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INSURANCE RISK MANAGEMENT TOOLS: VALUE AT RISK AND RISK ADJUSTED ECONOMIC VALUE

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ABSTRACT

This paper reviews the Value At Risk concept, a financial risk management tool, and discusses its application to insurance. Difficulties with this application are described and modifications suggested. An alternative tool, expressed as a risk adjustment to economic value, is suggested as an additional measure.

1. INTRODUCTION

The science of Risk Management has recently developed sophisticated analytical tools, largely in the context of the financial services industry. Driven by large losses by banking institutions and the clients of investment banking firms through derivative securities and "rogue" trader activities, this industry and its regulators have recognized the need for a more disciplined approach to the monitoring and controlling of its risk exposures.

The predominant concept that has emerged as the basic measure of risk exposure is "Value At Risk" (VAR), a statistically based, maximal loss concept. Popularized by such risk management processes as J.P. Morgan's *Riskmetrics*™ or Bankers Trust's *RAROC*™, it has become the common yardstick of risk measurement, generally accepted in concept if not always with regard to calculation specifics such as confidence intervals or time horizons.

As insurance usually involves risks with regard to financial assets, as well as liabilities which often behave much like financial assets, it would seem natural to extend the same concept to insurance risk management and some insurance companies have followed that approach.[1] On the other hand, there are significant differences between the businesses, and hence the risks, of insurance companies and banks that result in certain difficulties in applying the VAR concept whole-hog. At a minimum, some modifications may be necessary; to reflect fully the risk characteristics of insurance liabilities, additional different measures may be needed.

[1] See for example Prudential, [1996].

2. VALUE AT RISK CONCEPT

VAR is usually defined as "an estimate, with a predefined confidence interval, of how much one can lose from holding a position over a set horizon".[2]

Confidence intervals are typically 95% or 99%, and time periods are typically short: one month or less (e.g. there is a 99% probability that the loss in value over one month will be no greater than X). The choice of confidence interval is arbitrary but is meant to represent a maximal loss. The time horizon is determined by the time needed to unwind a position. As this can only a day, short horizons are the norm. "Value" is defined as market value.

VAR can be used to compare relative risk exposures between portfolios, over time, or to a company standard. It can also be used to determine a theoretical risk adjusted capital requirement, which can be used in turn to calculate return on risk adjusted capital, a risk adjusted performance measure.[3]

In calculating VAR, value (V) is usually assumed to change linearly with respect to changes in underlying risk drivers, r_i, (e.g. interest rates), and the various r_is are assumed to be normally distributed[4]; therefore, V has a normal distribution and:

$$VAR_i = \chi \cdot \sigma_{V\Delta i}$$

where $\chi = 1.65$ for a 95% confidence interval and 2.33 for a 99% confidence interval; and $\sigma_{V\Delta i}$ is value volatility, defined as one standard deviation change in V, over time period Δt, as a result of change in r_i.

In order to calculate total VAR:

- Uncorrelated Total VAR = $[\sum_i VAR_i^2]^{1/2}$, assumes risks are independent.
- Correlated Total VAR (adjusts for correlation among risks) = $\chi \cdot \sigma_{V\Delta T}$

Where $\sigma_{V\Delta T} = [\sum_i \sigma_{V\Delta i}^2 + 2 \sum\sum_{i<j} \sigma_{V\Delta i}\sigma_{V\Delta j}\rho_{V\Delta ij}]^{1/2}$
and ρ_{ij} is the correlation coefficient for risks i and j.[5]

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[2] J.P. Morgan, "Introduction to RiskMetrics™", [1995].
[3] e.g. Bankers Trust's *RAROC*™
[4] Bold font indicates statistical variable with associated probability distribution; regular font indicates single-valued deterministic variable.
[5] Alternatively, if Σ is the covariance matrix and e the column vector of ones, then $\sigma_{V\Delta T}^2 = e'\Sigma e$. See Wilson [1995].

- Correlation Effect = Correlated Total - Uncorrelated Total

  This represents the synergistic effect, either positive or negative, of the combination of these risks.

In order to calculate value volatility, $\sigma_{V\Delta t\,i}$, if V is linear with respect to r_i, then sensitivity, δ_i, to underlying risk driver, r_i, is defined as change in V per unit change in r_i per unit of V,[6]

$$\delta_i = \frac{\Delta V}{\Delta r_i \cdot V}$$

and if driver volatility, $\sigma_{r\Delta t\,i}$, is defined as one standard deviation change in underlying risk r_i over time period, Δt, then

$$\sigma_{V\Delta t\,i} = \delta_i \cdot V \cdot \sigma_{r\Delta t\,i}.$$

Sensitivity, δ_i, is usually calculated in one of two ways:

- Linear regression on sensitivity testing of change in V with respect to change in r_i, or
- For non-symmetrically distributed V resulting from non-linear relationships to underlying risk factors (e.g. assets and liabilities with embedded options), E(V) is calculated from the distribution of V generated by cash flow models using stochastically generated vectors of the underlying risk factor; the risk factor is then "shocked" by Δr_i, a new distribution generated, and a second E(V)′, calculated; sensitivity is then defined as:

$$\delta_i = \frac{E(V)' - E(V)}{\Delta r_i}.$$

Correlation coefficients for underlying risks, i and j, are calculated in the usual way as

$$\rho_{r\Delta t\,ij} = \frac{\sigma_{r\Delta t\,ij}}{\sigma_{r\Delta t\,i} \cdot \sigma_{r\Delta t\,j}}$$

and are readily available from financial data sources. As indicated, these will usually be based on volatilities of the risk drivers r_i, not V. If V is linear with respect to r_i, then $\rho_{V\Delta t\,ij} = \rho_{r\Delta t\,ij}$.

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[6] When r_i is interest rate change, δ_i is equivalent to the familiar Effective Duration.

When the key assumption of linearity of V with respect to r_i is invalid, then current methodology indicates the need for alternative approaches. $\Delta V/\Delta r_i$ is sometimes approximated by the "delta-gamma" method which uses second order terms but increased accuracy is not assured.[7] This may be because V will not be normally distributed and the resulting $\sigma_{V\Delta i}$ may not necessarily produce an accurate Value at Risk by multiplying by χ. In this case, it is recommended that the loss be calculated directly from the distribution of V produced by stochastic simulations.

3. APPLICATION OF VAR TO INSURANCE

The VAR concept can be applied directly to the asset side of an insurance company's balance sheet without much modification. As the assets of a typical insurance company tend to be dominated by high quality, liquid, stable securities, the precision and frequency of modeling employed for a portfolio of derivative securities may be overkill, but the concept is the same.

To apply VAR to the liability side, a "market value" of the liabilities must be determined. This is usually done by taking the present value of cash flows (usually defined as statutory earnings) using a model that incorporates such items as the premium and benefit characteristics of the insurance policies, required reserves, and anticipated expenses and taxes. As with assets, to the extent there are embedded options or other non-linearities leading to non-symmetrically distributed present values, E(V) may need to be calculated from the distribution of V generated by the models using stochastically generated vectors of underlying risk factors.

As for assets, the risk factor is then "shocked" by Δr_i and either a second deterministic V' calculated or a new distribution generated and a second $E(V)'$ calculated; delta sensitivity is then defined as before.

Assets and liabilities can be handled separately, or together if required by interacting cash flow dependencies. In either case it is possible to derive the VAR for assets less liabilities, i.e. surplus. This can be viewed as the maximal loss, over the time horizon, in the value of the company (or line of business).

4. DIFFICULTIES WITH INSURANCE APPLICATION

The nature of insurance assets and liabilities, particularly with regard to risk exposure, is significantly different from that of a bank or other financial institution. As a result, application of the VAR concept without modification can lead to difficulties.

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[7] See for example discussion on pages 35-37 of J.P. Morgan, *RiskMetrics™ - Technical Document*, [1995].

First with regard to the assets, as already noted, methodologies often employed for highly sensitive assets such as derivative securities may be overkill. For sensitive, actively traded securities, time horizons are often very short and daily VAR calculations, involving new volatilities and covariances, are common. The term structure of the yield curve is also very important and "Key Rate" durations (and possibly convexities) or other measures of sensitivities to different points on the yield curve are employed. "Arbitrage-free" interest rate paths are standardly used.

Most insurance company portfolios, on the other hand, consist of high quality, liquid, stable securities held to maturity. As VAR would not change radically over short time periods unless there were a dramatic change in financial markets, daily or even monthly recalculations may not be necessary. Also VAR may not be as sensitive to term structure, so one or two Key Rate durations may be sufficient. On the other hand, if the portfolio contains a significant mortgage loan or mortgage backed security position then extensive cash flow modeling of changes in values to changes in interest rates and other economic indices may be needed. The necessity for "arbitrage-free" interest rate paths can also be questioned, as they can produce an upwards bias in interest rates over long durations.

Modeling time saved may be more profitably employed on the liability side.

With regard to liabilities, the difficulties are more severe. First, liabilities for an ongoing insurance company have no readily determinable "market value" in the sense a marketable security does. Except in the case of sale or reinsurance of the block of business, such liabilities are not traded, so a calculated present value cannot be validated or "trued up" to the marketplace. Even in the case of sale or reinsurance, values are often driven by other considerations, perhaps irrelevant to the ongoing concern.

Even the theoretical present value calculation of "market value" is controversial.[8] First and foremost, the issue of what discount rate to use must be determined. Usually, Treasury rates plus a spread are used, but there is much debate over what spread is appropriate. While the spread should theoretically reflect the riskiness of the liabilities (an "option adjusted spread"), consistency with the asset side can be a problem. At this point this issue has not been satisfactorily resolved.[9]

Second, as the liabilities are not typically traded, the meaning of the time horizon as the time needed to unwind a position becomes questionable. The time needed to sell a block of liabilities can be six months to a year, with the value received, as mentioned, driven by other considerations and constrained by regulatory approval. Such sale cannot be considered a normal business practice for an insurance company. The "loss" will not normally even show up in the company's financial reports unless reserves are strengthened or assets written down. As a result, the concept of loss of value over a certain time horizon becomes even more arbitrary. At a minimum, longer periods, a quarter, or even a year, should be considered.

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[8] See Becker, [1996].
[9] See Altman and Vanderhoof [1996].

Third, insurance liabilities are typically of long duration, occasionally as long as thirty years or more. Short term fluctuations in value are not necessarily very important, unless they result in financial impairment (ratings downgrades, "runs on the bank", etc.). Of greater importance are long term trends in such risk factors as mortality, morbidity, expenses, etc. Again, a short term view of risk may have the wrong focus. However, a longer term view may invalidate the linearity assumption common to short term VAR calculations. This assumption may be a reasonable first approximation for such risk drivers as defaults, mortality, or stocks, but may not be appropriate for interest rates where there is significant convexity.

In the latter case, any change in interest rates may reduce value, V. Even if interest rate changes are assumed to be symmetrically distributed, the distribution of V will be highly skewed, with little to no upside potential and significant downside potential. The distribution of V may need to be developed from stochastic modeling and the loss calculated directly from the actual distribution.

Finally, many of the most serious insurance risk exposures are not quantifiable by standard statistical measures. Examples are regulatory/legislative changes, legal and public relations crises, or catastrophes. VAR cannot adequately incorporate these risks.

5. ECONOMIC VALUE CONCEPT

In the insurance company context, "value" does not usually mean market value, but rather economic value in the sense of present value of cash flows. In fact the insurance industry has started to incorporate this latter concept into management reporting, and occasionally external financial reporting, in the form of "Economic Value Added" (EVA) measures of financial performance.[10]

In its fundamental form EVA is defined as follows:

$$EVA = V_t - (1+ i_C)\cdot V_{t-1}$$

where V_t = Present Value of Distributable Earnings[11] discounted at the cost of capital rate, i_C.

As required (risk based) capital is reflected in each year's projected distributable earnings, EVA-based measures are said to be "risk adjusted", but the risk adjustment may be inappropriate for the following reasons:

[10] See Collins, [1995].

[11] Distributable earnings are usually defined as statutory earnings less change in required surplus (i.e. what could be distributed to shareholders, policyholders, or a parent company). Distributable earnings during year t have been left in V_t in this formula.

- In the case where risk based capital is assumed to be equal to actual capital, capital is an undisciplined risk quantifier. It is usually determined more by outside forces such as rating agencies, or management prerogative, than any disciplined analysis of risk applied on a consistent basis across different lines of business or companies. It changes slowly, if at all, as risk exposure changes due to different products, asset mix, or environmental shifts.

- In the case where risk based capital is determined separately from actual capital, perhaps based on a VAR-type calculation, there usually is capital left over. Companies then find themselves in the situation where each line of business could be earning the company "hurdle" rate on its assigned risk-adjusted capital but the company overall could be earning less than the hurdle rate.

- Risk based capital should be determined as an insolvency preventer. It therefore rightly concerns itself with maximal aggregate loss (VAR), the extreme left-hand tail of the distribution of possible surplus outcomes. Risk analysis for use in profitability measures should concern itself with the entire distribution of outcomes in order to determine the best balance of risk and reward, reflecting directly the degree of risk aversion of the business owner.

The fundamental problem with EVA from a risk management standpoint then is that when it calculates present value of distributable earnings, V, the present value is a single point estimate with no direct consideration of the <u>distribution</u> of future earnings. Preferably, V should reflect the full range of possible outcomes. Risk exposure should not be taken into account through use of a "hurdle rate" as the discount rate where the hurdle rate is predetermined (usually equivalent to cost of capital, i_c) and does not necessarily vary with the volatility of V.

6. RISK ADJUSTED ECONOMIC VALUE CONCEPT

6.1 DEFINITION

This problem can be rectified by considering the <u>distribution</u> of V rather than the single-point value, V. This distribution is the same as the one calculated in the VAR methodology, and used to derive E(V), before the risk driver is shocked to see the sensitivity of E(V) to change in r. Note that it is <u>not</u> the same as the distribution of the change in V over time horizon Δt, used in the final stage of the calculation of VAR. Note also that the discount rate used is <u>not</u> the hurdle rate used in EVA calculations but a market rate, ideally a risk-free rate as the risk adjustment will be applied separately.

Using Modern Portfolio Theory,[12] this distribution of V can be mapped through a utility function, and an expectation taken to get E[U(V)]. Equating this expectation to $U(V_r)$, produces a V_r that is the risk-free equivalent, or risk adjusted value.

[12] See for instance, Bodie, et al, PP 134-135.

$$U(V_r) = E[U(V)] \qquad (1)$$

The risk adjustment, A, to E(V) that reduces it to V_r is then:

$$A = E(V) - V_r\,. \qquad (2)$$

Risk adjustment, A_i, is then defined as the reduction in expected value that produces a risk-free equivalent value, based on the degree of risk aversion, a, due to risk driver, r_i.

Note that this methodology can be used directly to calculate risk adjusted economic value, or the appropriate discount rate can be calculated that equates the present value of distrubutable earnings to V_r and that discount rate used as the hurdle rate in normal EVA calculations.

6.2 APPLICATION TO EXPONENTIAL UTILITY

First, it is useful to consider value, V, as composed of a base value, V_0, and a randomly distributed element, ΔV, due to the effect of a risk driver.

$$V = V_0 + \Delta V.$$

If the utility function is assumed to be exponential,

$$U(X) = -\,e^{-aX},$$

then

$$E[U(V)] = E[-\,e^{-a(V_0+\Delta V)}]$$

$$= -\,e^{-aV_0}\,E[e^{-a\Delta V}]$$

$$= -\,e^{-aV_0}\,E[-U(\Delta V)]$$

$$= E[-\,e^{-aV_0+\ln E[-U(\Delta V)]}]$$

From (1)

$$U(V_r) = -\,e^{-aV_r} = E[-\,e^{-aV_0+\ln E[-U(\Delta V)]}]$$

$$V_r = V_0 - \tfrac{1}{a}\ln E[-U(\Delta V)]$$

From (2)

$$A = \tfrac{1}{a}\ln E[-U(\Delta V)]. \qquad (3)$$

6.3 APPLICATION TO NORMAL DISTRIBUTION

If value, ΔV, has a normal distribution with mean 0 and variance σ_V^2, then it can be shown that[13]

$$E[U(-\Delta V)] = e^{\frac{1}{2}a^2\sigma_V^2}. \tag{4}$$

From (3)

$$A = \tfrac{1}{2}a\,\sigma_V^2 \tag{5}$$

and for the risk adjustment for a particular risk driver r_i:

$$A_i = \tfrac{1}{2}a\,\sigma_{Vi}^2 \tag{6}$$

The nature of the risk adjustment is therefore a factor dependent on the degree of risk aversion, a, and the variance, σ_V^2; as opposed to VAR which, using similar assumptions, is independent of the degree of risk aversion, a, and dependent on standard deviation, $\sigma_{V\Delta t}$.

6.4 RISK AVERSION FACTOR

In the above formulas, a is the degree of risk aversion in terms of <u>values</u>. When used in Modern Portfolio Theory, a is usually defined as the degree of risk aversion in terms of <u>returns</u>, a_R. For instance, one way to derive a_R, would be to analyze stock market total returns over a significant period of time, fit a normal distribution to them, and see what value of a_R equates E[U(**R**)], where **R** is actual total return, to $E[U(R_f)]$, where R is a risk free return (e.g., the average yield on one year Treasury bills).

The equivalence relationship for such returns, comparable to the one for value, would be

$$R_f = \mu_R - \tfrac{1}{2}a_R\,\sigma_R^2$$

where μ_R and σ_R^2 are the mean and variance of the returns.This can be restated as

$$a_R = 2(\mu_R - R_f)\,/\sigma_R^2. \tag{7}$$

Values in the 2-6 range often result.[14]

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[13]See Bowers et al.:11.

The value for a_R thus calculated is the degree of risk aversion for $\Delta V/V$. Appendix A shows that a corresponding a_V for V is given by

$$a_V = a_R / V_0 .$$

Therefore the above expression for A_i can be recast as

$$A_i = \frac{a_R}{2V} \sigma_{Vi}^2$$

where σ_{Vi} = one standard deviation in the distribution of ΔV due to risk driver, r_i.[15] This situation usually results from the case in which ΔV is linear with respect to r_i (i.e. $\Delta V = k\, r_i$), and r_i is normally distributed, in which case σ_{Vi} can be calculated directly as $k\sigma_{ri}$. This set of assumptions will be referred to as "linear/normal".

A total Risk Adjustment, A_T, can then be calculated as

$$A_T = \frac{a_R}{2V} \sigma_{VT}^2$$

where $\sigma_{VT} = [\sum_i \sigma_{Vi}^2 + 2 \sum\sum_{i<j} \sigma_{Vi} \sigma_{Vj} \rho_{ij}]^{1/2}$

and ρ_{ij} is the correlation coefficient for risks i and j.

6.5 APPLICATION WHEN DISTRIBUTION IS GAMMA

There are many cases when V is not normally distributed, either because r_i is not normally distributed or V is not linear with respect to r_i. As a result, the degree of skewness may mean that a normal distribution would misrepresent the risk exposure.

In such cases a Gamma distribution may be appropriate.

$$f(X) = \frac{\beta^{-\alpha} X^{\alpha-1} e^{-X/\beta}}{\Gamma(\alpha)}.$$

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[14] Analysis of total annual returns on large company stocks via the S&P 500 Composite with dividends reinvested, compared to yields on one year Treasury bills, from 1950 through 1994, produced α=5.7. Bodie, et al, "Investments" mentions values of 2-4, see p179.

[15] Note that $\sigma_{Vi} \neq \sigma_{V\Delta i}$, which is derived from the VAR concept of change in value over Δt.

If ΔV has a Gamma distribution, then in most risk exposure situations the long tail exists for negative values of ΔV, and $X = -\Delta V$

$$f(\Delta V) = \frac{\beta^{-\alpha}(-\Delta V)^{\alpha-1}e^{\Delta V/\beta}}{\Gamma(\alpha)}.$$

Then

$$E[-U(\Delta V)] = \frac{\beta^{-\alpha}}{\Gamma(\alpha)}\int_{-\infty}^{0} e^{-a\Delta V}(-\Delta V)^{\alpha-1}e^{\Delta V/\beta}\, d(\Delta V)$$

$$= \frac{\beta^{-\alpha}}{\Gamma(\alpha)}\int_{-\infty}^{0} (-\Delta V)^{\alpha-1}e^{-\frac{1}{\beta}(a\beta-1)\Delta V}\, d(\Delta V)$$

$$= (1 - a\beta)^{-\alpha}.$$[16]

From (3)

$$A = \tfrac{1}{a}\ln(1 - a\beta)^{-\alpha}. \qquad (8)$$

In the special case of $V = k\,(\Delta r)^2$, $k<0$, (note: no "Δr" term, usually applicable for interest risk where there is convexity but little duration mismatch)[17], and Δr normally distributed, the distribution of ΔV is Gamma with $\alpha = 1/2$, and $\beta = 2k\sigma^2$. (See Appendix B).

In this case:

$$A_1 = \tfrac{1}{a}\ln(1+2ak\sigma^2)^{-\frac{1}{2}}. \qquad (9)$$

For V quadratic with respect to r_1 in the form of $V = a_1 r_1^2 + b_1 r_1 + c$ (significant duration mismatch), or some higher order relationship, V is not normal nor Gamma distributed. In this case E[U(V)] would need to be calculated directly from the actual distribution's histogram.

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[16] Obtained by substituting $u=(a\beta-1)(\Delta V)$.

[17] When asset/liability management is based on Duration, D, and Convexity, C, (first two moments only), then values are effectively approximated by:
$V_A = a_A x^2 + b_A x + c_A$ and $V_L = a_L x^2 + b_L x + c_L$, where b = D and a = C/2. Then $V = V_A - V_L = \frac{1}{2}(C_A - C_L)x^2 + (D_A - D_L)x + (c_A - c_L)$. So when $D_A = D_L$, this approximation is valid.

In each case, A_T may need to be calculated from the distribution of V with respect to all risk drivers combined, if the normal approximation is not adequate.

7. COMPARISON OF RAEV TO VAR

The following example may help illustrate how application of this concept to an insurance company's line of business would compare to a VAR analysis.

First it is important to clarify the different definitions of V distributions and resulting differences in σs between the two approaches.

As mentioned, the distribution of V used in Risk Adjusted Economic Value is the distribution of future earnings, <u>not</u> the same as the distribution of the change in V over time horizon Δt used to calculate VAR. Thus, in general, $\sigma_{V_i} \neq \sigma_{V_{\Delta t}i}$. In one case, however, $\sigma_{V_i} = \sigma_{V_{\Delta t}i}$, facilitating transfer from VAR calculations to Risk Adjusted Economic Value. This is explored below:

Assuming linearity/normality, VAR's $\sigma_{V_{\Delta t}i}$ is derived from

$$\sigma_{V_{\Delta t}i} = \delta_i \cdot V \cdot \sigma_{r_{\Delta t}i}$$

$$\text{where } \delta_i = \frac{\Delta V}{V \cdot \Delta r_i}, \qquad (10)$$

whereas the Risk Adjusted Economic Value's σ_{V_i} is derived from the distribution of the present value of distributable earnings, V. If V_i is the component of V due to risk driver r_i, then for many insurance liabilities,

$$V_i = \sum_{k=0}^{\infty} v^{k+1} \, {}_kp_i \, q_i \, A_k$$

$$V_i = q_i \sum_{k=0}^{\infty} v^{k+1} (1 - q_i)^k A_k$$

where:
q_i is the rate of decrement due to r_i (i.e. $q_i = r_i$), and A_k is an amount affected by r_i.
(e.g. examples of q_i and A_k are:
average mortality rate and amount at risk;
average bond default rate and bond assets).

Ignoring second order terms (as VAR does in this case)

$$\frac{\Delta V_i}{\Delta q_i} = C_i \qquad (11)$$

where C_i is a constant.

In which case

$$\sigma_{V1} = C_1 \sigma_d .$$

Therefore, if the time horizon, Δt, used in the VAR calculations is the same as the time period over which q_1 is operative (e.g. one year),[18] then

$$\sigma_d = \sigma_{c\Delta t 1}$$

and from (10) and (11)

$$V \delta_1 = C_1$$

therefore

$$\sigma_{V\Delta t 1} = C_1 \sigma_{c\Delta t 1} = C_1 \sigma_d = \sigma_{V1} .$$

Now, to pursue an example, assume that a VAR analysis of a block of life insurance policies, with Δt = one year, indicates that the following risk exposures are the largest:

SAMPLE VALUE AT RISK PROFILE
RISK FACTORS

	VALUE VOLATILITY	SENSITIVITY TO DRIVER	DRIVER VOLATILITY	CORRELATION COEFFICIENTS			
RISK	σ_{Vi}	δ_i	σ_d	DEFAULTS	INTEREST	MORTALITY	WITHDRWLS
DEFAULTS	50.0	25	2.0%	1	-0.1	0	0.2
INTEREST RATES	24.0	24	1.0%	-0.1	1	0	0
MORTALITY	10.0	100	0.1%	0.0	0	1	0
WITHDRAWALS	2.0	2	1.0%	0.2	0	0	1
UNCORRELATED TOTAL	56.4						
CORRELATED TOTAL	54.6						

Value volatilities and totals have been calculated by the value volatility formulas.

Non-quantifiable risks, such as regulatory changes, have to be identified separately. Withdrawals here are withdrawals other than those sensitive to interest rate changes, the effect of which are included in the "Interest Rates" line.

[18] In actuarial nomenclature, for $_m q_1$, m=Δt.

Applying the formulas for VAR_i assuming linearity/normality we get

SAMPLE VALUE AT RISK PROFILE
SUMMARY
($ mm)

RISK	VALUE AT RISK
DEFAULTS	117
INTEREST RATES	56
MORTALITY	23
STOCKS	5
UNCORRELATED TOTAL	131
CORRELATION EFFECT	-4
CORRELATED TOTAL	127

If the time horizon, Δt, used above is the same as the one used in modeling the distribution of V (e.g. one year), so that $\sigma_{V_i} = \sigma_{V_{\Delta t\, i}}$, and assuming linearity/normality, the formulas for Risk Adjusted Value, V_r, produce

SAMPLE RISK ADJUSTED ECONOMIC VALUE
SUMMARY
($mm)

UNADJUSTED TOTAL VALUE	120

RISK	VALUE AT RISK	RISK ADJUSTMENT
DEFAULTS	117	59
INTEREST RATES	56	14
MORTALITY	23	2
STOCKS	5	0
UNCORRELATED TOTAL	131	76
CORRELATION EFFECT	-4	-5
CORRELATED TOTAL	127	71

RISK ADJUSTED VALUE	49

In this illustrative case, a significant risk exposure implies a large risk adjustment, reducing the value of $120, calculated as present value of distributable earnings discounted at the risk-free rate, to an equivalent risk-free value (cash value) of only $49.

8. CONCLUSION

The financial concept of Value At Risk, with the right modifications, may be an adequate measure of risk exposure for an insurance line of business. It is certainly an appropriate first step in the risk exposure quantification and prioritization process.

Risk Adjusted Economic Value may prove to be a useful additional tool, particularly for long duration liabilities, and for comparing the value of different lines of business, or the same line under different strategies, on a risk-free equivalent basis.

This is an emerging field of study that will require a significant amount of practical application of existing or new methodologies before the usefulness of any of them can be assessed. In the meantime, merely engaging in the risk analysis process can help an insurance company understand its risk exposures and manage them prudently.

APPENDIX A

RELATIONSHIP OF a_R TO a_V

Assume V is normally distributed with mean μ_V and variance σ_V^2. Then[19]

$$E[U(V)] = -e^{-(a_V\mu_V - \frac{1}{2}a_V^2\sigma_V^2)}$$

If the risk free rate of return is R:

$$E[U(V_0\{1+R\})] = -e^{-a_V V_0(1+R)}$$

Equating, we get:

$$a_V V_0(1+R) = a_V\mu_V - \tfrac{1}{2}a_V^2\sigma_V^2$$

$$a_V = 2\frac{\mu_V - V_0(1+R)}{\sigma_V^2}$$

In deriving a_R from the distribution of total returns on stocks we have assumed that $R = \Delta V/V_0$ is normally distributed and have calculated μ_R and σ_R^2. As $V = V_0(1+R)$, these means and variances are related by $\mu_V = V_0(\mu_R + 1)$ and $\sigma_V^2 = V_0^2\sigma_R^2$; therefore,

$$a_V = 2\frac{V_0(\mu_R+1) - V_0(1+R)}{V_0^2\sigma_R^2} = 2\frac{\mu_R - R}{V_0\sigma_R^2}$$

And since by equation (7): $a_R = 2\dfrac{\mu_R - R}{\sigma_R^2}$

then

$$a_V = \frac{a_R}{V_0}.$$

[19] See Longley-Cook, [1983]: 328.

APPENDIX B

DISTRIBUTION OF V
WHEN $\Delta V = kX^2$, k<0

Assume:

$$\Delta V = k\,X^2,\ k<0$$

and

$$f(X) = \frac{1}{\sigma\sqrt{2\pi}}\ e^{-\frac{1}{2}\left(\frac{X}{\sigma}\right)^2}$$

Then a frequency function for ΔV, $g(\Delta V)$, can be derived using the change in variable technique: [20]

$$G(t) = P\{\Delta V<t\} = P\{kX^2 <t\} = P\{+\sqrt{t/k}\ < X < -\sqrt{t/k}\ \}\ ,\ \text{for } k<0$$

$$= 1- \frac{2}{\sigma\sqrt{2\pi}} \int_0^{\sqrt{t/k}} e^{-\frac{1}{2}\left(\frac{X}{\sigma}\right)^2} dx$$

Then

$$dG(t)/dt = \frac{-2}{\sigma\sqrt{2\pi}}\ e^{-\frac{1}{2}\,t/(k\sigma^2)}\ \cdot\frac{1}{2}\left(\frac{t}{k}\right)^{-\frac{1}{2}}\left(\frac{1}{k}\right)$$

$$g(\Delta V) = \frac{-1}{k\sigma^2\sqrt{2\pi}}\left(\frac{\Delta V}{k\sigma^2}\right)^{-\frac{1}{2}}\cdot e^{-\frac{1}{2}(\Delta V)/(k\sigma^2)} \qquad ,\ \Delta V \le 0.$$

$$g(\Delta V) = \frac{(-2k\sigma^2)^{-1/2}(\Delta V)^{-1/2}e^{-\Delta V/(2k\sigma^2)}}{\sqrt{\pi}}$$

Which is Gamma, for $\alpha = 1/2$, and $\beta = -2k\sigma^2$.

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[20] See Hoel, Paul G. *Introduction to Mathematical Statistics*, John Wiley & Sons, Inc., New York, 1966: 123.

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