ACTUARIAL RESEARCH CLEARING HOUSE
1995 VOL. 1

A GENERAL FRAMEWORK FOR FINANCIAL DECISIONS

by Oakley E. VanSlyke

Excerpts for ARC
8/25/94

I. INTRODUCTION

This paper presents a generalization of the principal current methods of financial decision-making. In the way physical sciences benefit from new general theories that encompass those theories they succeed, the science of finance can benefit from the development of a general framework that encompasses many of the current methods of financial analysis. This general framework shows the set of assumptions that underlies each specific method now in use encompassed by the theory. In addition, the general theory shows alternate methods to use when the assumptions of the method currently in use are not appropriate.

This paper develops a general framework for financial decision-making from a few simple axioms of rational economic behavior. The axioms are not new, but the mixing of traditional axioms of preference theory and the concept of scenarios (from the work of von Neumann and Morgenstern) leads to a general framework that we believe has not appeared before in financial literature.

In this paper's most sweeping example, it applies the proposed general framework for financial decision-making to the capital budgeting problem, showing that both Net Present Value and Internal Rate of Return rules reflect a set of assumptions that are perfectly reasonable in many problems, but inappropriate in others. The paper shows alternative methods to use when these methods are not appropriate. In this paper's most controversial example, it finds that methods used to determine the appropriateness of leveraged buyouts might have systematically underestimated the economic costs of long-term uncertainty about profit. The implication is that methods used to price leveraged buyouts have provided a sufficient cause for demand for long-term debt to be driven to excessively high levels. Thus too much debt has been used and too many highly leveraged deals have been done. The paper finds similar problems with methods adopted in the 1970's by the U. S. life insurance industry to price long-term contracts. These problems apparently caused the life insurance industry to underestimate the appropriate premium for long-term contracts and overestimate the value of long-term assets such as real estate and high-interest bonds. If this is true, poor pricing methods may be part of the explanation for the sharp drop in operating results for many firms selling such contracts. This paper also suggest standards for pricing in the sevice and insurance sectors of the economy, standards for price regulation, and standards for solvency regulation.

II. THE PROBLEMS

A. The Current Situation

Adequate solutions have been developed for many problems which financial decision-makers actually encounter. Often, however, the developed solutions are not applicable to a problem. In these cases, a generalized framework for financial decision-making may suggest a solution. In addition, a generalized framework for financial decision-making can clarify the relationship among well known financial decision tools.

1. *The Capital Budgeting Problem.* One of the most frequent of all financial decisions is whether or not to enter into a new venture-a new business, an investment in equipment, or any other new tradeoff between risk and return. The classical solutions, most often used today, are the Internal Rate of Return (IRR) model and the the Net Present Value (NPV) model.

2. *The Pricing Problem.* Another frequent decision is to set a price for a product or service in light of estimates of costs and levels of sales. Microeconomic theory shows that when prices are in equilibrium those prices will include a provision for profit equal to the cost of capital, and no more.

3. *The Leverage Problem.* Modigliani ( ) and Modigliani and Miller (1958) have shown that given certain reasonable assumptions a firm should be indifferent to leveraging through issuance of debt or issuance of stock. The solution is more complex when there are corporate and personal income taxes. Modigliani and Miller (1963) have shown that, under certain reasonable conditions, it is preferable to issue debt rather than stock when taxes are present.

4. *The Portfolio Investment Problem.* Markowitz (1952), Sharpe (1964), Ross (1976), Roll(1977), Lintner (1965) and Mossin (1966) have all contributed to the development of the Capital Asset Pricing Model, Arbitrage Pricing Theory, and their generalizations.

5. *The Rate Regulation Problem.* Regulatory authorities must decide, and the courts must adjudicate, the appropriate level of profit margin in rates for rate-regulated enterprises. Examples of regulated rates include utility rates, travel accident insurance rates, private passenger automobile insurance rates, and rates for services under government contracts.

6. *The Solvency Problem.* The cost to taxpayers of the savings and loan insolvencies of the late 1980's and early 1990's has highlighted the responsibilities of regulators to require adequate levels of net worth in firms that hold assets in trust, such as banks, insurance companies, and limited investment partnerships. The same problem can be viewed from another perspective, which is that regulators have a responsibility to limit the activites of such an enterprise to a level of risk consistent with the enterprise's net worth.

B. Notation

The notation of this paper will be familiar to most actuaries, both here and abroad. Because it may be new to many readers, however, we will define the notation in some detail. This will allow us to conclude this section with precise mathematical formulations of the various problems to which we will apply the general framework. We have altered standard actuarial notation where it seemed preferable to use standard notation of the literature of finance; we will make these departures from "standard" notation clear as well.

Many of the concepts underlying the notation are present in order to allow us to discuss risk in some detail. While defining the notation we will provide short descriptions of the underlying concepts.

Time is presumed to begin with t=0 at the time of the decision. The entire situation at hand (status quo ante), and all decision alternatives under consideration, (by definition) affect financial results for $0 \leq t < \infty$.

A *scenario* is a description of one possible future of the world, adequately exact to permit estimation of probabilities of events at all future times. The degree of exactness or precision needed to define a scenario depends on the problem at hand. In the classic work by Kahn, an example of a scenario is "limited nuclear war." All possible scenarios, taken together, encompass (by definition) all possible future events. Examples typical of financial work include:

- Three possible scenarios are recession, steady economic growth with moderate inflation, and high inflation
- Three possible scenarios are steady rise in consumer demand, erratic consumer demand, and steady decline in consumer demand.

Scenarios may need to be defined in terms of several variables in order to make clear the probabilities of all significant possible outcomes.[1]

---

[1]Scenarios are not alternatives among which the decision-maker must choose. The probabilities that the define the decision maker's understanding of the value of the firm and its options must reflect all information available at the time of the decision. In addition, those probabilities must reflect the application of the decision rule described in this section to all foreseeable decisions. Raiffa ( ) calls this process of trimming a decision tree "averaging out and folding back". After averaging out and folding back, the scenarios and their probabilities, and the probabilities of various cash flows arising under each scenario, all reflect only outcomes not under the decision maker's control.

Scenarios are denoted by

scenario j, $j=1,...,N$

All amounts must be expressed as *cash flows*. All cash flows must be expressed after-tax, in principle, although in many practical calculations corporate and personal income taxes will not affect decisions.[2] Cash held at the time of the decision has a value at $t=0$, but all other assets are expressed in terms of their anticipated cash flows and have *no* value at t=0.

Probabilities are associated with various possible cash flows at each point in time for each scenario. Probabilities can be expressed discretely (e.g., in terms of fractions) or continuously (in terms of cumulative probability density functions). Several simple examples of notation are:

$p(x)$ denotes the probability of receiving a cash flow x.

$p(x|t,j)$ denotes the probability of receiving a cash flow x at time t given scenario j.

$p(j)$ denotes the probability of the jth scenario.[3]

Every decision-maker is presumed to be able to make decisions consistently within the context of some *utility function* for money, denoted by $u(x)$. All decision-makers will be assumed to be risk-averse. The function $c(x)=-u'(x)/u''(x)$ will be interpreted in various ways as a measure of local risk capacity. This is the reciprocal of the traditional measure of risk

---

[2]This result--that cash flows are to be expressed after corporate or personal income taxes--has been independently derived by many writers. The reader unfamiliar with this conclusion may wish to develop a proof. *Hint:* Assume that after-tax and pre-tax cash flows are equally appropriate. Do the values computed using pre-tax cash flows give consistent decisions (in the von Neumann-Morgenstern sense)?

[3]As a further example of the notation, the probability of receiving a cash flow x at time t in the jth scenario is the joint probability of scenario j and cash flow x at time t given scenario j:

$$p(j)p(x\ j,t).$$

aversion, $r(x)$.[4] Additional subscripts and function arguments will be used to express constraints on the decision-maker's utility function. For example:

$u(x \mid w_0)$ is the utility of receiving a cash flow x given wealth w_0.

$u(x \mid t,j)$ is the utility of receiving a cash flow x at time t given scenario j.

Note that $u(x)$ is not the utility of an amount of wealth x. We assume the effects of the decision-maker's wealth on his or her utility function are encompassed in conditions and arguments of $u(x)$.[5]

---

[4]Pratt (1964) and Sharpe (1991) discuss closely related quantities. Pratt analyzes risk aversion, $r(x)$. His notation is in the mainstream of financial and economic literature, while ours is not. We beg the reader's patience, but there is an important reason for our choice of notation. The measure of risk capacity is in units of dollars. The units of risk aversion are in units of dollars^{-1}. This has created a considerable degree of confusion. For example, Nobel Laureate Harry Markowitz has criticized the application of a utility function with constant risk aversion of 10, applied to annual rates of compound return (i.e., 1+R) on stock investments (Markowitz (1991). The reciprocal of 10 is 0.1, so such a utility function scales returns by comparing $1+R$ to 0.1. Such a utility function can be dismissed without analytical review, for fluctuations in 1+R will always be large in relation to 0.1.

Sharpe (1991) refers to $2c(x)$ as the "risk tolerance" of the decision maker. This is close to the mark, as generally $2c(x)$ is the maximum amount of money a decision maker will put at risk without an impractically large risk charge. This is true for a wide range of utility functions including all those with constant or decreasing risk aversion other than $u(x)=x$. The term "risk tolerance" has, however, two drawbacks. First, it seems to be a measure of risk, rather than a measure of money. It is closer to common English to say, "The firm has a risk capacity of $100 million" than to say "the firm has a risk tolerance of $200 million," though the statements are equivalent. Second, the term "tolerance" suggests forebearing rather than foreboding. The dollar amount that is risk tolerance is in the neighborhood of values at which the decision maker shies away from--becomes, that is, *in*tolerant of--risk. Van Slyke ( ) refers to $2c(x)$ as the decision maker's "flinch point" to emphasize the correspondence between the dollar amount $2c(x)$ and the limit to the decision maker's willingness to assume risk in exchange for reward. Several qualities of $2c(x)$ are discussed in Appendix A, which also includes a graphical portrayal of the concept of the "flinch point."

[5]There is no distinction between the utility of wealth and the utility of cash flows for a decision maker whose local risk capacity $c(x)$ is a constant. We shall argue in Section IV that this constraint characterizes the core of practical problems. Elementary discussions of utility functions suggest that utility functions must be functions of wealth rather than cash flow in order that the utility of receiving amount X plus the utility of receiving amount Y be the utility of receiving both X and Y. This condition is met in the general framework by requiring that two cash flows or transactions received at the same time within the same scenario must be expressed in terms of their certain monetary equivalents before being added together.

We assume each utility function is such that it has an inverse, u^{-1}, defined by $u^{-1}(u(x))=x$.

The *certain monetary equivalent* of a set of possible cash flows at a single time is that amount of current cash that has the same utility as the uncertain cash flows. That is,

$$CME = u^{-1}\left[\int_{-\infty}^{\infty} p(x)u(x)dx\right] \qquad (2.1)$$

A potential *transaction* is an opportunity available to a decision-maker which will affect one or more of the probabilities he or she perceives. (The perception that only current cash will be affected by a transaction is a trivial case involving no change in probabilities of future cash flows.) Examples of transactions include capital budgeting opportunities, borrowing, lending, and underwriting the risk of another's fortunes. Transactions can be valued only in the context of objective or subjective probabilities of cash flows, and in the context of a particular utility function (a function of x, t, and j: the amount received, the time received, and the scenario in which the cash flow is to be realized). We will call the existing set of probabilities and utility functions the *firm*. The subscript F denotes a set of probabilities $p_F(x|t,j)$ and a utility function $u_F(x|t,j)$ for the firm.

To provide concise notation we will use the logical operator $\cup$ to denote the union of the firm and the transaction, as follows:

> $F\cup T$ denotes the firm <u>and</u> the transaction, that is, the revised probabilities for the firm after entering into the transaction.
>
> $p_{F\cup T}(x)$ denotes the probability of cash flow x if the firm enters into the transaction.

$u_{F \cup T}(x)$ denotes the utility of cash flow x if the firm enters into the transaction.

We assume each decision-maker holds a liquidity preference, a set of expectations about the undesirability of deferring inward certain cash flows. The existence of such liquidity preferences can be inferred from the price behavior of low-risk markets such as the market for United States Treasury securities, which sell at greater discount the longer the maturity.[6] The *present value* of a set of anticipated cash flows reflects this liquidity preference. The present value factor for a particular time is denoted $v(t)$. The factor for a particular time for a particular scenario is $v(t|j)$. Note that all of the information about risk perceptions and risk preferences is expressed in $p(x)$ and $u(x)$, and that none is expressed in $v(t)$.[7]

The *expected value* of current cash and a set of expectations about future cash flows is a weighted average of the possible cash flows, discounted by the present value factor, with weights equal to the perceived probabilities of their realization.[8] Considering the general case of several scenarios, an infinite amount of future time, and an infinite range of cash flows, we can write:

[6]We are not referring to the yield schedule, but only to the observation that a Treasury security maturing for $1000 in five years sells for less than a treasury security maturing for $1000 in one year. Actually, uncertainty over personal income taxes makes it impossible to attain a truly risk-free return in the United States. In earlier economic writing British consuls were used to denote risk-free perpetuities. The absence of completely risk-free investment opportunities in the major world markets does not damage the observation that the decision maker's liquidity preference schedule is, in principle, separate and separable from the decision maker's utility schedule.

[7]The schedule denoted by $v(t)$ is the schedule by which profit can be accumulated without risk. This part of total profit arises from the ability of enterprises to earn a profit using borrowed funds to pay wages and acquire control of capital equipment. The Capital Asset Pricing Model showed the relationship between market risk and profit. The existence of a rate of return on investment that is uncorrelated with the returns on the capital market is implied by the Capital Asset Pricing Model, and this rate of return leads to a bidding down of the price of risk-free securities to the point that their yields are approximately the same as the rate of return for an uncorrelated portfolio. Therefore the yields of government securities provide approximate values for the schedule of risk-free returns denoted by $v(t)$.

[8]This is sometimes called the actuarial value of the cash flows. We have used the term "expected value" because it seems more widely used and because many actuarially determined values make an explicit provision for risk, rather than explicitly ignoring risk.

Risk-Adjusted Value

The Risk-Adjusted Value (RAV) of a firm is the certain monetary equivalent of the firm over a range of scenerios:

$$RAV_F = u^{-1}\left[\sum_{j=1}^{n} p(j)\ u(RAV(j))\right]$$

The Risk-Adjusted Value of the firm under the j th scenario is the present value of the Risk-Adjusted Value of all future cash flows:

$$RAV(j) = \int_{t=0}^{\infty} v(t|j)\, RAV(j,t)\, dt$$

The Risk-Adjusted Value of the possible cash flows at time t in scenario j is the certain monetary equivalent of the possible cash flows:

$$RAV(t,j) = u^{-1}\left[\int_{x=-\infty}^{\infty} p(x|t,j)\ u(x|t,j) dx\right]$$

$$E_F = \sum_{j=1}^{N} p(j) \int_{t=0}^{\infty} v_F(t|j) \int_{-\infty}^{\infty} x p(x|t,j) dx dt \quad (2.2)$$

$$= \text{current cash} + \sum_{j=1} p_F(j) \int_{t>0} v_F(t|j) \int_{-\infty} x p_F(x|t,j) dx dt \quad (2.3)$$

The intrinsic value of a firm is closely related to its level of current cash and the possibilities and risks of increases and decreases in cash in the future. The *Risk-Adjusted Value* of a firm is one specific model of the intrinsic value of the firm. In a departure from common notation (except among actuaries) this will be denoted by the three letters *RAV*. *RAV* is in the same units as cash flows, that is, *RAV* is measured in dollars, marks or yen.

$$RAV_F = u^{-1}\left[\sum_{j=1}^{n} p(j)\ u(RAV(j))\right] \quad (2.4a)$$

$$RAV(j) = \int_{t=0}^{\infty} v(t|j)\, RAV(t,j)\, dt \quad (2.4b)$$

$$RAV(t,j) = u^{-1}\left[\int_{x=-\infty}^{\infty} p(x|t,j)\ u(x|t,j) dx\right] \quad (2.4c)$$

In practical problems, of course, the continuous functions and their integrals can be replaced by discrete functions and their sums. Computations like those outlined in equation (2.3) are straighforward with modern spreadsheet languages. A summary-level spreadsheet can determine *RAV* from information about the results of the various scenarios. For each scenario, a spreadsheet can take the present value of the risk-adjusted values of the possible events (at those times and in the appropriate scenario). For each point in time in each scenario, a spreadsheet can determine the risk-adjusted value (that is, the certain monetary equivalent) of the possible events. The spreadsheets can be designed to reflect a number of underlying parameters such as the firm's risk capacity, the probabilities of the various scenarios, and the uncertainty of outcomes at given times. This allows the decision-maker to compute the Risk-Adjusted Value using (2.4), and then to determine the sensitivity of that result to changes in assumptions.

The present-value operator operates on dollar amounts, not on utilities. The expectation operator, on the other hand, operates on utilities, not on dollar amounts. When risk is considered explicitly, a fundamental difference emerges bewteen a 90% chance of getting $1 million tomorrow and a 100% chance of getting $1 million in, say, two years.

The Risk-Adjusted Value of a particular firm can be denoted $RAV_{\mathbf{F}}$. We define the Risk-Adjusted Value of a transaction to be:

$$RAV_{\mathbf{T}} = RAV_{\mathbf{F} \cup \mathbf{T}} - RAV_{\mathbf{F}} . \tag{2.5}$$

From simple algebra,

$$E_{\mathbf{T}} = E_{\mathbf{F} \cup \mathbf{T}} - E_{\mathbf{F}} . \tag{2.6}$$

Following Pratt (1964), a *risk premium* can be defined as the amount such that the decision-maker will be indifferent to entering into the transaction *and* receiving a side payment of the risk premium regardless of the outcome. We can define the risk premium $\pi_{\mathbf{T}}$ by requiring the risk-adjusted value of the firm and the transaction, $RAV_{\mathbf{F} \cup \mathbf{T}}$, to be equal to the risk-adjusted value of the firm, plus the expected value of the cash flows of the transaction as defined above, less the risk premium:

$$RAV_{\mathbf{F} \cup \mathbf{T}} = RAV_{\mathbf{F}} + E_{\mathbf{T}} - \pi_{\mathbf{T}} . \tag{2.7}$$

Therefore

$$\pi_T = E_T - RAV_T \ . \qquad (2.8)$$

C. Decision Rule

With this notation we can express a general framework for financial decision-making as a single decision rule for the firm. The *decision rule* is:

> Enter into transaction T if and only if doing so increases the Risk-adjusted value of the firm; that is, if
>
> $$RAV_{F \cup T} > RAV_F$$
>
> or, equivalently,
>
> $$RAV_{F \cup T} - RAV_F > 0. \qquad (2.9)$$

Decision rules for regulators should be based on recognition of the equilibrium state or states this decision rule implies if employed by all decision-makers.

This decision rule gives us the tool we need to express each of the classic problems of finance in mathematical terms.

D. Mathematical Definition of the Problems

The thrust of this paper is that the classic solutions to the classic problems of finance are special cases of the application of the decision rule. The decision rule allows improved decisions when the assumptions underlying the special cases are not appropriate. We conclude this section on problem definition by putting the classic solutions in the notation of the decision rule.

a. *The Capital Budgeting Problem.* The classic solutions to the capital budgeting problem are:

NPV Method: Discount the expected value of the expected cash flow at each point in time by a factor that reflects both liquidity and risk, compounded until the time the cash flow is expected to be paid or received. Select a minimum acceptable rate of discount, denoted by (1+IRR), and enter into the transaction if and only if

$$\int_{t=0}^{\infty} \frac{1}{(1+IRR)^t} E[x|t]dt > 0 \qquad (2.10)$$

IRR Method: Determine the IRR that makes the integral above equal to zero. This will be a unique solution if the cash flows are a pattern of outward cash flows followed by a series of inward cash flows. Enter into the transaction if and only if the IRR is greater than some threshold value.

b. *The Pricing Problem.* Let **T** denote the implications on the $p(x \mid t,j)$ and $u(x \mid t,j)$ of a particular choice of pricing strategy for all time $(t \geq 0)$ in light of all current information. The selected pricing strategy $\mathbf{T}^*$ is the one that maximizes $RAV_{\mathbf{F} \cup \mathbf{T}}$. Also, $RAV_{\mathbf{F} \cup \mathbf{T}^*}$ is the maximum Risk-Adjusted Value of the firm over all possible pricing strategies. (This follows from repeated application of the decision rule to all possible pricing decisions at t=0.) Let R_f be a prospective measure of risk-free return and R_r be a prospective measure of return on risk, and select these definitions to encompass in R_r the entire cost of capital. The classic solution to the pricing problem is that $\mathbf{T}^*$ is that pricing strategy that maximizes

$$\int_{0}^{\infty} \frac{1}{(1+R_f+R_r)^t} E[x|t]dt \qquad (2.11)$$

That is, under certain assumptions the optimal pricing strategy maximizes the increase in net cash inflows if each expected cash flow is adjusted to its present value by an amount that reflects the cost of capital, as measured by the return on risk R_r.

c. *The Leverage Problem.* In the classic statement of the leverage problem the firm is faced with two opportunities to raise a certain sum of capital. The opportunities are to raise the capital by issuing debt, and to raise the capital by issuing stock. Let **D** denote the transaction of issuing debt. Let **S** denote the transaction of issuing stock. In this classic statement of the problem the firm is assumed to make all other decisions the same, regardless of whether it chooses alternative **D** or alternative **S** (providing it chooses one or the other).

Note that the transactions **D** and **S** both increase cash flows in the short run, as the firm receives the proceeds of the debt or stock offering, and change the probabilities of later cash flows. Although the precise effects on later cash flows depend on many details of the firm, generally speaking the choice of issuing debt will lead to fixed outward cash flows, while the choice of issuing stock will lead only to the presence of a greater number of shares among which to divide total dividend distributions.

The classic solutions developed by Modigliani and Miller ( ) are that in the absence of corporate and personal income taxes

$$RAV_{\mathbf{D}} = RAV_{\mathbf{S}}, \tag{2.12}$$

and in the presence of corporate and personal income taxes

$$RAV_{\mathbf{D}} > RAV_{\mathbf{S}}. \tag{2.13}$$

d. *The Portfolio Investment Problem.* Sharpe (1964) provided the classic solution known as the Capital Asset Pricing Model (CAPM). As summarized clearly in Sharpe (1991) the only assumptions of the CAPM are:

i. There exists a risk-free rate of interest at which all firms can borrow and lend. We will denote this by R_f.

ii. All firms can assess and do agree on the prospects for various investments, at least in terms of the investments' expected values, variances, and correlation coefficients.

iii. Each firm maximizes its utility subject to a full investment constraint. Let X_{ik} denote the share of the portfolio of firm k which is invested in investment i. The full investment constraint is that $\Sigma X_{ik} = 1$. Moreover, the markets have cleared, and all securities are held by all of the firms.

iv. Each firm maximizes the value of his portfolio as measured by $U_k = E_k - V_k/2c_k$, where V_k denotes the variance of the portfolio's returns and c_k denotes the investor's risk capacity. Risk capacity varies from investor to investor but is constant for any one investor over the feasible range of expected return and variance.

An alternative and equally important solution to the portfolio investment problem is Arbitrage Pricing Theory (APT) developed by Ross (1977). APT is not normative; it does not suggest how an investor should make portfolio decisions. It is descriptive, and suggests how the prices of securities should vary in an open securities market in equilibrium. As such, APT is unreachable by a normative approach such as a general framework for decision-making. Any general framework for decision-making should be consistent with APT, however, in the sense that the market equilibrium state predicted by assuming that all investors employ the general framework should be one of the states described by APT.

e. *The Rate Regulation Problem.* As elucidated by the Federal courts, the guiding principle of rate regulation is that a firm cannot be denied a fair rate of return, yet must not be granted an excessive rate of return.[9] The fair rate of return is that rate of return that would be reached in an equilibrium market; anything less would be confiscatory and therefore not a legal excercise of regulatory powers.[10]

Let **T** denote a particular price structure for the firm. Assume that the products or services are provided in a mature market economy in its equilibrium state; that is, that the firm must earn a return on risk, but not be given additional returns for innovation, or additional returns for making a market for a scarce good. Denote the risk-free rate of return by R_f and the return for risk as R_r. The classic solution is that the regulated rate can be expressed in terms of an appropriate rate of return on risk that can be added to and compounded with the risk-free return in the following way:

$$RAV_T = \int_0^{\infty} \frac{1}{(1+R_f+R_r)^t} E[x|t]dt \qquad (2.14)$$

f. *The Solvency Problem.* Let L denote the degree of leverage of the firm, so that all dollar amounts--both inward and outward--are increased by the factor L.[11] Then define:

$$RAV_L = u^{-1}\left[\sum_1^n p(j)u[\int_0^{\infty} v(t|j)u^{-1}[\int_{-\infty}^{\infty} p(x|t,j)u(L\cdot x|t,j)dx]dt]\right] \qquad (2.15)$$

In this section we have defined a general framework for financial decision-making and described a number of classical problems in finance in terms of that framework. In principle, there is no reason this framework could not be generalized further to applications other than

---

[9]Hope Natural Gas. give quote.

[10]CalFarm Insurance Company et al v. George Deukmejian et al, California, 1988. "The terms 'fair and reasonable' and 'confiscatory' are antonyms, not separate tests." footnote 5 at page 9.

[11]The quantity L is dimensionless. To make the concept of L more concrete, imagine that the scale is set so that the average frim has a leverage L of 1.0. A low leverage firm would have a value of L less than 1.0, and a high-leverage firm would have a value of L greater than 1.0. This can be done without loss of generality in the formula.

finance; this would require the decision-maker to express utilities for non-financial values. In the next section, however, we will limit our scope to the five areas of financial interest outlined above.

III. Results

IV. SUMMARY

The basic building blocks of a general framework for financial decision-making are at least as old as The Theory of Games and Economic Behavior, by von Neumann and Morgenstern (1944). These building blocks include the axioms that decision-makers should be risk-averse and that decision-makers should be consistent in all of their decisions, and include the important concept of scenarios. Perhaps as long as 100 years ago, actuaries were working with a generalized actuarial formula which recognized the risk-free rate of return, explicit probabilities of cash flows, and explicit provision for risk. This paper unites the two, adding the concept of scenarios to the generalized actuarial formula and adopting the methods of the Capital Asset Pricing Model to determine the relationship between uncertainty and risk loading, which is expressed as a dollar quantity and named *risk capacity*.

The paper illustrates the practical advantage of having a general framework for financial decision-making by considering a number of commonly used techniques of financial theory. The general framework suggests:

- Important and relatively easy improvements are made in capital budgeting theory to account for future cash flows that are more certain, such as debt obligations, differently from those that are less certain, such as profits from operations.

- The long-term costs of uncertainties have been undervalued by decision-makers in the markets for leveraged buyouts and life and annuity insurance policies.
- There is a limit to the extent to which debt leverage should replace equity leverage, even in the presence of high income taxes.
- Regulators in the fields of utilities, government contracts, and insurance can determine the minimum constitutional profit margin and maximum allowable leverage; the required values for the firms' risk capacity can be estimated from the "characteristic line" of the Capital Asset Pricing Model.

Perhaps most important, having a general framework for financial decision-making which encompasses the current methods as special cases provides a rich source of ideas for further research.

Partial Bibliography

Arrow, Kenneth J. "The Role of Securities in the Optimal Allocation of Risk Bearing" Review of Economic Studies 1964, v. 86, pp. 91-96

Borch, Karl. "Equilibrium, Optimum and Prejudices in Capital Markets"

Debreu, Gerard. Theory of Value: An Axiomatic Analysis of Economic Equilibrium. Yale University Press, New Haven. c. 1959 by Cowles Foundation for Research in Economics at Yale University.

Fama, Eugene. "The Behavior of Stock Market Prices" Journal of Business 1965, v. 38, pp. 34-105

Lintner, John. "The aggregation of investors' diverse judgements and preferences in purely competitive markets" Journal of Financial and Quantitative Analysis 1969, v. 4, pp. 346-382

Lintner, John. "The valuation of risk assets and the selection of risky investments in stock portfolios and at capital budgets" Review of Economics and Statistics, Feb., 1965

Loubergè, ed. Risk, Information and Insurance Kluwer Academic Publishers, Boston 1990.

Markowitz, Harry. "Portfolio selection" Journal of Finance 1952, v.7, pp. 77-91

Miller, Merton H. "The Modigliani-Miller Propositions after 30 years" Journal of Economic Perspectives 1988, v. 2, pp. 99-120

Modigliani, Franco and Miller, Merton H. "The Cost of Capital, Corporation Finance, and the Theory of Investment" The American Economic Review 1958, v. 48, no. 2

Modigliani, Franco. "Debt, dividend policy, taxes, inflation, & market valuation" Journal of Finance v. 38, pg 256

Modigliani, Franco and Merton Miller. "The cost of capital, corporation finance, and theory of investment" American Economic Review 1958, v. 48, pp. 261-297

Modigliani, Franco and Merton Miller. "Cost of Capital: Corrections" American Economic Review 1963, v. 53, pp. 433-443

Mossin, Jan. "Equilibrium in a capital asset market" Econometrica 1966, v. 35, pp. 768-783.

Pratt, John W. "Risk Aversion in the Small and in the Large" Econometrica 1964, v. 32, no. 1-2, pp. 122-136

Roll, Richard. "A Critique of the Asset Pricing Theory's Tests" Journal Financial Economics 1977, v. 4

Roll, Richard and Stephen Ross. "A empirical investigation of the Arbitrage Pricing Theory" Journal of Finance v. 35, pg 1073

Ross, Stephen. "The Arbitrage Theory of Capital Asset Pricing" Journal of Economic Theory 1976, v. 13, pp 341-360

Ross, Stephen. "The capital asset pricing model, short-sale restrictions and related issues" Journal of Finance v. 32, pg 177

Ross, Stephen. "Return, risk and arbitrage" in Irwin Friend and James L Bickler, eds. Risk & Return in Finance c. 1977, v. 1, pg 189-218

Ross, Westerfield, and Jaffe. Corporate Finance, Second Edition. Irwin, Boston 1990.

Sharpe, W. F. "A simplified model for portfolio analysis" Management Science 1963, v. 9, pp. 277-293

Sharpe, William. "Capital asset prices: a theory of market equilibrium under conditions of risk" Journal of Finance 1964, v. 19, pp. 425-442

Sharpe, William. "Factor models, CAPMs and the APT" Journal of Portfolio Management Fall, 1984, pp. 21-25

Stone, B. K. Risk, Return and Equilibrium MIT Press, 1970.

Risk Adjusted Value

$$E_F = \sum_{j=1}^{n} p(j)E(j)$$

$$E(j) = \int_{t=0}^{\infty} v(t|j)\ E(t,j)$$

$$E(t,j) = \int_{-\infty}^{\infty} p(x|t,j)\ x\,dx$$

Risk-Adjusted Value

$$RAV_F = u^{-1}\left[\sum_{j=1}^{n} p(j)\ u(RAV(j))\right]$$

$$RAV(j) = \int_{t=0}^{\infty} v(t|j)\, RAV(t,j)\, dt$$

$$RAV(t,j) = u^{-1}\left[\int_{x=-\infty}^{\infty} p(x|t,j)\ u(x|t,j)\,dx\right]$$

Risk-Adjusted Value

1. The present value function v(t|j) is risk-free after-tax.

2. The probabilites p(j) and p(x|t,j) reflect all risks.

3. The X variable encompasses all possible cash flows, including taxes.

4. The present value operator operates on currency values, not utilities.

5. The expectation operator operates on utilities, not cash flows.

Risk-Adjusted Value

Derivation of the IRR formula:

$$\int_{t>0}^{\infty} v^t RAV(t)dt > I$$

Substituting the specific values for this example:

$$\int_{t>0}^{\infty} v^t \left(E(t) - \frac{\sigma^2(t)}{2c} \right) dt > I$$

and

$$\int_{t>0}^{\infty} v^t E(t) \left(1 - \frac{c_1}{2c}(1 - c_t^2) \right) dt > I$$

if $c_1 = 2c$, then

$$\int_{t>0}^{\infty} v^t E(t) c_2^t dt > I$$

and

$$\int_{t>0}^{\infty} E(t) \left(\frac{c_2}{1 + R_f} \right)^t > I$$

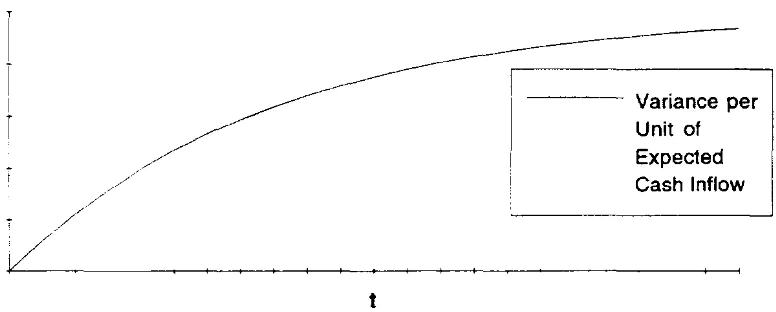
Define a variable IRR such that

$$c_2 = \frac{1 + R_f}{1 + IRR}$$

Then

$$\int_{t>0}^{\infty} (1 + IRR)^{-t} \; E(t) \; dt > I$$

CONDITION FOR IRR RULE



The IRR Rule holds when

$$\frac{\sigma^2(t)}{E(t)} = 2c\,(1 - \left[\frac{1+R_f}{1+IRR}\right]^t)$$