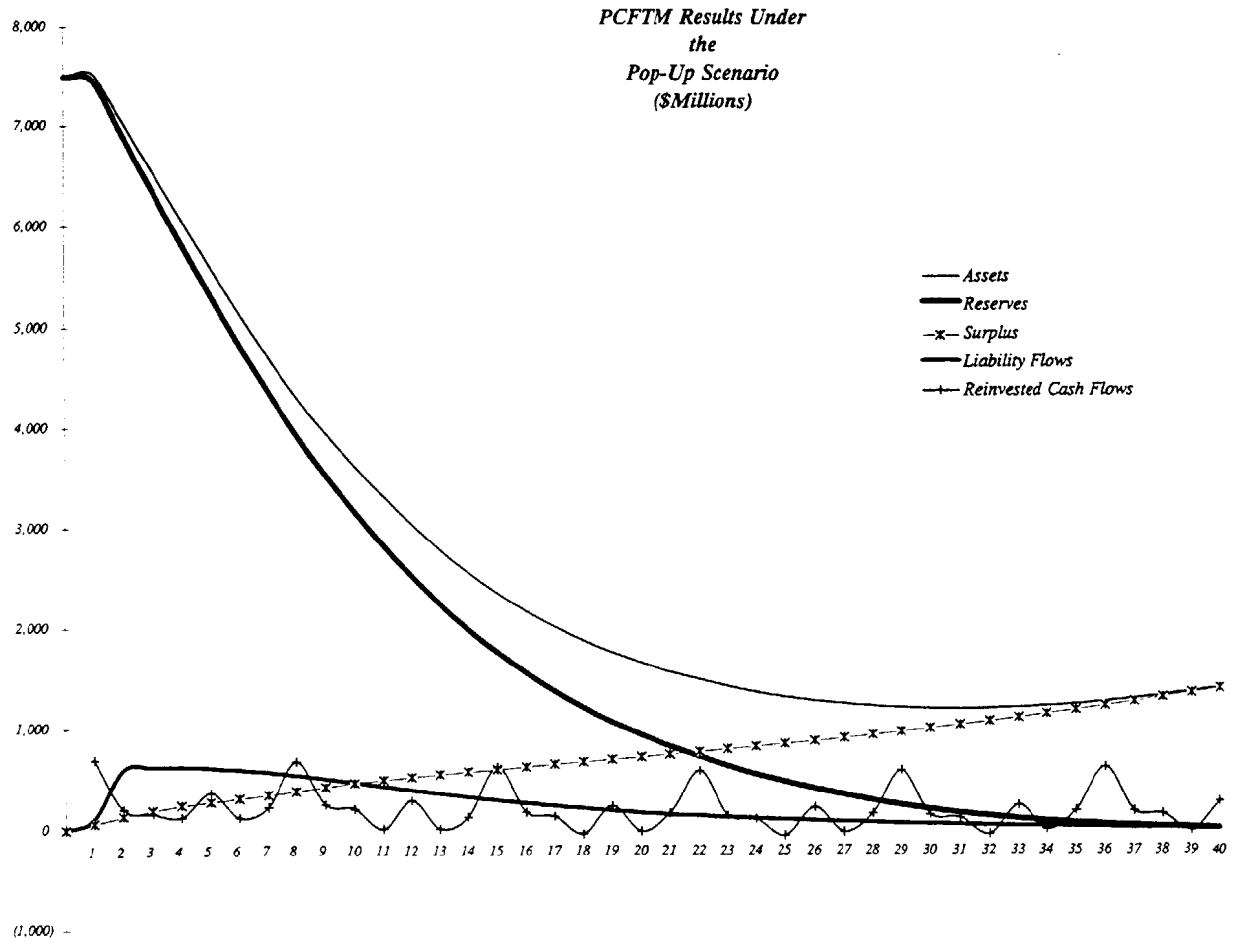


**PCFTM Results Under  
the  
Up Scenario  
(\$Millions)**



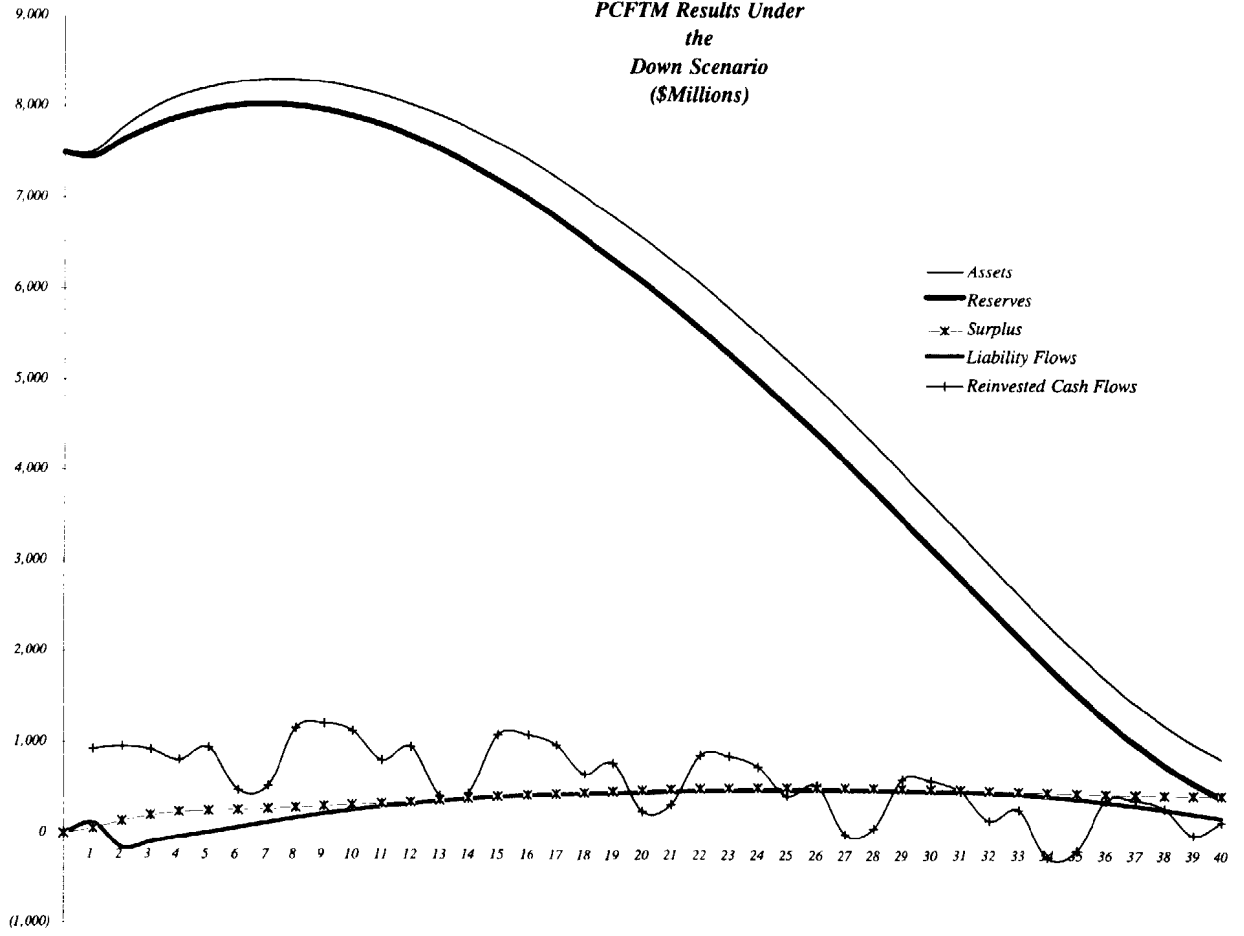
PCFTM Results Under  
the  
Up-Down Scenario  
(\$Millions)



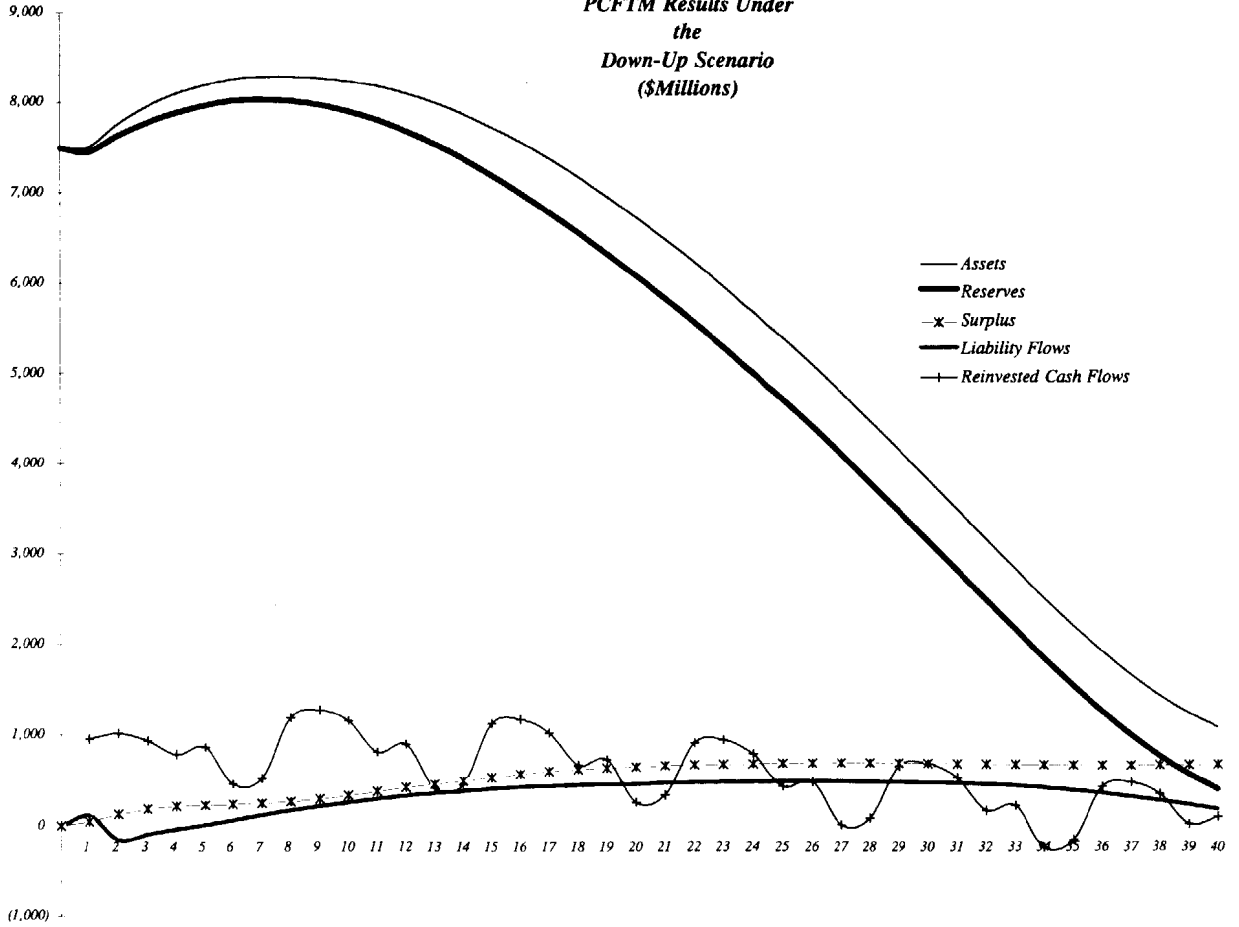


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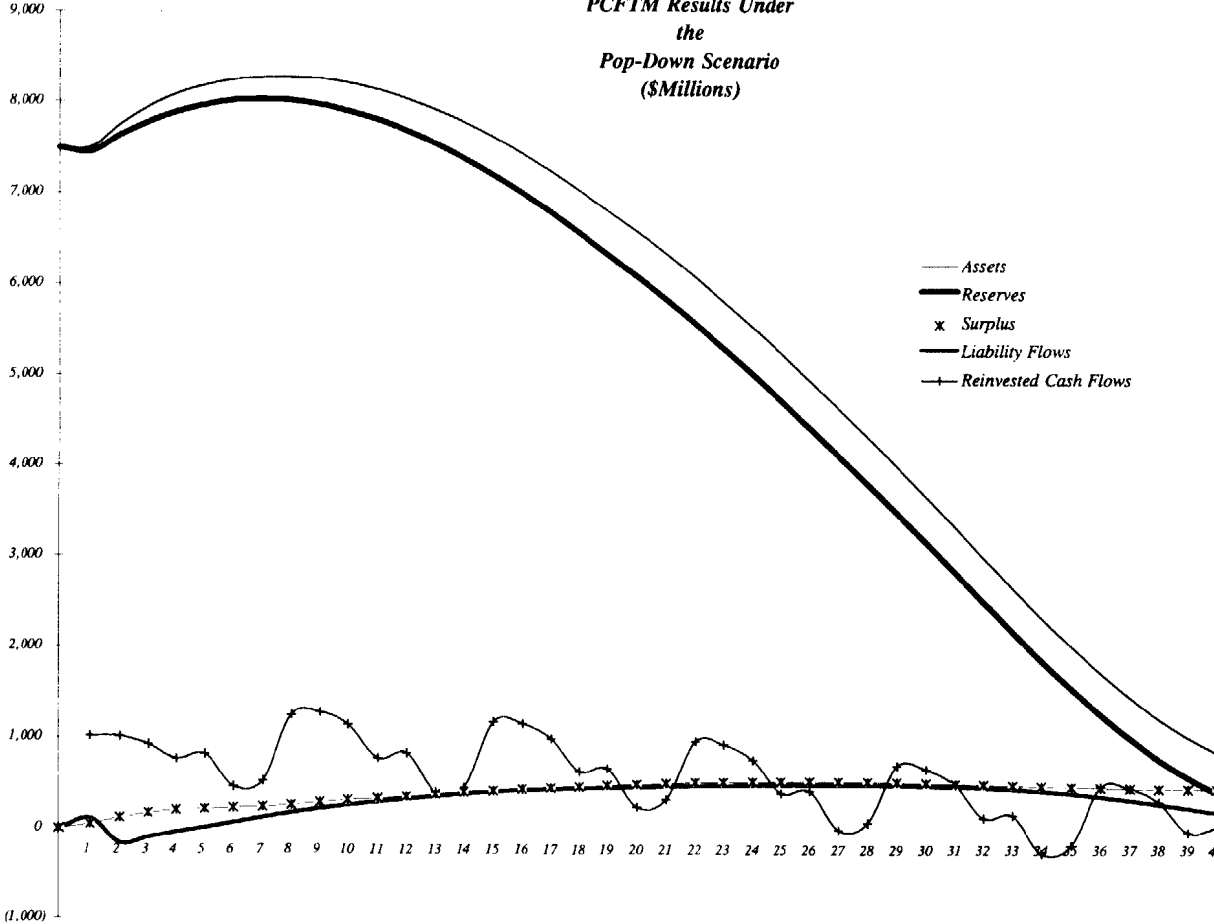
*PCFTM Results Under  
the  
Down Scenario  
(\$Millions)*



**PCFTM Results Under  
the  
Down-Up Scenario  
(\$Millions)**



*PCFTM Results Under  
the  
Pop-Down Scenario  
(\$Millions)*



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